THE CONSTRUCTION AND PRINCIPAL USES OF Mathematical Inſtruments.

Tranſlated from the FRENCH of

M. BION,

Chief Inſtrument-Maker to the *French* King.

To which are added,

The *Conſtruction* and *Uſes* of ſuch INSTRUMENTS as are omitted by *M. BION*, particularly of thoſe invented or improved by the ENGLISH.

By *EDMUND STONE.*

The whole illuſtrated with Thirty Folio Copper-Plates, containing the Figures, &c. of the ſeveral INSTRUMENTS.

Imitetur igitur Ars Naturam, & quod ea deſiderat inveniat; quod oſtendit ſequatur. Nihil enim aut Natura extremum invenerit, aut Doctrina primum: ſed Rerum Principia ab Ingenio profecta ſunt, & Exitus Diſciplina comparantur.

CICER. ad Heren. lib. iii.

THE SECOND EDITION.

To which is added,

A SUPPLEMENT:

Containing a further Account of ſome of the moſt uſeful Mathematical Inſtruments as now improved.

Construction and Principal Uses of Mathematical Instruments

to which are added the Construction and Uses of such Instruments as are omitted by M. Bion, particularly those invented or improved by the English, by Edmund Stone. 1758.

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FOREWORD

'Mathematicks are now become a popular Study, and make a part of the Education of almost every Gentleman.'

Thus Stone began the preface to his translation of Nicolas Bion's master-work over two hundred years ago, and thus today, but less neatly, speak our Ministers of Education, our Commissions, our Pundits and our Authorities.

Nicolas Bion (165?–1733) was a most successful, if not overly original, maker of scientific instruments. He had however the rare gift of explaining clearly to the general reader how things worked. His *Traite de la Construction et Principaux Usages des Instruments de Mathematique* was first published in 1709 and had run into five editions in French by 1752, it had also been published in German and, as late as 1799, the Directoire was seriously considering republishing the work.

Edmund Stone (1700–1768) published his first translation of Bion in 1723, and, thanks to this and the influence of his patron, became a Fellow of the Royal Society in 1725. Stone's patron was the Duke of Argyll on whose estates his father was employed as gardener. In 1758 he published a further edition of the work in response to popular demand with *A Supplement concerning a further account of some of the most useful Mathematickal Instruments as now Improved.*

This work gives the method of construction and the clearest possible description of the use of nearly every measuring and calculating instrument in use during the seventeenth and eighteenth centuries.

Stone added several important instruments to Bion's nearly exhaustive list, for example: the description and use of the Gunter's Scale, the Plain Scale, Everard's and Coggeshall's Slide Rules, the English Sector, the Proportional Compasses, the Sea Quadrant, the English Theodolite, the Circumferentor, the Surveying Wheel or Way-Wiser, the Gunners' Callipers and Gunter's Quandrant. Other specialised authors describe one or two such instruments, but nowhere else can be found such a wide-ranging, detailed, clear and never prolix description of the instruments that served the Age of Reason.

This is a book to be read together with the Voyages of Dampier and of Anson, with Robinson Crusoe, with the letters of Lord Chesterfield to his Son; it is a book that to my mind could be of great use in the class-room, to seize the imagination of the ingenious young, readily bored by arithmetic without apparent purpose; it is a book that in our Apocalyptic times would be of greater use as a manual of survival than many dealing with present day technology.

In my own experience it gives clearer and simpler solutions to problems of dialling, surveying and the applied mathematics than any other work to which I have easy access.

Its only disadvantage up to now has been its rarity and its price, and to that, this reprint provides the happy solution. It becomes once more a tool, a work-book that need not be kept sterilized in a library or a show-case for fear of theft or damage, it is a tool that can be used.

It is obviously essential to Museums to historians and students, to collectors of mathematical instruments, to artisans and to such dealers as have the leisure and inclination to study their stock in trade from other than a mercantile point of view.

PETER BROPHY

To his GRACE,

JOHN,

Duke of *Argyll* and *Greenwich*, &c.

Lord Steward of His Majesty's Houshold.

MY LORD,

THE Subject of the following Treatise seems of Importance enough to claim Your Grace's Patronage; and of Use enough to deserve it. It made it's first Appearance under that of his Highness the Duke of *Orleans*: and, to render it's second equally Magnificent, craves now to be introduced under that of Your Grace. Indeed, as the first Design of it's appearing in *English* was laid in Your Grace's Family; and as it was carried on and finished in the same, it seems to have some Title to Your Grace's Countenance: It naturally seeks Protection where it found it's Birth, and lays claim to the Privileges of a Native of your Family, as well as those of a Domestick. What I have said of my Book, holds almost equally good of myself. I have been, the greatest part of my Life, an humble Retainer to Your Grace. In Your Family it was, I first caught an Affection for MATHEMATICKS; and it was under Your Countenance, that I took occasion to Cultivate them. Your Grace therefore has a kind of Property in all I do of this kind, and it would be an Injustice to lay it at any other Feet.

ANOTHER Person would have here taken Occasion to expatiate on Your Grace's Character: Dedicators, Your Grace very well knows, are great Dealers in that Way; and look on it as one of the Privileges of their Place, to praise their Patrons without Offence. Accordingly, Your Grace's Lineage would have been traced up to the earliest Times, and the Virtues of Your Noble Ancestors drawn out to View. Your Grace's personal Merit, shining and conspicuous as it is, would have been set off in it's full Light, and Your Heroick and Virtuous Atchievements painted in all their Colours. *Flanders*, *Bavaria*, *Spain*, and *Scotland*, would have been called in, as Witnesses of Your Glory; of Your Prudence, as a General; and Your Bravery, as a Soldier: Nor would Your Integrity, as a Minister; Your Magnificence, as a Nobleman; or Your Love of Liberty and Your Country, as a Patriot, have been omitted. For myself, *My Lord*, it is my Business rather to admire than applaud You: Panegyrick is a thing out of my Province; and Your Grace would be a sufferer by the best Things I could say. Were I allowed to touch on any Thing, it should be Your Private rather than Your Popular Character, rather as you are a Gentleman, than as a General, or a Hero. If You

have

have every thing Great and Heroick in the latter; You have all that is Beautiful and Amiable in the former. To enumerate every thing of this Kind visible in your Grace, would be to give a detail of a whole System of Virtues; and to draw your Picture at full, would be little less than to collect into one Piece what is Great and Good in a thousand: A Work fitter for a Volume than a Dedication.

MY Zeal for Your Grace had like to have driven me beyond either my Duty or Design. It was my Resolution not to say any thing that might look like Praise; but I find one cannot do common Justice to Your Grace, without running into the Appearance of it. I am sensible there is no Topick less inoffensive to You, than that of Your own Merit: but the Misfortune is, there is none so engaging or so copious. 'Tis pity You should value Praise so little; when You deserve it so much: For hence, a Person, who would not be Ungrateful, is under a Necessity of becoming Troublesome. I have reason to fear Your Grace's Resentments, for having said thus much; and yet apprehend those of the Publick for having said no more. If I am Delinquent on either Side, Your Grace will do me the Justice, to believe it entirely owing to that Excess of Devotion wherewith I am,

MY LORD,

Your Grace's most Humble,

and most Obedient Servant,

Edmund Stone.

THE

TRANSLATOR's

PREFACE.

MATHEMATICKS *are now become a popular Study, and make a part of the Education of almost every Gentleman. Indeed, they are so useful, so entertaining and extensive a Branch of Knowledge, that it is no wonder they should gain Ground; and that uncommon Countenance they now find, must be esteemed as an Instance of the Felicity of the Age, and the good Sense of the People. Mathematicks have wherewith to gratify all Tastes, and to employ all Talents. Here the greatest Genius has room to exert his utmost Faculties, and the meanest will not fail to find something on a Level with his. Their Theory, affords a noble Field for the Speculative Part of Mankind; and, their Practice, an ample Province for the Men of Action and Business.*

THE Masters in Mathematicks have not been wanting in their Respect to the rest of Mankind: They have frankly communicated their Knowledge to the World; and have published Treatises on every Branch of their Art: Insomuch, that a Man who has a Disposition to this Study, will find himself abundantly supplied with Helps, to what Part soever he applies himself. There seems, then, but little wanting to Mathematicks, considered as a Science: If there be any Defect, it is when considered as an Art. I mean, Mathematicks appears more accessible, as well as more extensive, on the Side of their Theory than on that of their Practice. Not that the latter has been less laboured by Authors than the former, but because a sufficient Regard does not seem to have been had to the Instruments, *whereon it wholly depends.*

MATHEMATICAL INSTRUMENTS *are the Means by which those Sciences are rendered useful in the Affairs of Life. By their Assistance it is, that subtile and abstract Speculation is reduced into Act. They connect, as it were, the Theory to the Practice, and turn what was bare Contemplation, to the most substantial Uses. The Knowledge of these is the Knowledge of Practical Mathematicks: So that the Descriptions and Uses of* Mathematical Instruments, *make, perhaps, one of the most serviceable Branches of Learning in the World. The Way then to render the Knowledge of* Mathematicks *general and diffusive, is by making that of* Mathematical Instruments *so: With a View of which kind, our Author seems to have engaged in the following*

ing Treatise; at least, it was from a View of this kind, that I undertook to translate it.

THE Design of the Work, however useful, yet seems to be New among us. Particular Authors have indeed touched on particular Parts: One, for Instance, having described the Globe; *another the* Sector; *and a third the* Quadrant: *but for a general Course, or Collection of* Mathematical Instruments, *I know of none that has attempted it. It is true, in* Harris's Lexicon, *we have the Names of most of them; and in* Moxon's Dictionary *the Figures of many: But the Accounts given of them in both are so short, lame, and deficient, that there is but little to be learned from either of them.*

I chose M. BION's *Book for the Ground-Work of mine, as judging it better to make use of a good safe Model provided to my Hands, than run the Risque of proceeding upon my own Bottom. The* French *Instruments described by him, are, in the main, the same with those used among us. Such* English *Instruments as he has omitted, I have been careful to supply: And throughout, have taken the Liberty not only to make up his Deficiencies, but amend his Errors.*

THOSE who desire an Inventory of the Work, have it as follows:

IT is divided into Eight Books, and each of these subdivided into Chapters. To the whole are prefixed Preliminary Definitions necessary for the understanding of what follows.

IN the First Book are laid down the Construction and Principal Uses of the most simple and common Instruments, as Compasses, Ruler, Drawing-Pen, Porte-Craion, Square, Protractor. *And to these I have added five other Articles, of the* Carpenter's Joint-Rule, *the Four-foot* Gauging-Rod, Everard's Sliding-Rule, Coggeshall's Sliding-Rule, *the* Plotting-Scale, *an improved* Protractor, *the* Plain-Scale, *and* Gunter's Scale.

THE Second Book contains the Construction and Principal Uses of the French Sector, *(or* Compass *of* Proportion) *those of various* Gauging-Rods. *To this Book I have added the Construction and principal Uses of the* English Sector.

THE Subject of the Third Book is very much diversified. Under this are found the Construction and Uses of several curious and diverting as well as useful Instruments; particularly Compasses *of various kinds,* Parallel-Rules, *the* Parallelogram *or* Pentagraph, *&c. Under this Head are also laid down several Things not easily to be met with elsewhere: As, the Manner of arming* Load-Stones; *the Composition of divers* Microscopes, *with several other curious Amusements. To the first Chapter of this Book I have added the Descriptions and Uses of the Turn-up* Compasses *and* Proportional Compasses, *with the* Sector-Lines *upon them, as also the Manner of projecting them.*

IN the Fourth Book you have the Construction and Uses of the principal Instruments used in taking Plots, measuring or laying out Lands, taking Heights, Distances, accessible or inaccessible; Staffs, *for instance,* Fathoms [*or* Toises], Chains, Surveying-Crosses, Recipient-Angles, Theodolites, Semi-Circles, *the* Compass, *with their Uses in Fortification. To this Book I have added three Articles of the* English Theodolite, Plain-Table, Circumferentor, *and* Surveying-Wheel. *What I have there added of the Uses of those Instruments, though but short, yet I flatter myself will be found more Instructive than much larger Accounts of them in the common Books of Surveying.*

THE Fifth Book contains the Construction of several different kinds of Water-Levels; *with the Manner of rectifying and using them, for the Conveyance of Water from one Place to another. In this Book are also found the Construction and Uses of Instruments for* Gunnery: *And to these I have added the Construction and Use of the* English Callipers.

IN

IN the Sixth Book are contained the Conftruction and Ufes of Aftronomical *Inftruments; as the* Aftronomical Quadrant, *and* Micrometer, *with an Inftrument of* Mr de la Hire's *for fhewing the Eclipfes of the Sun and Moon, and* Mr Huygens's *Second* Pendulum-Clock *for Aftronomical Obfervations. In this is alfo fhewn the Manner of making Celeftial Obfervations according to* Mr de la Hire *and* Caffini. *To this Book I have added four Chapters, containing the Defcription and general Ufes of the* Globes, *with the manner of making them: The Defcription and Ufes of the* Ptolemaick *and a* Copernican Sphere, *the* Orrery, *and a* Micrometer, *better than that defcribed by the Author, and of* Gunter's Quadrant.

THE Seventh Book contains the Conftruction and Ufes of the Sea-Compafs, *the* Azimuth-Compafs, Sea-Quadrant, Fore-Staff, *and other Inftruments for taking Altitudes at Sea; as likewife the Conftruction and Ufes of the* Sinical-Quadrant, *and* Mercator's Charts.

IN the Eighth Book are found the Conftructions and Ufes of all kinds of Sun-Dials, *whether fixed or portable; with the Inftruments ufed in drawing them; as alfo a* Moon-Dial, Nocturnal, *&c. To this is fubjoined a fhort Defcription of the principal Tools ufed in making* Mathematical Inftruments: *And, laftly, I have added, by way of Appendix, the Conftruction of the great Eclipfe of the* Sun, *that will happen* May *the* 11th, 1724, *by the* Sector.

THE

THE

CONSTRUCTION

AND

Principal Uſes

OF

MATHEMATICAL INSTRUMENTS.

Definitions neceſſary for underſtanding this Treatiſe.

Plate 1.

A POINT is that which hath no Parts, and conſequently is indiviſible. Fig. 1.

A Line is Length without Breadth, whoſe Original is from a Point. Fig. 2.

There are three kinds of Lines; *viz.* Right Lines, Curve Lines, and Mixed Lines.

A Right Line is the ſhorteſt of all thoſe that can be drawn from one Point to another. Fig. 2.

A Curve Line is that which doth not go directly from one of its Extremes to the other, but winds about. Fig. 3.

A Mixed Line is that which hath one Part ſtrait, and the other crooked. Fig. 4.

Lines compared as to their Poſitions or Situations, are either parallel, perpendicular, or oblique.

Parallel Lines are ſuch that always keep the ſame Diſtance to each other, and which, if both ways infinitely produced, will never meet, whether they be Right Lines, or Curves. Fig. 5.

Perpendicular Lines are thoſe that meeting, incline no more to one ſide than to the other; and therefore they make two equal Angles, which conſequently will be Right Angles. Fig. 6.

Oblique Lines are thoſe, which meeting one another, form oblique and unequal Angles, that is, acute and obtuſe Angles. Fig. 7.

Moreover, Lines have other Denominations; which are as follow:

An upright, plumb, or vertical Line, is that which, if produced, would paſs through the Center of the Earth, as the String of a ſuſpended Plummet. Fig. 8.

A horizontal Line, or Line of apparent Level, is a right Line that touches the Surface of the Earth in one Point, or which is parallel to a Tangent in that Point. Fig. 9.

A Line of true Level is that, whoſe Points are all equally diſtant from the Center of the Earth, as the Circumference of the ſame.

A finite Line is that whoſe Length is determined.

There are also occult Lines, drawn with the Points of Compasses, or more properly with a Pencil, because then they may be easier rubbed out: These Lines must not be seen when the Work is finished, unless they are left to show how the Operation is performed; and then they are dotted, which is done with a Dotting-Wheel.

Fig. 10. The Lines that must remain, and which are called apparent Lines, are drawn with Ink, put into a drawing Pen, as plain and small as possible, by means of the Screw belonging to it.

Fig. 9. A Tangent is a Line touching a Figure, and not cutting it; as the Line A B.

Fig. 9. A Subtense, or chord Line, is that which joins the Extremes of an Arc; as the Line C D.

Fig. 11. An Arc is a Part of a Circumference; as the Arc D F E.

The different kinds of Curve Lines are infinite; but the simplest, most regular, and easiest to draw, is a Circle.

Fig. 11. A circular Line, or the Circumference of a Circle, is a Curve; all the Parts of which are equally distant from one Point in the middle of it, which is called the Center of the Circle.

Right Lines, drawn from the Center of a Circle to the Circumfererence, are called Radii, or Semidiameters; as N O.

Those Chords that pass through the Center of a Circle, are called Diameters; as M P.

The Circumference of every Circle is supposed to be divided into 360 equal Parts, called Degrees.

The Number 360 was chosen by Geometricians for the Division of a Circle, because it may be more exactly subdivided into many equal Parts, without any Remainder, than any other*: as for example; half of 360 is 180, $\frac{1}{3}$ is 120, $\frac{1}{4}$ is 90, $\frac{1}{5}$ is 72, $\frac{1}{6}$ is 60, and so of other of its aliquot Parts.

Every Degree is divided into 60 equal Parts, called Minutes, every Minute into 60 Seconds, and every Second into 60 Thirds, *&c.* which are thus distinguished $40^{\circ}\ 35'\ 49''\ 57'''$ signify forty Degrees, thirty-five Minutes, forty-nine Seconds, and fifty-seven Thirds. The aforesaid Division serves for measuring of Angles; but the Sub-Divisions into Seconds and Thirds are not used, unless in great Circumferences.

The Opening of two different Lines cutting one another, or meeting in the same Point, is called an Angle.

Fig. 12. When two Lines cut, or meet each other in one Point on a Plane, the Angle they make with each other, is called a plane-Angle, or a plain-Angle.

When the Lines that make a plain-Angle, are strait Lines, the Angle is called a Right-lined Angle.

Fig. 13. If the two Lines forming an Angle, are Curves, the Angle is called a Curve-lined Angle.

Fig. 14. If one of the Lines is a Curve, and the other a strait Line, the Angle is called a Mixed-lined Angle.

The two Lines that make an Angle, are called its Sides; the Point wherein they cut or meet each other, being the Vertex.

When an Angle is expressed by three Letters, that in the middle represents the Angle, and the other two the Sides.

In producing or lessening the Sides of an Angle, the Quantity of the said Angle is not at all altered thereby; for the Magnitude of an Angle is not measured by the Magnitude of its Sides.

The Measure of a Right-lined Angle is the Portion of a Circle comprehended between its Sides, whose Vertex is the Center of the Circle: It matters not how big the Radius of
Fig. 15. the Circle be; because whether the circular Arcs, comprehended between the Sides A B, A C, of the Angle be bigger or lesser, they still have the same Number of Degrees.

If, for example, the Arc of a small Circle be 60 Degrees, which is the sixth part of the whole Circumference, the Arc of a greater Circle will likewise be 60 Degrees, or the sixth part of the Circumference of the greater Circle, and the Angle B A C will be 60 Degrees.

Every Angle is either a right, acute, or obtuse Angle.

Fig. 16. The Measure of a right Angle is an Arc of 90 Degrees, which is $\frac{1}{4}$ of the Circumference of a Circle.

Fig. 17. An acute Angle is lesser than 90 Degrees.

Fig. 18. An obtuse Angle is more than 90 Degrees.

There can be no Angle of 180 Degrees, which is the Semi-Circumference of a Circle; for two right Lines so posited, cannot cut, but will meeat each other directly, and consequently will make but one right Line, which will be the Diameter of a Circle.

Fig. 15. The Sine of an Angle or Arc, is half the Chord of double the same Arc: as for example, to have the Sine of the Angle D A E, or of the Arc D E (which is the Measure of it) by doubling the Arc E D, you will have the Arc E D F, whose Chord is E F, whereof E H, its half, is the right Sine of the Angle D A E: the Line DG is the Tangent of the same Angle, and the Line A G is its Secant.

Two Arcs together making a whole Circle, have the same Chord; for it is manifest, that the Line E F is as well the Chord of the greater Arc E B C F, as of the lesser one E D F.

---

* *Our Author should have said,* Lesser Number.

For the same reason two Arcs, which together make a Semicircle, have but one right Sine; as the Line E H is as well the Sine of the obtuse Angle E A I, or of the Arc E B I, which is its Measure, as of the acute Angle E A D, or of the Arc E D.

The same may be said of Tangents and Secants.

The Sine of 90 Degrees, which is the Radius or Semidiameter, as D A, is called the Sinus Totus.

A Surface, or Superficies, is that which hath only Length and Breadth.

There are two kinds of Surfaces, *viz.* Plane and Curve.

A Plane Surface is that to which a right Line may be applied all manner of ways; as the Top of a very smooth Table. Fig. 19.

A Curve Surface is that to which a right Line cannot be applied all manner of ways; they are either Convex, or Concave; as the Outside of a Shell is Convex, and the Inside Concave. Fig. 20.

Term, or Bound, is that which limits any thing; as Points are the Bounds of Lines, Lines the Bounds of Surfaces, and Surfaces the Bounds of Solids.

A Figure is that which is bounded every way.

Figures that be terminated under only one Bound, are Circles, and Ellipses, or Ovals, which are bounded by only one Curve Line.

Figures terminated by several Bounds, or Lines, are the Triangle or Trigon, which hath three Sides, and three Angles. Fig. 21.

The Square, or Tetragon, which hath four. Fig. 22.

The Pentagon, five. Fig. 23.

The Hexagon, six. Fig. 24.

The Heptagon, seven.

The Octagon, eight.

The Nonagon, nine.

The Decagon, ten.

The Undecagon, eleven.

And the Dodecagon, twelve.

All the aforesaid Figures, and those having a greater Number of Sides: are called by the general Name of Polygon, which signifies Figures having many Angles; and for distinguishing them, there is added the Number of Sides: as a Decagon may be called a Polygon of ten Sides; likewise a Dodecagon is called a Polygon of twelve Sides, and so of others.

Figures, whose Sides and Angles are equal (as those before-named) are called regular Polygons.

Those Figures, whose Sides and Angles are unequal, are called Irregular Polygons.

Triangles are distinguished by their Sides or their Angles.

As to their Sides; that Triangle which hath its three Sides equal, is called an Equilateral Triangle, and is also equiangular. Fig. 25.

That Triangle which hath only two equal Sides, is called an Isosceles Triangle. Fig. 26.

And that which hath three unequal Sides, is called a Scalene Triangle. As to their Angles; a Triangle, which hath one right Angle, is called right-angled; and the Side opposite to the right Angle, is called the Hypothenuse. Fig. 27. Fig. 28.

That which hath one Angle obtuse, is called an obtuse angled Triangle. Fig. 29.

That which hath all the Angles acute, is called an acute angled Triangle.

Quadrilateral Figures, or Figures having four Sides, have different Appellations. Fig. 30.

If the opposite Sides are parallel, the quadrilateral Figure is called by the general name of Parallelogram.

If a Parallelogram hath four equal Sides, and the four Angles right ones, it is called a Square. Fig. 31.

If all the Sides are not equal, but the four Angles right ones, it is called an oblong, right angled Parallelogram, or simply a Rectangle. Fig. 32.

A right Line drawn in a Parallelogram, from one of the Angles to the opposite one, is called a Diagonal; as the Line A B.

If the four Sides be equal, and also the opposite Angles, but not right ones, it is called a Rhombus, or Lozangé. Fig. 33.

If two opposite of the four Sides are equal, and the opposite Angles also equal, but not right ones, the quadrilateral Figure is called a Rhomboides. Fig. 34.

Also a Square is equiangular and equilateral; an Oblong is equiangular, but not equilateral; a Rhombus is equilateral, but not equiangular:

And a Rhombodies is neither equilateral nor equiangular.

Every quadrilateral Figure, that hath neither its Opposite Sides, Parallel, or Equal, is called a Trapezium. Fig. 35.

A Circle is a plane Figure, comprehended under one Line, which is called its Circumference, which is equally distant from a Point in the middle, called the Center. Fig. 36.

A Semicircle is a Figure terminated by the Diameter and the Semicircumference. Fig. 37.

A Portion, or Segment of a Circle, is a Figure comprehended by a part of the Circumference, and a Chord lesser than the Diameter; there is a greater and lesser Segment.

A Sector

Fig. 38. A Sector of a Circle is a Figure made by a part of a Circle, terminated by two Radii, or Semidiameters, which do not make a right Line; there is a great and ſmall Sector.

Fig. 39. An Ellipſis is a Figure longer than it is broad, comprehended but by one Curve Line, in which the * two greateſt Lines that can be drawn at right Angles to one another, are called the Axes of the Ellipſis; the greateſt of which is called the great Axis, and the leſſer the leaſt Axis.

The Center of an Ellipſis is that Point wherein the two Axes cut each other.

Fig. 40. Thoſe Figures that have the ſame Center, are called Concentrick Figures.

Fig 41. Excentrick Figures are thoſe that have not the ſame Center.

Fig. 42. Similar Figures are thoſe which have their Angles equal each to each; that is, which have each Angle of one Figure equal to the correſpondent Angle in the other Figure, and have the Sides about the equal Angles proportional. As ſuppoſe the Side *a b* is one half, or one third of the Side A B; then all the other Sides of the leſſer Figure *a b c d*, will be likewiſe one half, or one third of the Sides of the greater Figure A B C D.

The correſpondent Sides in this Figure are called homologous Sides; as the Side A B of the greater Figure, and the Side *a b* of the leſſer Figure, are called homologous Sides.

Equal Figures are thoſe that equally contain an equal Number of equal Quantities.

There are Figures that are ſimiliar and equal.

Others are equal, and not ſimilar;

And, finally, others are ſimilar, but not equal.

Fig. 43. Iſoperimetrical Figures are thoſe whoſe Circuits are equal: As, for Example, the Triangle A B C, and the Square A B C D, are Iſoperimetrical Figures; becauſe each Side of the Triangle being 8, its Circuit is 24, and every Side of the Square being 6, its Circuit is alſo 24 of thoſe equal Parts that make the Circuit of the Triangle.

Body, or a Solid, is that which hath Length, Breadth, and Thickneſs.

Fig. 44. A Sphere, Globe, or Ball, is made by the entire Revolution of a Semicircle about its Diameter, which is at Reſt, and which is called the Sphere's Axis.

Fig. 45. A Spheroid is a Solid, made by the entire Revolution of a Semi-Ellipſis about its Axis remaining at Reſt.

Fig. 46. A Pyramid is a Solid contained under ſeveral Triangular Planes meeting in one Point, and having a Polygon for its Baſe.

Fig. 47. A Cone is a Species of a Pyramid, having a circular Baſe: This Solid is made by the entire Revolution of a right-angled Triangle about one of the Sides, forming the right Angle, which Side is called the Axis of the Cone.

Fig. 48. A Cylinder is a Solid, whoſe Baſes are two equal Circles. This Solid is generated by the entire Revolution of a right angled Parallelogram about one of its Sides, which is called the Cylinder's Axis.

Fig. 49. A Priſm is a Solid, whoſe two Baſes are two ſimilar, equal, and parallel Planes; and when the parallel Planes are Triangles, the Priſm is called a Triangular Priſm.

Fig. 50. When the two Baſes of a Priſm are Parallelograms, it is called a Parallelepipedon.

If the Sides of the aforeſaid Bodies are perpendicular to the Baſe, they are called right, or Iſoſceles Solids.

If they are inclined, they are called Oblique, or Scalenous Solids.

A regular Body is that which is contained under regular and equal Figures, all the ſolid Angles of which are likewiſe equal.

A ſolid Angle is the meeting of ſeveral Planes in one Point; as the Point of a Diamond.

There are required more than two Planes to conſtitute a ſolid Angle.

There are five regular Bodies repreſented in the ſame Plate, together with the Unfoldings of their Planes, *viz.*

Fig. 51. The Tetrahedron, contained under four equal and equilateral Triangles.

Fig. 52. The Hexahedron, or Cube, contained under ſix equal Squares.

Fig. 53. The Octahedron is contained under eight equal and equilateral Triangles.

Fig. 54. The Dodecahedron, contained under twelve equilateral and equal Pentagons.

Fig. 55. The Icoſahedron, contained under twenty equal and equilateral Triangles.

The Unfoldings nigh to each of the aforenamed regular Bodies, ſhew how to draw them on Braſs or Paſteboard, in order to cut them out; which when done, if they are duly folded up, there will be formed the regular Bodies.

All other Solids are called by the general Name of Polyhedron, which ſignifies a Body terminated by many Superficies.

If in the following Work, Terms be uſed that are not here defined, they ſhall be defined and explained in their proper Places.

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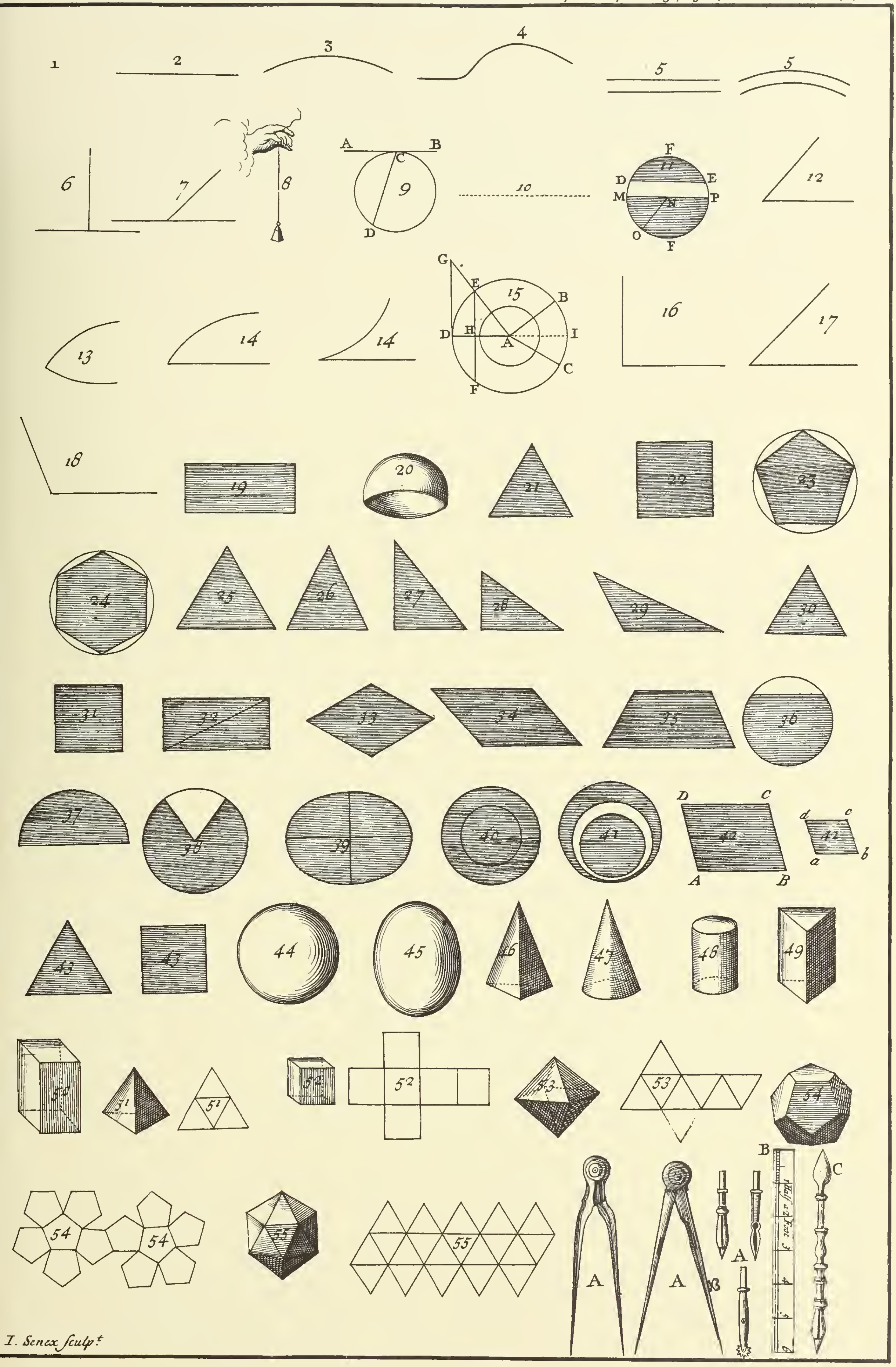
* *Our Author ſhould have ſaid,* the greateſt and leaſt Lines. *This is not a good Definition of an Ellipſis. See another of mine in the Appendix that I have added.*

BOOK

Plate I. *Place this to fold out fronting page 4. Note all the rest must fold out*



I. Senex sculp.t

BOOK I.

Of the Conſtruction and Uſe of Mathematical Inſtruments; containing the common Inſtruments, as Compaſſes, *the* Ruler, *the* Drawing-Pen, *the* Porte-Craion *or* Pencil-Holder, *the* Square, *and the* Protractor.

CHAP. I.

Of the Conſtruction and Uſe of the Compaſſes, *the* Ruler, *the* Drawing-Pen, *and the* Porte-Craion *or* Pencil-holder.

THERE are ſeveral Sorts of Compaſſes, of which we ſhall ſpeak more fully hereafter; but that whoſe Uſes we intend to lay down in this Chapter, is the common Pair of Compaſſes. Of theſe Compaſſes there are two kinds, *viz.* Simple Ones, which have their Points fixed, and others whoſe Points may be taken off; both kinds being of different Bigneſſes, but they are commonly in Length from three to ſix Inches. To theſe Compaſſes, that ſhift their Points, there belongs a Drawing-Pen-Point, a Pencil-Point, and ſometimes a Dotting-Wheel, to make dotted Lines.

The Goodneſs of Compaſſes conſiſts chiefly in this, That the Motion of their Head be very equable, that ſo they may not leap in opening and ſhutting; that the Joints be well fitted; that they are well filed and poliſhed; and, laſtly, that the Steel-Points are well joined and equal. The Figure A ſheweth theſe kinds of Compaſſes, whoſe Conſtruction we ſhall give in the third Book. Fig. A.

Rulers, which are of Braſs, or Wood, ought to be very ſtrait every way; they are made ſtrait with Files and a Planner, whoſe Bottom is Steel; as alſo by rubbing them and another very ſtrait Ruler together: one Side of theſe Rulers is ſloped, to keep the Ink from blotting the Paper. Fig. B.

When Lines are drawn with Ink, they ought to be very fine.

To know whether a Ruler be very ſtrait or not, draw a right Line upon a Plane; then turn the Ruler about, and apply the ſame Edge to the Line; and if the Edge of the Ruler exactly agrees with the right Line, it is a Sign the Ruler is very ſtrait.

The Drawing-Pen is made of two Steel Blades joined together, and faſtened to a little Pillar, at the other End of which is a Porte-Craion; there is a Cavity between the aforeſaid Blades, in which Ink is put with a Pen: alſo the Blades muſt join each other in Points that be very equal. There is likewiſe a ſmall Screw, ſerving more or leſs to open the Blades, that ſo Lines may be drawn fine or coarſe, according to neceſſity. Fig. C.

The Porte-Craion ought to be of equal Bigneſs every where, and very ſtraitly ſlit down the middle with a fine Saw; alſo the Porte-Craion is bent towards the end, in order to faſten a Pencil in it, by means of a little Ring.

USE I. *To divide a right Line into two equal Parts.*

Plate 2. Fig. 1. Let the given Line be A B, which is to be divided into two equal Parts: About the Point A, as a Center, or one of the Ends of the Line, describe the circular Arc C D, with your Compasses opened to any Distance, but nevertheless greater than one half of A B. Likewise about the other end B, as a Center, describe, with the same Opening of your Compasses, the circular Arc E F, cutting the former Arc in the Points G H; then place a Ruler upon these two Intersections, and draw the Line G H, which will divide the Line A B into two equal Parts.

Note, The two Arcs will not intersect each other, if the Opening of the Compasses be not greater than half of the given Line.

USE II. *Upon a right Line, and from a Point give in it, to raise a Perpendicular.*

Fig. 2. Let the given right Line be A B, and the Point given in it C, upon which it is required to raise a Perpendicular.

From the given Point C, mark both ways with your Compasses, on the given Line, the equal Distances C A, C B; then about the Points A B, as Centers, and with any opening of your Compasses (greater than half the given Line) describe the Arcs D E, F G, intersecting each other in the Point H, and draw the Line H C, which will be perpendicular to A B.

Fig. 3. If the given Point C be at the End of the Line, describe about the Point C, as a Centre, any Arc of a Circle; on which take twice the same opening of your Compasses, *viz.* from B to D, and from D to E: then about the Points D, E, describe two Arcs, intersecting one another in the Point F; lay a Ruler upon the Points F and C, and draw the Line F C, which will be a Perpendicular upon the End of the Line C B.

If there is not room to take the Length of D E, divide the Arc B D in two equal Parts in the Point G, and make D H equal to D G; then the Line H C will be a Perpendicular.

Fig. 4. Or otherwise, having drawn the indefinite Line B D F, through the Points D, F, and made D F equal to B D; F C will be a Perpendicular.

Fig. 5. Or again in this Manner: having taken the Point P at pleasure above the given Line, about the said Point, as a Center; and with the Interval P C, describe the Arc B C D, then draw the Line B P, and produce it 'till it cuts the aforesaid Arc in the Point D, and from the Point D to the Point C, draw the Perpendicular D C.

USE III. *From a Point given without a Line, to let fall a Perpendicular to the said Line.*

Let the given Point be C, from which, to the given Line A B, it is required to draw a Perpendicular.

Fig. 6. About the Point C, as a Center, describe an Arc of a Circle cutting the Line A B in the two Points D E; then from the Points D E, make the Intersection F; lay a Ruler upon the Points C and F, and draw the Perpendicular C G.

Note, The Intersection F may be made above or below the given Line; but it is best to have it below it; because when the Points C F are at a good Distance, the Perpendicular may be drawn truer than when they are nigh.

When the Portion of the Circle described about the Point C, does not cut the Line A B in two Points, the Line must be continued if it can; if it cannot, Recourse must be had to the Method of *Fig.* 5. for raising a Perpendicular on the End of a Line: as suppose a Perpendicular is to be let fall from the Point D, on the Line C D, draw, at pleasure, the Line D B, which bisect in the Point P; then about this Point, as a Centre, and with the Distance P D, describe the Arc D C B, cutting the Line A B in the Point C. Lastly, lay a Ruler upon the Points C and D, and draw the Line C D, which will be the Perpendicular required.

Fig. 7. Otherwise, let A B be the given Line, and C the Point without it; take two Points 1 and 2 at pleasure, on the the said Line A B; then about the Points 1 and 2, and with the Distances 1 C, 2 C, describe Arcs of Circles, intersecting each other in two Points, as in C and D; then lay a Ruler on the two Intersections, and draw a Line, which will be the Perpendicular required.

USE IV. *To cut a right-lined Angle into two equal Parts.*

Let A C B be the Angle to be cut into two equal Parts.

Fig. 8. About the Point C, as a Center, describe the Arc D E at pleasure; then about the Points D and E, describe two other Arcs, cutting each other in the Point F, and draw the Line F C through the Points F, C, which will cut the given Angle into two equal Parts.

If it be required to divide the Angle A C B into three equal Parts, the Arc D E must tentatively be divided by your Compasses into three equal Parts*; because the Trisection of an Angle by right Lines, hath not yet been geometrically found.

USE

* Our Author should have said here, because the Trisection of an Angle *cannot* be geometrically performed by right Lines or Circles either. See more particularly how to do this in the Appendix.

USE V. *To raiſe a right Line on a given Line, that may incline no more on one Side than the other.*

Make the ſame Operation as before, and produce the Line F C G. Fig. 8.

USE VI. *Upon a given right Line, and from a Point given in it, to make an Angle equal to a given Angle.*

Let A B be the given Line, and A the given Point upon which it is required to make an Angle equal to the given Angle E F G.

About the Point F, as a Center, deſcribe the Portion of a Circle; and with the ſame opening of your Compaſſes, deſcribe about the Point A another Portion; then take the Bigneſs of the Arc E G between your Compaſſes, which Diſtance lay off on the Arc B C: now through the Points A, C, draw the Line A C, and the Angle B A C will be equal to the Angle E F G. Fig 9.

USE VII. *To draw a Line from a given Point, parallel to a given Line.*

Let A B be the given Line, and C the Point through which it is required to draw a Line parallel to A B.

About the Point C, as a Center, and with any opening of your Compaſſes, taken at pleaſure, deſcribe the Arc D B cutting the given Line in the Point B: alſo about the ſame Point B, as a Center, and with the ſame opening of your Compaſſes, deſcribe the Arc C A; then take the Diſtance of the Points C, A, and lay it off from B to D, and through the Points C and D, draw the Line C D, which will be parallel to A B. Fig. 10.

Otherwiſe, about the Point C, as a Center, deſcribe an Arc touching the given Line; and about another Point, taken at pleaſure in the Line A B, deſcribe, with the ſame opening of your Compaſſes, the Arc D: then through the Point C, draw a Line touching the Arc D, and the Line C D will be parallel to A B. Fig. 11.

But as it is difficult to find whereabouts the Point of Contact will be, there is another way which is better, and is thus:

About the Point C, as a Center, and with any Diſtance, deſcribe an Arc cutting the Line A B in A. Fig 12.

And about another Point in the ſame Line, as B, deſcribe another Arc, with the ſame opening of your Compaſſes; then open the Compaſſes to the Diſtance A B, and about the Point C, as a Center, deſcribe an Arc cutting the former one in the Point D; and through the Points C and D draw a Line, which will be parallel to A B.

USE VIII. *To divide a given Line into any number of equal Parts.*

Let the Line given be A B, which is required to be divided into eight equal Parts: firſt, draw the Line B C, at pleaſure, making any Angle with the Line A B. Likewiſe draw the Line A D parallel to B C; then divide B C into eight equal Parts, taken at pleaſure, and make the ſame Parts on the Line A D, and through the Diviſions of them, draw Lines, which will divide the Line A B into eight equal Parts. Fig. 13.

Or otherwiſe, draw the Line *a b* parallel to A B, which is propoſed to be divided; then take 8 equal Parts on the Line *a b*. Now through the Ends of the two Parallels draw two Lines, which form Triangles with the Parallels, and interſect each other in the Point C; then from the Point C, draw Lines to the Diviſions made on the Line *a b*, which will cut the Line A B in the Number of equal Parts required. Fig. 14.

This Diviſion of Lines ſerves to make Diagonal Scales; as ſuppoſe the Line A B is to make a Scale of eighty Parts, or eighty Fathom; each Part of this Line, divided into eight, contains ten Fathom: but ſince it is difficult to divide each of the aforeſaid Parts into ten others, you muſt raiſe from the Ends of the Line A B, the Perpendiculars A D and B C, on which take ten Parts at pleaſure; from every of which, you muſt draw Parallels to the Line A B; then the ſame Diviſions muſt be made on the Line D C, as on A B; and the tranſverſal Lines A E, 10 F, 20 G, *&c.* muſt be drawn. Fig. 15.

Now it is eaſy to take off any Number of Fathoms from this Scale: as, for Example, to take off 23 Fathoms; Take the Concourſe of the Tranſverſal 20 G, with the Parallel 3, that is at the Point Z, and Z 3 will be 23 Fathom. Moreover, if 58 Fathom is required, take the Concourſe of the Tranſverſal 50 H, with the Parallel 8, which is Y, and Y 8 will repreſent 58 Fathom, and ſo of others. Feet might be put upon this Scale, by making a greater Diſtance between the Parallels; and by ſub-dividing them into 12 equal Parts, there would be obtained Inches.

But now to divide a very ſhort Line into a great Number of equal Parts, as into 100 or 1000: For Example; Suppoſe the Line A D is to be divided into 1000 equal Parts; firſt, from the Ends A D, raiſe the Perpendiculars A B, D C, and divide each of theſe Perpendiculars into 10 equal Parts, and draw through the Diviſions the like Number of Parallels to A D; then divide each of the Lines A D, B C, into 10 equal Parts, which join by the like Number of Perpendiculars. Again, ſubdivide the firſt Space A E, and it's Parallel, into 10 more Parts, which join by tranſverſal or oblique Lines, as the Line E 1, *&c.* Fig. 16.

By

By this Means the first Interval A E, will be divided into 100 equal Parts; for which Reason, the Numbers 200, 300, 400, 500, *&c.* to 1000, are placed on this Scale, as may be seen in *Fig.* 16.

The Manner of taking off any Number of equal Parts from the aforesaid Scale, is the same as that which hath been already shewn in the precedent Figure. We shall again mention this Scale in the Chapter of the Sector. There are also Sines, Tangents, and Secants, projected upon Scales, in the following Manner: If from each Degree of the Quadrant I F, beginning from the Point I, Perpendiculars are let fall to the Radius A I, these will be the Sines of each Degree, the greatest of which is the Radius of the Circle, or Sinus Totus A F; and the Lengths of all these Sines may be projected upon the Radius, in order to make a Scale, beginning from the Point A; as the Sine D K is laid off from A towards G, *&c.*

And if the Tangent I E, be indefinitely produced towards E, and from the Center A, Fig. 17. Lines, as A E, be drawn through each Degree of the Quadrant, to the Tangent I E produced, these will be the Secants of each Degree of the Quadrant. Whence it is manifest, that any one of the Secants is greater than the Radius A I. It is likewise plain, that every Tangent I E, is terminated by it's Secant A E, in the Line I E, which will be a Scale of Tangents: and it is in this manner, that the simple Scales of Sines, Tangents, and Secants, are made in taking between your Compasses each of those Distances, and laying them off upon a Ruler. The Tables of Sines, Tangents, and Secants, are likewise made on this Principle: for the Radius of a Circle, or Sine of a right Angle, is supposed to be divided into 10000, and then there is found how many of these Parts every right Sine contains; as also the Tangents and Secants from one Minute to 90 Degrees; which, when put in order, are called the Tables of Sines, Tangents, and Secants.

Logarithms are Numbers in an Arithmetical Progression, to which answer other Numbers in a Geometrical Progression, as the two following Progressions.

Prog. Geom. Numb. 1, 2, 4, 8, 16, 32, 64, 128, 256, *&c.*

Prog. Arith. Log. 0, 1, 2, 3, 4, 5, 6, 7, 8, *&c.* Logarithms were invented to perform Multiplication by only the help of Addition, and Division by Substraction; by which Operations are infinitely shortened, and so they are of excellent Use in Astronomical Calculations.

Note, The Use of these Tables is explained in Books of the Tables of Sines, Tangents, and Secants.

USE IX. *To cut off from a given Line any Part assigned.*

Let the Line A B be the given Line from which it is required to cut off the fourth Part.

Fig. 18. Draw the indefinite Line A C, making any Angle with the Line A B, which divide into four equal Parts at pleasure; then from the last Division, draw the Line B 4, and afterwards the Line D I, parallel to B 4, which will be a fourth Part of A B.

USE X. *To draw a right Line through a given Point, that shall touch a Circle.*

If the given Point be in the Circumference, draw the Radius A B, and on the Point B raise the Perpendicular B C, which will be a Tangent in the Point B. But if the given Fig. 19. & 20. Point B be without the Circle, draw a right Line from the Center A, to the Point B, which bisect in the Point D: then about the said Point D, as a Center, and with the Distance B D, describe a Semi-Circle cutting the Circle in the Point E, and draw B E, which will be a Tangent.

If a Circle be given with it's Tangent, and the Point of Contact be required, let fall the Perpendicular A B from the Center of the Circle, and the Intersection of the Tangent with the said Perpendicular, will be in the Point of Contract.

USE XI. *Upon a given Line to describe a Spiral, making any Number of Revolutions.*

Let the given Line be A B, upon which it is required to describe a Spiral of 3 Revolutions. First, bisect that Line in the Point C, about which Point, as a Center, describe a Semi-Circle, whose Diameter may be equal to the given Line A B; then trisect the Semi-Diame- Fig. 21. ter A C in the Points D E, and about the same Center describe, on the same Side the Line A B, two other Semi-Circles passing through the Points D E: again, subdivide the Space C E, into two equal Parts in the Point F; about which, as a Center, describe on the other Side of the given Line, three other Semi-Circles, and a Spiral of three Revolutions will be had. If the Spiral is required to make four Revolutions, you must divide the Semi-Diameter A C into 4 equal Parts *.

USE XII. Num. I. *Upon a given right Line, to describe an equilateral Triangle.*

Let A B be the given Line on which it is required to describe an equilateral Triangle.

Fig. 22. About the Point A, and with the Distance A B, describe an Arc of a Circle; and about the Point B, as a Center, with the Distance B A, describe another Arc cutting the precedent one in the Point C; then draw the Lines C A, C B; and the Triangle A B C, will be an equilateral Triangle.

USE

* *Although* this is *called a Spiral,* yet in *reality, it is only several* unequal *Semi-Circles*; of *no manner of use in any geometrical Effection, a Spiral being vastly different.* See the *Appendix.*

USE XII. NUM. II. *Upon a given right Line A B to describe an Isoscelles Triangle, one of whose equal Legs is given.*

About the Point A, as a Center, with your Compasses opened to that given Distance, describe a small Arc of a Circle, and about the Point B with the same Opening, describe another small Arc cutting the former one in the Point C. Draw A C and B C, then will the Triangle A B C be an Isoscelles Triangle, described upon the given Base A B, and having either of its equal Legs A C, B C of a given Length. Fig. 23.

USE XIII. *Upon a given right Line, to make a Triangle equal and similar to a given one.*

Let the given Triangle be A B C, to which it is required to make another similar, as D E F. Fig. 24. and 25.

Make the Line D E equal to A B; then about the Point D, as a Center, and with the Radius A C describe an Arc; also about the Point E, as a Center, and with the Radius B C describe another Arc, cutting the former one in the Point F; then draw the Lines D F, E F, and there will be a Triangle made equal and similar to the given one.

USE XIV. *Upon a given right Line to make a Triangle similar to a given one.*

Let the given Line be H I, upon which it is required to make a Triangle similar (but not equal) to the Triangle A B C. Fig. 26. and 27.

Make the Angle H equal to the Angle B, and the Angle I equal to the Angle A; then draw the Lines H G, I G, 'till they meet each other, and the Triangle H I G will be that required.

USE XV. *To make a Triangle of three right Lines given; but any two of them must be longer than the third.*

Let the three given Lines be A, B, C; first make the Line D E equal to the Line A, and about the Point E as a Center, with an Interval, equal to the Line B, describe the Portion of a Circle; also about D, as a Center, with an Interval equal to C, describe another Portion of a Circle, cutting the former one in the Point F; then draw the right Lines F D, F E, and the Triangle D F E will be that required. Fig. 28.

USE XVI. *Upon a given right Line to make a Square.*

Let the given Line be A B, on which it is required to describe a Square, whose Side may be equal to the given Line, first about the Point A, as a Center; and with the Distance A B, describe the Arc B D, and about the Point B the Arc A E, intersecting it in the Point C, and divide the Arc C A, or C B, into two equal Parts in the Point F: now make the Intervals C E, and C D, equal to C F, and draw the Lines A D, B E, D E, and the Square will be made. Fig. 29.

Or, otherwise, upon the End of the Line A B, raise the Perpendicular A D equal to A B, and about the Point D, as a Center, with the Distance A D, describe an Arc; likewise, with the same Opening of your Compasses about the Point B, describe another Arc, cutting the first in the Point E, and draw the Lines A D, D E, E B, and the Square will be made. Fig. 30. and 31.

I shall shew, in the Uses of the Protractor and Sector, how to make any regular Polygon upon a given Line; but, by the way, I shall give one general Method for constructing them, by means only of a Ruler and Compasses.

USE XVII. *To inscribe any regular Polygon in a Circle.*

Suppose, for Example, a Pentagon is to be made: Now if the Circle be given, divide it's Diameter into five equal Parts (by *Use* VIII.), but if it be not given, draw with your Pencil an indefinite Line for a Diameter; which being divided into five equal Parts, open your Compasses the whole Extent of the Diameter, and setting one Foot of them upon the Ends of the Diameter, describe two Arcs intersecting each other in the Point C, that thereby an equilateral Triangle may be formed; then having drawn a Circle about the Diameter, lay a Ruler upon the said Point C, and upon the second Division of the Diameter, and draw a Line, cutting the concave Part of the Circumference in the Point D; then the Arc A D will be nighly a fifth Part of the Circumference: therefore the Extent A D will divide the Circle into five equal Parts, and drawing five Lines, the proposed Polygon will be made. Fig. 32.

This is a general Method to make all regular Polygons: As, to make a Heptagon, there is no more to do but divide the Diameter A B into seven equal Parts (that is, into as many Parts as the Figure hath Sides), and always drawing a Line from the Point C, thro' the second Division of the Diameter.

The Construction of a Hexagon is simpler; because, without any Preparation, the Radius, or Semidiameter of the Circle, will divide the Circumference into six equal Parts.

And the Dodecagon is made in only bisecting each Arc of the Hexagon; therefore to make a Decagon, every Arc of the Pentagon must be bisected.

This Problem is almost the same as that described in *cap.* 17. *lib.* 1. of the Chevalier *de Ville*'s Fortification, except, that for dividing the Circle, he draws a Line from the exterior Angle of the equilateral Triangle, thro' the first Point of Division of the Diameter, and af-

 terwards

terwards he doubles the Arc of the Circle; but his Method is far from being exact: for, in the Description of a Pentagon, the Angle at the Center is too great by forty-four Minutes; in the Heptagon, it is too great one Degree and five Minutes; and so the Error will be augmented in Polygons of a greater number of Sides. But by making the Line pass thro' the second Point of Division of the Diameter, the Angle at the Center of the Pentagon will be but about six Minutes too little, and in the Heptagon it is too great by about six Minutes; which are much less Errors, and almost insensible in the Description of the Polygons.

The Truth of the aforesaid Method of inscribing any regular Polygon in a Circle, which is mentioned in Sturmy's Mathesis Enucleata, *may, by the help of Trigonometry, be easily examined. For, suppose* A C G *to be a Circle,* D *the Center,* A C *the Diameter,* A B C *an equilateral Triangle,* E *the second Point of Division of the Diameter divided into any Number of equal Parts,* B F *drawn thro' the Points* B, E, Fig. 33. D B *perpendicular to* A C, *and the Points* D, F, *joined: Now because the Semidiameter* D C, *and the whole Diameter* B C *are given, the Perpendicular* D B (per Prop. 47. lib. 1. Eucl.) *will be had.*

Again, because the Number of equal Parts the Diameter is divided into, is given, the Line C E, *which is two of those equal Parts, will be given, and consequently the Part* D E; *then in the right-angled Triangle* B D E, *the Sides* B D, D E *being given, the Angle* D B E *may be found, by saying, as* D B *is to* D E, *so is Radius to the Tangent of the Angle* D B E.

Moreover, because in the Triangle B D F, *the Sides* D B, *and* D F *(equal to* D C*), are given, and the Angle* F B D *(which is now found), the Angle* B F D *may be fonnd, by saying, as* D F *is to* D B, *so is the Sine of the Angle* D B F, *to the Sine of the Angle* D F B: *which being found, add it to the Angle* D B F, *and substract the Sum from* 180 *Degrees; then the Remainder will be the Angle* B D F, *from which take the right Angle* B D C, *and the Remainder will be the Angle* F D C *of the Center of the Polygon.*

I have calculated, according to the aforesaid Directions, the Quantity of the Angle F D C *for a Pentagon, which I find to want about* 14 *Minutes of* 72 *Degrees, the Angle of the Center for a Pentagon (though our Author says it wants but six), likewise the Hexagon wants* 12 *Minutes of* 60 *Degrees, the Angle at the Center; that of the Octagon is one Degree too great, and that of the Dodecagon* 29 *Minutes too great: therefore this Method is very erroneous, and not to be used; it being only true for inscribing a Square.*

USE XVIII. *To draw a Circle thro' three given Points, provided they be not in a right Line.*

Fig. 34. Let the given Points be A, B, C: first draw a Line from the Point A to the Point B, and another from the Point B to the Point C; both of which divide into two equal Parts by the Lines D E, F G, drawn at right Angles to them, and meeting each other in the Point H, which will be the Center of the Circle: Now about the Point H as a Center, and with the Distance H A, H B, or H C, describe a Circle, and what was required will be done.

By this means the Circumference of a Circle begun, may be finished, in taking three Points in it, and proceeding as before.

USE XIX. *To find the Center of a Circle.*

Fig. 35. Let A B D be the given Circle, whose Center is required to be found; draw the Line A B, which bisect by the Line C D at right Angles: likewise bisect the Line B D by the Line E F, cutting the Line C D in the Point G, which will be the Center of the Circle.

USE XX. *To draw a right Line equal to the Circumference of a Circle; and, contrariwise, to make the Circumference of a Circle equal to a given Line.*

Let the given Circle A B C D be that whose Circumference it is required to make a right Line equal to: First draw a right Line, and lay off upon it three times and $\frac{1}{7}$ of the Diameter, as from G to H; then this right Line G H will be almost equal to the Circumference of the Circle: I say almost; for if it could be exactly had equal to the Circumference, the Quadrature of the Circle would also be had *, which hath not yet been Geometrically found. Fig 36.

USE XXI. *To describe an Oval upon a given right Line.*

Let A B be the given Line, upon which it is required to describe an Oval; trisect it in the Points C and D; then upon the Part C D describe two equilateral Triangles, whose Sides produce; and about the Points C, D, with the Distance C A, or D B, describe Portions of a Circle to the Sides of the Triangles, produced to the Points E, F, G, H; then about the Points I, K, as Centers, and with the Radius I E, or I G, describe the Arc E G on one Side, and the Arc F H on the other, and the Oval will be made. Note, *This is not properly an Oval, but something like one.* Fig. 37.

USE XXII. *To describe an Ellipsis, having the two Axes given.*

Let the great Axis be A B, and the small one C D, intersecting each other at right Angles in the Point G.

First

* No, *nor never will, it being impossible to be done.*

Firſt take with your Compaſſes, or a String, half the Length of the great Axis, that is, A G, or G B; and with this Length ſetting one foot of your Compaſſes in the Point C, deſcribe a Circle cutting the great Axis in the Points E, F, which will be the Foci of the Ellipſis. This being done, place Pins in theſe Foci; or, if the Ellipſis to be deſcribed be required large, and to be on the Ground, as in a Garden, drive Pegs into them: Then take a Thread, or String, equal in Length to the great Axis A B, and after having doubled it, put it about the two Pins or Pegs placed in the Foci E, F; ſo that the two Ends which you hold in your Hand may be in the End C of the ſmall Axis: then holding a Pencil, or ſomething elſe proper to make a Mark, in your Hand at C, move it round, keeping the String always tight, 'till it, together with the Ends of the Thread or String, comes again to the Point C, and the Ellipſis A D B C will be deſcribed by the Pencil. Fig. 38.

Note, This Method of deſcribing an Ellipſis is the beſt of any; as alſo if the Thread, or String, be in Length augmented or diminiſhed, without changing the Diſtance of the Foci, there will be had Ellipſes of another kind. Moreover, if without changing the Length of the Thread, or String, the Diſtance of the Foci be diminiſhed, there will ſtill be had another Species of Ellipſes; and when the Foci's Diſtance is infinitely diminiſhed, a Circle will be deſcribed: But if the Length of the great Axis be augmented or diminiſhed, together with the String (which is equal to it), in the ſame Proportion as the Diſtance of the Foci, all the Ellipſes will be ſimilar, but of different Magnitudes.

To draw an Ellipſis another way.

The two Foci E, F, being found (as in the precedent Figure), any Number of Points, thro' which the Ellipſis muſt paſs, may in this manner be found. Open your Compaſſes at pleaſure to any Diſtance greater than A F, as to the Diſtance A I; then ſet one of their Points in the Focus F, and with the other deſcribe the Arc O R; afterwards open the Compaſſes the Diſtance I B, which is the remaining part of the great Axis, and ſetting one of it's Points in the other Focus E, with the Diſtance I B deſcribe the Arc S T, and the Point P of Interſection will be in the Periphery of the Ellipſis. In like manner, the Diſtances A L, L B, deſcribed about the Foci, will interſect each other in the Point H: and, finally, by opening your Compaſſes to different Diſtances, any Number of Points may be found; which being joined, an Ellipſis will be had. Fig. 39.

Note, Every Opening of your Compaſſes ſerves to find four Points equally diſtant from the Axes; as alſo if, from any Point taken at pleaſure in the Periphery of an Ellipſis, two right Lines, as P F, P E, are drawn to the Foci; theſe will be both together equal to the great Axis.

USE XXIII. *To make one Figure equal and ſimilar to another Figure.*

Let the propoſed Figure be A B C D E, to which another is to be made ſimilar and equal.

Firſt divide it into Triangles by the Lines A C, A D; then draw the Line *a b* equal to A B; and about the Point *b*, with the Diſtance B C, deſcribe an Arc: alſo about the Point *a*, and with the Diſtance A C, deſcribe another Arc, cutting the former one in the Point *c*, and draw the Line *b c*: In like manner proceed for the other Sides, and the Figure *a b c d e* will be ſimilar to the propoſed Figure A B C D E. Fig. 40.

USE XXIV. *To reduce great Figures to leſſer ones, and contrariwiſe.*

Becauſe the Reduction of Figures is uſeful, there is here three ways given to reduce them.

Firſt, a Figure may be reduced in taking a Point within it, and drawing of Lines to all the Angles: for Example, let the Figure A B C D E be propoſed to be reduced to a leſſer. Fig. 41.

Take the Point F, about the middle of the Figure, and draw Lines to all the Angles A B C D E; then draw the Line *a b* parallel to the Line A B, the Line *b c* parallel to B C, *&c* and the Figure *a b c d e* will be ſimilar to the Figure A B C D E.

If a greater Figure be required, there is no more to do but produce the Lines drawn from the Center of the Figure, and then drawing Parallels to it's Sides.

To reduce a Figure by the Scale.

Meaſure all the Sides of the propoſed Figure A B C D E, with the Scale G H; then take another leſſer Scale K L, containing as many equal Parts as the greater. Now make the Side *a b* as many Parts of the leſſer Scale, as the Side A B contains of the greater one's Parts; alſo make *b c* as many Parts as B C, and *a c* as many as A C, *&c.* by which means the Figure will be reduced to a leſſer one. Fig. 42.

To reduce a leſſer Figure to a greater one, a greater Scale muſt be had, and proceed as before.

To reduce Figures by the Angle of Proportion.

Let the propoſed Figure A B C D E be that which is to be diminiſhed in the proportion of the Line A B, to the Line *a b*.

Firſt draw the indefinite Line G H, and take the Length A B, and lay off from G to H; then about the Point G, deſcribe the Arc H I. Again, take the Length of the given Side *a b*, as a Chord of the Arc H I, draw the Line G I, and the Angle I G H will give all the Sides of the Figure to be reduced. Fig. 43.

As

As to have the Point *c*, take the Interval B C, and about the Point G describe the Arc K L; also about the Point *b*, with the Distance L K, describe a small Arc. Now take the Distance A C, and about the Point G describe the Arc M N; likewise about the Point *a*, with the Distance M N, describe an Arc, cutting the precedent one in the Point *c*, which will be that which must be had to draw the Side *b c*: in like manner proceed for all the other Sides and Angels of the Figure.

If by this means a small Figure is to be reduced to a greater, the same manner of proceeding will do it; but the Side of the Figure to be augmented must be lesser than double of that answering to it. As for Example; to reduce the Figure *a b c d e* to a greater, the Side A B of the greater one, must be lesser than double the Side *a b* of the smaller one: for if it was double, the two Lines forming the Angle I G H, would directly meet each other, and make but one right Line.

To reduce Figures by means of Squares.

This way particularly serves to copy, augment, or diminish a Map.

Fig. 44. Let, for Example, the Map A B C D be proposed to be reduced to a lesser one. First, divide it into Squares, then make a lesser similar Figure *a b c d*, which likewise divide into the same Number of Squares as you did A B C D. This being done, draw in every Square of the lesser Figure, what is contained in the correspondent Square of the greater Figure, and there will be a lesser Map. *Note*, The greater the Number of Squares are, the juster will the Figure be.

CHAP. II.

Of the Construction and Use of the Square.

A Square is an Instrument serving to raise Perpendiculars, and to know whether one Line be perpendicular to another. It is made of two Rulers of Brass, or other Metal, joined in such manner as to make a right Angle with each other. There are some Squares, whose two Rulers, or Branches, are firmly fixed; and others that open and shut by help of a Joint, that ought to be well fitted to hinder the Square from shaking; and that it may preserve it's right Angle. To do which, there is adjusted in a small Gutter made at the Angle (which is 45 Degrees) of one of the Branches of the Square, three Knuckles proportionable in Length and Breadth, to the Length and Breadth of the Square. These Knuckles ought to be so far distant from each other, that they may exactly receive between them two other Knuckles, which are adjusted to the other Branch of the Square. The Knuckles being thus placed, are soldered to the Branches, and afterwards are united to one another by means of a Pin, which must exactly fill the Cavity of the Knuckles, that thereby the Motion of the Branches may be steady. Fig. D.

Note. There are some Squares to which a Thread and Plummet is hung, which serves for levelling; that is, to make horizontal Planes: also upon one of the Sides of the Square are sometimes sundry Lines and Scales placed; and upon the other, half a Foot divided into 6 Inches, every one of which is subdivided into 12 Lines: moreover, there are sometimes added to it other Country Measures compared with the *Paris* Foot.

USE I. *To let fall from a given Point, a Perpendicular upon a given Line.*

Let the Line given be A B, and C the given Point, either in or without the Line.

Fig. 45. Apply one of the Sides of the Square to the given Line, in such manner that the other Side touches the given Point; then draw the Line C D, which will be a Perpendicular. *Note*, If the Square be turned about, and that Side which before was applied to the Line, is made to pass thro' the Point C, and, as before, another Line be drawn, as C D: by this means you may know whether the Square be true. For when it is true, the two Lines drawn thro' the Point C, will make but one Line.

USE II. *To know if one Line be perpendicular to another; that is, whether they make right Angles with each other.*

Apply one of the Sides of the Square to one of the Lines, and see if the other Side exactly agrees with the other Line. All this is so extreme easy, there needs but a few words to explain it.

CHAP. III.

Of the Construction and Uses of the Protractor.

THE Protractor is a Semi-Circle divided into 180 Degrees, or half of 360, which every whole Circle is supposed to be divided into, as was said in the Definitions. One Side of this Instrument is filed flat, for better applying it on the Paper; and the other Side is Fig. E. sloped;

floped; that is, made thin towards the Edge whereon the Divifions are: and for better difcovering the Points wherein Angles terminate, there is a fmall femicircular Notch made in the Center of the Inftrument.

How to divide the Limb of the Protractor.

Upon the Line A B, and about the Center O, defcribe a Semicircle; then carry the Radius A O round the Circumference, which will divide the Semicircle into three equal Parts, in the Points C, D, each of which is 60 Degrees. Again, divide the Arc B C into two equal Parts, in the Point E, and the Arc B E, will be 30 Degrees: then turning this Opening of your Compaffes round the Semicircle, it will divide it into fix equal Parts. Moreover, divide them again into three equal Parts, and each will be 10 Degrees; and dividing every one of thefe 10 Degrees into two equal Parts more, Arcs of 5 Degrees will be had. And laftly, in fubdividing each of thefe Arcs of 5 Degrees, into five equal Parts, Arcs of one Degree will be had.

In the fame manner may a whole Circle be divided into 360 Degrees, which we fhall fpeak of hereafter.

Note, Protractors are fometimes made of Horn, which, becaufe they are tranfparent, are commodious enough; but they ought to be kept in a Book when they are not ufing, becaufe the Horn is apt to wrinkle.

USE I. *To make an Angle of any Number of Degrees.*

For Example; to make at the Point A, an Angle of 50 Degrees on the Line C A B, lay the Center of the Protractor, marked by a femicircular Cavity, upon the Point A, fo that the Diameter of the Semicircle be upon the Line A B; then make a Dot over againft the 50th Fig. 45.
Degree of the Limb of the Protractor, and thro' it draw a Line to the Point A, which will make an Angle of 50 Degrees with the Line A B.

USE II. *The Angle* B A D *being given, to find how many Degrees it contains.*

Lay the Center of the Protractor upon the Point A, and it's Diameter upon the Line B C; then fee what Degree the Line A B cuts the Limb of the Protractor in, which will be the Fig. 46.
Angle B A D of 50 Degrees.

USE III. *To infcribe any regular Polygon in a Circle.*

To do this, you muft firft know how many Degrees the Angle of the Center of each of the regular Polygons contains; which may be found in dividing 360 Degrees, by the Number of Sides of a propofed Polygon: as, for Example, dividing 360 by 5, the Quotient 72, fheweth that the Angle of the Center of a Pentagon is 72 Degrees: again, in dividing 360 by 8, the Quotient 45, gives the Quantity of the Angle of the Center of an Octagon, and fo for others.

In knowing the Angle of the Center, the Angle formed by the Sides of the Polygon may likewife be known, in fubftracting the Angle of the Center of the Polygon from 180 Degrees; as taking 72 Degrees, the Angle of the Center of a Pentagon from 180 Degrees, there remains 108, the Angle of the Polygon. Moreover, taking from 180 Degrees, the Angle of the Center of an Octagon, which is 45 Degrees, there remains 135 Degrees, the Angle of the Octagon.

Therefore to infcribe a Pentagon in a Circle, lay the Center of the Protractor upon the Fig. 47.
Center of the Circle, and apply the Diameter of the Protractor, to the Diameter of the Circle; then make a Dot againft the 72^d Degree of the Limb of the Protractor; and thro' this Dot, and the Center of the Circle, draw a Line cutting the Circumference of the Circle in the Point C. Now take between your Compaffes the Diftance of the Points B and C, which will divide the Circumference of the Circle into 5 equal Parts, and drawing 5 right Lines, the Polygon will be made.

If a Heptagon is to be infcribed, divide 360 Degrees by 7, and the Quotient $51\frac{3}{7}^d$ fheweth, that the Angle of the Center is almoft $51\frac{1}{2}^d$; therefore having placed the Protractor, as before, Note, $51\frac{1}{2}$ Degrees on the Limb of the Protractor, thro' which draw a Line from the Center of the Circle, and you will have the Side of the Heptagon.

Note, Upon fome Protractors are placed the Numbers, denoting regular Polygons, to avoid the trouble of Divifion, in finding the Angles at the Center: as the Number 5, for a Pentagon, is fet againft 72 Degrees on the Limb of the Protractor; the Number 6 for a Hexagon, is fet over-againft 60 Degrees, the Number 7 againft $51\frac{1}{2}^d$, *&c.*

USE IV. *To defcribe any regular Polygon upon a given Line.*

Let the given Line be C D, upon which it is required to defcribe a regular Pentagon.

We have fhewn in the precedent Ufe, how to find the Angles of any regular Polygon; and Fig. 48.
fince the Angle made by the two Sides of the Polygon is 108 Degrees, 54 Degrees it's half will be the Semi-Angle of the Polygon; by means of which, you may defcribe it in the following manner:

Apply the Diameter of the Protractor to the Line C D, and it's Center to the End D; then make a Dot againft the 54th Degree of the Limb, and draw the Line D F, making an

Angle of 54^d with the Line C D. Moreover, remove the Center of the Protractor to the other End C, and there likewise make an Angle of 54 Degrees, by drawing the Line C F; then about the Point of Concourse F, describe a Circle with the Distance C F. Lastly, take the Length of the given Line C D, and carry it round the Circumference of the Circle, and drawing four right Lines, the Pentagon will be made.

If an Octagon is to be described upon a given right Line, take half the Angle of the Polygon, which is $67\frac{1}{2}$ Degrees, and make an Angle of the like Number of Degrees upon each End of the given Line, by which an Isosceles Triangle will be formed, whose Vertex will be the Center of a Circle, which will be divided into eight equal Parts, by carrying the Compasses round it with the Extent of the given Line.

There may be made many more Operations with the Instruments already spoken of; but we shall content ourselves with those already mentioned, as being the most common, and useful.

ADDITIONS *of* English *Instruments.*

Of the Construction and Uses of the Carpenter's Joint-Rule, *the* Four-foot Gauging-Rod; *Everard's* Sliding-Rule; *Coggeshall's* Sliding-Rule, *the* Plotting-Scale, *an* Improved Protractor, *the* Plain-Scale, *and* Gunter's Scale.

CHAP. I.

Of the Construction and Uses of the Carpenter's Joint-Rule, *together with the Line of Numbers commonly placed thereon.*

Plate 3. Fig. 1. THIS Rule is usually made of Box, 24 Inches long, an Inch and a half, or an Inch and a quarter broad, and of a Thickness at pleasure; one Side of it is divided into 24 equal Inches, according to the Standard at *Guildhall*, *London*, and every one of these 24 Inches are divided into 8 equal Parts; that is, into halfs, quarters, and half-quarters: The half-inches are distinguished from the quarters, and the quarters from the half-quarters, by Strokes of different Lengths, and at every whole Inch are set Figures, proceeding from 1 to 24.

On the same Side of this Rule, is commonly placed *Gunter*'s Line of Numbers, of which more hereafter.

Fig. 2. The other Side of the Rule hath upon it the Lines of Timber and Board-Measure, the Construction of which is as follows:

The Line of Timber-Measure begins at 8 and a half; that is, when the Figures of the Timber-Line stand upright to you, it begins at the left End at 8 and a half, and proceeds to 36, within an Inch, and $\frac{3}{8}$ of an Inch of the other End. It is made from a Consideration, that 1728 Inches make a solid Foot: for any Division; suppose 9, which signifies the Side of a Square is so placed against some one of the Divisions of Inches or Parts on the other Side, beginning from the right Hand, that it's Square, which is 81 Inches, multiplied by that Number of Inches and Parts, must make 1728 Inches, or a solid Foot; which in dividing 1728 by 81, must be placed against $21\frac{1}{3}$ Inches from the right Hand. In like manner the Division for the Number 10, on the Line of Timber-Measure, must be placed against $17\frac{28}{100}$ Inches on the other Side; because 1728, divided by the Square of 10, which is 100, gives $17\frac{28}{100}$, and in like manner for all the other Divisions. But because a Square, whose Side is either 1, 2, *&c.* to 8 Inches, requires more than 24 Inches in Length to multiply it by, in order to make a solid Foot, or 1728 Inches; and since 24 Inches is the whole Length of the Rule, therefore there is a Table put upon the left end of the Rule, supplying a greater Length.

The upper Row of Figures, numbered 1, 2, 3, 4, 5, 6, 7, 8, are Inches, or the Lengths of the Sides of Squares; and the second and third Rows are the correspondent Feet and Inches to make up a solid Foot. It is made by dividing 144 Inches by the Squares of 1, 2, 3, 4, 5, 6, 7, 8; as the Square of 1 Inch is 1, by which dividing 144, the Quotient will be 144 Feet for the first Number of the second Row of Figures, and in like manner for the rest.

On

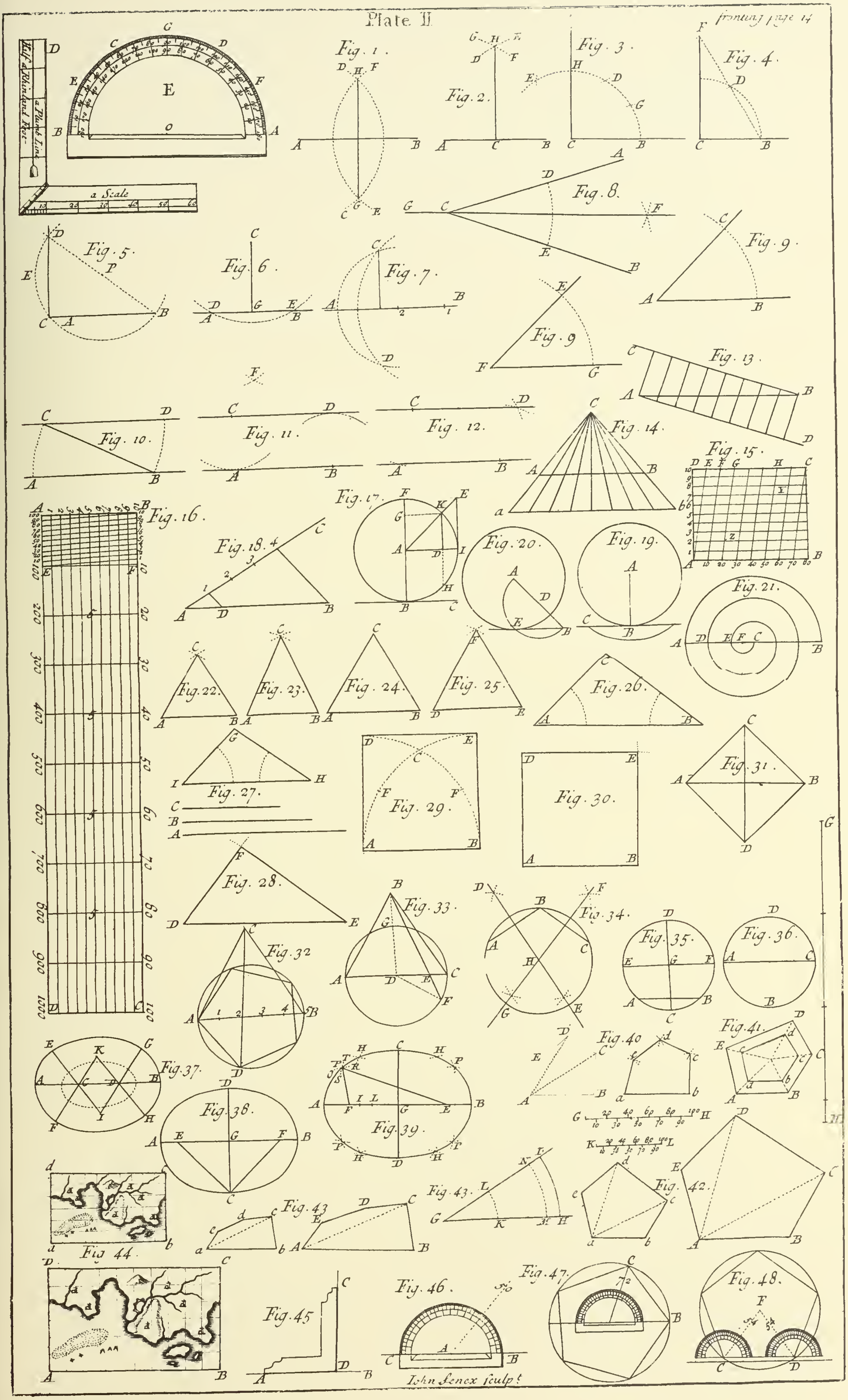


Plate II
fronting page 14
Half a Rhimbard Rect.
a Plumb Line
a Scale
John Senex sculpt.

On, or next the other Edge of the Rule, you have the Line of Board-Meafure; and when the Figures ftand upright, you fee it numbered 7, 8, 9, *&c.* to 36. which is juft 4 Inches from the right Hand. It is thus divided; fuppofe the Divifion 7 is to be marked, divide 144, which is the Number of Inches in a fquare Foot, by 7, and the Quotient will be 20 $\frac{4}{7}$ Inches; whence the Divifion 7 muft be againft 20 $\frac{4}{7}$ Inches on the other Side of the Rule. Again, to mark the Divifion 8, divide 144 by 8, and the Quotient, which is 18 Inches, muft be placed on the Line of Board-Meafure againft 18 Inches on the other Side: proceed thus for the other Divifions of the faid Line. But becaufe the Side of a long Square, that is either 1, 2, 3, 4, 5 Inches, requires the other Side to be more than 24 Inches, which is the whole Length of the Rule; therefore there is a Table placed at the other end of the Rule, made in dividing 144 Inches by each of the Numbers in the upper Row, and then each of the Quotients by 12, to bring them into Feet.

USE *of the Carpenter's Joint-Rule.*

The Inches on this Rule are to meafure the Length or Breadth of any given Superficies or Solid, and the manner of doing it is fuperfluous to mention, it being not only eafy, but even natural to any Man; for holding the Rule in the left Hand, and applying it to the Board, or any thing to be meafured, you have your Defire. But now for the Ufe of the other Side, I fhall fhew in two or three Examples in each Meafure, that is, Superficial and Solid.

Example I. *The Breadth of any Superficies; as Board, Glafs, or the like, being given: to find how much in Length makes a Square Foot.*

To do which, look for the Number of Inches your Superficies is broad, in the Line of Board Meafure, and keep your Finger there; and right againft it, on the Inches Side, you have the Number of Inches that makes up a Foot of Board, Glafs, or any other Superficies. Suppofe you have a Piece 8 Inches broad, how many Inches make a Foot? Look for 8 on the Board Meafure, and juft againft your Finger (being fet to 8) on the Inch-Side, you will find 18, and fo many Inches long, at that Breadth, goes to make a fuperficial Foot.

Again, fuppofe a Superficies is 18 Inches broad, then you will find that 8 Inches in Length will make a fuperficial Foot; and if a Superficies is 36 Inches broad, then 4 Inches in Length makes a Foot.

Or you may do it more eafy thus: Take your Rule, holding it in your left Hand, and apply it to the Breadth of the Board or Glafs, making the End, which is next 36, even with one Edge of the Board or Glafs, and the other Edge of the Board will fhew how many Inches, or Quarters of an Inch, go to make a fquare Foot of Board or Glafs. This is but the Converfe of the former, and needs no Example; for laying the Rule to it, and looking on the Board-Meafure, you have your Defire.

Or elfe you may do it thus, in all narrow Pieces under 6 Inches broad: As fuppofe 3 $\frac{1}{4}$ Inches, double 3 $\frac{1}{4}$, it makes 6 $\frac{1}{2}$; then twice the Length from 6 $\frac{1}{2}$ to the End of the Rule, will make a fuperficial Foot, or fo much in Length makes a Foot.

Example II. *A Superficies of any Length or Breadth being given, to find the Content.*

Having found the Breadth, and how much makes one Foot, turn that over as many times as you can upon the Length of the Superficies, for fo many Feet are in that Superficies: But if it is a great Breadth, you may turn it over two or three times, and then take that together; and fo fay 2, 4, 6, 8, 10, *&c.* or 3, 6, 9, 12, 15, 18, 21, 'till you come to the End of the Superficies.

The USE *of the Table at the End of the Board-Meafure.*

If a Superficies is 1 Inch broad, how many Inches in Length muft there go to make a fuperficial Foot? Look in the upper Row of Figures for 1 Inch, and under it, in the fecond Row, you will find 12 Feet; which fhews that 12 Feet in Length, and 1 Inch in Breadth, will make a fuperficial Foot.

Again, a Superficies 5 Inches broad, will be found, in the faid Table, to have 2 Feet and about 5 Inches in Length to make a fuperficial Foot; and a Piece 8 Inches broad, will have a Length of 1 Foot 6 Inches to make a fuperficial Foot.

USE *of the Line of Timber-Meafure.*

The Ufe of this Line is much like the former: for firft you muft learn how much your Piece is fquare, and then look for the fame Number on the Line of Timber-Meafure, and the Space from thence to the End of the Rule, is the true Length at that Squarenefs to make a Foot of Timber.

Example. There is a Piece that is 9 Inches fquare, look for 9 on the Line of Timber-Meafure, and then the Space from 9, to the End of the Rule, is the true Length to make a folid Foot of Timber, and it is 21 $\frac{1}{3}$ Inches.

Again, fuppofe a Piece of Timber is 24 Inches fquare, then 3 Inches in Length will make a Foot, for you will find three Inches on the other Side againft 24: But if it is fmall Timber, as under 9 Inches fquare, you muft feek the Square in the upper Rank in the Table, and

and right under you have the Feet and Inches that go to make a fold Foot, as was in the Table of Board Meafure: As fuppofe a Piece of Timber is 7 Inches fquare, look in the Table for 7, in the upper Row of Numbers, and you will find directly under 2 Feet, 11 Inches, which is the Length of the Piece of Timber that goes to make a folid Foot: But if a Piece be not exactly fquare, *viz.* is broader at one Side than the other, then the ufual way is to add them both together, and take half the Sum for the Side of the Square; but if they differ much, this way is very erroneous: for that half is always too great, which from hence will eafily be manifeft.

Fig. 3. Let A C be the longeft Side, C D the fhorteft, and B D, or A B, half their Sum, which is taken for the Side of the Square, that is, for the Side of a Square whofe Area is equal to the Product of the two Sides A C, and C D, into one another, or the Rectangle under them: Now with the Diftance B D, and on the Center B, defcribe a Semicircle; draw the Diameter E B, at right Angles, to A D, and from the Point C raife the Perpendicular F C; then it is manifeft, *per Prop.* 13. *lib.* 6. *Eucl.* that F C is a mean Proportional between the Sides A C, C D; that is, F C is the true Side of the Square, which, *per Prop.* 15. *lib.* 3. *Eucl.* is much lefs than E B, or it's Equal A B, or B D.

The ufual way likewife for round Timber, is to take a String, and girt it about, and the fourth part of it is commonly allowed for the Side of the Square, that is, for the Side of a Square equal to the circular Bafe, and then you deal with it as if it was juft Square. But this way is alfo erroneous; for by this Method you lofe above $\frac{1}{5}$ of the true Solidity. But for maintaining this ill Cuftom, they plead, The Overplus Meafure may well be allowed, becaufe the Chips cut off are of little Value, and will not near countervail the Labour of bringing the Timber to a Square, to which Form it muft be brought before it be fit to ufe.

The Defcription of Gunter's *Line, or the Line of Numbers.*

The Line of Numbers is only the Logarithms transferred on a Ruler from the Tables, by means of a Scale divided into a great Number of equal Parts; and whereas in the Logarithms, by adding or fubftracting them from one another, the *Quæfita* is produced; fo here, by turning a Pair of Compaffes forwards or backwards, according to due Order on this Line, the *Quæfita* will in like manner be produced. The Conftruction of this Line I fhall give in fpeaking of *Gunter*'s Scale.

As to the Length of the Line of Numbers, the longer it is, the better it is; whence it hath been contrived feveral ways: As firft upon a Rule of two Foot, and a Rule of three Foot long, by *Gunter*, which (as I fuppofe) is the Reafon why it is called *Gunter*'s Line; then that Line was doubled, or laid fo together, that you might work either right on, or crofs from one to another, by Mr *Windgate*; afterwards projected in a Circle, by Mr *Oughtred*, and alfo to flide one by another, by the the fame Author; and laft of all, projected into a kind of Spiral, of 5, 10, or 20 Turns, more or lefs, by Mr *Brown*, the Ufes being in all of them in a manner the fame, only fome with Compaffes, as Mr *Gunter*'s and Mr *Windgate*'s; and fome with flat Compaffes, or an opening Index, as Mr *Oughtred*'s and Mr *Brown*'s; and fome without either, as the Sliding-Rules.

The Order of the Divifions on this Line of Numbers, and commonly on moft others, is thus; it begins with 1, and fo proceeds with 2, 3, 4, 5, 6, 7, 8, 9; and then 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, whofe Order of Numeration is thus: The firft 1 fignifies one Tenth of any whole Number or Integer, and confequently the next 2 is two Tenths; 3, three Tenths; and all the fmall intermediate Divifions are 100 Parts of an Integer, or a Tenth of one of the former Tenths; fo that 1 in the middle is one whole Integer; the next 2, two Integers; and 10 at the end, 10 Integers: Thus the Line is in it's moft proper Acceptation, or natural Divifion.

But if you are to deal with a Number greater than 10, then 1 at the beginning muft fignify 1 Integer, and 1 in the middle 10 Integers, and 10 at the end 100 Integers. But if you would have it to a Figure more, then the firft 1 is 10, the fecond 100, and the laft 10 a 1000. If you proceed further, then the firft 1 is 100, the middle 1 a 1000, and the 10 at the end 10000, which is as great a Number as can well be difcovered, on this or moft ordinary Lines of Numbers; and fo far, with convenient Care, you may refolve a Queftion tolerably exact.

Numeration on the Line of Numbers.

Any whole Number being given under four Figures, to find the Point on the Line of Numbers that reprefents the fame.

Firft look for the firft Figure of your Number amongft the long Divifions that are figured, and that leads you to the firft Figure of your Number; then for the fecond Figure, count fo many Tenths from that long Divifion forwards, as that fecond Figure amounts to; then for the third Figure, count from the laft Tenth fo many Centefmes as the third Figure contains; and fo for the fourth Figure, count, from the laft Centefme, fo many Millions as that fourth Figure has Units, or is in Value, and that will be the Point where the Number propounded is on the Line of Numbers. Two or three Examples will make this manifeft.

Firft, to find the Point upon the Line of Numbers reprefenting the Number 12. Now becaufe the firft Figure of this Number is 1, you muft take the 1 in the middle for the firft Figure;

Figure; then the next Figure being 2, count two Tenths from that 1, and there will be the Point repreſenting 12.

Secondly, To find the Point repreſenting 144. Firſt, as before, take for 1 the firſt Figure of the Number 144, the middle Figure 1; then for the ſecond (*viz.* 4.) count four Tenths forwards; laſtly, for the other 4, count four Centeſms further, and that is the Point for 144.

Thirdly, To find the Point repreſenting 1728. Firſt, as before, for 1000 take the middle 1 on the Line. Secondly, for 7 reckon ſeven Tenths forwards, and that is 700. Thirdly, for 2, reckon two Centeſms, from that 7th Tenth, for 20. And, Laſtly, for 8 you muſt reaſonably eſtimate that following Centeſm to be divided into 10 Parts (if it be not expreſſed, which in Lines of ordinary Length cannot be done), and 8 of that ſuppoſed 10 Parts is the preciſe Point for 1728, the Number propounded to be found; and the like of any other Number.

But if you was to find a Fraction, you muſt conſider, that properly, or abſolutely, the Line only expreſſes Decimal Fractions; as thus, $\frac{1}{10}$, or $\frac{1}{100}$, or $\frac{1}{1000}$, and more near the Rule in common Acceptation cannot expreſs; as one Inch, one Tenth, one Hundredth, or one Thouſandth Part of an Inch, it being capable to be applied to any thing in a decimal way: (But if you would uſe other Fractions, as Quarters, Half-Quarters, *&c.* you muſt reaſonably read them, or elſe reduce them into Decimals.)

The fundamental Uſes of the Line of Numbers.

USE I. *Two Numbers being given, to find a third Geometrically proportional to them, and to three a fourth, and to four a fifth,* &c.

Extend your Compaſſes upon the Line of Numbers, from one Number to another; which done, if that Extent is applied (upwards or downwards, as you would either increaſe or diminiſh the Number), from either of the Numbers, the moveable Point will fall upon the third proportional Number required. Alſo the ſame Extent, applied the ſame way from the third, will give you a fourth, and from the fourth a fifth, *&c.* For Example, let the Numbers 2 and 4 be propoſed, to find a third Proportional, *&c.* to them: Extend the Compaſſes upon the firſt Part of the Line of Numbers, from 2 to 4; which done, if the ſame Extent is applied upwards from 4, the moveable Point will fall upon 8, the third Proportional required; and then from 8 it will reach to 16, the fourth Proportional; and from 16 to 32 the fifth, *&c.* Contrariwiſe, if you would diminiſh, as from 4 to 2, the moveable Point will fall on 1, and from 1 to $\frac{5}{10}$, or .5, and from .5 to .25, *&c.* as is manifeſt from the Nature of the Logarithms, and *Prop.* 20. *lib.* 7. *Eucl.*

But generally in this, and moſt other Work, make uſe of the ſmall Diviſions in the middle of the Line, that you may the better eſtimate the Fractions of the Numbers you make uſe of; for how much you miſs in ſetting the Compaſſes to the firſt and ſecond Term, ſo much the more you will err in the fourth; therefore the middle Part will be moſt uſeful: As for Example, as 8 to 11, ſo is 12 to 16.50, if you imagine one Integer to be divided but into 10 Parts, as they are on the Line on a two-foot Rule.

USE II. *One Number being given to be multiplied by another given Number, to find the Product.*

Extend your Compaſſes from 1 to the Multiplicator, and the ſame Extent, applied the ſame way from the Multiplicand, will cauſe the moveable Point to fall upon the Product; as is manifeſt from the Nature of the Logarithms, and *Defin.* 15. *lib.* 7. *Eucl.*

Example. Let 6 be given to be multiplied by 5; extend your Compaſſes from 1 to 5, and the ſame Extent will reach from 6 to 30, the Product ſought. Again, ſuppoſe 125 is to be multiplied by 144; extend you Compaſſes from 1 to 125, and the moveable Point will fall from 144 on 18000 the Product.

USE III. *One Number being given to be divided by another, to find the Quotient.*

Extend your Compaſſes from the Diviſor to 1, and the ſame Extent will reach from the Dividend to the Quotient; or, extend the Compaſſes from the Diviſor to the Dividend, the ſame Extent will reach the ſame way from 1 to the Quotient, as is manifeſt from the Nature of the Logarithms, and this Property, that as the Diviſor is to Unity, ſo is the Dividend to the Quotient.

Example. Let 750 be a Number given, to be divided by 25 (the Diviſor), extend your Compaſſes downwards from 25 to 1; then applying that Extent the ſame way from 750, and the other Point of the Compaſſes will fall upon 30, the Quotient ſought. Again, let 1728 be given to be divided by 12; extend your Compaſſes from 12 to 1, and the ſame Extent will reach the ſame way from 1728 to 144.

If the Number is a Decimal Fraction, then you muſt work as if it was an abſolute whole Number; but if it is a whole Number joined to a decimal Fraction, it is worked here as properly as a whole Number: As ſuppoſe 111.4 is to be divided by 1.728, extend your Compaſſes from 1.728 to 1, the ſame Extent, applied from 111.4, will reach to 64.5. So again, 56.4 being to be divided by 8.75, and the Quotient will be found to be 6.45.

Now to know of how many Figures any Quotient ought to conſiſt, it is neceſſary to write down the Dividend, and the Diviſor under it, and ſee how often it may be written under it; for ſo many Figures muſt there be in the Quotient: As in dividing this Number 12231 by 27, according to the Rules of Diviſion, 27 may be written 3 times under the Dividend; therefore there muſt be 3 Figures in the Quotient: for if you extend the Compaſſes from 27 to 1, it will reach from 12231 to 453, the Quotient ſought.

Note, That in this Uſe, or any other, it is beſt to order it ſo, that your Compaſſes may be at the cloſeſt Extent; for you may take a cloſe Extent more eaſy and exact than a large Extent, as by Experience you will find.

USE IV. *Three Numbers being given, to find a fourth in a direct Proportion.*

Extend your Compaſſes from the firſt Number to the ſecond; that done, the ſame Extent applied the ſame way from the third, will reach to the fourth Proportional ſought, as is manifeſt from the Nature of the Logarithms, and *Prop.* 19. *lib.* 7. *Eucl.* from whence it may be gathered, that the third Number multiply'd by the ſecond, divided by the firſt, will give the fourth ſought.

Example. If 7 give *e* 22, what will 14 give? Extend your Compaſſes upwards from 7 to 14, and that Extent applied the ſame way, will reach from 22 to 44, the fourth Proportional required. Again, if 38 gives 76, what will 96 give? Extend your Compaſſes from 38 to 96, and the ſame Extent will reach from 76 to 192, the fourth Proportional ſought.

USE V. *Three Numbers being given, to find a fourth in an Inverſe Proportion.*

Extend your Compaſſes from the firſt of the given Numbers to the ſecond of the ſame Denomination; if that Diſtance be applied from the third Number backwards, it will reach to the fourth Number ſought.

Example. If 60 give *e* 5, what will 30 give? Extend your Compaſſes from 60 to 30, and that Extent applied the contrary way from 5, will give 2.5 the Anſwer. Again, If 60 gives 48, what will 40 give? Extend your Compaſſes from 60 to 40; that Extent applied the contrary way from 48, will reach to 32, the fourth Number ſought.

USE VI. *Three Numbers being given, to find a fourth in a duplicate Proportion.*

This Uſe concerns Queſtions of Proportions between Lines and between Superficies; now if the Denominations of the firſt and ſecond Terms are Lines, then extend your Compaſſes from the firſt Term to the ſecond (of the ſame kind of Denomination): this done, that Extent applied twice the ſame way from the third Term, and the moveable Point will fall upon the fourth Term required, which is manifeſt from the nature of the Logarithms, and from hence, *viz.* Becauſe the fourth Number to be found is only a fourth Proportional to the Square of the firſt, the Square of the ſecond, and the third, it is plain that the third, multiplied by the Square of the ſecond, divided by the third, will be the fourth Number ſought.

Example. If the Area of a Circle, whoſe Diameter is 14, be 154, what will the Content of a Circle be, whoſe Diameter is 28? Here 14 and 28 having the ſame Denomination, *viz.* both Lines, extend the Compaſſes from 14 to 28, then applying that Extent the ſame way from 154 twice, the moveable Point will fall upon 616, the fourth Proportional or Area ſought: Becauſe Circles are to each other as the Squares of their Diameters, *per Prop.* 2. *lib.* 12. *Eucl.*

USE VII. *Three Numbers being given, to find a fourth in a triplicate Proportion.*

This Uſe is to find the Proportion between the Powers of Lines and Solids; that is, two Lines being given and a Solid, to find a fourth Solid, that has the ſame Proportion to the given Solid, as the given Lines have to one another. Therefore extend the Compaſſes from the firſt Line to the ſecond, and that Extent, applied three times from the given Solid or third Number, will give the fourth ſought: Becauſe the third multiplied by the Cube of the ſecond, divided by the Cube of the firſt, will give the fourth.

Example. If an Iron Bullet, whoſe Diameter is 4 Inches, weighs 9 Pounds, what will another Iron Bullet weigh, whoſe Diameter is 8 Inches? Extend your Compaſſes from 4 to 8, that Extent applied the ſame way three times from 9, will give 72, the Weight of the Bullet ſought. Becauſe the Weight of homogeneal Bodies are as their Magnitudes, and Spheres are to one another as the Cubes of their Diameters, *per Prop.* 16. *lib.* 12. *Eucl.*

USE VIII. *To find a mean Proportional between two given Numbers.*

Biſect the Diſtance between the given Numbers, which Point of Biſection will fall on the mean Proportional ſought: Becauſe the ſquare Root of the Quotient of the two Extremes divided by one another, mutiplied by the leſſer, is equal to the Mean.

Example. The Extremes being 8 and 32, the middle Point between them will be found to be 16.

USE IX. *To find two mean Proportionals between two given Lines.*

Triſect the Space between the two given Extremes, and the two Points of Triſection will give the two Means. Becauſe the Cube Root of the Quotient of the Extremes divided by one another, multiplied by the leſſer Extreme, will give the firſt of the Mean Proportionals ſought, and that firſt Mean multiplied by the aforeſaid Cube Root, will give the ſecond.

Example.

Example. Let 8 and 27 be the two given Extremes, the two Means will be found to be 12 and 18, which are the two Means sought.

USE X. *To find the Square Root of any Number under* 1000000.

The Square Root of any Number is always a mean Proportional between 1, and the Number whose Root you would find; but yet with this general Caution, *viz.* If the Figures of the Number are even, that is, 2, 4, 6, 8, 10, *&c.* then you must look for the Unit at the Beginning of the Line, and the Number in the second Part or Radius, and the Root in the first Part; or rather reckon 10 at the end to be Unity, and then both Root and Square will fall backwards towards the middle in the second Length or Part of the Line: But if they be odd, then the middle 1 will be most convenient to be counted Unity, and both Root and Square will be found from thence forwards towards 10; so that according to this Rule, the Square Root of 9 will be found to be 3, the Square Root of 64 will be found to be 8, the Square Root of 144 to be 12, *&c.*

USE XI. *To find the Cube Root of any Number under* 1000000000.

The Cube Root is always the first of two mean Proportionals between 1 and the Number given, and therefore to be found by trisecting the Space between them; whence the Cube Root of 1728 will be found 12, the Root of 17280 is near 26, the Root of 172800 is almost 56. Although the Point on the Line representing all the square Numbers is in one place, yet by altering the Unit, it produceth various Points and Numbers for their respective proper Roots. The Rule to find this, is in this manner: You must set Dots (or suppose them to be set) over the first Figure to the Left-hand, the fourth Figure, the seventh, and the tenth; now if by this means the last Dot to the Left-hand falls on the last Figure, as it doth in 1728, then the Unit must be placed at 1 in the middle of the Line, and the Root, the Square, and Cube, will all fall forwards towards the end of the Line.

But if it falls on the last but 1, as it doth in 17280, then the Unit may be placed at 1 in the Beginning of the Line, and the Cube in the second Length; or else the Unit may be placed at 10 in the End of the Line, and the Cube in the first Part of the Line. But, if the last Dot falls under the last Figure but two, as in 172800, the Unit must always be placed at 10 in the End of the Line, and then the Root, the Square, and Cube, will all fall backwards, and be found in the second Part, between the Middle 1, and the End of the Line. By these Rules it appears, that the Cube Root of 8 is 2, the Cube Root of 27 is 3, the Cube Root of 64 is 4, of 125 is 5, of 216 is 6, of 345 is 7, of 512 is 8, of 729 is 9, of 1000 is 10, *&c.*

CHAP. II.

Of the Construction and Use of the Four-Foot Gauging-Rod.

THIS Rod, whose Use is to find the Quantities of Liquors contained in any kinds of Vessels, is usually made of Box-Wood, and consists of four Rules, each a Foot long, and about $\frac{3}{8}$ of an Inch square, joined together by three Brass Joints; by which means the Rod is rendered four Foot long, when the four Rules are quite opened, and but one Foot when they are folded together.

On the first Face of this Rod are placed two Diagonal Lines, one for Beer, and the other for Wine; by means of which the Content of any common Vessel in Beer or Wine Gallons may be readily found, in putting the Rod in at the Bung-hole of the Vessel until it meets the Interfection of the Head of the Vessel with the opposite Staves to the Bung-hole. For distinction of this Line, there is writ thereon *Beer* and *Wine Gallons.* Fig. 4.

On the second Face of this Rod, are, a Line of Inches, and the Gauge Line, which is a Line expressing the Area's of Circles, whose Diameters are the correspondent Inches in Ale Gallons. At the Beginning of it is writ, *Ale Area.* Fig. 5.

On the third Face are three Scales of Lines; the first, at the end of which is writ *Hogshead*, is for finding how many Gallons there is in a Hogshead, when it is not full, lying with it's Axis parallel to the Horizon. The second Line, at the End of which is writ B. L. signifying a Butt lying, is for the same Use as that for the Hogshead. The third Line is to find how much Liquor is wanting to fill up a Butt when it is standing. At the End of it is writ B. S. signifying a Butt standing. Fig. 6

Half way the fourth Face of the Gauging-Rod are three Scales of Lines, to find the Wants in a Firkin, Kilderkin, and Barrel, lying with their Axes parallel to the Horizon. They are distinguished by the Letters F. K. B. signifying a Firkin, Kilderkin, and Barrel. Fig. 7.

Construction

Construction of the two Diagonal Lines.

These two Diagonal Lines are put upon this Gauging-Rod, in the fame manner that our Author, in the laft Ufe of the Line of Solids in the fecond Book directs, for putting on the Diagonals on his Gauging-Rod, *viz.* by taking the Diagonal of fome Veffel that is fimilar, or nighly fimilar to the Veffels, whofe Contents in Beer, or Wine Gallons, are afterwards, by means of them, to be found; and then knowing how many Gallons in Beer and Wine the aforefaid Veffel contains, which Gallons muft be fet againft the Inches, or Parts of Inches, of their Diagonals Length, on the Diagonal-Face of the Gauging-Rod. Now to find how many Inches, or Parts, the Diagonal of any other fimilar Veffel muft be, when it's Content in Beer and Wine-Gallons is given; you muft fay, As the Content of the firft Veffel, which is known, is to the Cube of the Length of it's Diagonal; So is the Content of that other fimilar Veffel, in Beer or Wine-Gallons, to the Cube of the Length of it's Diagonal: the Cube-Root of which extracted, will give the Length of the Diagonal fought. As for Example, fuppofe a little Veffel fimilar, or nighly fimilar to *Englifh* Veffels of a ufual Form, contains 1 Beer Gallon, or about 1 $\frac{2}{5}$ Wine Gallon, and the Diagonal is found to be 7.75 Inches; what will be the Diagonal of a fimilar Veffel, containing 2 Beer Gallons, or 2.8 Wine Gallons? Say, As 1 Gallon is to the Cube of 7.75, which is 465.48437, So is 2 Gallons to the Cube of the Diagonal fought, 930.96875, whofe Root will be 9.72 Inches, and fo much will be the Length of the Diagonal: therefore fet 2 Beer Gallons on the Diagonal Face of the Rod, againft 9.72 Inches. In this manner may the Diagonal Face of the Rod be divided from 1 Beer Gallon to 240, and from 1 Wine Gallon to 300, and fubdivided in half Gallons, as on the Rod.

Construction of the Gauge-Line on the fecond Face of the Rod.

On this Line is fet the Gallons, and hundred Parts of Gallons, that any Cylinder, an Inch deep, and any Inches and Parts, from 1 to 46 in Diameter, contains of Ale. As for Example; againft 1.9 Inches ftands .01 of a Gallon, denoted by a Dot; againft 2.63 Inches ftands .02 of a Gallon. The Tenths of the Gallons are denoted by the Figures 1, 2, 3, 4, *&c.* as .1 of a Gallon is fet againft 5.96 Inches; .2 againft 8.44 Inches, and 1 Gallon againft 18.95 Inches, as *per* Figure. The Conftruction of this Line is thus: Becaufe 282 folid Inches make an Ale Gallon, therefore the Diameter of a Cylinder, one Inch deep, whofe Content is an Ale Gallon, or 282 folid Inches, will be 18.95 Inches; whence againft 18.95 Inches, on the fame Face of the Gauging-Rod, fet, on the Line drawn to contain the Divifions of the Gauge-Line, 1 Gallon. Now to find the Diameter of a Cylinder one Inch deep, that fhall contain the .01 Part of a Gallon, fay, As 1 Gallon is to the .01 Part of a Gallon, So is the the Square of 18.95 Inches, which is 359, to the Square of the Diameter of the Cylinder, containing the hundredth Part of a Gallon, which will be found by extracting the fquare Root of that Quantity 1.9 Inch: therefore fet the firft Dot againft 1.9 of an Inch. Again, to find againft what Inches, or Parts, .02 of a Gallon muft be placed, fay, As 1 is to .02, So is 359 to the Square of the Number of Inches, or Parts, whofe Root extracted will give 2.63 Inches; againft which make a fecond Dot for .02 of a Gallon. In this manner proceed for all the other Divifions on the Gauge-Line, always making 1 and 359 the two firft Terms of the Proportion, and the Gallons or Parts the third; fo fhall the fourth be the Square of the Inches, or Parts, that the Gallons, or Parts expreffed in the third Term, are to be fet againft. The Reafon of the aforefaid Porportion is, that Cylinders, of equal Altitudes, are to each other as their Bafes, and Circles as the Squares of their Diameters.

Construction of the Scales on the third and fourth Faces.

The firft Scale of Lines on the third Face, which ferves for finding the Gallons wanting in a Hogfhead pofited with it's Axis parallel to the Horizon, or lying down, contains the Divifions from 1 Gallon to 54 Gallons, which is the Number of Ale-Gallons a Hogfhead contains when full.

The fecond Scale of Lines, on the fame Face, containing the Divifions from 1 Gallon to 108 Gallons, which are the Number of Ale-Gallons contained in a Butt, is for the fame Ufe as the firft Scale of Lines when the Butt is lying.

The third Scale, likewife numbered from 1 Gallon to 108, is for finding how many Gallons is wanting in a Butt ftanding upright.

The three Scales of Lines, on part of the fourth Face, are, as I have already faid, for finding the Wants in a Firkin, Kilderkin, and Barrel lying down, in Ale-Gallons. The readieft way to make the Divifions of either of thefe Scales of Lines for their correfpondent Veffels, when lying down, as for a Hogfhead, is to pour in firft one Gallon of Water, and then put the Rod downright into the Bung-hole to the oppofite Staves; then where the Surface of the Water cuts the third Face of the Rod (becaufe the Scale of Lines for the Hogfhead is on that Face) make the Divifion for 1 Gallon; then pour in another Gallon, and where the Surface of the Water cuts the Rod, make the Divifion for 2 Gallons. Again, pour in another Gallon, and where the Surface of the Water cuts the Rod, make the Divifion for three Gallons. Proceed thus, by pouring in of one Gallon fucceffively after another, and mak-

ing

ing of Diviſions at every Place in the Face of the Rod, to which the Water ariſes, until the Hogſhead be full, and then the Scale for a Hogſhead, on the third Face, will be divided. Proceed, in the ſame manner, in making the Diviſions for the other Scales of Lines uſed in finding the Wants in the ſeveral Veſſels aforementioned lying down. And taking off the Head of a Butt that is ſtanding, and pouring of Water in the ſame manner as in the Hogſhead, putting the Rod downright into the Butt, and making Diviſions on the Rod, as was done for the Hogſhead, the Line will be finiſhed, when figured.

Note, The Diviſions for Half-Gallons, marked by long Dots on the fourth Face, are made by pouring in of Half-Gallons ſucceſſively, *&c.*

USE *of the Diagonal Lines on the Gauging-Rod.*

To find the Content of a Veſſel in Beer or Wine-Gallons.

Put the braſed End of the Gauging-Rod into the Bung-hole of the Caſk, with the Diagonal Lines upwards, and thruſt the braſed End to the meeting of the Head and Staves.

Then with Chalk make a Mark on the middle of the Bung-hole of the Veſſel, and alſo on the Diagonal Lines of the Rod, right againſt, or over one another, when the braſed End is thruſt home to the Head and Staves.

Then turn the Gauging-Rod to the other End of the Veſſel, and thruſt the braſed End home to the End as before.

And ſee if the Mark made on the Gauging-Rod come even with the Mark made on the Bung-Hole, when the Rod was thruſt to the other End; which if it be, the Mark made on the Diagonal Lines, will, on the ſame Lines, ſhew the whole Content of the Caſk in Beer or Wine-Gallons.

But if the Mark firſt made on the Bung-hole be not right againſt that made on the Rod, when put the other way; then right againſt the Mark made on the Bung-hole, make another on the Diagonal Lines: then the Diviſion on the Diagonal Line, between the two Chalks, will ſhew the Veſſel's whole Content in Beer or Wine-Gallons. As for Example; if the Diagonal Line of a Veſſel be 28 Inches 4 Tenths, it's Content in Beer-Gallons will be near 51, and in Wine-Gallons 62.

But if a Veſſel be open, as a Half-Barrel, Tun, or Copper, and the Meaſure from the middle on one Side, to the Head and Staves, be 38 Inches, the Diagonal Line gives 122 Beer-Gallons; half of which, *viz.* 61, is the Content of the open Half-Tub.

But if you have a large Veſſel, as a Tun, or Copper, and the Diagonal Line, taken by a long Rule, prove 70 Inches; then the Content of that Veſſel may be found thus:

Every Inch, at the Beginning-End of the Diagonal Line, call 10 Inches, then 10 Inches becomes 100 Inches.

And every Tenth of a Gallon call 100 Gallons; and every whole Gallon, with a Figure, call 1000 Gallons. Example, at 44.8 Inches, on the Diagonal Beer-Line, is 200 Gallons; ſo alſo at 4 Inches 48 Parts, now called 44 Inches 8 Tenths, is juſt two Tenths of a Gallon, now called 200 Gallons.

Alſo if the Diagonal Line be 76 Inches and 7 Tenths, a cloſe Caſk, of ſo great a Diagonal, will hold 1000 Beer-Gallons: but an open Caſk but half ſo much, *viz.* 500 Beer-Gallons.

For reducing of Wine-Gallons to Beer-Gallons, or, *vice verſa*, by Inſpection, this may be done.

Thus 30 Wine-Gallons, is 24 $\frac{1}{2}$ Beer-Gallons, *&c.*

USE *of the Gauge-Line.*

USE I. *To find the Content of any Cylindrical Veſſel in Ale-Gallons.*

Seek the Diameter of the Veſſel in the Inches, and juſt againſt it, on the Gauge-Line, is the Quantity of Ale-Gallons contained in one Inch deep: then this multiplied by the Length of the Cylinder, will give it's Content in Ale-Gallons. For Example; ſuppoſe the Length of the Veſſel be 32.06, and the Diameter of it's Baſe 25 Inches, what is the Content in Ale-Gallons? Right againſt 25 Inches, on the Gauge-Line, is 1 Gallon, and .745 of a Gallon; which multiplied by 32.06, the Length, gives 55.9447 Gallons for the Content of the Veſſel.

USE II. *The Bung-Diameter of a Hogſhead is 25 Inches, the Head-Diameter 22 Inches, and the Length 32.06 Inches; to find the Quantity of Ale-Gallons contained in it.*

Seek 25, the Bung-Diameter, on the Line of Inches, and right againſt it, on the Gauge-Line, you will find 1.745; take $\frac{1}{3}$ of it, which is .580, and ſet it down twice. Seek 22 Inches, the Head Diameter, and againſt it you will find, on the Gauge-Line, 1.356; $\frac{1}{3}$ of which added to twice .580, gives 1.6096; which multiplied by the Length 32.06, the Product will be 51.603776, the Content in Ale-Gallons. This Operation ſuppoſes, that the aforeſaid Hogſhead is in the Figure of the middle Fruſtum of a Spheroid.

The Uſe of the Lines on the two other Faces of the Rod, is very eaſy; for you need but put it downright into the Bung-hole (if the Veſſel you deſire to know the Quantity of Ale-Gallons contained therein be lying), to the oppoſite Staves; and then where the Surface of the Liquor cuts any one of the Lines appropriated for that Veſſel, will be the Number of Gallons contained in that Veſſel.

CHAP. III.

Of the Construction and Use of Everard's *Sliding-Rule for Gauging.*

THIS Instrument is commonly made of Box, exactly a Foot long, one Inch broad, and about six Tenths of an Inch thick. It consists of three Parts, *viz.* A Rule, and two small Scales or Sliding-Pieces to slide in it; one on one Side, and the other on the other: So that when both the Sliding-Pieces are drawn out to their full Extent, the whole will be three Foot long.

Fig. 8. On the first broad Face of this Instrument are four Lines of Numbers; the first Line of Numbers consists of two Radius's, and is numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 1. and then 2, 3, 4, 5, *&c.* to 10. On this Line are placed four Brass Center Pins, the first in the first Radius, at 2150.42, and the third likewise at the same Number taken in the second Radius, having M B set to them; signifying, that the aforesaid Number represents the Cubic Inches in a Malt Bushel: the second and fourth Center Pins are set at the Numbers 282 on each Radius; they have the Letter A set to them, signifying that the aforesaid Number 282 is the Cubic Inches in an Ale-Gallon. *Note*, The little long black Dots, over the Center Pins, are put directly over the proper Numbers. This Line of Numbers hath A placed at the End thereof, and is called A for Distinction-sake.

The second and third Lines of Numbers which are on the Sliding-Piece (and which may be called but one Line), are exactly the same with the first Line of Numbers: They are both, for Distinction, called B. The little black Dot, that is hard by the Division 7, on the first Radius, having S *i* set after it, is put directly over .707, which is the Side of a Square inscribed in a Circle, whose Diameter is Unity. The black Dot hard by 9, after which is writ S *e*, is set directly over .886, which is the Side of a Square equal to the Area of a Circle, whose Diameter is Unity. The black Dot that is nigh W, is set directly over 231, which is the Number of Cubic Inches in a Wine-Gallon. Lastly, the black Dot by C, is set directly over 3.14, which is the Circumference of a Circle, whose Diameter is Unity.

The fourth Line, on the first Face, is a broken Line of Numbers of two Radius's, numbered 2, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, 6, 5, 4, 3, the Number 1 is set against M B on the first Radius. This Line of Numbers hath M D set to it, signifying *Malt Depth.*

Fig. 9. On the second broad Face of this Rule, are,

I. A Line of Numbers of but one Radius, which is numbered 1, 2, 3, *&c.* to 10, and hath D set at the End thereof for distinguishing it. There are upon it four Brass Center Pins: the first, to which is set W G, is the Gauge-Point for a Wine-Gallon; that is, the Diameter of a Cylinder, whose Height is an Inch, and Content 231 Cubic Inches, or a Wine-Gallon, which is 17.15 Inches. The second Center-Pin A G stands at the Gauge-Point for an Ale-Gallon, which is 18.95 Inches. The third Center-Pin M S stands at 46.3, which is the Side of a Square, whose Content is equal to the Inches in a solid Bushel. The fourth Center-Pin M R is the Gauge-Point for a Malt Bushel, which is 52.32 Inches.

II. Two Lines of Numbers on the Sliding-Piece, which are exactly the same as on the Sliding-Piece on the other Side the Rule, they are called C. The first black Dot something on this Side the Division of the Number 8, to which is set ⊙ *c*, is set to .795, which is the Area of a Circle whose Circumference is Unity; and the second, to which is set ⊙ *d*, stands at .785, the Area of a Circle, whose Diameter is Unity.

III. Two Lines of Segments, each numbered 1, 2, 3 to 100; the first is for finding the Ullage of a Cask, taken as the middle Frustum of a Spheroid, lying with it's Axis parallel to the Horizon, and the other for finding the Ullage of a Cask standing.

Fig. 10. Again, on one of the narrow Faces of this Rule, is, (1.) A Line of Inches, numbered 1, 2, 3, 4, *&c.* to 12. each of which is subdivided into ten equal Parts. (2.) A Line, by means of which, and the Line of Inches, is found a mean Diameter for a Cask in the Figure of the middle Frustum of a Spheroid; it is figured 1, 2, 3, *&c.* to 7. at the End thereof is writ *Spheroid.* (3.) A Line for finding the mean Diameter of a Cask in the Figure of the middle Frustum of a parabolic Spindle, which by Gaugers is called *the second Variety of Casks*; it is numbered 1, 2, 3, 4, 5, 6, and at it's End is writ, 2 *Variety.* (4.) A Line, by means of which may be found the mean Diameter of a Cask of the third Variety; that is, a Cask in the Figure of two parabolic Conoids abutting upon a common Base: it is numbered 1, 2, 3, 4, 5, at the End thereof is writ 3 *Variety.*

Fig. 11. And on the other narrow Face, is, (1.) A Foot divided into 100 equal Parts, every ten of which are numbered; F M stands at the beginning of it, signifying Foot-Measure. (2.) A Line of Inches, like that before spoken of, having I M set to the Beginning thereof, signifying Inch-Measure. (3.) A Line for finding the mean Diameter for the fourth Variety of Casks, which is the middle Frustum of two Cones, abutting upon one common Base; it is numbered 1, 2, 3, 4, 5, 6. and at the Beginning thereof is writ F C, signifying Frustum of a Cone.

These

Theſe are all the Lines on the four Faces of the Rule; but on the Backſide of the two Sliding-Pieces are a Line of Inches from 13 to 36, when the two Sliding-Pieces are put Endways together, and againſt that the correſpondent Gallons, or hundred Parts, that any ſmall Tub, or ſuch like open Veſſel (from 13 to 36 Inches Diameter) will contain at one Inch deep: it's Conſtruction is the ſame as before delivered, in ſpeaking of the Line of Ale Area on the Four-foot Gauging-Rod.

All the Lines of Numbers, before-deſcribed, may be put upon the Faces of this Sliding-Rule, as directed in the Conſtruction of the Line of Numbers on *Gunter*'s Scale; only you muſt obſerve, that the firſt Radius of the broken Line of Numbers M D, begins directly under M B, and ends directly under the other M B; and that when either of the Lines of Numbers A or B are made, the Line M D from them may alſo be made. *Example*, The Diſtance from 1 to 2, on the Line A, laid off from 1 (towards the Left Hand) to 2, on the Line M D, will give the Diviſion 2; the Diſtance from 1 to 3, on the Line A, will be equal to the Diſtance from 1 to 3; the contrary way on the Line M D: underſtand the ſame of other Diviſions and Subdiviſions. The reaſon of thus breaking this Line of Numbers, I ſhall ſhew in it's Uſe.

The Line of Segments for the middle Fruſtum of a Spheroid lying, may be put upon the Sliding-Rule in the following manner: Take ſome Veſſel lying, as a Butt, and fill it full of Water, then find it's Content in Ale or Wine-Gallons (for it matters not which), take alſo it's Bung-Diameter very exactly in Inches, or Tenths of Inches. Now to find againſt what Number, on the Line of Numbers of the Sliding-Piece, any Diviſion of the Line of Segments muſt ſtand; ſuppoſe the Diviſion 1, ſay, As Unity is to .01, So is the Content of the aforeſaid Veſſel in Gallons to a fourth Number (which will be the Gallons, or Gallons and Parts that are contained in ſuch a Segment of the Veſſel, as .01 is of a ſimilar Veſſel, whoſe Area is ſuppoſed Unity;) then let out of the Veſſel as many Gallons of Water as that fourth Proportional directs, and having taken the Dry Inches, ſay, By the Rule of Three, As the Bung-Diameter is to thoſe Dry Inches found, So is 100 to a fourth Number; which will be the Number on the Line C, againſt which the Diviſion 1 on the Segment-Line muſt ſtand.

Again, to find where the Diviſion 2 muſt ſtand on the Line of Segments, ſay, As 1 is to .02, So is the Content of the aforeſaid Veſſel to the Gallons that muſt be taken out of it; then ſay, As the Bung-Diameter is to the Dry Inches, So is 100 to the Number on the Line C, againſt which the Diviſion 2 muſt ſtand. Proceed in this manner for finding the Diviſions 3, 4, 5, 6, 7, 8, 9, and when you come to find where the Diviſion 10 muſt ſtand, you muſt ſay, As Unity is to the Veſſel's Content, So is .1 to the Number of Gallons to be taken out of the Veſſel, and go on as before. Moreover, to find where the Diviſion 20 muſt ſtand, ſay, As 1 is to the Content, So is .2 to the Number of Gallons to be taken out of the Veſſel, *&c.* In this manner may the Diviſions to 100 be found.

To find where the firſt Subdiviſion before 1 muſt ſtand, ſay, As 1 is to the Veſſel's Content, So is .002 to the Number of Gallons to be let out of the Veſſel, and proceed as at firſt directed. And for the ſecond Subdiviſion, make .003 the third Term of the Rule of Three, and proceed as before.

For the Subdiviſions between 1 and 2, 2 and 3, *&c.* ſuppoſe 1 to be .0100, then the firſt Diviſion from 1 will be .011, the ſecond .012, the third .013, *&c.* which muſt be made the third Terms of the firſt Rule of Three, for finding where any of thoſe Subdiviſions muſt ſtand. And for the Subdiviſions between 10 and 20, 20 and 30, you muſt ſuppoſe 10 to be .10, and 20 to be .20; then the firſt Subdiviſion from 10 will be .11, the ſecond .12, the third .13, *&c.* which will be the third Terms in the firſt Rule of Three, for finding whereabouts theſe Diviſions muſt ſtand.

The other Segment-Line, on the ſame Face of the Rule, may be made in the ſame manner as this, by ſetting the aforeſaid Veſſel upright, and making uſe of the Length inſtead of the Bung-Diameter.

The Conſtruction of the four Lines on the narrow Faces of this Rule, is from the Rules that *Everard* hath laid down for finding the Contents of the four Varieties of Caſks. For, (1.) If there is a Caſk in the Form of the middle Fruſtum of a Spheroid, half the Difference of the Squares of the Bung and Head-Diameter, added to the Sum and half Sum of the ſaid Squares, divided by 3, will be the Square of the mean Diameter for a ſpheroidal Veſſel; the Root of which will be the mean Diameter. (2.) Three Tenths of the Differences of the Squares of the Bung and Head-Diameters, added to the Sum and half Sum of the ſaid Squares, and the whole divided by 3, will be the Square of the mean Diameter of a Caſk of the ſecond Variety. (3.) To the Sum and half Sum of the Squares of the Bung and Head-Diameters, add one Tenth of the Difference of the ſaid Squares, which Sum, divided by 3, gives the Square of the mean Diameter of a Caſk of the third Variety. (4.) And Laſtly, from the Sum and half Sum of the Squares of the Bung and Head-Diameters, ſubſtract half the Square of the Difference of Diameters, and the Remainder, divided by 3, will be the Square of the mean Diameter for the fourth Variety of Caſks.

USE

USE *of* Everard's *Sliding-Rule*.

USE I. *One Number being given to be multiplied by another, to find the Product.*

Notation on the Lines of Numbers upon this Rule, is the same as before was shewn in the Use of the Carpenter's Rule; therefore I shall not here repeat it, but proceed to solve this Use by the following Examples: Suppose 4 is to be multiplied by 6: Set 1 upon the Line of Numbers B, to 4 upon the Line A, and then against 6 upon B, is 24, the Product sought upon A. Again, to multiply 26 by 68, set 1 upon B to 26 upon A; then against 68 upon B, is 1768 on A.

Note, The Product of any two Numbers will have so many Places as there are in both the Numbers given, except when the lesser of them does not exceed so many of the first Figures of the Product, for then it will have one less.

USE II. *One Number being given to be divided by another, to find the Quotient.*

Suppose 24 is to be divided by 4, what is the Quotient? Set 4 upon B, to 1 upon A; then against 24 upon B, is 6 upon A, which will be the Quotient.

Again, let 952 be divided by 14: To find the Quotient, set 14 upon A, to 1 upon B, and against 952 upon A, you will have 68 the Quotient upon B.

Note, The Quotient will always consist of so many Figures as the Dividend hath more than the Divisor, except when the Divisor does not exceed so many of the first Figures of the Dividend; for then it will have one Place more.

USE III. *Three Numbers being given, to find a fourth in a direct Proportion.*

If 8 gives 20, what will 22 give? Set 8 upon B, to 20 upon A; and then against 22 on B, stands 55 upon A, which is the fourth Number sought.

USE IV. *To find a mean Proportional between two given Numbers.*

Example. Let the two Numbers be 50 and 72; set 50 upon C, to 72 upon D; and then against 72 upon C, is 60 upon D, which is the Geometrical Mean between 50 and 72.

USE V. *To find the square Root of any Number under* 1000000

The Extraction of the square Root, by help of this Instrument, is easier than any of the aforesaid Uses: for if the Lines C and D be applied one to another, so that 10 at the End of D, be even with 10 at the End of C; then those two Lines, thus applied, are like a Table of square Roots, shewing the square Root of any Number by Inspection only: for against any Number upon C, you have the square Root thereof upon D.

Note, When the Number given consists of 1, 3, 5, or 7 Places of Integers, seek it in the first Radius on the Line C, and against it you have the Root required upon D. *Example*, Let the Number given be 144, I find this on the first Radius of the Line C, and against it is 12, the Root sought upon the Line D.

USE VI. *The Diameter or Circumference of a Circle being given, to find either.*

Set 1 on the Line A against 3.141, (where is writ C) on the Line B, and against any Diameter, on the Line A, you have the Circumference on the Line B, and contrariwise: As suppose the Diameter of a Circle be 20 Inches, the Circumference will be 62.831; and if the Circumference be 94.247, the Diameter will be 30.

USE VII. *The Diameter of any Circle being given; to find the Area, in Inches, or in Ale or Wine-Gallons.*

Example. Let the Diameter be 20 Inches, what is the Area? Set 1 upon D to .785, (where is set ⊙ *d*) and then against 20 upon D, is 314.159, the Area required. Now to find that Circle's Area in Ale-Gallons, set 18.95 (marked A G) upon D to 1 upon C; then against the Diameter 20, upon D, is the Number of Ale-Gallons upon C, which is 1.11 Gallons. Understand the same for Wine-Gallons, by the proper Gauge-Point.

USE VIII. *The transverse and conjugate Diameters of an Ellipsis being given, to find the Area in Ale-Gallons.*

Example. Let the transverse Diameter be 72 Inches, and the Conjugate 50: Set 359.05, the Square of the Gauge-Point, upon B, to one of the Diameters (suppose 50 upon A); then against the other Diameter 72 upon B, you will have the Area upon A, which, in this Example, will be 10.02 Ale-Gallons, the Content of this Ellipsis at one Inch deep. The like may be done for Wine-Gallons, if instead of 359.05, you use 249.11, the Square of the Gauge-Point for Wine-Gallons.

USE IX. *To find the Area or Content of a Triangular Superficies in Ale Gallons.*

Let the Base of the Triangle be 260 Inches, and the Perpendicular, let fall from the opposite Angle, be 110 Inches; set 282 (marked A) upon B, to 130, half the Base upon A; then against 110 upon B, is 50.7 Gallons upon A.

USE

USE X. *To find the Content of an Oblong in Ale Gallons.*

Suppose one of the Sides is 130 Inches, and the other 180; set 282 upon B, to 180 upon A; then against 130 upon B, is 82.97 Ale Gallons, the Area required.

USE XI. *The Side of any regular Polygon being given, to find the Content thereof in Ale Gallons.*

In any regular Polygon, the Perpendicular let fall from the Center to one of the Sides, being found and multiplied by half the Sum of the Sides, gives the Area. Example, in a Pentagon, suppose the Side is an Inch, then the Perpendicular let fall from the Center, will be found .837, in saying, As the Sine of half the Angle at the Center, which in this Polygon is 36 Degrees, is to half the given Side .5; So is the Sine of 36 Degrees taken from 90, which is 54 Degrees, to the Perpendicular aforesaid: whence the Area of a Pentagon Polygon, each of whose Sides is Unity, will be 1.72 Inches; which, divided by 282, gives .0061 the Ale Gallons in that Polygon. By the same Method you may find the Area of any other Polygon, whose Side is Unity in Ale Gallons. Now, suppose the Side of a Pentagon is 50 Inches, What is the Content thereof in Ale Gallons? Set 1 upon D, to .0061 upon C; then against 50 upon D, you have the Area 15.252 Ale Gallons upon C.

USE XII. *To find the Content of a Cylinder in Ale Gallons.*

Suppose the Diameter of the Base of a Cylinder is 120 Inches, and the perpendicular Height 36 Inches. Set the Gauge-Point (A G) to the Height 36 upon C; then against 120 the Diameter, upon D, is 1443.6 the Content in Ale Gallons.

USE XIII. *The Bung and Head Diameters, together with the Length of any Cask, being given, to find it's Content in Ale or Wine Gallons.*

Suppose the Length of a Cask taken, as the middle Frustum of a Spheroid be 40 Inches, it's Head-Diameter 24 Inches, and Bung-Diameter 32 Inches. Substract the Head-Diameter from the Bung-Diameter, and the Difference is 8: then look for 8 Inches on the Line of Inches, upon the first narrow Face of the Rule; and against it on the Line Spheroid stands 5.6 Inches, which added to the Head-Diameter 24, gives 29.6 Inches for that Cask's mean Diameter: then set the Gauge-Point for Ale (marked A G) upon D, to 40 upon C; and against 29.6 upon D, is 97 45 the Content of that Cask in Ale-Gallons. If the Gauge-Point for Wine (marked W G) is used instead of that for Ale, you will have the Vessel's Content in Wine-Gallons.

If a Cask, suppose of the same Dimensions as the former, be taken as the middle Frustum of a parabolick Spindle, which is of the second Variety, you must see what Inches and Parts on the Line marked *Second Variety*, stand against the Difference of the Bung and Head-Diameters, which, in this Example, is 8; and you will find 5.1 Inches, which added to 24 the Head-Diameter, makes 29.1 Inches the mean Diameter of the Cask; then set the Rule as before, and against 29.1 Inches, you will have 94.12 Ale-Gallons for the Content of the Cask.

Again; if a Cask, suppose of the same Dimensions with either of the former ones, be taken as the middle Frustum of 2 parabolick Conoids, which is one of the third Variety, you will find against 8 Inches (the Difference of the Bung and Head-Diameters), on the Line of Inches, stands 4.57 Inches, on the Line called 3d Variety, which added to 24, the Head-Diameter, gives 28.57 Inches for the Cask's mean Diameter: proceed as at first, and you will find the Content of this Cask to be 90.8 Ale Gallons.

Lastly, If a Cask, suppose of the same Dimensions as before, is taken as the Frustums of 2 Cones, which is the fourth Variety, look on the other narrow Face of the Rule for 8 Inches, upon the Line of Inches; and against it, on the Line F. C, you will find 4.1 Inches, which added to 24, gives 28.1 for the mean Diameter of this Cask: proceeding as at first, and you will find the Content of this Cask, in Ale-Gallons, to be 87.93.

USE XIV. *There is a Cask posited with it's Axis parallel to the Horizon, or Lying, in part empty; suppose it's Content is* 97.455 *Ale-Gallons, the Bung-Diameter* 32 *Inches, and the dry Inches* 8, *to find the Quantity of Liquor in the Cask.*

As the Bung-Diameter upon C, is to 100 upon the Line of Segments L, So is the dry Inches on C, to a fourth Number on the Line of Segments: then As 100 upon B, is to the Cask's whole Content upon A, So is that fourth Number to the Liquor wanting to fill up the Cask; which, substracted from the Liquor that the Cask holds, gives the Liquor in the Cask. Example; Set 32, the Bung-Diameter, on C, to 100 on the Segment Line L; then against 8, the Dry-Inches on C, stands 17.6 on the Segment Line. Now set 100 upon B, to the Cask's whole Content upon A; and against 17.6 upon B, you have 16.5 Gallons upon A; and substracting the said Gallons from 97.45, the Vessel's whole Content, the Liquor in the Cask will be 80.95 Gallons.

USE XV. *Suppose the aforesaid Cask's Axis be perpendicular to the Horizon, or upright, and the Length of it be* 40 *Inches: to find how much Liquor there will be in the Cask, when* 10 *of those Inches are dry.*

Set 40 Inches, the Length, on the Line C, to 100 on the Segment Line S; and against 10, the Dry-Inches, on the Line C, stands 24.2 on the Segment Line S. Now set 100 upon B, to 97.455, the Cask's whole Content, upon A; and against 24.2 on B, you will have 23.5 Gallons, which are the Gallons wanting to fill up the Cask, and being substracted from the whole Content 97.455, gives 73.955 Gallons for the Quantity of Liquor remaining in the Cask.

USE XVI. *To find the Content of any right-angled Parallelepipedon (which may represent a Cistern, or* Uting-Fat) *in Malt-Bushels.*

Suppose the Length of the Base is 80 Inches, the Breadth 50, and the Depth 9 Inches. Set the Breadth 50 on B, to the Depth 9 on C; then against the Length 80 on A, stands 16.8 Bushels on the Line B, which are the Number of Bushels of Malt contained in the aforesaid Cistern.

The broken Line of Numbers M. D, is so set under the Lines A or B, that any Number on A or B multiplied by the Number directly under it on the Line M D, will always be equal to 2150.42, the Number of Inches in a Malt-Bushel: from whence the Reason of the aforesaid Operation for finding the Number of Malt-Bushels, may be thus deduced. Let us call the Breadth a, the Length b, the Depth c, and the Number of Inches in a Malt-Bushel f; then the Malt-Bushels in any Utensil of the aforesaid Figure, will be expressed by $\frac{abc}{f}$. But by the Sliding-Rule the Operation is, to set the Breadth a, to the Depth c; that is (from the aforementioned Property of the broken Line of Numbers M D), to $\frac{f}{c}$ on the Line A; and then against the Length b, on the Line A, will the Number of Malt-Bushels stand: therefore the Operation is but finding the fourth Term of this Analogy, by means of the Lines A and B, *viz.* $\frac{f}{c} : a :: b : \frac{abc}{f}$.

CHAP. IV.

Of the Construction and Use of Coggeshall's *Sliding-Rule for Measuring.*

THIS Rule is framed three Ways; for some have the two Rulers composing them sliding by one another, like Glaziers Rules; and sometimes there is a Groove made in one Side of a Two-Foot Joint-Rule, in which a thin sliding Piece being put, the Lines put upon this Rule, are placed upon the said Side. And lastly, one Part sliding in a Groove made along the Middle of the other, the Length of each of which is a Foot: the Form of this last being represented by *Fig.* 12.

Fig. 12. Upon the sliding Side of the Rule are four Lines of Numbers; three are double Lines, or Lines of Numbers to two Radius's, and one a single broken Line of Numbers, marked by the Letters A, B, C, and D.

The three double Lines of Numbers A, B, C, are figured 1, 2, 3, 4, 5, 6, 7, 8, 9; and then 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; they being the same as the Line A, and the two Lines on the Sliding-Piece C, upon *Everard*'s Sliding-Rule; and their Construction, Use, and Manner of using, are also the same.

The single Line of Numbers D, whose Radius is exactly equal to the two Radius's of either of the Lines of Numbers A, B, C, is broke, for easier measuring of Timber, and figured thus, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40; this Line is called the *Girt* Line: from 4 to 5 it is divided into 10 Parts, and each *Tenth* into two Parts, and so on from 5 to 10; then from 10 to 20, it is divided into 10 Parts, and each Tenth into 4 Parts, and so on from 20 to 40, at the End, which is right against 10, at the End of either of the double Lines of Numbers.

The Lines on the Back-side of this Rule, are these; a Line of Inch-Measure from 1 to 12, each Inch being subdivided into Halfs, Quarters, and Half-quarters: another Line of Inch-Measure from 1 to 12, and each Inch subdivided into 10 equal Parts: a Line of Foot-Measure, being one Foot divided into 100 equal Parts, and figured, 10, 20, 30, *&c.* to 100.

The Back-side of the Sliding-Piece is divided into Inches, Halfs, and Half-quarters, and figured from 12 to 24; so that it may be slid out to 2 Foot, to measure the Length of any thing.

The Lines of Numbers, A, B, or C, being either of them constructed (which see in the Chapter concerning *Gunter*'s Scale) the Line D, from thence, may easily be constructed.

For

For having set 4 directly under 1, for the Beginning of the Line; to find where any Division, suppose 5, must be placed, take twice the Distance from 4 to 5, on either of the Radius's of either of the Lines of Numbers A, B, C, and lay off from 4 that Extent, which will give the Division 5. Proceed thus for all the other Divisions and Subdivisions, by always taking the double of them on the Lines A, B, or C.

Note, For the manner of Notation, on this Rule, see the Line of Numbers on the Carpenters.

The Use of this Rule in measuring plain Superficies.

SECTION I.

USE I. *To measure a Geometrical Square.*

Let there be a Square whose Sides are each 5 Feet; set 1 on the Line B, to 5 on the Line A; then against 5 on the Line B, is 25 Feet the Content of the Square on the Line A.

USE II. *To measure a right angled Parallelogram, or Long-Square.*

Let there be a Parallelogram, whose longest Side is 18 Feet, and shortest 10; set 1 on the Line B, to 10 on the Line A; then against 18 Feet on the Line B, is 180 Feet the Content on the Line A.

USE III. *To measure a Rhombus.*

Let the Side of a Rhombus be 12 Feet, and the Length of a Perpendicular let fall from one of the obtuse Angles, to the opposite Side, 9 Feet; set 1 on the Line B, to 12, the Length of the Side, on the Line A: then against 9, the Length of the Perpendicular on the Line B, is 108 Feet the Content.

USE IV. *To measure a Rhomboides.*

Suppose the Length of either of the longest Sides of a Rhomboides to be 25 Feet, and the Length of the Perpendicular let fall from one of the obtuse Angles to the opposite longest Side, is 8 Feet; set 1 on the Line B, to 25, the Length, on the Line A; then against 8 Feet on the Line B, stands 200 Feet the Content.

USE V. *To measure a Triangle.*

Let the Base of a Triangle be 7 Feet, and the Length of the Perpendicular let fall from the opposite Angle to the Base, 4 Feet. Set 1 on the Line B, to 7 on the Line A; then against half the Perpendicular, which is 2, on the Line B, is 14 on the Line A, for the Content of the Triangle.

USE VI. *The Diameter of a Circle being given, to find it's Content.*

Let the Diameter of a Circle be 3.5 Feet: set 11 on the Girt-Line D, to 95 on the Line C; then against 3.5 Feet on D, is 9.6 on the Line C, which is the Content in Feet of the said Circle.

The Reason of this Operation, is, that As the Square of 11, which is 121, is to 95; So is the Square of the Diameter of any Circle, to it's Content. Also, from the Nature of the Logarithms, it is manifest, if any Number, taken on a single Line of Numbers (whether whole or broken, in the manner that the Line D is), be set to another Number, taken on a double Line of Numbers of the same Length; that the Square of the Number taken on the single Line of Numbers, will be to the Number it is set against, on the double Line of Numbers, as the Square of any other Number, taken on the single Line of Numbers, to the Number against it on the double Line of Numbers.

USE VII. *To find the Content of an Oval or Ellipsis.*

Let the Transverse, or longest Diameter, be 9 Feet, and the Conjugate, or shortest Diameter, 4 Feet, to find the Content of this Ellipsis.

Theorem. The Content of every Ellipsis, is a mean Proportional between a Circle, whose Diameter is equal to the longest Diameter of the Ellipsis, and a Circle whose Diameter is equal to the shortest Diameter of the same Ellipsis; as is manifest *per Cor.* 3. *Prop.* XI. *Lib.* 11. of *Sturmy*'s *Mathesis Enucleata.*

Therefore a mean Proportional must first be found between 4 and 9, the longest and shortest Diameters; to do which by the Sliding-Rule, set the greater of the two Numbers 9 on the *Girt Line*, to the same Number on the Line C; then against the lesser Number 4, on the same Line C, is 6 the mean Proportional sought on the Girt-Line. Now we have only the Content of a Circle to find, whose Diameter is 6 Feet; which, when found, will be the Content of the Ellipsis sought: therefore (by the last Problem) set 11 on the Girt-Line D, to 95 on the Line C; then against 6 Feet on the Girt-Line D, stands on the Line C, 28.28 Feet for the Content of the aforesaid Ellipsis.

The Reason of the Operation for finding a mean Proportional between two Numbers, as 4 and 9, is manifest from what I said in the last Use of the Property of a double and single Line

Line of Numbers sliding by one another. And from this Theorem, *viz.* That if there are three Numbers continually proportional (as 4, 6, and 9), the Square of the greatest (as 81) is to the greatest (9), as the Square of the middle one (6), or the Rectangle under the Extremes (which is equal to it, *per Prop.* 20. *Lib.* 7. *Eucl.*) is to the lesser Extreme (4).

This Use may be easier solved at one Operation by the Lines A and B, thus; set 1.27 on the Line B, to the transverse Axis 9 Feet, on the Line A: then against the Conjugate Axis 4, on the Line B, stands 28.28 Feet on the Line A, for the Content.

Note, The standing Number 1.27, is the Quotient of 14 divided by 11; also as 14 is to 11, so is the Rectangle under the transverse and conjugate Axes of any Ellipsis to it's Area; whence the Reason of this Operation is easily manifest.

SECTION II.

Of measuring Timber.

USE I. *To measure Timber the common Way.*

Take the Length in Feet, Half-feet (and if desired), in Quarters; then measure half-way back again, where girt the Tree with a small Cord or Chalk-Line; double this Line twice very even, and this fourth Part of the Girt, or Circumference, which is called the *Girt*, measure in Inches, Halfs, and Quarters of Inches; but the Length must be given in Feet, and the *Girt* in Inches. The Dimensions being thus taken, the Tree is to be measured as square Timber, the Girt, or $\frac{1}{4}$ of the Circumference being taken for the Side of the Square, in the following manner.

Always set 12 on the Girt-Line D, to the Length in Feet on the Line C; then against the Side of the Square, on the Girt-Line D, taken in Inches, you will find on the Line C the Content of the Tree in Feet.

Example I. Suppose the Girt of a Tree, in the middle, be 60 Inches, and the Length 30 Feet, What is the Content? Set 12 on the Girt-Line D, to 30 Feet on the Line C; then against 15, the one fourth of 60, on the Girt-Line D, is 46.8 Feet the Content on the Line C.

Example II. A Piece of Timber is 15 Feet long, and $\frac{1}{4}$ of the Girt 42 Inches: Set 12 on the *Girt-Line* D, to 15 on the second Radius of the Line C; then against 42, at the Beginning of the Girt-Line D, is, on the Line C, 184 Feet, the Content sought.

Example III. The Length of a Piece is 9 Inches, and a Quarter of the Girt 35 Inches, What is the Content? Now, because the Length is not a Foot, measure it by your Line of Foot Measure, and see what decimal Part of a Foot it makes, which will be .75; then set 12 on the Girt-Line, to 75 on the first Radius of the Line C; and against 35 on the Girt-Line D, is 6.4 Feet on the Line C, for the Content.

Example IV. A Rail is 18 Feet long, and the Quarter of the Girt 3 Inches: set 12 on the Girt-Line D, to 18 on the first Radius of the Line C; then against 30, which must be taken for 3, on the Girt-Line D, is just 1.12 Feet for the Content.

The Reason of the Operations of this Use, is manifest from what I said about the Property of the Lines D and C, in Use VI. and this Theorem, *viz.* that as 144, the square Inches in a Foot, is to the Content of the Square Base of a Parallelopipedon taken in Inches; that is, to the Square of $\frac{1}{4}$ of the Girt: So is the Length of a Parallelopipedon taken in Feet, to the Solidity of the said Parallelopipedon in Feet.

This Use may be sooner done by taking all the Dimensions in Foot Measure thus, count 10, 20, 30, 40, &c. on the Girt-Line to be 1, 2, 3, 4, &c. and then place 10 on the Girt-Line D (now called 1) to the Length of the Tree on the Line C, and against the Girt, in Foot Measure, on the Girt-Line D, stands the Content on the Line C.

Example I. Let the Length of a Tree be, as in the first Example foregoing, *viz.* 30 Feet, and the Girt 60 Inches, or 5 Feet, What is the Content? Set 10 (now called 1) on the Girt-Line D, to 30 Feet on the Line C; then against 1.25 Feet, the one fourth of the Girt, on the Girt-Line D, stands 46.8 Feet on the Line C, for the Content, as before.

Example II. A Piece of Timber is 15 Feet long, and one fourth of the Girt is 42 Inches, or 3.5 Feet, What is the Content?

Set 10 on the Girt-Line, to 15 on the first Radius of the Line C; then against 3.5 Feet on the Girt-Line, is 184 Feet on the Line C, the Content required.

Example III. A Length is 9.75 Feet, and $\frac{1}{4}$ of the Girt 39 Inches, or 3 Feet $\frac{25}{100}$ set 10 on the Girt-Line to 9.75 on the Line C; and against 3.25 Feet, on the Girt-Line D, is beyond 100 on the Line C: in this Case take half the Length, and then the Content found must be doubled, as here:

Set 10 on the Girt-Line, to (half of 9.75) 4.87; and then against 3.25 is 51.5; the double of which is 103 Feet, the Content required.

Note, If the Content of any Piece of Timber in Feet, be divided by 50, you have the Content in Loads: but some will have a Load to be but 40 solid Feet; therefore you may take which of the two is most customary with you.

USE

USE II. *To meaſure Round Timber the true way.*

The manner of meaſuring Round Timber in the laſt Uſe, being the common way, but not the true one, as I have already ſaid in ſpeaking of the Carpenter's Rule: I ſhall now give you a Point on the Girt-Line D, which muſt be uſed inſtead of 12, which is 10.635, at which there ought to be placed a little Braſs Center-Pin: this 10.635 is the Side of a Square, equal to a Circle, whoſe Diameter is 12 Inches.

Example. Let a Length be (as in the ſecond Example of the laſt Uſe) 15 Feet, and the $\frac{1}{4}$ of the Girt 42 Inches: ſet the ſaid Point 10.635, to 15 the Length; then againſt 42, at the Beginning of the Girt Line, is 233 Feet for the Content ſought: but by the common way, there ariſes only 184 Feet.

Note, As the Area, or Content of a Circle (in Inches) whoſe Diameter is 12 Inches, is to the Length of any Cylinder in Feet; So is the Square of of the Circumference of the Baſe of the Cylinder, in Inches, to the ſolid Content of the Cylinder in Feet.

Alſo the common Meaſure is to the true Meaſure, as 11 is to 14; that is, as the Area, or Content of a Circle, to the Square of it's Diameter; which, from hence, will be eaſily manifeſt: Call the Diameter of any Circle D, and $\frac{1}{4}$ the Circumference C; then the Content of the ſaid Circle will be equal to $D \times C$; therefore $D \times C$ is to $D \times D$, as 11 is to 14. But the common Meaſure (becauſe the Length of the Piece is the ſame) will be to the true Meaſure, as C C, the Square of the Circumference, to $D \times C$ the Content of the ſaid Circle; whence $D \times C$ muſt be to D^q, as C^q is to $D \times C$; and by comparing the Rectangles under the Means and Extremes, they will be found equal; therefore what I propoſed is true.

If the Girt of a Piece of Timber be taken in Feet, the Point for true Meaſure is .886, or .89, which is the Side of a Square, equal to the Content of a Circle, whoſe Diameter is Unity. And then, for the foregoing Example, the Length being 15 Feet, and ¼ of the Girt 42 Inches; ſet the aforeſaid Point 89 on the Girt-Line, to the Length 15 Feet on the Line C, (in the firſt Radius) then againſt 3.5 Feet (which is 35) on the Girt-Line D, is 233 Feet on the Line C, the true Content required.

USE III. *To meaſure a Cube.*

Let there be a Cube whoſe Sides are 6 Feet; to find the Content: ſet 12 on the Girt-Line D, to 6 on the Line C; then againſt 72 Inches (the Inches in 6 Feet) on the Girt Line D, is 216 Feet on the Line C, which is the Content required.

USE IV. *To meaſure unequal ſquared Timber; that is, if the Breadth and Depth are not equal.*

Meaſure the Length of the Piece, and the Breadth and Depth (at the End) in Inches; then find a mean Proportional between the Breadth and Depth of the Piece; which mean Proportional is the Side of a Square equal to the End of the Piece: which being found, the Piece may be meaſured as ſquare Timber.

Example I. Let there be a Piece of Timber whoſe Length is 13 Feet, the Breadth 23 Inches, and the Depth 13 Inches: ſet 23 on the Girt-Line D, to 23 on the Line C; then againſt 13 on the Line C, is 17.35 on the Girt-Line D for the mean Proportional. Now again; ſetting 12 on the Girt Line D, to 13 Feet, the Length, on the Line C; then againſt 17.35 on the Girt-Line D, is 27 Feet the Content required.

Example II. Let there be a Piece of Stone 7.4 Feet in Length, 30 Inches in Breadth, and 23.5 Deep: ſet 30 Inches on the Girt-Line D, to 30 on the Line C; then againſt 23.5, on the Line C, is 26.5 on the Girt-Line D; then ſet 12 on the Girt-Line D, to 7.4 on the Line C; and againſt 26.5, on the Girt Line, is 36 Feet the Content ſought.

USE V. *To find the Content of a Piece of Timber in Form of a triangular Priſm.*

You muſt firſt find a mean Proportional between the Baſe, and half the Perpendicular of the triangular End, or between the Perpendicular and half the Baſe, both meaſured in Inches, and that mean Proportional will be the Side of a Square equal to the Triangle.

Then to find the Content, ſet 12 on the Girt-Line D, to the Length in Feet on the Line of Numbers C; and againſt the mean Proportional on the Girt-Line D, is the Content on the Line of Numbers C.

But the Dimenſions being all taken in Foot-Meaſure, and the mean Proportional found in the ſame; then ſet 1 on the Girt-Line, to the Length on the Line C; and againſt the mean Proportional in the Girt-Line, is the Content in the Line C.

Example. There is a Piece of Timber 19 Feet 6 Inches in Length, the Baſe of the Triangle at each End 21 Inches, and the Perpendicular 16 Inches: to find the Content.

Set 21 Inches on the Girt-Line D, to 21 on the Line C; then againſt 8 on the Line C, is 12.95 on the Line D, the mean Proportional; then ſet 12 on the Line D, to 19.5 Feet the Length, on the Line C; and againſt 12.95 (the mean Proportional) on the Girt-Line D, is 22.8 Feet the Content on the Line C. Or thus, take all the Dimenſions in Foot-Meaſure, and then the Length 19 Feet 6 Inches, is 19.5, the Baſe 21 Inches, is 1.75, and the Perpendicular 16 Inches, is 1.33. Now ſet 1 on the Girt-Line D, to the Length 19.5 on the double Line C; and againſt 1.08 on the Girt-Line D, is 22.8 Feet on the Line C, for the Content.

USE VI. *To meaſure Taper Timber.*

The Length being meaſured in Feet, note one third of it, which may be found thus: ſet 3 on the Line A, to the Length on the Line B; then againſt 1 on the Line A, is the third Part on the Line B: then if the Solid be round, meaſure the Diameter at each End in Inches, and ſubſtract the leſſer Diameter from the greater, and add half the Difference to the leſſer Diameter, the Sum is the Diameter in the middle of the Piece; then ſet 13.54 on the Girt-Line D, to the Length on the Line C; and againſt the Diameter in the middle, on the Girt-Line, is a fourth Number on the Line C. Again; ſet 13.54 on the Girt-Line, to the third part of the Length on the Line C: then againſt half the Difference on the Girt-Line, is another fourth Number on the Line C; theſe two fourth Numbers added together, will give the Content.

Example. Let the Length be 27 Feet (one third of which will be 9), the greater Diameter 22 Inches, and the leſſer 18, the Sum of the greater and leſſer Diameters will be 40; their Difference 4, half their Difference 2, which added to the leſſer Diameter, gives 20 Inches for the Diameter in the middle of the Piece. Now ſet 13.54 on the Girt-Line D, to 27 on the Line C; and againſt 20 on the Line D, is 58.9 Feet. Again, ſet 13.54 of the Girt-Line, to 9 on the Line C; then againſt 2 on the Girt-Line (repreſented by 20), is .196 Parts: therefore, by adding 58.9 Feet, to .196 Feet, the Sum is 59.096 Feet the Content. If all the Dimenſions are taken in Foot-Meaſure, then you muſt add the greater and leſſer Diameters together, which in this Example make 3.33 Feet; half of which is the Diameter in the middle of the Piece, *viz.* 1.67 Feet, the Difference of the Diameters is 0.33 Feet, half of which Difference is 0.17 Feet.

Then ſet 1.13 on the Girt-Line, to the Length 27 Feet on the Line C; and againſt 1.67 on the Line D, is 58.9 Feet: then again, ſet 1.13 on the Line D, to 9 Feet on the Line C; and then againſt 0.17 on the Line D, is 196 Parts of a Foot, and both added together is the Content; that is, 58.9 and .196 added, makes 59.096 Feet as before.

If the Solid is ſquare, and has the ſame Dimenſions; that is, the Length 27 Feet, the Side of the greater End 22 Inches, and the Side of the leſſer End 18 Inches, to find the Content in Inch-Meaſure: ſet 12 on the Girt-Line, to 27 the Length of the Solid, on the Line C; and againſt 20 Inches, the Side of the mean Square on the Girt-Line, is 75.4 Feet. Again; ſet 12 on the Girt-Line, to 9 Feet, one third of the Length, on the Line C; and againſt 2 Inches, half the Difference of the Sides of the Squares of the Ends, on the Girt-Line, is .25 Parts of a Foot; both together is 75.65 Feet the Content of the Solid: or thus, When all the Dimenſions are taken in Foot-Meaſure, ſet 1 on the Girt-Line, to the Length 27 Feet on the Line C; then againſt 1.67 Feet, the Side of the middle Square on the Girt-Line, ſtands 75.4 Feet; and ſetting 1 on the Girt-Line to 9 Feet, one third of the Length on the Line C, againſt 0.167, half the Difference of the Sides of the Squares of the Ends on the Girt-Line, is on the Line C, .25 Parts of a Foot; which added to the other, makes 75.65 Feet, as before, for the Content.

Note, The fixed Numbers 13.54, and 1.13 are, the firſt, the Diameter of a Circle whoſe Area, or Content is 144; that is, the Number of ſquare Inches in a ſuperficial Foot; and the other, the Diameter of a Circle whoſe Area is Unity.

USE VII. *To find how many Inches in Length will make a Foot-Solid, at any Girt, being the Side of a Square not exceeding* 40 *Inches.*

Let the Girt, or Side of the Square, taken upon the Girt-Line, be ſet to 1 on the Line C: then againſt 41.57 of the Girt-Line, is the Number of Inches on the Line C, that will make a Solid-Foot.

Example. Let the Side of a Square be 8 Inches: ſet 8 on the Girt-Line D, to 1 on the Line C; then againſt 41.57 on the Girt-Line D, is 27 Inches for the Length of one ſolid Foot. To do this in Foot-Meaſure; the Side of the Square 8 Inches, in Foot-Meaſure, is .66 Parts, which taken on the Girt-Line, and being ſet to 1 on the Line C, againſt 1 on the Girt-Line, is 2.25 Feet, for the Length to make one Foot of Timber.

Note, 41.57 is the Square-Root of 1728, the Number of Cubic Inches in a ſolid Foot.

USE VIII. *The Diameter of a Circle, or round Piece of Timber, being given: to find the Side of a Square within the Circle; or to know how many Inches the Side of the Square will be, when the round Timber is ſquared.*

Rule. Set 8.5 on the Line A, to 6 on the Line B; then againſt the Diameter on the Line A, is the Side of the Square on the Line B.

Example. Let the Diameter be 18 Inches: ſet 8.5 on A, to 6 on B; then againſt 18 on A, is 12 $\frac{7}{}$ on the Line B, for the Side of a Square within the Circle. The ſame done in Foot-Meaſure: the Diameter being 18 Inches, is in Foot-Meaſure 1.5; then ſet 1 on the Line A, to .707 on the Line B; and againſt the Diameter 1.5 on the Line A, is 1.7 on the Line B; that is, 1.7 Foot is the Side of an inſcribed Square in a Circle, whoſe Diameter is 1.5 Foot.

Note,

Note, The given Numbers 8.5 and 6, or more exacter, 1 and .707, are, the one the Diameter of a Circle, and the other the Side of a Square inscribed in that Circle.

USE IX. *The Girt of a Tree, or round Piece of Timber being given; to find the Side of a Square within.*

Rule. Set 10 to 9 on the Lines A and B; then against the Girt on the Line A, are the Inches for the Side of the Square on the Line B.

Let the Girt be 12 Inches; set 10 on the Line A, to 9 on the Line B; then against 12 on the Line A, is 10.8 on the Line B, for the Side of the Square. By Foot-Measure it is thus; the Girt 12 Inches is one Foot; then set 10 on the Line A, to 9 on the Line B; and against the Girt 1 Foot, on the Line A, is .89 Parts of a Foot for the Side of the Square within.

Note, The Numbers 10 and 9, or 1 and .9, shew when the Square within the Circle is 1, the fourth Part of the Circumference is .9 Parts of the same. Also, by this and the last Use, you may know, before a Piece of Timber be hewn, how many Boards or Planks of any Thickness it will make.

USE X. *The fourth Part of the Girt of a round Piece of Timber being given; to find the Side of a Square equal to it.*

Rule. Set 1 on the Line A, to 1.128, on the Line B; then against the one fourth of the Girt, on the Line A, is on the Line B, the Side of the Square equal to it.

Example. Let the Girt (that is, one fourth of the whole Girt), be 16 Inches; What is the Side of a Square equal to it? Set 1 to 1.13, on the Lines A and B; then against 16 on the Line A, is 18 on the Line B; which shews, that a Square, whose Side is 18 Inches, is equal to a Circle, whose Girt is 64 Inches, and $\frac{1}{4}$ of it's Girt 16 Inches.

USE XI. *To find the Solidity of a Cone.*

Let the Diameter of the Base of a Cone be 12 Feet, and it's Altitude or Height, 24; to find the Content.

This Use may be solved at one Opearation, thus; set 1.95 on the Girt-Line, to the Height of the Cone 24, on the Line C; then against the Diameter of the Base of the Cone 12, on the Girt Line, stands on the Line C, 904.8 Feet, for the Content.

Note, 1.95 is the Square Root of the Quotient of 42 divided by 11: and As the Quotient of 42 divided by 11, is to the Height of any Cone, So is the Square of the Diameter of it's Base to the solid Content.

USE XII. *To find the Solidity of a Square Pyramid.*

Suppose the Side of the Base is 8 Inches, and the Height 30, set 7 on the Girt-Line, to $\frac{1}{3}$ of the Length, *viz.* 10, on the Line C; then against the Side of the Base 8, on the Girt-Line, is 640 Inches, on the Line C, for the Solidity.

USE XIII. *To find the Solidity of a Sphere, by having the Circumference given.*

Let the Circumference of a Sphere be 22 Inches; to find the Content. As 2904 is to 49, So is the Cube of the Circumference of a Sphere to it's solid Content: therefore set 53.8 (the Square Root of 2904) on the Girt Line, to 49 on the Line C; then against the Circumference 22 Inches on the Girt-Line, is a fourth Number, *viz.* 8.09. Again, set 1, on the Line B, to 22 on the Line A; then against 8.09, on the Line C, stands 179.6 on the Line A, for the Content of the said Sphere in solid Inches. If the Diameter had been given, you must have used the fixed Numbers 4.57 and 11, instead of 53.8 and 49, and then have proceeded as before: because As 21 is to 11, So is the Cube of the Diameter of a Sphere to the solid Content thereof.

This Use may be otherwise solved at one Operation, thus: set 7.69 on the Girt-Line D, to the Circumference of the Sphere 22 Inches, on the Line C; then against 22 Inches, on the Girt-Line D, stands, on the Line C, the solid Content 179.6 Inches. If the Diameter be given to find the Solidity at one Operation, you must set 1.38, on the Girt-Line to the Diameter on the Line C; then against the same Diameter, on the Girt-Line, stands, on the Line C, the Content.

Note, 7.69, and 1.38 are, the one, the Square Root of the Quotient of 2904 divided by 49; and the other, the Square Root of the Quotient of 21, divided by 11.

USE XIV. *The Circumference of a Sphere being given, to find it's Superficies.*

Suppose the Circumference of a Sphere be 20 Inches, What is the Area of it's Superficies? Set 4.69 (the Square Root of 22) on the Girt-Line D, to 7 on the Line C; then against 20 Inches on the Girt-Line, stands upon the Line C 136.5, the Area of the Superficies of the Sphere.

The reason of this is, because as 22 is to 7; So is the Square of the Circumference of a Sphere to the superficial Area thereof.

USE

USE XV. *To find the Solidity of the Segment of a Sphere.*

Say, As 21 is to the Sine; So is 11 times the Square of the said Sine, added to 33 times the Square of half the Chord, to the solid Content of the Segment. As suppose the Sine be 10 Inches, and half the Chord 16 Inches; to find the Content: say, As 21 is to 10; So is 9548, the Sum of 11 times the Square of 10, added to 33 times the Square of 16, to the Content 4546.6 Inches.

USE XVI. *To find the Area of the Convex Superficies of the Segment of a Sphere.*

Say, As 14 is to 44 times the Diameter of a Sphere; So is the Length of the Sine of any Segment thereof, to the convex Superficies of the said Segment. Suppose the Sine be 12 Inches, and the Diameter 30; say, As 14 is to 1320; So is 12 to 1131.4 Inches, the Content sought.

CHAP. V.

Of the Construction and Uses of the Plotting Scale, and an improved Protractor

THE Plotting-Scale is generally made of Box-Wood, and sometimes of Brass, Ivory, or Silver, exactly a Foot, or half a Foot in Length, about an Inch and a half broad, and of a convenient Thickness: Those that are but half a Foot long, have that Length given them, that thereby they may be put into Cases of Instruments.

Plate 4. Fig. 1. On one Side of this Scale is placed seven several Scales of Lines, five of which are divided into as many equal Parts as the Length of the Plotting-Scale will permit. The other two are likewise equal Parts, but have two Lines of Chords of different Lengths joined to them. The first of the equal Divisions, on the first Scale of Lines, is subdivided into 10 equal Parts, at the Beginning of which is set the Number 10; signifying, that ten of those Subdivisions make an Inch: that is, in this Case, every of the Divisions on the first Scale, is exactly an Inch; at the End of the first of which, is set 0; at the End of the second 1; at the End of the third 2; and so on to the End of the Scale. The first of the equal Divisions, on the second Scale of Lines, which are lesser than the Divisions on the first Scale, is likewise subdivided into 10 equal Parts, and hath the Number 16 set at the Beginning of it, signifying, that 16 of those Subdivisions make an Inch, or one of the Divisions $\frac{10}{16}$ of an Inch; at the End of the first of which is placed 0; at the End of the second 1; at the End of the third 2: and so on to the End of the Scale. The first of the equal Divisions on the third Scale of Lines, which are lesser than the Divisions of the precedent Scale's, is also subdivided into 10 equal Parts; at the Beginning of which is set the Number 20; signifying, that 20 of those Subdivisions go to make an Inch, or that one of the Divisions is $\frac{10}{20}$ or $\frac{1}{2}$ of an Inch, which Divisions are marked, 0, 1, 2, 3, and so on to the End of the Scale. Understand the same for the other four Scales, at the Beginnings of which are writ, 24, 32, 40, 48; only the Divisions of the two last Scales of Lines are not continued to the End of the Scale, because of two Lines of Chords of different Lengths, the Beginnings of which are marked by the Letters C, C, signifying Chords. The Construction of which see in the next Chapter.

Note, Each of the aforesaid Scales of Lines are aptly distinguished from one another, by being called Scales of 10, 16, 20, 24, 32, or 48, in an Inch; as the first Scale, is a Scale of 10 in an Inch; the second, 16 in an Inch; the third, 20 in an Inch; the fourth, 24; and so on.

Fig. 2. On the back Side of this Scale, is placed a Diagonal Scale; the first of whose Divisions, which is half an Inch, if the Scale is a Foot long; and one fourth, if the Scale is but half a Foot long, is diagonally subdivided into 100 equal Parts. Also at the other End of the Scale is another Diagonal Subdivision of an Inch into 100 equal Parts, if the Scale is a Foot long; but if it is half a Foot, the Subdivision is of half an Inch into 100 equal Parts. The Figure of this Diagonal Scale, and what our Author has already said of it, in Use 8, is sufficient to shew it's Construction and Use.

There is also next to the Diagonal Scale, a Foot divided into 100 equal Parts, if the Scale is a Foot long, every 10 of which are numbered 10, 20, 30, *&c.* There is likewise next to that the Divisions of Inches, numbered, 1, 2, 3, *&c.* each of which is subdivided into ten equal Parts.

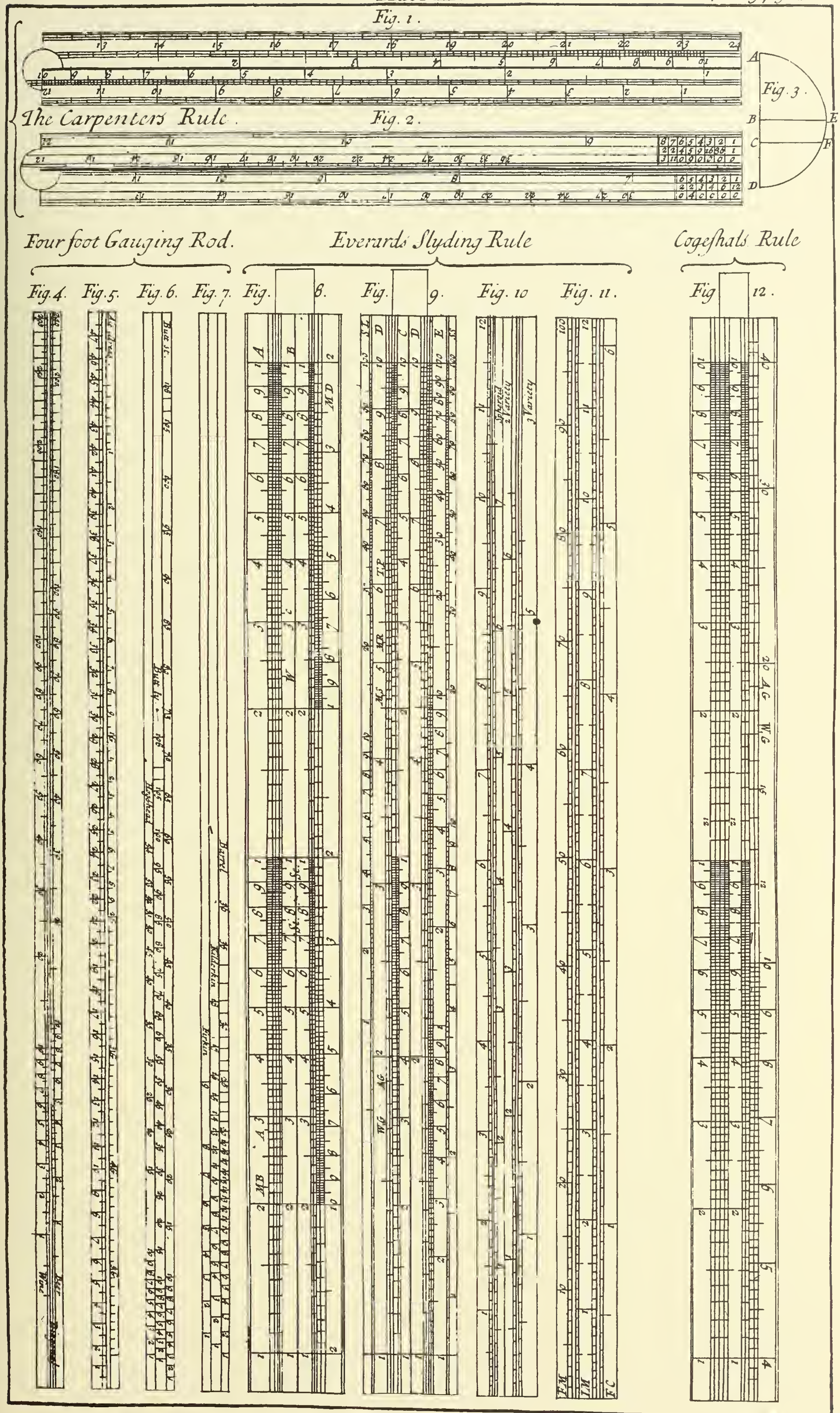
Use of the Plotting-Scale.

This Scale's principal Use is to lay down Chains and Links taken in surveying Land.

USE

Plate III.

fronting page 5



I. Senex sculp.

USE I. *Any Distance being measured by your Chain, to lay it down upon Paper.*

Suppose, that measuring along a Hedge, or the Distance between any two Marks, or Places, with your Chain, you find the Length thereof to contain 6 Chains, 50 Links. Now to take this Distance from your Scale, and lay it down upon Paper, do thus: Fig. 3.

First draw the Line A B, then place one Foot of your Compasses upon your Scale at the Figure 6, for the 6 Chains, and extend the other Foot to 5 of the Subdivisions (which represents the 50 Links), then set this Distance upon the Line drawn from A to B, and the Line A B will contain 6 Chains, 50 Links, if you take the Distance from the Scale of 10 in an Inch.

But if you would have the Line shorter, and yet to contain 6 Chains 50 Links, then take your Distance from a smaller Scale, as of 16, 20, 24, &c. in an Inch, and then the 6 Chains, 50 Links, will end at C: if taken from the Scale of 16 in an Inch; or at D, if taken from the Scale of 20 in an Inch, &c. either of which Lines will contain 6 Chains, 50 Links, and be proportional one to another, as the Scales from which they were taken. And in this manner any Number of Chains and Links may be taken from any of the Scales.

USE II. *A right Line being given, to find how many Chains and Links are therein contained, according to any assigned Scale.*

Suppose A B was a given Line, and it is required to find how many Chains and Links are contained therein, according to the Scale of 10 in an Inch: Take in your Compasses the Length of the Line A B, and applying it to the Scale of 10 in an Inch, you will find that the Extent of your Compasses will reach from 6 of the great Divisions, to 5 of the small ones; whence the Line A B, contains 6 Chains, 50 Links. The like must be done for any Line, and also by any of the other Scales. Fig. 3.

But note, that in laying down the Lengths of Lines by your Scales, whatsoever Scale you begin your Work with, with the same Scale you must continue it to the End, not laying down one Line by one Scale, and another by another; but if you would have a large Work in a little room, then use a small Scale, as of 32, 40, or 48 in an Inch. But contrariwise, if you would express every small Particular, then it is best to use the Scales of 10, or 16 in an Inch.

The Use of the Lines of Chords on the Plotting-Scale, is to protract or lay down Angles, when a Protractor is wanting, which is much more convenient in laying off Angles: *vide* Uses of the Plain-Scale. To take off Parts from the Diagonal Scale, see Use VIII. of our Author's.

Of the Construction and Use of an improved Protractor.

This Protractor is made of Brass, as the others commonly are, and has likewise it's Semicircular Limb divided into 180 Degrees; there is an Index adjusted in the Center of this Protractor, by means of which, an Angle of any Number of Degrees and Minutes, may be protracted: there is a Circle cut out in the Piece, whose Edge, next to the Limb, serves for the Diameter of the Semicircle; the Center of this Circle is in the Center of the Limb, and it is cut sloping, so that it makes the Frustum of a Cone, the greatest Base being underneath. In this Circle is adjusted a Ring, to which the Ring of the End of the Index is riveted; by which means the Index will move freely about the Limb. There is a little Steel Point fixed to the Ring, adjusted in the aforesaid Circle, the End of which terminates in the Center of the Circle; the End of this Point must be laid to the angular Point to be protracted. Fig. 4.

The Index consists of two Pieces, one End of that which comes out beyond the Limb of the Protractor is cut slopewise, so as exactly to fit the Edge of the Limb of the Protractor, which is likewise sloped underneath, and is fastened to the other Piece; by which means the Index is kept down close to the Limb.

The Divisions on both Edges of that Part of the Index beyond the Limb, are 60 equal Parts of the Portions of Circles (passing thro' the Center of the Protractor, and two Points assumed in the outward Edge of the Limb of that Piece of the Index nighest the Center), intercepted by two other right Lines drawn from the Center; so that they each make, with Lines drawn to the assumed Points from the Center, Angles of one Degree.

To lay off any Number of Degrees and Minutes by this Protractor, you must move the Index, so that one of the Lines drawn upon the Limb, from one of the aforementioned Points, may be upon the Number of Degrees sought; and then pricking off as many of the equal Parts on the proper Edge of the Index, as there are Minutes given, and drawing a Line from the Center, to that Point so pricked off, you will have an Angle, with the Diameter of the Protractor, of the proposed Number of Degrees or Minutes. The reason of this Contrivance is from *Prop.* 27. *Lib.* 3. *Eucl.* where it is proved that Angles insisting upon the same Arcs, in equal Circles, or in the same Circle (for it is the same thing), are equal.

CHAP. VI.

The Projection of the Plain-Scale.

Fig. 5, 6. FIRST, draw a Circle A B D C, which cross at right Angles with the Diameters A D, C B; then continue out A D to G, and upon the Point B, raise B F perpendicular to C B. Now draw the Chord A B, and divide the Quadrant A B into 9 equal Parts, setting the Figures 10, 20, 30, *&c.* to 90 to them; each of which 9 Parts again subdivide into 10 more equal Parts, and then the Quadrant will be divided into 90 Degrees. Now setting one Foot of your Compasses in the Point A, transfer the said Divisions to the Chord Line A B, and set thereto the Figures 10, 20, 30, *&c.* and the Line of Chords A B, will be divided, and then may be put upon your Scale, represented in *Fig.* 6. Now to project the Sines, divide the Arc B D into 90 Degrees, as before you did A B; from every of which Degrees, let fall Perpendiculars on the Semidiameter E B; which Perpendiculars will divide E B into a Line of Sines, to which you must set 10, 20, 30, *&c.* beginning from the Center, and then you may transfer the Line of Sines to your Scale.

Again, to project the Line of Tangents; from the Center E, and thro' every Division of the Arc B D, draw right Lines cutting B F, which will divide it into a Line of Tangents, setting thereto the Numbers 10, 20, 30, *&c.* which you must transfer to your Scale.

To project the Line of Secants, transfer the Distances E 10, E 20, E 30, *&c.* that is, the Distance from E to 10, 20, 30, *&c.* on the Tangent Line, upon the Line E G, and setting thereto the Numbers 10, 20, 30, *&c.* the Line E G will be divided into a Line of Secants, which must be transferred on the Scale.

To project the Semi-tangents; draw Lines from the Point C, thro' every Degree of the Quadrant A B, and they will divide the Diameter A E into a Line of Semi-tangents: but because the Semi-tangents, or Plane-Scales of a Foot in Length, run to 160 Degrees, continue out the Line A E, and draw Lines from the Point C, thro' the Degrees of the Quadrant C A, cutting the said continued Portion of A E, and you will have a Line of Half-tangents to 160 Degrees, or further, if you please.

Note, The Semi-tangent of any Arc, is but the Tangent of half that Arc, as will easily appear from it's manner of Projection, and *Prop.* 20. *Lib.* 3. *Eucl.* where it is proved, that an Angle at the Center, is double to one at the Circumference.

Moreover, to draw the Rhumb-Line; from every 8th part of the Quadrant A C, setting one Foot of your Compasses in A, describe Arcs cutting the Chord A C, which will divide A C into a Line of whole Rhumbs, and in the same Manner may the Subdivisions of half and quarter Rhumbs be made.

Lastly, to project the Line of Longitude; draw the Line H D, equal and parallel to the Radius C E, which divide into 60 equal Parts (because 60 Miles make a Degree of Longitude under the Equator), every 10 of which Number set Figures to. Now from every of those Parts, let fall Perpendiculars to C E, cutting the Arc C D; and having drawn the Chord C D, with one Foot of your Compasses in D, transfer the Distances from D, to each of the Points in the Arc C D, on the Chord C D, and set thereto the Numbers 10, 20, *&c.* and the Line of Longitude will be divided.

The Reason of this Construction is, that As Radius is to the Sine Complement of any Latitude, So is the Length of a Degree of Longitude under the Equator, which is 60 Miles, to the Length of a Degree of Longitude in that Latitude.

These being all the Lines commonly put upon the Rulers, called Plain-Scales, excepting equal Parts; therefore I shall proceed to shew their manner of using in Trigonometry, and Spherical Geometry.

But, by the way, note, That Plain-Scales are commonly of these two Lengths, *viz.* some one foot long, and others, which are put into Cases of Instruments, but half a foot in Length; and on one Side is a Diagonal Scale: they are generally made of Box, and sometimes of Brass or Ivory.

USE I. *To make an Angle in the Point* A, *at the End of the Line* A B, *of any Number of Degrees, suppose* 40.

Fig. 7. Take in your Compasses 60 Degrees from the Line of Chords, and setting one Foot in the Point A, describe the Arc C B; then take 40 Degrees, which is the Number proposed, from the same Line of Chords, and lay them off on the Arc from B to E; draw the Line A E, and the Angle B A E will be 40 Degrees, as is manifest from the Construction of the Line of Chords, and *Prop.* 15. *Lib.* 4. *Eucl.* which shews that the Semidiameter of any Circle, is equal to the Side of a Hexagon inscribed in the same Circle; that is, to the Chord of 60 Degrees.

USE II. *The Angle* E A B *being given, to find the Quantity of Degrees it contains.*

Take in your Compaſſes 60 Degrees from the Line of Chords, and deſcribe the Arc B C; then take the Extent from B to E in the Compaſſes; which Extent apply on the Line of Chords, and the Quantity of the Angle will be ſhewn. This Uſe, which is only the Reverſe of the former, may be likewiſe done by the Lines of Sines and Tangents, the Method of doing which is enough manifeſt from Uſe I. Fig. 7.

USE III. *The Baſe of a Triangle being given* 40 *Leagues, the Angle* ABC 36 *Degrees, and the Angle* B A C 41 *Degrees; to make the Triangle, find the Lengths of the Sides* A C, C B, *and alſo the other Angle.*

Draw the indefinite right Line A D, and take the Extent of 40 Leagues, from the Line of Leagues, between your Compaſſes, which lay off upon the ſaid Line from A to B for the Baſe of the Triangle; at the Points A and B make, by Uſe I. the Angles A B C, B C A; the firſt 36 Degrees, and the laſt 41 Degrees, and the Triangle A C B will be formed; then take in your Compaſſes the Length of the Side A C, and apply it to the ſame Scale of Leagues, and you will find it's Length to be 24 Leagues. Do thus for the other Side B C, and you will find it 27 Leagues and a half; and, by Uſe II. the Angle A C B will be found 103 Degrees. Fig. 8.

By this Uſe the following Problem in Navigation may be ſolved, *viz.* Two Ports, both lying under the ſame Meridian, being any Number of Miles diſtant from each other, ſuppoſe 30, and the Pilot of a Ship, out at Sea on a certain time, finds the Bearing of one of the Ports is S W by S, and the Bearing of the other N W: the Ship's Diſtance from each of the Ports at that time is required?

To ſolve this Problem; draw the right Line A B equal to 3 Inches, or 3 of the largeſt equal Parts on the Diagonal Scale, which is to repreſent the 30 Miles, or the Diſtance from one of the Ports, as A to the other B; at the Point B make an Angle, equal to the bearing of the Port B from the Ship, which muſt be 33 Degrees, 45 Minutes; likewiſe make another Angle at the Point A, equal to the Bearing of the Port A from the Ship, which muſt be 45 Degrees, then the Point C will be the Place the Ship was in at the time of Obſervation. Fig. 9.

Now to find the Diſtance of the Ship from the Port A, take the Length of the Side A C in your Compaſſes, and applying it to the Diagonal Scale, you will find it to be $17\frac{1}{10}$ Miles. In the ſame manner the Diſtance of the Ship, from the Port B, will be found $21\frac{1}{2}$ Miles.

Note, The Reaſon why the Angles A and B are equal to the Bearing of the Ship from each of thoſe Ports, depends on *Prop.* 29. *lib.* 1. *Eucl.*

USE IV. *The Baſe* A B *of a Triangle being given* 60 *Leagues, the oppoſite Angle* A C B 108 *Degrees, and the Side* C B 40 *Leagues; to make the ſaid Triangle, and find the Length of the other Side* A C. Fig. 10.

Draw the Line *a b* equal to A B, the given Baſe; and becauſe in any Triangle the Sines of the Sides are proportionable to the Sines of the oppoſite Angles (as is demonſtrated by Trigonometrical Writers), it follows, that As A B is to the Sine of the given Angle C, which is of 72 Degrees, *viz.* the Complement of 108 Degrees to 180; So is the given Side B C, to the Sine of the Angle C A B: therefore make *b c* equal to the given Side B C of 40 Leagues. Take in your Compaſſes, upon the Line of Sines, the Sine of 72 Degrees, to which Length make *b e* equal, and draw the Line *a c*; likewiſe draw *e d* parallel to *a c*, and (by *Prop.* 4. *lib.* 6. *Eucl.*) *b d* will be the Sine of the Angle C A B, which will be found, by applying it to the Line of Sines, about 39 Degrees: therefore make an Angle at the Point A of 39 Degrees, then take in your Compaſſes the Length 40 Leagues, and ſetting one Foot in the Point B, with the other deſcribe an Arc, which will cut the Side A C in the Point C, and conſequently the Triangle A B C will be made, and the Length of the Side A C will be found 34 Leagues. Fig. 11.

USE V. *Concerning the Line of Rhumbs.*

The Uſe of the Line of Rhumbs is only to lay off, or meaſure, the Angles of a Ship's Courſe in Navigation, more expeditiouſly than can be done by the Line of Chords: As ſuppoſe a Ship's Courſe is N N E, it is required to lay it down.

Draw the Line A B, repreſenting the Meridian; take 60 Degrees from the Line of Chords, and about the Point A deſcribe the Arc B C. Now becauſe N N E is the third Rhumb from the North, therefore take the third Rhumb in your Compaſſes, on the Line of Rhumbs, and lay it off upon the Arc from B to C; draw the Line A C, and the Angle B A C will be the Courſe. Fig. 12.

USE VI. *Of the Line of Longitude.*

The Uſe of this Line is to find in what Degrees of Latitude a Degree of Longitude is 1, 2, 3, 4, *&c.* Miles, which is eaſily done by means of the Line of Chords next to it: for it is only ſeeing what Degree of the Line of Chords anſwers to a propoſed Number of Miles, and that Degree will be the Latitude, in which a Degree of Longitude is equal to that propoſed

posed Number of Miles. As for Example; against 10 Miles, on the Line of Longitude, stand 80 Degrees, and something more; whence, in the Latitude of about 80 Degrees, a Degree of Longitude is 10 Miles. Again, 30 Miles on the Line of Longitude, answers to 60 Degrees on the Line of Chords; therefore in the Latitude of 60 Degrees, a Degree of Longitude is 30 Miles. Moreover, against 58 Miles, on the Line of Longitude, stands 15 Degrees of the Line of Chords, which shews that a Degree of Longitude, in the Latitude of 15 Deg. is 58 Miles; and so for others.

USE *of the Plain-Scale in Spherical Geometry.*

USE I. *To find the Pole of any Great Circle.*

If the Pole of the Primitive Circle be required, it is it's Center.

If the Pole of a right or perpendicular Circle be sought, it is 90 Degrees distant, reckoned upon the Limb from the Points, where this Circle, which is a Diameter, cuts it.

If the Pole of an oblique Circle be required,

(1.) Consider that this Circle must cut the primitive in two Points, that will be distant from each other just a Diameter, as is the Case of the Intersection of all great Circles.

(2.) The Pole of this Circle must be in a right Line, perpendicular to it's Plane.

(3.) This Circle's Pole cannot but lie between the Center of the primitive one, and it's own.

Fig. 13. *Example.* Let the Pole of the oblique Circle A B C be required.

1. Draw the Diameter A C, and then another, as D E, perpendicular to it.

2. Lay the Edge of your Scale from A to B, it will cut the Limb in F; then take the Chord of 90 Degrees, and set it from F to *h*.

3. Lay the Edge of your Scale from *h* to A, it will cut D E in *g*, which Point *g* is the Pole required.

Note, To find the Points F and *h*, is called reducing B to the primitive Circle, and to the Diameter. Also, *Note*, that every of the primitive Circles in this Use, and the following ones, are supposed to be described from 60 Degrees, taken off from the lesser Line of Chords on the Scale.

USE II. *To describe a Spherical Angle of any Number of given Degrees.*

1. If the angular Point be at the Center of the primitive Circle, then it is at any plane Angle, numbering the Degrees in the Limb from the Line of Chords; for all Circles passing through the Center, and which are at right Angles with the Limb, must be projected into right Lines.

2. If the Angle given is to be described at the Periphery of the primitive Circle, draw a Diameter, as A C; then take the Secant of the Angle given in your Compasses, and setting one Foot in A, cross the Diameter in *e*: or if no Diameter be drawn, placing one Foot in C, and crossing the former Arc, you will find the same Point *e*, which is the Center of the Circle A *a* C, which, with the primitive Circle, makes the Angle D A *a* required.

Note, If the Angle given be obtuse, take the Secant of it's Supplement to 180 Degrees.

3. If a Point, as *a*, were assigned, through which the Arc of the Circle constituting the Angle must pass, draw the Diameter A C (as before) then take the Secant of the given Angle, and setting one Foot in A or C, strike an Arc as at *e*; and then with the Secant of the given Angle, setting one Foot in *a*, cross the other Arc in *e*; which will be the Center of the oblique Circle required.

USE III. *To draw a great Circle through any two Points given, as* a *and* b, *within the primitive one.*

Fig. 14. Draw a Diameter through that Point which is furthest from the Center, as D R, producing it beyond the Limb if there be Occasion; set 90 Degrees of Chords from D or R, to O, and draw O *a*.

Then erect O H perpendicular to *a* O, and produce it 'till it cuts the Diameter prolonged in H; that Intersection H is a third Point, through which, as also *a* and *b*, if a Circle be drawn, it will be a great Circle, as *e a b g*.

Which is easily proved, by drawing the Lines *e* C *g*; for that Line is a Diameter, because it's Parts, multiplied into one another, are equal to *a c* × C H, equal to O C squared. *Per Prop.* 35. *lib.* 3. & *Coroll.* 8. *lib.* 6. *Eucl.*

USE IV. *To draw a great Circle perpendicular to, or at right Angles to another.*

Let it pass through it's Poles, and it is done.

Of which there will be four Cases:

1. To draw a Circle perpendicular to the Primitive, which is done by any strait Line passing through the Center.

2. To draw a Circle perpendicular to a right Circle, is only to draw a Diameter at right Angles with that right Circle.

3. To draw an oblique Circle perpendicular to a right one, only draw a right Circle that shall pass through both the Poles of such a right Circle.

Thus

Thus the oblique Circle D C R is perpendicular to the right one O Q, becauſe it paſſes through it's Poles D and R.

4. To draw an oblique Circle perpendicular to another : *Plate* 5.

Firſt find P, the Pole of the given oblique Circle C *e* B, and then draw any-how the Diameter D R : So a Circle, drawn through the three Points D, P, and R, will be the Circle required ; for paſſing through the Poles of the oblique Circle C *e* B, it muſt be perpendicular to it. Fig 1.

U S E V. *To meaſure the Quantity of the Degrees of any Arc of a great Circle.*

1. If the Arc be part of the Primitive, it is meaſured on the Line of Chords.

2. If the Arc be any part of a right Circle, the Degrees of it are meaſured on the Scale of Semi-Tangents, ſuppoſing the Center of the primitive Circle to be in the Beginning of the Scale ; ſo that if the Degrees are to be reckoned from the Center, you muſt account according to the Order of the Scale of Half-Tangents.

But if the Degrees are to be accounted from the Periphery of the Primitive, as will often happen, then you muſt begin to account from the End of the Scale of Half-Tangents, calling 80, 10 ; 70, 20, *&c.*

3. To meaſure any part of an oblique Circle ; firſt find it's Pole, and there laying the Ruler, reduce the two Extremities of the Arc required to the primitive Circle, and then meaſure the Diſtance between thoſe Points on the Line of Chords.

Thus, in the laſt Figure, if the Quantity of *e* B, an Arc of the oblique Circle C *e* B be required, lay a Ruler to P the Pole, and reduce the Points *e* B to the primitive Circle ; ſo ſhall the Diſtance between O and B, meaſured on the Line of Chords, be the Quantity of Degrees contained in the Arc *e* B. Fig. 1.

U S E VI. *To meaſure any Spherical Angle.*

1. If the angular Point be at the Center of the primitive Circle, then the Diſtance between the Legs taken from the Limb, and meaſured on the Chords, is the Quantity of the Angle ſought.

2. If the angular Point be at the Periphery, as A C B ; here the Poles of both Circles being in the ſame Diameter, find the Pole of the oblique Circle C B O, which let be P ; then the Diſtance of B P, meaſured on the Scale of Half-Tangents, is the Meaſure of the Angle A C B. Fig. 2.

For the Poles of all Circles muſt be as far diſtant from each other, as are the Angles of the Inclinations of their Planes.

But if the two Poles are not in the ſame Diameter, being both found in their proper Diameter, reduce thoſe Points to the primitive Circle ; and then the Diſtance between them there, accounted on the Line of Chords, is the Quantity of the Angle ſought.

When the angular Point is ſomewhere within the primitive Circle, and yet not at the Center, proceed thus : Suppoſe the Angle *a b* C be ſought ; find the Pole P of the Circle *a b d*, and then the Pole of the Circle *e b c* ; after which lay a Ruler to the angular Point, and the two Poles P and Q, and reduce them to the primitive Circle by the Points *x* and *z* ; So is the Arc *x z*, meaſured on the Line of Chords, the Meaſure of the Angle *a b* C required. Fig. 3.

U S E VII. *To draw a Parallel Circle.*

1. If it be to be drawn parallel to the primitive Circle, at any given Diſtance, draw it from the Center of the Primitive, with the Complement of that Diſtance taken from the Scale of Half-Tangents.

2. If it be to be drawn parallel to a right Circle ; as ſuppoſe *a b*, parallel to A B, was to be drawn at 23 Deg. 30 Min. Diſtance from it ; from the Line of Chords take 23 Deg. 30 Min. and ſet it both-ways on the Limb from A to *a*, and B to *b* (or ſet it's Complement 66 Deg. 30 Min. both-ways from P the Pole of A B) to the Points *a* and *b*. Fig. 4.

Then take the Tangent of the Parallel's Diſtance from the Pole of the right Circle A B, which is here 66 Deg. 30 Min. and ſetting one foot in *a* and *b*, with the other ſtrike two little Arcs, to interſect each other ſomewhere above P, which will give C, the Center of the parallel Circle *a b d* required.

3. If it be drawn parallel to an oblique Circle, and at the Diſtance ſuppoſe of 40 Degrees : Fig. 5.

Firſt find P, the Pole of the oblique Circle A B C, and then meaſure, on the Scale of Half-Tangants, the Diſtance *g* P, which ſuppoſe to be 34 Degrees ; then add to it 50 Degrees, the Complement of the Circle's Diſtance, it will make 84 Degrees ; and alſo ſubſtracting 50 from it, or it from 50, it will make 16 Degrees : Then this Sum and Difference taken from the Scale of Half-Tangents, and ſet each way from P the Pole of the oblique Circle, will give the two Extremes *a b* of the Diameter, or the Points of the Interſection of the Parallel ; and then the middle Diſtance between *a* and *b*, is the Center of the true parallel Circle P *a b*, which is parallel to the given oblique Circle A B C ; and at the given Diſtance of 40 Degrees : or the Half-Tangent of 84, ſet from *g*, will give *b* ; and the Half-Tangent of 16 Degrees, ſet alſo from *g*, and the Points *a* and *b*, the two Ends of the parallel Circle's Diameter will be had.

USE VIII. *To measure any projected Arc of a parallel Circle.*

1. If it be parallel to the Primitive, then a Ruler, laid through the Center and the Division of the Limb, will divide the Parallel into the same Degrees, or determine, in the Limb, the Quantity of any Arc parallel to it.

2. If the Circle be parallel to a right one, as *a d b* is, in case the second of the last Use, and it were required to measure that Arc *a b*, or to divide it into proper Degrees: Since that parallel Circle is 66 Deg. 30 Min. distant from P, the nearer Pole of the right Circle A B, and consequently 113 Deg. 30 Min. distant from it's other Pole; take the Half-Tangent of 113 Deg. 30 Min. or the Tangent of it's half, 56 Deg. 45 Min. and with that Distance, and on the Center of the Primitive, draw a Circle parallel to the Limb; and divide that half of it, which lies towards the opposite Pole of A B, into it's Degrees: Then a Ruler laid from P, and the equal Divisions of that Semicircle, will divide *a b*, or measure any part thereof.

3. To measure or divide the Arc of a Circle which is projected, parallel to an oblique one. As suppose the Circle *a b*, which is parallel to the oblique one A B C, *Fig. Case* 3. of the precedent Use, and at the Distance of 40 Degrees; this parallel Circle being 40 Degrees distant from the Plane of the Circle A B C, must be 50 Degrees distant from it's Pole, and consequently 130 Degrees from it's opposite Pole: therefore take the Semi-Tangent of 130 Degrees, or the Tangent of it's half, 65 Degrees, and with that, as a Radius, draw a Circle parallel to the Limb of the Primitive, which Circle divide into proper Degrees; then shall a Ruler laid through P, and the equal Division of that Circle, cut the little Circle *a b* into it's proper Degrees, or truly give the Measure of any part thereof.

These being most of the general Uses of the Scales of Lines commonly put upon Plain-Scales, their particular Applications in Navigation, Spherical Trigonometry, and Astronomy, would take up too much room; therefore I proceed to *Gunter*'s Scale.

As for it's Use in the Projection of the Sphere, see the *Uses of the* English *Sector.*

CHAP. VII.

Of the Construction and Uses of Gunter's-*Scale.*

Fig. 6. THIS Scale is commonly made of Box, and sometimes of Brass, exactly two Foot long (though there are others but a Foot long, which are not so exact) about an Inch and $\frac{3}{4}$ broad, and of a convenient Thickness.

The Lines that are put on one Side of it are the Line of Numbers, marked on the Scale *Numbers*; the Line of artificial Sines, marked *Sines*; the Line of artificial Tangents, marked *Tangents*; the Line of artificial versed Sines, marked V. S. signifying Versed Sines; the artificial Sines of the Rhumbs, marked S. R. signifying the Sines of the Rhumbs; the artificial Tangents of the Rhumbs, marked T. R. signifying Tangents of the Rhumbs; the Meridian-Line in *Mercator*'s Chart, marked *Merid.* signifying Meridian-Line; and equal Parts, marked E. P. signifying equal Parts.

There are commonly placed on these Scales, that are but a Foot long, the Lines of Latitudes, Hours, and Inclinations of Meridians.

On the Back-side of this Scale are placed all the Lines that are put upon a Plain-Scale.

The Lines of artificial Sines, Tangents, and Numbers are so fitted on this Scale, that, by means of a Pair of Compasses, any Problem, whether in right-lined, or spherical Trigonometry, may be solved by them very expeditiously, with tolerable Exactness; and therefore the Contrivance of these Lines on a Scale is extremely useful in all Parts of Mathematicks that Trigonometry hath to do with; as Navigation, Dialling, Astronomy, &c.

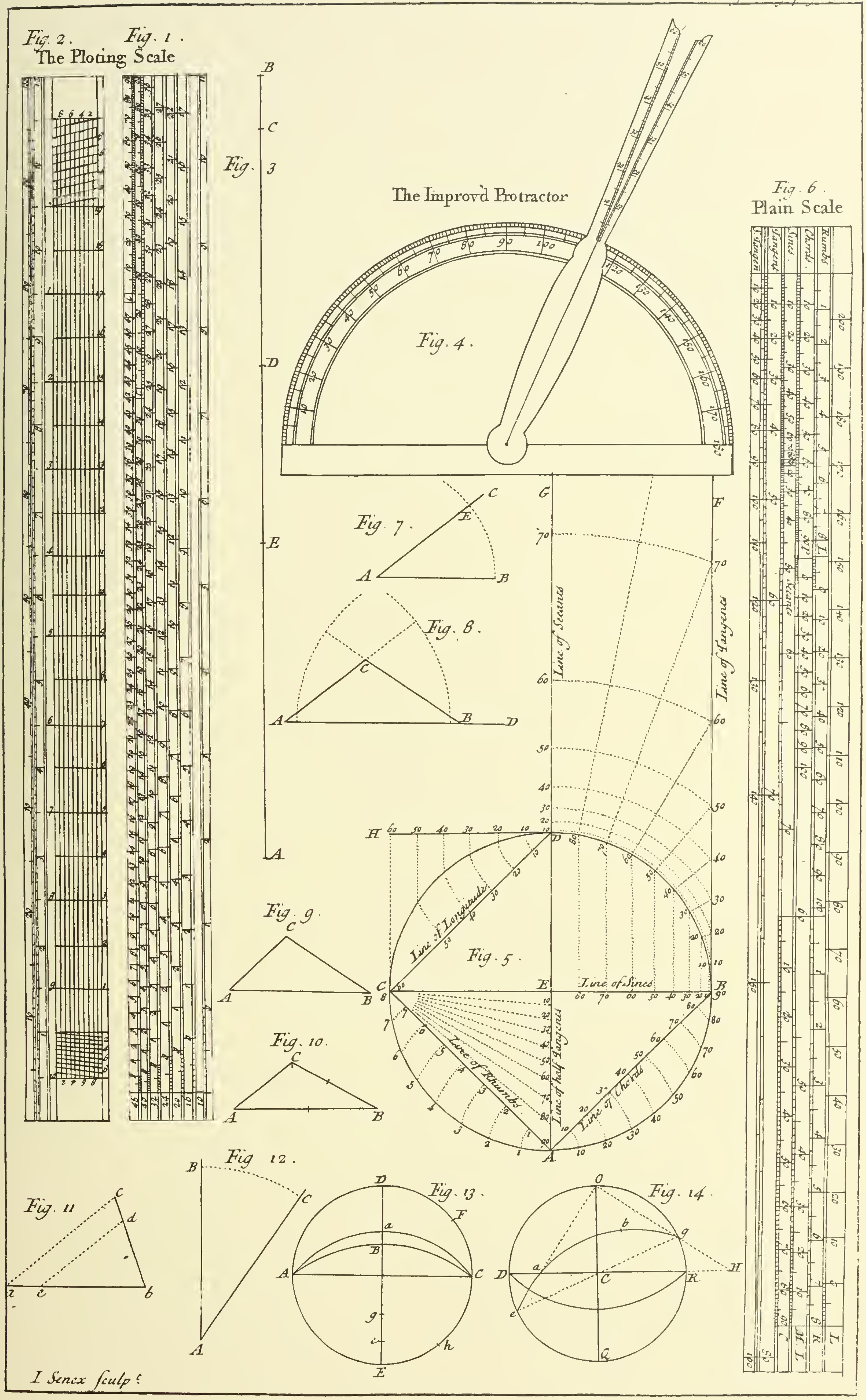
Construction of the Line of Numbers.

The Construction of the Line of Numbers is thus: Having pitched upon it's Length, which, on *Gunter*'s Scale, let be 23 Inches, take exactly half that Length, which will be the Length of either of the Radius's; then take that half Length, and divide into 10 equal Parts, one of which diagonally subdivide into 100 equal Parts, that is, make a Diagonal Scale of 1000 equal Parts of the aforesaid Half-Length, which may easily be done from our Author's 8th Use.

Now having drawn, on *Gunter*'s Scale, three Parallels, for better distinguishing the Divisions of the Line of Numbers, and made a Mark for the Beginning of it, half an Inch from the Beginning-end of the Scale, look in a Table of Logarithms for the Number 200, and against it you will find 2.301030; and rejecting the Characteristick 2, and also the three last Figures 030, because the Length of the Radius is divided but into 1000 equal Parts, take 301 of those 1000 Parts in your Compasses, and lay off that Distance from the Beginning of the

Plate IV.

fronting page 38.



the Line, at the End of which write 2 for the firſt Prime. Again, to find the Diviſion for the ſecond Prime, look in the Table of Logarithms for the Number 300, and againſt it you will find 2.477121; and rejecting the Characteriſtick 2, and the three laſt Figures 121, as before, take 477 from your Diagonal Scale, and lay off that Diſtance from 1 at the Beginning, at the End of which write 3 for the ſecond Prime. In this manner proceed for all the Primes of the firſt Radius to 1, which will be the whole Length of your Diagonal Scale, or 1000 equal Parts. And becauſe each of the Primes of the ſecond Radius are at the ſame Diſtance from 1, at the End of the firſt Radius, as the ſame Primes, on the firſt Radius, are diſtant from 1 at the Beginning, the Primes on the ſecond Radius are eaſily found.

The Diviſions of the Tenths, between each of the Primes in both Radius's, are found thus: Look in the Table of Logarithms for 110, and againſt it you will find 2.041393, and rejecting the Characteriſtick 2, and the three laſt Figures, there will remain 41; which taken from the aforeſaid Diagonal Scale of 1000, will give the firſt Tenth in the firſt Prime. Again, look in the Table for 120, and againſt it you will find 2.079181; and by rejecting the Characteriſtick, and the three laſt Figures, there will remain 79; which taken from the Diagonal Scale, will give the ſecond Tenth in the firſt Prime. Proceed thus for all the Tenths in the firſt Primes of both Radius's. And to find the Tenths in the ſecond Primes of both Radius's, look in the Table for the Number 210, and againſt it you will find 2.322219, whence rejecting as before, you will have 322, which laid off from the Beginning of the firſt Prime, will give the firſt Tenth in the ſecond Prime. Again, to find the ſecond Tenth in the ſecond Prime, look for the Number 220, and againſt it you will find 2.342423, whence by rejecting, as before, you will have 342 for the ſecond Tenth, in the ſecond Prime. In like manner may the Tenths in all the Primes of both Radius's be found.

To find every two Centeſms in the firſt Prime of the ſecond Radius, look for the Number 102 in the Table of Logarithms, and againſt it you will find 2.008600, and by rejecting, as in the Table of Logarithms, and againſt it you will find 2.008600, and by rejecting, as at firſt, you will have 8 for the ſecond Centeſm. Again, look in the Table for 104, and proceed as before, and you will have 17 for the third Centeſm. In like manner you may have every ſecond Centeſm in the firſt, and alſo the Second Primes of the ſecond Radius.

Note, In biſecting every of the two Centeſms in the firſt Prime, Centeſms will be had. Note alſo, That the third, fourth, and fifth Primes, cannot be divided into every two Centeſms, but only into every five, becauſe of the Smallneſs of the Diviſions.

Conſtruction of the Line of artificial Sines and Tangents.

The Line of artificial Sines on *Gunter*'s Scale, is nothing but the Logarithms of the natural Sines, tranſlated from the Tables of artificial Sines and Tangents, almoſt in the ſame Manner as the Logarithms of the natural Numbers were; the Method of doing which is thus:

Having drawn three Parallels under the Line of Numbers for diſtinguiſhing the Diviſions of the Line, and marked a Point exactly half an Inch from the Beginning-end of the Scale, repreſenting the Beginning of the Line of Sines, look in the Tables of artificial Sines and Tangents, for the Sine of 40 Minutes, which is the firſt Subdiviſion of the Line, and it will be found 8.065776: then rejecting the Characteriſtick 8, and the three laſt Figures 776, as in the Conſtruction of the Line of Numbers, the 65 remaining, muſt be taken on the ſame Scale of 1000 Parts, as ſerved before for the Line of Numbers; this 65 laid off from the Beginning of the Line of Sines, will give the Diviſion on the Line of Sines for 40 Minutes. Again, To make the next Diviſion, which is for 50 Minutes, ſeek in the Table for the Sine of 50 Minutes, which will be found 8.162681; then rejecting the Characteriſtick 8, and the three laſt Figures 681, take the Remainder 162, from your Scale of 1000 Parts, and lay it off from the Beginning of the Line, and that will give the Diviſion for 50 Minutes. Moreover, to make the Diviſion for 1 Degree, ſeek the Sine of 1 Degree, which is 8.241855, and rejecting as before, take the Remainder 241 from the Scale of 1000, and lay it off from the Beginning on the Line of Sines, which will give the Diviſion for 1 Degree. Proceed thus for the other Degrees and Minutes to 90; only take notice, that when you come to 5 Degrees, 50 Minutes, the Parts to be taken off the Scale are more than 1000, and, conſequently, longer than the Scale itſelf. In that Caſe you muſt make a Mark in the Middle of the Line of Sines; from which lay off all the Parts found above 1000, for the Degrees and Minutes: As, to make the Diviſion for 6 Degrees, the Sine of which is 9.019235, the Parts to be taken off the Scale will be 1019; therefore lay off 19 from the middle Point, repreſenting 1000, and the Diviſion for 6 Degrees will be had. Proceed in the ſame manner for the Line of artificial Tangents, 'till you come to 45 Degrees, whoſe Length is equal to Radius; and the Diviſions for the Degrees and Minutes above 45, which ſhould go beyond 45, are ſet down by their Complements to 90. For Example, the Diviſion of 40 Degrees hath it's Complement 50 ſet to it, becauſe the proper equal Parts taken off the Scale of 1000 to make the Diviſion, for the Tangent of 50 Deg. will be as much above 1000 (which are the equal Parts for the Tangent of 45 Degrees, to be laid off from the middle of the Line of Tangents) as the equal Parts for the Diviſion of the Tangent of 40 Degrees wants of 1000; an Example of which will make it manifeſt: The Tangent of 40 Degrees is 9.924813, and by rejecting the Characteriſtick, and the three laſt Figures, the Parts of 1000, *viz.* 924 taken from 1000, and there remains 76, which are the Parts that the Tangent of 40 Degrees is diſtant from the

Tangent

Tangent of 45 Degrees. Again, the Tangent of 50 Degrees is 10.076186, and by rejecting the Characteristick, and the three last Figures, the Parts 76 above 1000, for the Division of the Tangent of 50 Degrees, which must fall beyond 45 Degrees, are equal to the Parts that the Division of 40 Degrees wants of 1000. Understand the same for the Tangent of any other Degree, or Minute, and it's Complement: the Reason of this is, because Radius is a mean Proportional between any Tangent and it's Complement.

The Construction of the artificial Sines of the Rhumbs, and quarter Rhumbs, is deduced from a Consideration that the first Rhumb makes an Angle of 11 Deg. 15 Min. with the Meridian; the second, 22 Deg. 30 Min. the third, 33 Deg. 45 Min the fourth, 45 Deg. &c. therefore to make the Division on *Gunter*'s Scale, for the first Rhumb, take the Extent of the artificial Sine of 11 Deg. 15 Min. on the Scale, and lay it off upon the Line drawn to contain the Divisions of the Line of Rhumbs, and that will give the Division for the first Rhumb. Again; take the Extent, on the Line of artificial Sines, of the Sine of 22.30 Min. and lay it off in the same manner as before, and you will have the second Rhumb: proceed thus for all the other Rhumbs. The Divisions for the half Rhumbs, and quarter Rhumbs, are also made in the same manner: the Divisions of the artificial Tangents of the Rhumbs, are made in the same manner as the Divisions of the artificial Sines of the Rhumbs, by taking the artificial Tangents of the several Angles that the Rhumbs and quarter Rhumbs make with the Meridian.

The Construction of the Line of artificial versed Sines.

This Line, which begins at about 11 Deg. 45 Min. and runs to 180 Deg which is exactly under 90 of the Line of Sines (though on the Scale they are numbered backwards; that is, to the versed Sine of each 10 Degrees above 20, are set the Numbers of their Complements to 182, for a Reason hereafter shewn), may be thus made, by means of the Table of Sines, and the aforesaid equal Parts. Suppose the Division for the versed Sine of 15 Degrees be to be made. Take half 15 Degrees, which will be $7^{\mathrm{d}}.\ 30^{\mathrm{m}}$; the Sine of which doubled will be 18.231396, and by substracting the Radius therefrom, you will have 8.231396; and rejecting the three last Figures, and the Characteristick, there will remain 231; this 231 taken from your Scale of 1000, and laid off from a Point directly under the Beginning of the Line of Sines, will give the Division for the versed Sine of 15 Degrees, at which is set 165, *viz.* the Complement of 15^{d} to 180^{d}. Again; to make the Division for 20 Degrees; twice the Sine of 10 Degrees (it's half) will be 18.479340; from which, substracting Radius, and rejecting the Characteristick, and the three last Figures, you will have 479; which taken from your Scale, and laid off from the Beginning of the Line, will give the Division for the versed Sine of 20 Degrees. And in this manner may the Line of versed Sines be divided to 180 Degrees, by observing what I have said in the Construction of the Line of Sines.

The Manner of projecting the Lines of Numbers, artificial Sines and Tangents, in Circles, and Spirals of any Number of Revolutions.

Fig. 7. Suppose the Circle BC is to be divided into a Line of Numbers of but one Radius; first, divide the Limb into 1000 equal Parts, beginning from the Point G; then take 301 of those Parts, which suppose to be at p, and lay a Ruler from the Center A, on the said Point p, and that will cut the Periphery of the Circle BC in the Point for the Log. of 2. Again, Take 477 Parts upon the Limb, and a Ruler laid from the Center upon the said Division, will cut the Circle BC in the Point for the Log. of the Number 3: and thus by taking the proper Parts upon the Limb, from the Point G, which were before directed to be used in dividing this Line upon the Scale; and laying a Ruler from the Center, may the Line of Numbers be projected upon the Circle BC. And in the same manner may the Lines of artificial Sines and Tangents be projected, from the Sine of $5^{\mathrm{d}}\ 45^{\mathrm{m}}$, and Tangent of $5^{\mathrm{d}}\ 42^{\mathrm{m}}$, to the Sine of 90^{d}, and the Tangent of 45^{d}, by taking (as before directed in the Construction of the straight Lines of Sines and Tangents) the Parts of 1000 for the Degrees and Minutes, and laying them off upon the Limb from the Point G, and then laying a Ruler from the Center, which will divide the Circles into Lines of Sines and Tangents.

Fig. 8. Now to project a Line of Numbers upon the Spiral of *Fig.* 8. having four Revolutions, or Turns; first, divide the Limb into 1000 equal Parts, beginning from the Point G; then take 301, which is the Log. of the Number 2 (when the Characteristick, and the three last Figures are rejected), and multiply it by 4, because the Spiral hath four Revolutions, and the Product is 1204: then if 204 of the Parts of 1000, be taken upon the Limb from G to p, and a Ruler be laid from the Center A to p, it will cut the second Revolution of the Spiral in the Point for the Number 2. Again; having multiplied 477, the Log. of the Number 3, by 4, the Product will be 1908; whence taking 908 Parts from the Point G on the Limb, to the Point q, lay a Ruler from A to q, and that will cut the second Revolution of the Spiral, in the Point for the Number 3. Moreover, multiply 602 by 4, and the Product will be 2408; whence take 408 Parts upon the Limb from G, and laying a Ruler from A, it will cut the third Revolution of the Spiral in the Point, for the Number 4: and in thus proceeding may the Spiral be divided into a Line of Numbers, whose Beginning is at the Point C, and end

End at the Point B. This being underftood, it will be no difficult Matter to project the Sines and Tangents in a Spiral of any Number of Revolutions.

In ufing either the Circular or Spiral Lines of Numbers, Sines, and Tangents, there is an opening Index placed in the Center A, confifting of two Arms; the one called the antecedent Arm, and the other the confequent Arm; then three Numbers, Sines, or Tangents being given, to find a fourth. If you move the antecedent Arm to the firft, and open the other Arm to the fecond (the two Arms keeping the fame Opening), and afterwards the antecedent Arm be moved to the third, the confequent Arm will fall upon the fourth required.

But, *Note*, That as many Revolutions of the Spiral as the fecond Term is diftant from the firft, fo many Revolutions will the fourth Term be diftant from the third.

Of the Meridian Line

The Meridian Line, on *Gunter*'s Scale, is nothing but the Table of Meridional Parts in *Mercator*'s Projection transferred on a Line, which may be done in the following Manner, by help of the Line of equal Parts fet under it, and a Table of Meridional Parts.

Take any one of the large Divifions of the aforefaid Line of equal Parts, whofe Length let be A B, and divide it into fix equal Parts upon fome Plane; at the Points A B raife the Perpendiculars A C, B D, equal to A B, and compleat the Parallelogram A B D C; divide the Sides A C, B D, into ten equal Parts, and the Side D C into fix, draw the Diagonals A F, 10, 20, *&c.* as *per* Figure, and you will have a Diagonal Scale, by which any part of the aforefaid Divifion under 60 may readily be taken. Fig. 9.

Now to make the Divifions of the Meridian Line, look in the Table of meridional Parts for 1 Degree, and againft it you will find 60: and rejecting the laft Figure, which in this Cafe is 0, take fix equal Parts from the aforementioned Diagonal Scale, and lay it off on the Meridian Line, which will give the Divifion for one Degree. Again, to find the Divifion for 2 Degrees, feek in the Table of Meridional Parts, for the Parts againft 2 Degrees, and they will be found 120: whence rejecting the laft Figure (which always muft be done), take 12 from your Scale, and lay it off from the Beginning of the Meridian Line, and the Divifion for 2 Degrees will be had. Moreover, to find the Divifion for 11 Degrees, you will find anfwering to it 664; and rejecting the laft Figure, the Remainder will be 66, which muft be laid off from the Beginning of the Meridian Line to have the Divifion for 11 Degrees. But becaufe 66 cannot be taken from the Diagonal Scale, you muft take only 6 from it; and for the 60, take it's whole Length, or elfe lay off the 6 from the End of the firft Divifion of the Line of equal Parts, and the Divifion for 11 Degrees will be had. In this manner may the Meridian Line be divided into Degrees, and every thirty Minutes, as it is upon the Scale.

There are feveral other ways of dividing this Meridian Line, but let this fuffice.

The Ufe of this Line is to project a *Mercator*'s Chart.

Projection of the Line of Latitudes and Hours.

Upon the End A, of the Diameter of the Circle, erect a Line of Sines at right Angles, of the Length of the Diameter; then from the Point B, the other End of the Diameter, draw right Lines to each Degree of that Line of Sines, cutting the Quadrant A C. Now having drawn the Chord-Line A C, which is to be the Line of Latitudes, fet one Foot of your Compaffes upon the Point A, and with the other transfer the Interfections made by the Lines drawn from B, on the Quadrant, to the Chord-Line A C, by means of which it will be divided into a Line of Latitudes. Or the Line of Latitudes may be made by this Canon, *viz.* As Radius is to the Chord of 90 Deg. So is the Tangent of any Degree, to another Tangent, the natural Sine of whofe Arc, taken from a Diagonal Scale of equal Parts, will give the Divifion, for that Degree, on the Line of Latitudes, and fo for any other Degree. Fig. 10.

Again, To graduate the Line of Hours, draw the Tangent G H equal to the Diameter A B, and parallel thereto; then divide each of the Arcs of half the Quadrants A K, K B, into three Parts, for the Degrees of every Hour from 12 to 6, which muft again be each fubdivided into Halfs, Quarters, *&c.* then, if thro' each of the aforefaid Divifions and Subdivifions, Lines be drawn from the Center, cutting the Tangent Line G H, they will divide the faid Line into a Line of Hours.

As for the Line of Inclination of Meridians, ufually put upon Scales, it is nothing but the Line of Hours numbered with Degrees inftead of Time; and the Lines of the Style's Height, and Angle of 12 and 6, fometimes put upon Scales, are made from Tables of the Style's Height, *&c.* and no otherwife ufed.

Whence the Line of Hours is but two Lines of natural Tangents to 45 Degrees, each fet together at the Center, and from thence Beginning and continued to each End of the Diameter, and from one End thereof, numbered with 90 Deg. to the other End; and may otherwife be thus divided: Let A B be the Radius of a Line of Tangents, C D another Radius equal and parallel thereto, and C B the Diameter to either of the faid Radius's, which is to be divided into a Line of Hours. Now if right Lines are drawn from the Point D, to every Degree of the Tangent-Line A B, thofe Lines will divide G B, half of the Line of Fig. 11.

 Hours,

Hours, as required; and Lines drawn from the Point A, to every Degree of the Tangent C D, will divide the other half of C B: therefore from the similar Triangles C D F, E F B, it will be as the Radius C D is to the Tangent E B of any Arc under 45: So is C F to F B; that is, As Radius is to the Tangent of any Arc under 45 Degrees, So is Radius + the Co-
Fig. 12. tangent of the said Arc to 45 Degrees, to Radius — the said Cotangent, as in *Fig.* 12. As the Radius A B, to the Tangent B C of any Arc, So is A B + E G, to A B — E G: for call A B, r; and B C, b; and from the Point C, draw C F parallel to E G, and make B D equal to A B. Then $DF (=FC) = \frac{\sqrt{rr - 2rb + bb}}{2}$, and $AF = \frac{\sqrt{rr + 2rb + bb}}{2}$.

Whence as $AF: \left(\frac{\sqrt{rr + 2rb + bb}}{2}\right) : FC \left(\frac{\sqrt{rr - 2rb + bb}}{2}\right) :: AB(r) : EG \left(\frac{rr - rb}{r + b}\right)$, therefore it will be $AB : (r) : BC(b) :: AB + EG\left(\frac{2rr}{r+b}\right) : AB - EG\left(\frac{2rb}{r+b}\right)$.

Thus having given the Construction of the Lines on *Gunter*'s Scale, I now proceed to shew their manner of using; but, *Note*, These Lines are also put upon Rulers to slide by each other, and are therefore called *Sliding-Gunters*, so that you may use them without Compasses; but any Person that understands how to use them with Compasses, may also, by what I have said of *Everard*'s and *Coggeshall*'s Sliding-Rules, use them without.

USE *of the Lines of Numbers, Sines, and Tangents.*

USE I. *The Base of a right-angled right-lined Triangle being given* 30 *Miles, and the opposite Angle to it* 26 *Degrees, to find the Length of the Hypothenuse.*

As the Sine of the Angle, 26 Degrees, is to the Base, 30 Miles, So is Radius to the Length of the Hypothenuse. Set one Foot of your Compasses upon the 26th Degree of the Line of Sines, and extend the other to 30 on the Line of Numbers; the Compasses remaining thus opened, set one Foot on 90 Degrees, or the End of the Line of Sines, and cause the other to fall on the Line of Numbers, which will give 68 Miles and about a half, for the Length of the Hypothenuse sought.

USE II. *The Base of a right-angled Triangle being given* 25 *Miles, and the Perpendicular* 15, *to find the Angle opposite to the Perpendicular.*

As the Base 25 Miles is to the Perpendicular 15 Miles, So is Radius to the Tangent of the Angle sought; because if the Base is made Radius, the Perpendicular will be the Tangent of the Angle opposite to the Perpendicular. Extend your Compasses on the Line of Numbers, from 15, the Perpendicular given, to 25, the Base given, and the same Extent will reach the contrary way, on the Line of Tangents, from 45 Degrees, to 31 Degrees, the Angle sought.

USE III. *The Base of a right-angled Triangle being given, suppose* 20 *Miles, and the Angle opposite to the Perpendicular* 50 *Degrees, to find the Perpendicular.*

As Radius is to the Tangent of the given Angle 50 Degrees, So is the Base 20 Miles to the Perpendicular sought. Extend your Compasses on the Line of Tangents, from the Tangent of 45 Degrees, to the Tangent of 50 Degrees, and the same Extent will reach on the Line of Numbers the contrary way, from the given Base 20 Miles, to the required Perpendicular, about 23 $\frac{3}{4}$ Miles.

Note, The Reason why the Extent on the Line of Numbers was taken from 20 to 23 $\frac{3}{4}$ forwards, is, because the Tangent of 50 Degrees (as I have already mentioned in the Construction of the Line of Tangents) should be as far beyond the Tangent of 45 Degrees, as it's Complement 40 Degrees wants of 45 Degrees.

USE IV. *The Base of a right-angled Triangle being given, suppose* 35 *Miles, and the Perpendicular* 48 *Miles; to find the Angle opposite to the Perpendicular.*

As the Base 35 Miles is to the Perpendicular 48 Miles, So is Radius to the Tangent of the Angle sought. Extend your Compasses from 35, on the Line of Numbers, to 48; the same Extent will reach the contrary way on the Line of Tangents, from the Tangent of 45 Degrees, to the Tangent of 36 Degrees 5 Minutes, or 53 Degrees 55 Minutes; and to know which of those Angles the Angle sought is equal to, consider that the Perpendicular of the Triangle is greater than the Base; therefore (because both the Angles opposite to the Perpendicular and Base together make 90 Degrees) the Angle opposite to the Perpendicular will be greater than the Angle opposite to the Base, and consequently the Angle 53 Degrees 55 Minutes, will be the Angle sought.

USE V. *The Hypothenuse of a right-angled Spherical Triangle being given, suppose* 60 *Degrees, and one of the Sides* 20 *Degrees; to find the Angle opposite to that Side.*

As the Sine of the Hypothenuse 60 Degrees is to Radius, So is the Sine of the given Side 20 Degrees, to the Sine of the Angle sought. Extend your Compasses, on the Line of Sines, from 60 Degrees to Radius, or 90 Degrees, and the same Extent will reach on the Line of Sines the same way, from 20 Degrees, the given Side, to 23 Degrees 10 Minutes, the Quantity of the Angle sought.

USE VI. *The Course and Distance of a Ship given; to find the Difference of Latitude and Departure.*

Suppose a Ship sails from the Latitude of 50 Deg. 10 Min. North, S. S. W. 48.5 Miles: As Radius is to the Distance sailed 48.5 Miles, So is the Sine of the Course, which is two Points, or the second Rhumb, from the Meridian, to the Departure. Extend your Compasses from 8, on the artificial Sine Rhumb-Line, to 48.5 on the Line of Numbers; the same Extent will reach the same way from the second Rhumb, on the Line of artificial Sines of the Rhumbs, to the Departure Westing 18.6 Miles. Again, as Radius is to the Distance sailed 48.5 Miles, So is the Co-Sine of the Course 67 Deg. 30 Min. to the Difference of Latitude. Extend your Compasses from Radius, on the Line of Sines, to 48.5 Miles on the Line of Numbers; the same Extent will reach the same way, from 67 Deg. 30 Min. on the Line of Sines, to 44.8 on the Line of Numbers; which converted into Degrees, by allowing 60 Miles to a Degree, and substracted from the given North-Latitude 50 Deg. 10 Min. leaves the Remainder 49 Deg. 25 Min. the present Latitude.

USE VII. *The Difference of Latitude and Departure from the Meridian being given; to find the Course and Distance.*

A Ship, from the Latitude of 59 Deg. North, sails North-Eastward 'till she has altered her Latitude 1 Deg. 10 Min. or 70 Miles, and is departed from the Meridian 57.5 Miles; to find the Course and Distance.

As the Difference of Latitude 70 Miles is to Radius, So is the Departure 57.5 Miles to the Tangent of the Course 39 Deg. 20 Min. or three Points and a half from the Meridian. Extend your Compasses from the fourth Rhumb, on the Line of artificial Tangents of the Rhumbs, to 70 Miles on the Line of Numbers: the same Extent will reach from 57.5 on the Line of Numbers, to the third Rhumb and a half on the Line of artificial Tangents of the Rhumbs. Again; As the Sine of the Course 39 Deg. 20 Min. is to the Departure 57.5 Miles, So is Radius to the Distance 90.6 Miles. Extend your Compasses from the third Rhumb and a half, on the artificial Sines of the Rhumbs, to 57.5 Miles on the Line of Numbers, and that Extent will reach from the Sine of the eighth Rhumb, on the Sines of the Rhumbs, to 90.6 Miles on the Line of Numbers.

USE *of the Line of Versed Sines.*

The three Sides of an oblique Spherical Triangle being given, to find the Angle opposite to the greatest Side.

Suppose the Side A B be 40 Degrees, the Side B C 60 Degrees, and the Side A C 96 Degrees, to find the Angle A B C. First add the three Sides together, and from half the Sum substract the greater Side A C, and note the Remainder; the Sum will be 196 Degrees, half of which is 98 Degrees; from which substracting 96 Degrees, the Remainder will be two Degrees. Fig. 13.

This done, extend your Compasses from the Sine of 90 Degrees, to the Sine of the Side A B 40 Degrees; and applying this Extent to the Sine of the other Side B C 60 Degrees, you will find it to reach to a fourth Sine about 34 Degrees. Again; from this fourth Sine extend your Compasses to the Sine of half the Sum, that is, to the Sine of 72 Degrees, the Complement of 98 Degrees to 180, and this second Extent will reach from the Sine of the Difference 2 Degrees, to the Sine of 3 Deg. 24 Min. against which, on the Versed Sines, stands 151 Deg. 50 Min. which is the Quantity of the Angle sought.

That the Reason of this Operation may appear, it is demonstrated in most Books of Trigonometry, that As Radius is to the Sine of A B, So is the Sine of B C to a fourth Sine; then As this fourth Sine is to Radius, So is the Difference of the versed Sines of A C and A B + B C to the versed Sine of the Complement of the Angle A B C to 180 Degrees. It is also demonstrated, that As Radius is to the Sine of half the Sum of any two Arcs, So is the Sine of half their Difference to half the Difference of the versed Sines of these two Arcs whence, if the Sine of A B be called a, the Sine of B C, b, and the Sine of A C, c, the fourth Sine in the first Analogy will be had; in saying, as $r : a :: b : \frac{ab}{r}$. Now to get the Difference of versed Sines of A C, and A B + B C, let us call the Sine of $\underline{AB + BC + AC}\, p$,

and

and the Sine of $\frac{AB+BC-AC}{2}$ q, then as $r:p::q:\frac{pq}{r}$, which laſt Term will be half the Difference of the verſed Sines of A C, and A B + B C: therefore, if we again ſay, as $\frac{ab}{r}:r::\frac{2pq}{r}:\frac{2rpq}{ab}$ this laſt Term will be the verſed Sine of the Complement of the Angle A B C: To find which at two Operations, you muſt ſay, As $r:a::b:\frac{ab}{r}$; then as $\frac{ab}{r}:p::q:\frac{rpq}{ab}$; which laſt Term, multiplied by 2, will be the verſed Sine Complement ſought. But, to avoid multiplying by 2, the verſed Sines on Scales are fitted from this Proportion, *viz.* As Radius is to half the Sine of an Arc, So is half the Sine of the ſame Arc, to half the verſed Sine of that Arc.

USE *of the Line of Latitudes and Hours.*

Theſe Lines are conjointly uſed, in readily pricking down the Hour-Lines from the Subſtyle, in an Iſoſceles Triangle, on any kind of upright Dials, having Centers in any given Latitude; that is, by means of them there will be this Proportion worked, *viz.* As Radius is to the Sine of the Style's Height, So is the Tangent of the Angle at the Pole, to the Tangent of the Hour-Lines Diſtance from the Subſtyle.

Now ſuppoſe the Hour-Lines are to be pricked down upon an upright Declining-Plane, declining 25 Deg. Eaſtwards: Firſt draw C 12 the Meridian, perpendicular to the Horizontal Line of the Plane, and make the Angle F C 12 equal to the Subſtyle's Diſtance from the Meridian, and draw the Line F C for the Subſtyle. This being done, draw the Line B A perpendicular to the ſaid Subſtyle, paſſing thro' the Center C; then out of your Line of Latitudes ſet off C A, C B, each equal to the Style's Height, and fit in the Hour-Scale, ſo that one End being at A, the other may meet with the Subſtyle Line at F. (Fig. 14.)

Now get the Difference between 30 Deg. 47 Min. the Inclination of Meridians, and 30 Degrees, the next Hour's Diſtance leſſer than the ſaid 30 Deg. 47 Min. and the Difference is 47 Minutes, that is, 3 Minutes in time; then count upon the Line of Hours,

Hours.	Min.			
0	3	from F to	10	And make Points at the Terminations, to which drawing Lines from the Center C, they ſhall be the Hour-Lines on one Side.
1	3		11	
2	3		12	
3	3		1	
4	3		2	
5	3		3	

Again, fitting in the Hour-Scale from B to F, count from that End at B, the former Arcs of Time.

Hours.	Min.			
0	3	from B to	4	And make Points at the Terminations, thro' which draw Lines from the Center C, and they will be the Hour-Lines on the other Side the Subſtyle.
1	3		5	
2	3		6	
3	3		7	
4	3		8	
5	3		9	

You muſt proceed thus for the Halfs and Quarters, in getting the Difference between the Half-Hour next leſſer (in this Example 22 Deg. 30 Min.) under the Arc of Inclination of Meridians; the Difference is 1 Deg. 17 Min. which in time is 33 Minutes, to be continually augmented an Hour at a Time, and ſo be pricked off, as before was done for the whole Hours.

If the Hour-Scale reach above the Plane, as at B, ſo that B C cannot be pricked down; then may an Angle be made on the upper Side of the Subſtyle, equal to the Angle F C A on the under Side, and thereby the Hour-Scale laid in it's due Poſition, having firſt found the Point F on the Subſtyle.

That the Reaſon of the conjoint Uſe of theſe Lines, in pricking off the Hour-Lines from the Subſtylar-Line may appear; let us ſuppoſe A C to be the Subſtylar-Line, A the Center of a Dial, B A a Portion of the Line of Latitudes, at right Angles to A C, and B C the Line of Hours fitted thereto. Now if C D be the Quantity of any Arc taken on the Line of Hours, and a right Line be drawn from the Center A through the Point D, the Angle F A C will be the ſame, as that found by ſaying, As Radius is to the Sine of the Number of Degrees pricked off upon the Line of Latitudes (that is, to the Sine of the Style's Height), from A to B; So is the Tangent of that Number of Degrees pricked off from C to D on the Line of Hours (that is, the Tangent of the Angle at the Pole), to another Tangent, whoſe (Fig. 15.)

whose Arc will be equal to F A C (that is, to the Tangent of the Diſtance of the Hour-Line A F from the Subſtyle).

Now to prove this, it is evident, from the Conſtruction of the Line of Latitudes, that as the Radius B C is to the Sine B G of an Arc; So is A C to A B: whence if A C be ſuppoſed Radius, B A is the Sine of the Arc pricked down from the Line of Latitudes.

Again, from the Nature of the Line of Hours; if C D be taken for the Tangent of an Arc, B D will be the Radius thereto. This being evident, let C E be the Tangent of the Angle F A C, then the Triangles B A D, D E C, will be ſimilar; whence as the Radius B D is to B A, the Sine of an Arc; ſo is C D, the Tangent of an Arc, to E C, the Tangent of the Angle F A C.

BOOK II.

Of the Construction and Uses of the SECTOR.

CHAP. I.

Of the Construction of the Sector.

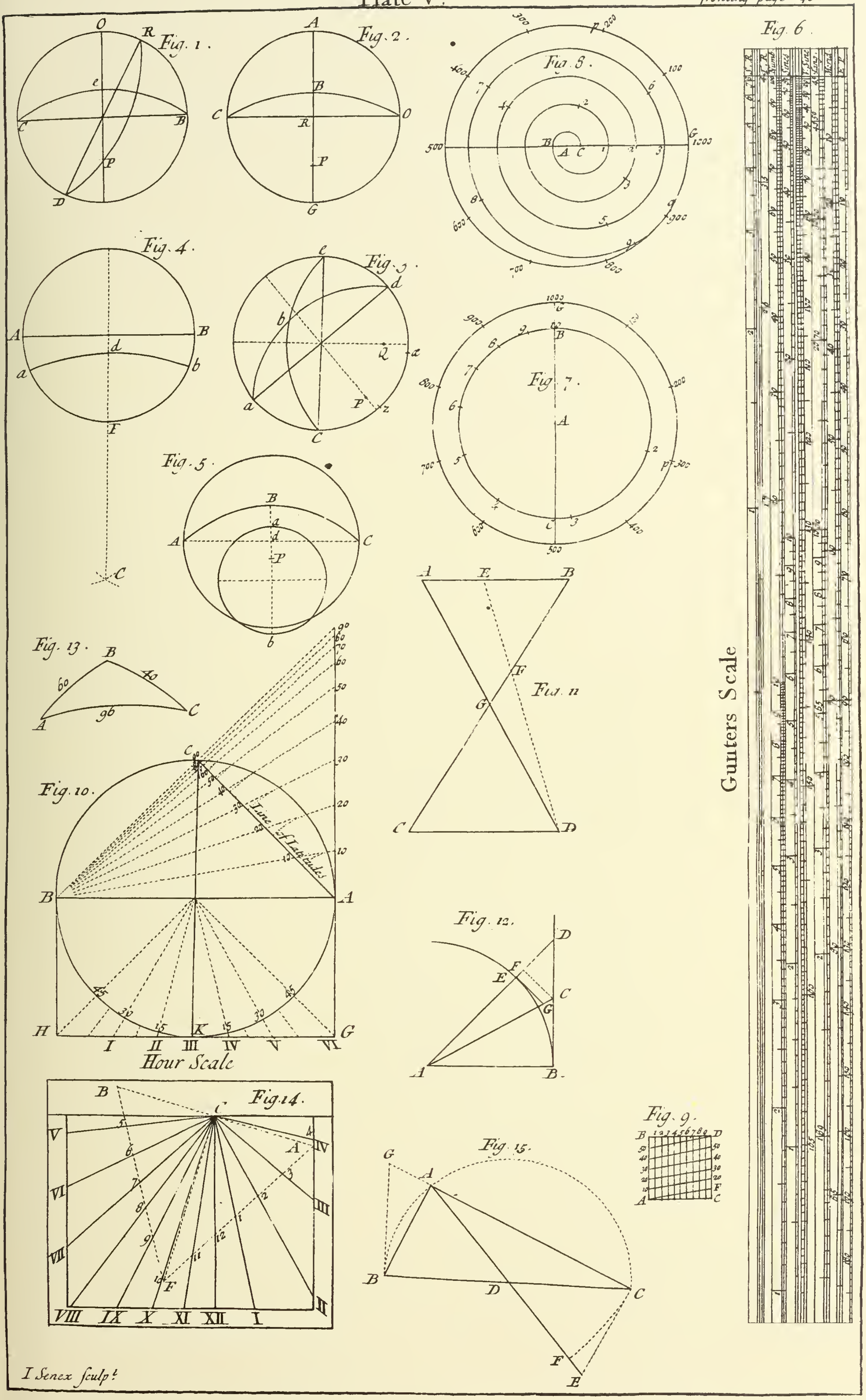
THE Sector is a Mathematical Instrument, whose Use is to find the Proportion between Quantities of the same kind; as between one Line and another, between one Superficies and another, between one Solid and another, *&c.*

This Instrument is made of two equal Rules, or Legs, of Silver, Brass, Ivory, or Wood, joined to each other by a Rivet, so worked, as to render it's Motion regular and uniform. To do which, first make two Slits with a Saw, about an Inch deep, at one End of one of the Rules, in order to fit therein the Head-Pieces, which must be well rivetted. Afterwards the Head must be rounded, by filing off the Superfluities, in such manner, that the Middle-Piece and Head-Pieces may be even with each other. Then to find the Center of the Rivet, set one Foot of your Compasses at the Bottom of the Middle-Piece, and mark with the other Foot four Sections in the middle of the Rivet, by opening the Middle-Piece of the Joint to four or more different Angles, and the Middle-Point of those Sections will be the Center of the Rivet, and, consequently, also the Center of the Sector. This being done, a Line must be drawn upon the Rule from the Center, near the inward Edge, by which Line the inward Edge of the Rule must be filed strait; the inward Edge of the other Rule being also made strait, and slit, to receive the Middle-Piece, you must cut away it's Corner in an Arc, so as it may well fit the Joint, and then rivet, with three or four little Rivets, the Rule to the Middle-Piece; by which means the two Legs may easily open and shut, and keep at any Opening required. But Care must be taken that the Legs are filed very flat, and do not twist; Care must also be taken that the Sector be well centered, that is, that being entirely opened, both Inside and Outside, may make a right Line, and that the Legs be very equal in Length and Breadth; in a word, that it be very strait every way. *Note*, The Length and Breadth of the aforesaid Rules are not determinate, but they are commonly six Inches long, three quarters of an Inch broad, and about one quarter in Thickness.

There are commonly drawn upon the Faces of this Instrument six kind of Lines; *viz.* the Line of equal Parts, the Line of Planes, and the Line of Polygons on one Side; the Line of Chords, the Line of Solids, and the Line of Metals on the other.

There is generally placed, near the Edge of the Sector, on one Side, a divided Line, whose Use is to find the Bores of Cannons; and on the other Side, a Line shewing the Diameters and Weights of Iron-Bullets, from one Quarter to 64 Pounds, whose Construction and Uses we shall give, in speaking of the Instruments belonging to the Artillery.

Plate V. fronting page 46.



SECTION I.

Of the Line of Equal Parts.

THIS Line is ſo called, becauſe it is divided into equal Parts, whoſe Number is commonly 200, when the Sector is ſix Inches long. *Plate 6.* Fig. 1.

Having drawn upon one of the Faces of each Leg the equal Lines A B, A B, from the Center of the Joint A: Firſt, divide them into two equal Parts, each of which will conſequently be 100; then each of thoſe Parts being again divided into two equal Parts, and each Part ariſing will be 50; then divide each of theſe laſt Parts into five others, and each Part produced will be 10; and finally dividing each of theſe new Parts into 2, and each of theſe laſt into five equal Parts, and by this means the Lines A B, A B, will be each divided into 200 equal Parts, every 5 of which muſt be diſtinguiſhed by ſhort Strokes, and every 10 numbered from the Center A to 200, at the other End.

Now becauſe the two other Lines, drawn upon the ſame Faces of each Leg, muſt terminate in the Center A, the Extremity B of the Line of equal Parts muſt be drawn as near as poſſible the outward Edges of each Leg, that ſo there may be Space enough left to draw the Line of Planes in the middle of the Breadth of the ſaid Legs, and the Line of Polygons near their inward Edges; but Care muſt be taken, in drawing of theſe Lines, that each one, and it's Fellow, be equally diſtant from the interior Edges of each Leg, as may be ſeen in the Figure.

SECTION II.

Of the Line of Planes.

THIS Line is ſo called, becauſe it contains the homologous Sides of a certain Number of ſimilar Planes, Multiples of a ſmall one, beginning from the Center A; that is, whoſe Surfaces are double, triple, quadruple, *&c.* that ſmall Plane, from Unity, according to the natural Order of Numbers, to 64, which is commonly the greateſt Term of the Diviſions, denoted upon the Line A C.

This Line may be divided two ways, both of which are founded upon *Prop.* 20. *lib. 6. Eucl* which demonſtrates, That ſimilar Plane Figures are to each other, as the Squares of their homologous Sides. The firſt way of dividing this Line is by Numbers, and the ſecond without Numbers, as follows:

Having drawn the Line A C, from the Center A, upon each Leg of the Sector; firſt divide it into eight equal Parts, the firſt of which, next to the Center A, which repreſents the Side of the leaſt Plane, hath no need of being drawn. The ſecond Diviſion from the Center, which is double the firſt, is the Side of a ſimilar Plane quadruple the leaſt Plane, (whoſe Side is ſuppoſed one of the eight Parts the Line A C is divided into), becauſe the Square of 2 is 4. The third Diviſion, which is three times the firſt, is the Side of a ſimilar Plane, nine times greater than the firſt, becauſe the Square of 3 is 9. The fourth Diviſion, which is four times the firſt, and conſequently half of the whole Scale, is the Side of a ſimilar Plane, ſixteen times greater than the firſt, becauſe the Square of 4 is 16. Laſtly, The eighth Diviſion which is eight times the firſt, is the Side of a ſimilar Plane, ſixty-four times greater, becauſe the Square of 8 is 64.

There is ſomething more to do to find the homologous Sides of Planes that are double, triple, quadruple, *&c.* of the firſt. For you muſt have a Scale divided into 1000 equal Parts (as that whoſe Conſtruction we have already given in Book I.), whoſe Length muſt be equal to the Line A C; and becauſe the Side of the leaſt Plane is $\frac{1}{8}$ of the Line A C, it will conſequently be $\frac{1}{8}$ of 1000, which is 125. Again, to have in Numbers the Side of a Plane double the leaſt, the ſquare Root of a Number twice the Square of 125 muſt be found. This Square is 15625, which doubled, is 31250, the ſquare Root of which is about 177, the Side of a ſimilar Plane double the leaſt, whoſe Side is ſuppoſed to be 125. Moreover, to have the Side of a Plane three times the firſt, the ſquare Root of a Number three times the Square of the firſt muſt be found. The Number is 46875, and it's Root, which is about 216, is the Side of a ſimilar Plane three times the leaſt, and ſo of others; therefore by laying off from the Center A, upon the Line of Planes, 177 Parts of the aforeſaid Scale, you will have the Length of the Side of a ſimilar Plane double the leaſt Plane. Again, laying off 216 Parts of the ſame Scale from the Center A, the Length of the Side of a ſimilar Plane will be had, which is three times the leaſt Plane. Fig. 2.

According to the aforeſaid Directions, the following Table is calculated, that ſhews the Number of equal Parts which are contained in the homologous Sides of all the ſimilar Planes that are double, triple, quadruple, *&c.* of a Plane whoſe Side is 125, to the Plane 64, that is, which contains it 64 times, and whoſe Side is 1000.

A TABLE

A TABLE *for dividing the Line of Planes.*

1	125	17	515	33	718	49	875
2	177	18	530	34	729	50	884
3	216	19	545	35	739	51	892
4	250	20	559	36	750	52	901
5	279	21	573	37	760	53	910
6	306	22	586	38	770	54	918
7	330	23	599	39	780	55	927
8	353	24	612	40	790	56	935
9	375	25	625	41	800	57	944
10	395	26	637	42	810	58	952
11	414	27	650	43	819	59	960
12	433	28	661	44	829	60	968
13	450	29	673	45	839	61	976
14	467	30	684	46	848	62	984
15	484	31	696	47	857	63	992
16	500	32	707	48	866	64	1000

Fig. 2. Each of the ten Diviſions which the Scale of 1000 Parts contains, is 100; and each of the Subdiviſions of the Line A B is 10: therefore if it is to be uſed for dividing any of the Lines of the Sector; as, for Example, the Line of Planes; take on the Scale a Line denoting the Hundreds, and the Exceſs above muſt be taken in the Space between the Points A B: As to denote the firſt Plane, to which the Number 125 anſwers, place your Compaſſes on the fifth Line of the Space marked 100, and open them to the Diſtance O P; in the ſame manner, if the Plane 50 is to be denoted, to which the Number 884 anſwers, for 800 take the 8th Space of the Scale, and for 84 take in the Space A B, the Interſection of the 8th Tranſverſal, with the fourth Parallel, which will be the Diſtance N L.

Fig. 5. The Line of Planes may otherwiſe be divided in the following manner, without Calculation, founded on *Prop.* 47. *lib.* 1. *Eucl.* Make the right-angled Iſoſceles Triangle K M N, whoſe Side K M, or K N, let be equal to the Side of the leaſt Plane, and then the Hypothenuſe M N will be the Side of a ſimilar Plane double to it; therefore having laid off with your Compaſſes the Diſtance M N, on the Side K L produced, from K to 2, the Length K 2 will be the Side of a Plane double the leaſt Plane. In like manner lay off the Diſtance M 2, from K to 3, the Line K 3 will be the Side of a Plane triple the firſt. Again, lay off the Diſtance M 3, from K to 4, the Line K 4 (twice K M) will be the Side of a Plane four times greater, that is, which will contain the leaſt Plane four times; and ſo of others, as may be ſeen in the Figure.

SECTION III.

Of the Line of Polygons.

This Line is ſo called, becauſe it contains the homologous Sides of the firſt twelve regular Polygons inſcribed in the ſame Circle, that is, from an equilateral Triangle to a Dodecagon.

The Side of the Triangle being the greateſt of all, muſt be the whole Length of each of the Legs of the Sector; and becauſe the Sides of the other regular Polygons, inſcribed in the ſame Circle, ſtill diminiſh as the Number of Sides increaſe, the Side of the Dodecagon is leaſt, and conſequently muſt be nigheſt the Center of the Sector.

Now ſuppoſing the Side of a Triangle to be a thouſand Parts, the Length of the Sides of every of the other Polygons muſt be found; and becauſe the Sides of regular Polygons, inſcribed in the ſame Circle, are in the ſame Proportion as the Chords of the Angles of the Center of each of the Polygons, it is neceſſary to ſhew here how to find the ſaid Angles.

To do which, divide 360 Deg. by the Number of the Sides of any Polygon, and the Quotient will give the Angle of the Center.

If, for Example, the Angle of the Center of a Hexagon is required, divide 360 Deg. by 6, and the Quotient will be 60; which ſhews that the Angle of the Center of a Hexagon is 60 Deg. If likewiſe the Angle of the Center of a Pentagon be required, divide 360 Deg. by 5, the Number of Sides, and the Quotient will be 72; which ſhews that the Angle of the Center of a Pentagon is 72 Deg. and ſo of others.

The Angle of the Center being known, if it be ſubſtracted from 180 Degrees, the Remainder will be the Angle of the Polygon: As, for Example, the Angle of the Center of a Pentagon

Pentagon being 72 Degrees, the Angle of the Circumference will be 108 Degrees, and so of others, as may be seen in the following Table.

Regular Polygons.	*Angles of the Center.*		*Angles at the Circumference.*	
	Degrees.		Degrees.	
Triangle.	120.		60.	
Square.	90.		90.	
Pentagon.	72.		108.	
Hexagon.	60.	Min.	120.	Min.
Heptagon.	51.	26.	128.	34.
Octogon.	45.		135.	
Nonagon.	40.		140.	
Decagon.	36.		144.	
Undecagon.	32.	44.	147.	16.
Dodecagon.	30.		150.	

Now to find in Numbers the Sides of the regular Polygons inscribed in the same Circle: Having supposed that the Side of the equilateral Triangle is 1000 equal Parts, instead of the Chords of the Angles of the Center, take their Halves, which are the Sines of half the Angles at their Centers, and make the following Analogy.

For Example, to find the Side of the Square, say, As the Sine of 60 Degrees, half the Angle of the Center of the equilateral Triangle, is to the Side of the same Triangle, supposed 1000; So is the Sine of 45 Degrees half the Angle of the Center of the Square, to the Side of the same Square, which, by calculating, will be found 816.

And in this manner are the following Tables of Polygons constructed.

The Side of an equilateral Triangle, denoted on the Sector by the Number		*Equal Parts.*
The Side of an equilateral Triangle, denoted on the Sector by the Number	3.	1000.
Of a Square by the Number	4.	816.
Of the Pentagon by the Numb.	5.	678.
Of the Hexagon by the Numb.	6.	577.
Of the Heptagon by the Numb.	7.	501.
Of the Octagon by the Numb.	8.	442.
Of the Nonagon by the Numb.	9.	395.
Of the Decagon by the Numb.	10.	357.
Of the Undecagon by the Numb.	11.	325.
Of the Dodecagon by the Numb.	12.	299.

We have neglected the Fractions remaining after the Calculation in this Table, as in all others; as being but thousandth Parts, which are not considerable.

Those that will not denote an equilateral Triangle upon the Sector, because of the Facility of describing it, and which consequently begin at the Square, use the following Table, wherein the Side of the Square is supposed 1000 Parts.

Another Table of Polygons.	*Parts.*
Square.	1000.
Pentagon.	831.
Hexagon.	707.
Heptagon.	613.
Octagon.	540.
Nonagon.	484.
Decagon.	437.
Undecagon.	398.
Dodecagon.	366.

To make the Line of Polygons upon the Sector (the same Scale of 1000 equal Parts being used, as that for making the Line of Planes), you must lay off from the Center A, upon both the Lines A D, the Number of Parts expressed in the Table, that thereby the Numbers 3, 4, 5, *&c.* may be graved upon the Sector, signifying the Numbers of the Sides of the regular Polygons.

SECTION IV.

Of the Line of Chords.

THIS Line is so named, because it contains the Chords of all the Degrees of a Semicircle, whose Diameter is the Length of that Line, which is denoted upon the other Surface of each Leg of the Sector, from the Point A, which is the Center of the Joint, to the End F of each Leg; so that the two Lines A F are exactly equal, and equidistant from the interior Edges of the Sector. Fig. 4.

Note, The Line of Chords must be drawn directly under the Line of equal Parts, because of some Operations that require a Correspondence between those two Lines.

It is also proper for the Line of Solids to be drawn under the Line of Planes, and the Line of Metals under the Line of Polygons.

Fig. 3. For the Division of the aforesaid Line A F, describe a Semicircle, whose Diameter let be equal to it, which divide into 180 Degrees; afterwards lay off the Lengths of the Chords of all those Degrees upon the Diameter of the Semicircle; then lay the Diameter of the Semicircle upon the Legs of the Sector, and mark upon them Points that represent the Degrees of the Semicircle, every fifth of which, distinguish by short Strokes, and every tenth by Numbers, beginning from the Point A, and going on to F.

The same Degrees may otherwise be denoted, upon the Line of Chords, by help of Numbers, in supposing the Semidiameter of a Circle, or the Chord of 180 Degrees, to be 1000 equal Parts; all of which Numbers may be found ready calculated in the common Tables of Sines: for instead of the Chords, there is no more to do but to take their halves, which are the Sines of half their Arcs. As for Example; instead of the Chord of 10 Degrees, the Sine of 5 must be taken; and because the Calculation in Tables is made for a Radius of 100000 Parts, the two last Numbers must be taken away, as may be seen in the following Table, where the Chords of all the Degrees to 180 are denoted.

Note, This Division is made with a Scale of 1000 Parts.

A TABLE *for the Line of Chords.*

D.	Ch.	D.	Ch.	D.	Ch.	D.	Ch.	D.	Ch.	D.	Ch.
1	8	31	267	61	507	91	713	121	870	151	968
2	17	32	275	62	515	92	719	122	874	152	970
3	26	33	284	63	522	93	725	123	879	153	972
4	35	34	292	64	530	94	731	124	883	154	974
5	43	35	300	65	537	95	737	125	887	155	976
6	52	36	309	66	544	96	743	126	891	156	978
7	61	37	317	67	552	97	749	127	895	157	980
8	70	38	325	68	559	98	754	128	899	158	981
9	78	39	334	69	566	99	760	129	902	159	983
10	87	40	342	70	573	100	766	130	906	160	985
11	96	41	350	71	580	101	771	131	910	161	986
12	104	42	358	72	588	102	777	132	913	162	987
13	113	43	366	73	595	103	782	133	917	163	989
14	122	44	374	74	602	104	788	134	920	164	990
15	130	45	382	75	609	105	793	135	924	165	991
16	139	46	390	76	615	106	798	136	927	166	992
17	145	47	399	77	622	107	804	137	930	167	993
18	156	48	406	78	629	108	809	138	933	168	994
19	165	49	414	79	636	109	814	139	936	169	995
20	173	50	422	80	643	110	819	140	939	170	996
21	182	51	430	81	649	111	824	141	941	171	997
22	191	52	438	82	656	112	829	142	945	172	997
23	199	53	446	83	662	113	834	143	948	173	998
24	208	54	454	84	669	114	838	144	951	174	998
25	216	55	462	85	675	115	843	145	954	175	999
26	225	56	469	86	682	116	848	146	956	176	999
27	233	57	477	87	688	117	852	147	959	177	999
28	242	58	485	88	694	118	857	148	961	178	1000
29	250	59	492	89	701	119	861	149	963	179	1000
30	259	60	500	90	707	120	866	150	966	180	1000

SECTION V.

Of the Line of Solids.

THIS Line is fo called, becaufe it contains the homologous Sides of a certain Number of fimilar Solids, Multiples of a leffer from Unity, according to the natural Order of Numbers, to 64, which is commonly the greateft of the Divifions of this Line, which is marked A H, next to the Line of Chords. Fig. 4.

To make the Divifions upon it, the Scale of 1000 Parts muft be ufed, and the Side of the 64th and greateft Solid muft be fuppofed 1000 equal Parts; then becaufe the Cube-Root of 64 is 4, and the Cube Root of 1 is 1, it follows that the Side of the 64th Solid is quadruple the Side of the firft and leaft Solid, which confequently will be 250, becaufe *(per Prop.* 33. *lib.* 11. *Eucl.)* fimilar Solids are to each other, as the Cubes of their homologous Sides.

The Number 500 (twice 250) is the Side of the eighth Solid, that is, of a Solid eight times as great as the firft: becaufe the Cube of 2, which is 8, is eight times the Cube of Unity.

Likewife the Number 750, which is three times 250, is the Side of the 27th Solid; becaufe the Cube of 3, which is 27, is 27 times the Cube of Unity.

There are more Calculations required to find the Sides of Solids double, triple, quadruple, *&c.* the firft, which cannot exactly be expreffed in Numbers, becaufe their Roots are incommenfurable; neverthelefs they may be fufficiently approached for Ufe, by the following Method.

For Example; To find the Number expreffing the Side of a Solid, twice the firft and leaft: it's Side 250 muft be cubed, which is 15625000; then this Number muft be doubled, and the Cube-Root of it extracted, which will be almoft 315, for the Side of a Solid double the firft. To have the Side of a Solid triple the firft, the faid Cube muft be tripled, and it's Cube-Root, which is 360, will be the Side of a Solid triple the firft; and fo of others, as may be feen in the following Table.

A TABLE *for the Line of Solids.*

1	250	17	643	33	802	49	914
2	315	18	655	34	810	50	921
3	360	19	667	35	818	51	927
4	397	20	678	36	825	52	933
5	427	21	689	37	833	53	939
6	454	22	700	38	840	54	945
7	478	23	711	39	848	55	951
8	500	24	721	40	855	56	956
9	520	25	731	41	862	57	962
10	538	26	740	42	869	58	967
11	556	27	750	43	876	59	973
12	572	28	759	44	882	60	978
13	588	29	768	45	889	61	984
14	602	30	777	46	896	62	989
15	616	31	785	47	902	63	995
16	630	32	794	48	908	64	1000

The Sides of all thefe Solids being thus found in Numbers, they are denoted on the Line of Solids, by laying off from the Center A the Parts which they contain, taken upon the Scale.

SECTION VI.

Of the Line of Metals.

THIS Line is fo named, becaufe it is ufed to find the Proportion between the fix Metals, of which Solids may be made.

It is placed upon the Legs of the Sector, hard by the Line of Solids, and the Metals are figured thereon by the Characters, which have been appropriated to them by Chymifts and Naturalifts. Fig. 4.

The Divifion of this Line is founded upon Experiments that have been made of the different Weights of equal Maffes of each of thefe Metals, from whence their Proportions are calculated, as in the following Table.

A TABLE

A TABLE *for the Line of Metals.*

Gold	☉	730.
Lead	♄	863.
Silver	☽	895.
Brass	♀	937.
Iron	♂	974.
Tin	♃	1000.

Advertisment.

That of all the six Metals which has the least Weight, which is Tin, is marked at the End of each Leg (as A G) at a Distance from the Center, equal to the Length of the Scale of 1000 Parts; and the other Metals nigher the said Center (each according to the Numbers which correspond with them), taken upon the same Scale.

Because most of the aforementioned Lines, marked on the Sector, are divided by means of the Scale of 1000 equal Parts, it is requisite that they be exactly equal between themselves and to the said Scale; therefore, because they all center in one Point (which is the Center of the Joint), they must all be terminated at the other End by an Arc, made upon the Surface of each of the Legs.

It is not always necessary to divide the Sector by the Methods we have given; for, to make them sooner, prepare a Ruler of the same Length, Breadth, and Thickness as the Sector, and draw upon it the same Lines we have already prescribed: then with a Beam-Compass transfer the same Divisions upon the Sector, having first drawn upon it the Lines to contain them.

SECTION VII.

Containing the Proofs of the Six Lines commonly put upon the Sector.

The Proof of the Line of equal Parts.

The Division of this Line is so easy, that there is no need of any other Proof, but to examine, with your Compasses, whether the two correspondent Lines, drawn upon the Legs of the Sector, are very equal, and equally divided; which may be known by taking between your Compasses (whose Points let be very sharp) any Number at pleasure of those equal Parts, beginning any where: for if the Line of equal Parts be well divided, by carrying that same Opening of your Compasses on the said Line, the two Points will always contain between them the same Number of equal Parts upon either of the Legs, reckoning from the Center, or from any other Point of Division.

The Proof of the Line of Chords.

The Method before explained will not serve to know whether the Line of Chords be well divided, because the Divisions are not equal: the Chord of 10 Degrees, for Example, is greater than half that of 20; likewise the Chord of 20 Degrees is greater than the half of that of 40 Degrees, and so on: so that the Divisions are greater towards the Center of the Sector, than towards the Ends of it's Legs, as is manifest from the Nature of the Circle. But because we have given two Methods for dividing the Line of Chords, one by help of Numbers, and the other by means of the Chords of Arcs, one of these Methods will serve to prove the other.

But there is still another Method, which is this: Take at pleasure, on the Line of Chords, two Numbers equally distant from 120 Degrees; as for Example, 110 and 130, which are each 10 Degrees distant from it; the first in Defect, and the last in Excess: Then take in your Compasses the Distance of the two Numbers 110 and 130, which must be equal to the Chord of 10 Degrees, or to the Distance of the Point 10, upon the Line of Chords, from the Center of the Sector.

You will find, by the same Means, that the Distance between 100 and 140 Degrees, is equal to the Chord of 20 Degrees; as likewise the Distance between 90 and 150 is equal to the Chord of 30 Degrees, which is the Number by which 120 exceeds 90, and by which 150 exceeds 120, and so of others, as may easily be noted by the aforegoing Table of Chords, where you may see (for Example) the Number 44, which is the Chord of 5 Degrees, is the Difference between 843, which is the Chord of 115 Degrees; and 887, which is the Chord of 125; as likewise 87, the Chord of 10 Degrees, is the Difference between the Chord of 110 Degrees and 130, *&c.* which are equally distant from 120 Degrees.

Proof of the Line of Polygons.

You may know whether this Line be well divided, by help of the Line of Chords, in the following manner.

Take in your Compasses, upon the Line of Polygons, the Distance of the Number 6, denoting a Hexagon, from the Center of the Joint; then carry this Distance upon the Line of Chords, putting each Point of your Compasses upon the correspondent Points, from 60 to 60, denoting the Angle of the Center of an Hexagon.

The

The Sector being thus opened, take upon each Line of Chords the Diſtance of the two Points, marked 72 from the Center, and lay it off upon the Line of Polygons, placing one Foot in the Center of the Joint; then the other Foot muſt reach to the Point 5, which appertains to a Pentagon, whoſe Angle at the Center is 72 Degrees.

Likewiſe in taking upon the Line of Chords the Diſtance of the two Points, denoting 90, and laying it off upon the Line of Polygons, the Foot of your Compaſſes muſt meet the Point 4, appertaining to a Square, whoſe Angle of the Center is 90 Deg. and ſo of others.

Proof of the Line of Planes.

Becauſe we have given two Methods for dividing the Line of Planes, one may ſerve to prove the other; but ſtill you may eaſier know whether the Diviſions be well made, in the following manner: Take between your Compaſſes the Diſtance of any Point upon this Line from the Center of the Joint, and lay it off from the ſame Point on the other Side of the ſame Line of Planes; then the Foot of your Compaſſes will fall upon the Number of a Plane four times greater than that which was taken towards the Center: and if again your Compaſſes thus opened ſhould be once more turned over, towards the End of the ſaid Line, the Point would fall upon the Number of a Plane nine times greater. As, for Example; if you take the Diſtance from the Center to the Plane 2, in placing one Point of your Compaſſes on 2, the other ought to fall upon 8; and by turning the Compaſſes once more, one of it's Points muſt fall upon 18, which contains 9 times 2. Moreover, in turning the Compaſſes once more over, the other Point ought to fall upon the Number 32, containing 2, 16 times. If, laſtly, you turn over the Compaſſes again, it muſt fall upon 50, and ſo of other ſimilar Planes, becauſe they are to each other as the Squares of their homologous Sides. It is this that facilitates the Diviſion of the Line of Planes; for having the firſt, theſe are likewiſe had, *viz.* the 4th, the 9th, the 16th, the 20th, the 25th, the 36th, the 49th, and the 64th. Having found the 2d, the 8th, the 18th; the 32d, and the 50th will be had: likewiſe having found the 3d, the 12th, the 27th, and the 48th will be had; and ſo of others.

Proof of the Line of Solids.

You may know whether this Line be well divided, in the following manner: Take between your Compaſſes the Diſtance of ſome Point on this Line from the Center of the Joint; then place one of it's Points, thus opened, upon this Point of Diviſion, and turn the other Point over towards the End of the Line. Now this Point muſt fall upon the Number of a Solid 8 times greater than that which was taken. Again, if the Compaſſes be once more turned over, it will fall upon a Solid 27 times greater than that which was firſt taken. As, for Example; the Diſtance of the firſt Solid from the Center, will be equal to the Diſtance from 8 to 27, and from 27 to 64. Likewiſe, twice the Diſtance from the Center to 3, will be equal to the Diſtance from 3 to 24. By the 4th Solid, the 32d will be had. Moreover, the 5th Solid will give the 40th; by the 6th the 48th Solid will be had; and, in a word, by help of the 7th, the 56th Solid will be had; becauſe ſimilar Solids are to each other, as the Cubes of their homologous Sides, which facilitates the Diviſion of the Line of Solids.

Proof of the Line of Metals.

We have already mentioned, that the Diviſion of this Line is founded upon Experiments made of the different Weights of a Cubic Foot of each of the ſix Metals, as they are here denoted.

Metals.		*Weights of a Cubic Foot.*	
Gold.	- -	1326 Pounds,	4 Ounces.
Lead.	- -	802.	2.
Silver.	- -	720.	12.
Braſs.	- -	627.	12.
Iron.	- -	558.	00.
Tin.	- -	516.	2.

From theſe different Weights of the ſix Metals the beforementioned Table was calculated, by means of which the Line of Metals was divided.

Now becauſe Tin is the lighteſt of the ſaid ſix Metals, it is manifeſt that if, for Example, a Ball of Tin is to be made of the ſame Weight as a Ball of Iron, or Braſs, the Ball of Tin muſt be greater than either of them; as alſo the Ball of Iron ought to be greater than that of Braſs, and ſo on to that which will be the leaſt. Therefore ſuppoſing the Diameter of a Ball of Tin to be 1000, the Queſtion is to find the Lengths of the Diameters of Iron and Braſs-Balls, that may be of the ſame Weight as the Ball of Tin.

Now to do this, you muſt make a Rule of Three, whoſe firſt Term let always be the heavieſt of the two Metals to be compared; the ſecond Term muſt be the Weight of the Tin, and the third muſt be the Number 64, which is the greateſt Solid of the Table of Solids, to which the Number 1000 anſwers. As, for Example; to compare Iron, a Cubic Foot of which weighs 558 Pounds, with Tin, a Cubic Foot of which weighs 516 Pounds, 2

P Ounces:

Ounces: Having reduced them all into Ounces, the 558 Pounds make 8928 Ounces; and the 516 Pounds, 2 Ounces, make 8258 Ounces: then say, if 8928 gives 8258, What will 64 give? The Rule being finished, the fourth Term will be 59 and a small Remainder; then look for the Number 59 in the Table of Solids, and the Number answering thereto is 973; instead of which take 974, because of the remaining Fraction: therefore, I say, that the Diameter of the Ball of Iron must be 974. In the same manner, by making four other Rules of Proportion, you may know whether the Numbers, marked against the four other Metals, are well calculated, and consequently, whether the Line of Metals be well divided.

CHAP. II.

Of the Use of the Sector.

THE Uses we shall here lay down, are only those that most appertain to the Sector, and which by it can be better performed, than by any other Instrument.

SECTION I.

Of the USE *of the Line of equal Parts.*

USE I. *To divide a given Line into any Number of equal Parts; for Example, into seven.*

Plate 7. Fig. 1. TAKE between your Compasses the proposed Line, as A B, and carry it, upon the Line of equal Parts, to a Number on both Sides, that may easily be divided by 7, as 70, whose 7th Part is 10; or else the Number 140, whose 7th Part is 20. Then keeping the Sector thus opened, shut the Feet of your Compasses, so that they may fall on the Numbers 10 on each Leg of the Sector, if the Number 70 be used; or upon the Numbers 20, if 140 be taken for the Length of the proposed Line; and this opening of your Compasses will be the 7th Part of the proposed Line.

Note, If the Line to be divided be too long to be applied to the Legs of the Sector, only divide one half, or one fourth of it by 7, and the double, or quadruple, of this 7th Part, will be the 7th Part of the whole Line.

USE II. *Several right Lines, constituting the Perimeter of a Polygon, being given, one of which is supposed to contain any Number of equal Parts: to find how many of these Parts are contained in each of the other Lines.*

Take that Line's Length, whose Measure is known, between your Compasses, and set it over, upon the Line of equal Parts, to the Number on each Side, expressing it's Length. The Sector remaining thus opened, carry upon it the Lengths of each of the other Lines, parallel to the beforementioned Line, and the Numbers that each of them falls on will shew their different Lengths: But if any one of the said Lines doth not exactly fall upon the same Number of the Lines of equal Parts, upon both Legs of the Sector; but, for Instance, one of the Points of the Compasses falls upon 29, and the other upon 30; the Length of the said Line will be 29 and a half.

USE III. *A right Line being given, and the Number of equal Parts it contains; to take from it a lesser Line, containing any Number of it's Parts.*

Let, for Example, the proposed Line be 120 equal Parts, from which it is required to take a Line of 25. First take the proposed Line between your Compasses, and then open the Sector, so that the Feet of your Compasses may fall upon 120, on the Line of equal Parts, upon each Leg of the Sector: The Sector remaining thus opened, take the Distance from 25 to 25, and that will give the Line desired. It is manifest, from the three aforementioned Uses, that the Line of equal Parts, upon the Legs of the Sector, may very fitly serve as a Scale for all kinds of plane Figures, provided that one of their Sides be known; and that, by means of this Line, they may be augmented or diminished.

USE IV. *Two right Lines being given, to find a third Proportional: and three being given, to find a fourth.*

If there be but two Lines proposed, then take the Length of the first between your Compasses, and lay it off upon the Line of equal Parts from the Center, in order to know the Number whereon it terminates; then open the Sector, so that the Length of the second Line may be terminated by the Length of the first. The Sector remaining thus opened, lay off the Length of the second Line upon one of the Legs from the Center; and, *Note*, the Number whereon it terminates, and the Distance between that Number, on both Legs of the Sector, will give the third Proportional required.

Let,

Let, for Example, the firſt Line propoſed be A B, 40 equal Parts; and the ſecond C D, 20. Firſt take the Length of 20 between your Compaſſes, and opening the Sector, ſet over this Diſtance upon 40, and 40 on each Leg of the Sector. The Sector remaining thus opened, take the Diſtance from 20 to 20, which will be the Length of the third Proportional ſought; which being meaſured, on the Line of equal Parts, from the Center, you will find it 10; for As 40 is to 20, So is 20 to 10. Fig. 3.

But if three Lines be given, and a fourth Proportional to them be required; take the ſecond Line between your Compaſſes, and, opening the Sector, apply this Extent to the Ends of the firſt, laid off from the Center, on both Legs of the Sector. The Sector being thus opened, lay off the third Line from the Center, and the Extent between the Number, whereon it terminates on both Legs of the Sector, will be the fourth Proportional required.

Let the firſt of the three Lines be 60, the ſecond 30, and the third 50; carry the Length of 30 to the Extent from 60 to 60; and the Sector remaining thus opened, take the Diſtance from 50 to 50, which is 25, and this will be the fourth Proportional ſought: for 60 Is to 30 As 50 to 25.

USE V. *To divide a Line into any given Proportion.*

As for Example; to divide a Line into two Parts, which may be to each other as 40 is to 70: Firſt add the two Numbers together, and their Sum will be 110; then take between your Compaſſes the Length of the Line propoſed, which ſuppoſe 165, and carry this Length to the Diſtance, from 110 to 110, on both Legs of the Sector. The Sector remaining thus opened, take the Extent from 40 to 40, and alſo from 70 to 70; the firſt of the two will give 60, and the latter 105, which will be the Parts of the Line propoſed; for 40 Is to 70, As 60 is to 105.

USE VI. *To open the Sector, ſo that the two Lines of equal Parts may make a right Angle.*

Find three Numbers, that may expreſs the Sides of a right-angled Triangle, as 3, 4, or 5, or their Equimultiples; but ſince it is better to have greater Numbers, let us take 60, 80, and 100. Now having taken, between your Compaſſes, the Diſtance from the Center of the Sector to 100, open the Sector, ſo that one Point of your Compaſſes, ſet upon 80 on one Leg, may fall upon 60, of the Line of equal Parts, upon the other Leg; and then the Sector will be ſo opened, that the two Lines of equal Parts make a right Angle.

USE VII. *To find a right Line equal to the Circumference of a given Circle.*

The Diameter of a Circle is to the Circumference almoſt as 50 to 157; therefore take, between your Compaſſes, the Diameter of the Circle, and ſet it over, upon the Legs of the Sector, from 50 to 50, on both Lines of equal Parts. The Sector remaining thus opened, take the Diſtance from 157 to 157, between your Compaſſes, and that will be almoſt equal to the Circumference of the propoſed Circle; I ſay almoſt, for the exact Proportion of the Diameter of a Circle to it's Circumference hath not yet been Geometrically found *.

SECTION II.

Of the USE *of the Line of Planes.*

USE I. *To augment or diminiſh any Plane Figures in a given Ratio.*

LET, for Example, the Triangle A B C be given, and it is required to make another Triangle ſimilar, and triple to it. Fig. 4.

Take the Length of the Side A B between your Compaſſes, and open the Sector, ſo that the Points of your Compaſſes fall upon 1 and 1, on each Line of Planes; the Sector remaining thus opened, take the Diſtance from the third Plane to the third, on each Leg of the Sector, which will be the Length of the homologous Side to the Side A B. After the ſame manner may the homologous Sides to the other two Sides of the given Triangle be found, and of theſe three Sides may be formed a Triangle triple to the propoſed one. *Note*, If the propoſed Plane Figure hath more than three Sides, it muſt be reduced into Triangles, by drawing of Diagonals.

If a Circle is to be augmented or diminiſhed, you muſt proceed in the ſame manner with it's Diameter.

USE II. *Two ſimilar Plane Figures being given; to find the Ratio between them.*

Take either of the Sides of one of the Figures, and open the Sector, ſo that it may fall upon the ſame Number or Diviſion, on the Line of Planes, on both Legs of the Sector. Then take the homologous Side of the other Figure, and apply that to ſome Number or Diviſion on both Legs of the Sector; and then the two Numbers, on which the homologous Sides fall, will expreſs the Ratio of the two Figures. As ſuppoſe the Side *a b*, of the leſſer Figure, falls upon the fourth Plane; and the homologous Side A B, of the greater, falls upon the ſixth Plane, the two Planes are to each other as 4 to 6. But if the Side of a Figure is applied to the Extent of ſome Plane, on both Legs of the Sector, and the homologous Side cannot Fig. 5.

* *No, nor never will, it being impoſſible. The moſt exact Proportion in ſmall Numbers, is that of* Adrian Metius, *viz.* 113 *to* 355.

not be adjusted parallel to it, so as it may fall on a whole Number on both Legs of the Sector; then you must place the Side of the first Figure upon some other Number, on each Leg; 'till a whole Number is found on both Legs of the Sector, whose Extent is equal to the Length of the homologous Side of the other Figure, to avoid Fractions.

If the proposed Figures are so great, that their Sides cannot be applied to the Opening of the Legs of the Sector, take the half, third, or fourth Parts of any of the two homologous Sides of the said Figures, and compare them together, as before, and you will have the Proportion of the said Figures.

USE III. *To open the Sector, so that the two Lines of Planes may make a right Angle.*

Take between your Compasses the Extent of any Plane from the Center of the Sector; as, for Example, the 40th: then apply this opening of your Compasses, upon the Line of Planes, on both Sides, to a Number equal to half the precedent one, which, in this Example, is 20; then the two Lines of Planes will be at right Angles: because, by the Construction of the Line of Planes, the Number 40, which may represent the longest Side of a Triangle, signifies a Plane equal to two other similar Planes, denoted by the Number 20 upon the Legs of the Sector: Whence, from *Prop.* 48. *lib.* 1. *Eucl.* the aforenamed Angle is a right one.

USE IV. *To make a plane Figure similar and equal to two other given similar plane Figures.*

Open the Sector (by the precedent Use) so that the Lines of Planes be at right Angles, and carry any two homologous Sides, of the two proposed Figures, upon the Line of Planes, from the Center, the one upon one Leg, and the other upon the other Leg; and then the Distance of the two Numbers found will give the homologous Side of a plane Figure similar and equal to the two given ones.

As, for Example, the Side of the lesser Figure being laid off from the Center, will reach to the fourth Plane; and the homologous Side of the greater Figure, likewise laid off upon the other Leg, will extend to the ninth Plane: then the Distance from 4 to 9 is the homologous Side of a Figure equal to the two proposed ones, by means of which it will be easy to make a Figure similar to them.

By means of this Use may be added together any Number of similar plane Figures, *viz.* in adding together the two first, and then adding their Sum to the third, and so on.

USE V. *Two similar unequal plane Figures being given; to find a third equal to their Difference.*

Open the Sector, so that the two Lines of Planes may make a right Angle; then lay off one Side of the lesser Figure from the Center of the Sector. This being done, take the homologous Side of the greater Figure, and set one Foot of your Compasses upon the Number whereon the first Side terminates, and the other Point will fall on the other Leg, upon the Number required.

As, for Example; having laid off the Side of the lesser Figure from the Center, which falls upon the Number 9, take the Length of the homologous Side of the greater Figure, and setting one Foot of your Compasses upon the Number 9, the other will fall on the Number 4 of the other Leg; therefore taking the Distance of the Number 4 from the Center of the Sector, that will be the homologous Side of a Figure similar and equal to the Difference of the two given Figures, whose Ratio is as 9 to 13.

USE VI. *To find a mean Proportional between two given Lines.*

Lay off both the given Lines upon the Line of equal Parts, in order to have their Lengths expressed in Numbers; the lesser of which suppose 20, and the greater 45: Then open the Sector, so that the Distance from 45 to 45, of the Lines of Planes, be equal in Length to the greater Line. The Sector remaining thus opened, take the Distance from 20 to 20 of the Line of Planes, which will be the mean Proportional sought; and having measured it upon the Line of equal Parts from the Center, you will find it to be 30: for As 20 is to 30, So is 30 to 45.

But because the greatest Number on the Line of Planes is 64, if any one of the Lines proposed be greater than 64, the Operation must be made with their half, third, or fourth Parts, in the following manner: Suppose the lesser Number be 32, and the greater 72; open the Sector, so that half of the greater Number, *viz.* 36, may be equal to the Distance from 36 to 36, of the Line of Planes, upon both Legs of the Sector; and then the Distance from 16 to 16 doubled, will be the mean Proportional sought.

SECTION III.

Of the USES *of the Line of Polygons.*

USE I. *To inscribe a regular Polygon in a given Circle.*

Fig. 6. TAKE the Semidiameter A C, of the given Circle, between your Compasses, and adjust it to the Number 6, upon the Line of Polygons, on each Leg of the Sector; and the Sector remaining thus opened, take the Distance of the two equal Numbers, expressing the Number

ber of Sides the Polygon is to have: for Example; take the Diftance from 5 to 5, to infcribe a Pentagon; from 7 to 7 for a Heptagon, and fo of others: either of thefe Diftances, carried about the Circumference of the Circle, will divide it into fo many equal Parts. And thus you may eafily defcribe any regular Polygon, from the equilateral Triangle to the Dodecagon.

USE II. *To defcribe a regular Polygon upon a given right Line.*

If, for Example, the Pentagon of *Fig.* 6. is to be defcribed upon the Line AB: Take the Length of the faid Line between your Compaffes, and apply it to the Extent of the Numbers 5, 5, on the Line of Polygons: The Sector remaining thus opened, take, upon the fame Lines, the Extent from 6 to 6, which will be the Semidiameter of the Circle the Polygon is to be infcribed in; therefore if, with this Diftance, you defcribe, from the Ends of the given Line AB, two Arcs of a Circle, their Interfection will be the Center of the Circle. Fig. 6.

If an Heptagon was propofed, apply the Length of the given Line to the Extent of the Numbers 7 and 7, on both Legs of the Sector, and always take the Extent from 6 to 6, to find the Center of the Circle; in which it will be eafy to infcribe an Heptagon, each Side of which will be equal to the given Line.

USE III. *To cut a given Line, as* DE, *into extreme and mean Proportion.*

Apply the Length of the given Line to the Extent of the Numbers 6 and 6, on both Sides, upon the Line of Polygons; and the Sector remaining thus opened, take the Extent of the Numbers 10 and 10, on both Legs of the Sector, which are thofe for a Decagon. This Extent will give DF, the greateft Segment of the propofed Line, becaufe the greateft Segment of the Radius of a Circle, cut into mean and extreme Proportion, is the Chord of 36 Degrees, which is the 10th Part of the Circumference. Fig. 7.

If the greater Segment is added to the Radius of the Circle, fo as to make but one Line, the Radius will be the greater Segment, and the Chord of 36 Degrees will be the leffer Segment.

USE IV. *Upon a given Line* DF, *to defcribe an Ifofceles Triangle, having the Angles at the Bafe double to that at the Vertex.*

Open the Sector, fo that the Ends of the given Line may fall upon 10 and 10, of the Line of Polygons, upon each Leg of the Sector. The Sector remaining thus opened, take the Diftance from 6 to 6, and this will be the Length of the two equal Sides of the Triangle to be made. Fig. 8.

It is manifeft that the Angle, at the Vertex of this Triangle, is 36 Degrees, and that each of the Angles at the Bafe is 72 Degrees; but the Angle of 36 Degrees, is the Angle of the Center of a Decagon.

USE V. *To open the Sector fo, that the two Lines of Polygons may make a right Angle*

Take between your Compaffes the Diftance of the Number 5, from the Center, on the Line of Polygons; then open the Sector, fo that this Diftance may be applied to the Number 6 on one Side, and to the Number 10 on the other, and then the two Lines of Polygons will make a right Angle; becaufe the Square of the Side of a Pentagon is equal to the Square of the Side of a Hexagon, together with the Square of the Side of a Decagon.

SECTION IV.

Of the USES *of the Line of Chords.*

USE I. *To open the Sector, fo that the two Lines of Chords may make an Angle of any Number of Degrees.*

FIRST take the Diftance, upon the Line of Chords, from the Center of the Joint, to the Number of Degrees propofed; then open the Sector, fo that the Diftance, from 60 to 60 on each Leg, be equal to the aforefaid Diftance, and then the Lines of Chords will make the Angle required.

As, to make an Angle of 40 Degrees; take the Diftance of the Number 40 from the Center, then open the Sector, 'till the Diftance from 60 to 60, be equal to the faid Diftance of 40 Degrees. If a right Angle be required, take the Diftance of 90 Degrees from the Center, and then let the Diftance from 60 to 60 be equal to that, and fo of others. Fig. 9.

USE II. *The Sector being opened, to find the Degrees of it's Opening.*

Take the Extent from 60 Degrees to 60 Degrees, and lay it off upon the Line of Chords from the Center; then the Number, whereon it terminates, fheweth the Degrees of it's Opening.

Sights are fometimes placed upon the Line of Chords, by means of which Angles are taken, in adding to the Sector a Ball and Socket, and placing it upon a Foot, to elevate it to the Height of the Eye: but thefe Operations are better performed with other Inftruments.

USE III. *To make a right-lined Angle, upon a given Line, of any Number of Degrees.*

Describe, upon the given Line, a circular Arc, whose Center let be the Point whereon the Angle is to be made; then set off the Radius, from 60 to 60, on the Lines of Chords. The Sector remaining thus opened, take the Distance of the two Numbers upon each Leg, expressing the proposed Degrees, and lay it from the Line upon the Arc described. Lastly, draw a right Line from the Center, through the End of the Arc, and it will make the Angle proposed.

Fig. 10. Suppose, for Example, an Angle of 40 Degrees be to be made at the End B, of the Line A B; having described any Arc about the Point B, always lay off the said Radius from 60 to 60 on the Line of Chords (because the Radius of a Circle is always equal to the Chord of 60 Degrees), and lay off the Distance of 40 Deg. and 40 Deg. from C to D. Lastly, drawing a Line through the Points B and D, the Angle of 40 Degrees will be had. *Vid. Fig.* 10.

By this Use a Figure, whose Sides and Angles are known, may be drawn.

USE IV. *A right-lined Angle being given; to find the Number of Degrees it contains.*

About the Vertex of the given Angle describe the Arc of a Circle, and open the Sector, so that the Distance from 60 to 60, on each Leg, be equal to the Radius of the Circle. Then take the Chord of the Arc between your Compasses, and carrying it upon the Legs of the Sector, see what equal Number, on each Leg, the Points of your Compasses fall on, and that will be the Quantity of Degrees the given Angle contains.

USE V. *To take the Quantity of an Arc, of any Number of Degrees, upon the Circumference of a given Circle.*

Open the Sector, so that the Distance from 60 to 60, on each Line of Chords, be equal to the Radius of the given Circle. The Sector remaining thus opened, take the Extent of the Chord of the Number of Degrees upon each Leg of the Sector, and lay it off upon the Circumference of the given Circle.

By this Use may any regular Polygon be inscribed in a given Circle, as well as by the Line of Polygons, *viz.* in knowing the Angle of the Center, by the Method and Table before expressed, in the Construction of the Line of Polygons.

For Example; to make a Pentagon by means of the Line of Chords: Having found the Angle of the Center, which is 72 Degrees, open the Sector, so that the Distance from 60 to 60, on each Leg of the Sector, be equal to the Radius of the given Circle; and then take the Extent from 72 to 72, on each Leg, between your Compasses, which carried round the Circumference, will divide it into five equal Parts, and the five Chords being drawn, the Polygon will be made.

USE VI. *To describe a regular Polygon upon the given right Line* F G.

Fig. 11. As, for Example, to make a Pentagon, whose Angle of the Center is 72 Degrees; open the Sector, so that the Distance from 72 Degrees to 72 Degrees, on each Line of Polygons, be equal to the Length of the given Line. The Sector remaining thus opened, take the Distance from 60 to 60, on each Leg, between your Compasses; with this Distance, about the Ends of the given Line, as Centers, describe two Arcs intersecting each other in D; and this D will be the Center of a Circle, whose Circumference will be divided, by the given Line, into five equal Parts.

SECTION V.

Of the USES *of the Line of Solids.*

USE I. *To augment or diminish any similar Solids in a given Ratio.*

Fig. 12. LET, for Example, a Cube be given, and it is required to make another double to it. Carry the Side of the given Cube to the Distance of some equal Number, on both Lines of Solids, at pleasure; as, for Example, to 20 and 20. The Sector being thus opened, take the Extent, on both Legs of the Sector, of a Number double to it, that is, of 40 and 40; and this is the Side of a Cube double the proposed one.

If a Ball or Globe be proposed, and it be required to make another thrice as big; carry the Diameter of the Ball to the Distance of some equal Number, on both Lines of Solids, at pleasure, as to 20 and 20; then take the Distance from 60 to 60 (because 60 is thrice 20), and that will be the Diameter of a Ball three times greater than the proposed one, because Balls are to each other as the Cubes of their Diameters.

If, again, a Chest, in figure of a right-angled Parallelepipedon, contains three Measures of Grain, and it be required to make another similar Chest to contain five Measures; open the Sector, so that the Distance from 30 to 30, on each Line of Solids, be equal to the Length of the Base of the Chest; then the Distance from 50 to 50, on each Leg, will be the homologous Side of that Solid to be made. Again, apply the Breadth of the Base to the Distance of the said Numbers 30 and 30, and then the Distance from 50 to 50 will be the homologous

mologous Side to the ſaid Breadth. Now having made a Parallelogram with theſe two Lengths, your next thing will be to find the Depth: To do which, open the Sector, ſo that the Diſtance from 30 to 30 be equal to the Depth of the given Cheſt; then the Diſtance from 50 to 50 will be the Depth of the Cheſt to be made. This being done, it will be eaſy to make the Parallelepipedon, containing the five propoſed Meaſures.

If the Lines are ſo long, that they cannot be applied to the Legs of the Sector, take any of their Parts, and with them proceed as before; then the reſpective Parts of the required Dimenſions will be had.

USE II. *Two ſimilar Bodies being given; to find their Ratio.*

Take either of the Sides of one of the propoſed Bodies between your Compaſſes, and having carried it to the Diſtance of ſome equal Number, on each Line of Solids, take the homologous Side of the other Solid, and note the Number on each Leg it falls upon; and then the ſaid Numbers will ſhew the Ratio of the two ſimilar Solids.

But if the Side of the firſt Solid be ſo applied to ſome Number on each Leg of the Sector, that the homologous Side of the other cannot be applied to the Extent of ſome Number on each Leg; then you muſt apply the Side of the firſt Solid to ſuch a Number on each Line, that the Length of the Side of the ſecond Solid may fall upon ſome whole Number on each Line of Solids, to avoid Fractions.

USE III. *To conſtruct and divide a Line, whoſe Uſe is to find the Diameters of Cannon-Balls.*

It is found, by Experience, that an Iron Ball, three Inches in Diameter, weighs 40 Pounds; whence it will be eaſy to find the Diameters of other Balls of different Weights, and the ſame Metal, in the following manner: Open the Sector, ſo that the Diſtance from the 4th Solid to the 4th Solid, on each Line of Solids, be equal to three Inches The Sector remaining thus opened, take upon the Lines of Solids the Diſtances of all the Numbers, from 1 to 64, on one Leg, to the ſame Numbers on the other Leg; then lay off all theſe Lengths upon a right Line drawn on a Ruler, or upon one of the Legs of the Sector, and where the Diameters terminate, denote the Weights of the Balls.

But now to mark the Fractions of a Pound, as $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, open the Sector, ſo that the Diſtance of the 4th Solids on each Leg of the Sector, be equal to the Diameter of a Ball of one Pound. The Sector remaining thus opened, the Diſtance from the 1ſt Solid to the 1ſt on each Leg of the Sector, will give the Diameter for $\frac{1}{4}$ of a Pound; from the 2d to the 2d, for $\frac{2}{4}$ of a Pound; and from the 3d to the 3d, for $\frac{3}{4}$ of a Pound, and ſo of others. When the Diameters of Balls are known, the Diameters or Bores of Cannon, to which they are proper, will likewiſe be known: but there are commonly two or three Lines given for the Vent of great Balls, and for leſſer ones in proportion. The Diameters of Balls are meaſured with ſpherick Compaſſes, as will be more fully explained among the Inſtruments for Artillery.

USE IV. *To make a Solid ſimilar and equal to the Sum of any Number of ſimilar given Solids.*

Open the Sector, and apply either of the Sides of either of the Bodies to the ſame Number on each Line of Solids; then note on what equal Numbers, on both Legs of the Sector, the homologous Sides of the other Solids fall. This being done, add together the ſaid Numbers, and take the Extent, on both Lines of Solids, of the Number ariſing from that Addition; and this Extent will be the homologous Side of a Body, equal and ſimilar to the Sum of the given Bodies.

Example; Suppoſe the Side choſen of the firſt Solid be applied to the fifth Solid, on each Leg of the Sector, and the homologous Sides of the others fall, the one on the 7th, and the other on the 8th Solid, on each Line of Solids; add the three Numbers 5, 7, and 8 together, and their Sum is 20; therefore the Diſtance from 20 to 20, on each Line of Solids, will be the homologous Side of a Body, equal and ſimilar to the three others.

USE V. *Two ſimilar and unequal Bodies being given; to find a third ſimilar and equal to their Difference.*

Open the Sector, and apply either of the Sides of either of the Bodies to ſome equal Number on each Leg of the Sector, and ſee what equal Numbers, on both Legs, the homologous Sides of the other Solids fall upon; then ſubſtract the leſſer Number from the greater, and take the Diſtance from the remaining Number, on one Line of Solids, to the ſame on the other; and this will be the homologous Side of a Body, equal to the Difference of the two given ones.

As, for Example; the Side of the greateſt being ſet over, upon the Line of Solids, from 15 to 15, the homologous Side of the leſſer will be equal to the Diſtance from 9 to 9; then taking 9 from 15, there remains 6: therefore the Diſtance from 6 to 6 will be the homologous Side of the Solid ſought.

USE VI. *To find two mean Proportionals between two given Lines.*

For Example; ſuppoſe there are two Lines, one of which is 54, and the other 16: open the Sector, ſo that the Diſtance from 54 to 54, on each Leg of the Sector, be equal to the

Length of the longest Line. The Sector remaining thus opened, the Distance from 16 to 16, on each Leg, will be equal to the greater of the mean Proportionals, and will be found to be 36. Again, shutting the Legs of the Sector closer, 'till the Distance between 54 and 54, on each Leg, be equal to 36; then the Distance from 16 to 16 will be the lesser of the mean Proportionals, and will be found to be 24: Whence these four Lines will be in continual Proportion, 54, 36, 24, 16.

If the Lines be too long, or the Numbers of their equal Parts too great, you must take their halfs, thirds, or fourths, &c. and proceed as before. For Example; to find two mean Proportionals between two Lines, one of which is 32, and the other 256, take the fourth Parts of both the Lines, which are 8 and 64. This being done, open the Sector, so that the Distance from 8 to 8, on each Line of Solids, be equal to 8; then take the Distance from 64 to 64, and that gives 16, for $\frac{1}{4}$ of the first of the two mean Proportionals. Again, open the Sector, so that the Distance from 8 to 8 be equal to 16; the Sector being thus opened, the Distance from 64 to 64 will give 16, for $\frac{1}{4}$ of the second of the mean Proportionals sought: whence the mean Proportionals are 64 and 128; for 32, 64, 128, 256, are proportional.

USE VII. *To find the Side of a Cube equal to the Side of a given Parallelepipedon.*

First, find a mean Proportional between the two Sides of the Base of the Parallelepipedon; then between the Number found, and the Height of the Parallelepipedon, find the first of two mean Proportionals, which will be the Side of the Cube sought.

For Example, let the two Sides of the Parallelepipedon be 24 and 54, and it's Height 63; the Side of a Cube equal to it is sought.

Open the Sector, so that the Distance between 54 and 54, on the Line of Planes, be equal to the Side of 54; then take the Distance from 24 to 24 on the same Line, which, measured upon the Line of equal Parts, will give 36 for a mean Proportional. This being done, take 36 between your Compasses, and open the Sector, so that the Points of the Compasses may fall upon 36 and 36, on each Line of Solids; then take the Distance from 63 to 63 on the Lines of Solids, which will be found almost 44 $\frac{1}{2}$, for the Side of a Cube equal to the given Parallelepipedon.

USE VIII. *To construct and divide a Gauging-Rod to measure Casks, and other the like Vessels, proper to hold Liquors.*

Fig. 13 The Gauging-Rod, of which we are now going to speak, is a Ruler made of Metal, divided into certain Parts, whereby the Number of Pints contained in a Vessel may be found, in putting it in at the Bung-hole, 'till it's End touches the Angle, made by the Bottom, with that part of the Side opposite to the Bung-hole, as the Line A C diagonally situated.

The Gauging-Rod being thus posited, the Division, answering to the middle of the Bung-hole, shews the Quantity of Liquor, or Number of Pints the Vessel, when full, holds.

But it is necessary to change the Position of the aforesaid Rod, so that it's End C may touch the Angle of the other Bottom B, in order to see whether the middle of the Bung-hole be in the middle of the Vessel; for if there is any Difference, half of it must be taken.

The Use of this Gauging-Rod is very easy: for, without any Calculation by it, the Dimensions of Casks may immediately be taken; all the Difficulty consist only in well dividing it.

Now, in order to divide it, a little Cask, holding a *Septier*, or *Gallon*, must be made similar to the Vessels that are commonly used; for this Rod will not exactly give the Dimensions of dissimilar Vessels, that is, such that have the Diameters of the Heads, those of the Bungs, and the Lengths not proportional to the Diameters of the Head, Bung, and Length of that which the Divisions of the Rod are made by.

Now suppose the Diameter, at the Head of a Cask, be 20 Inches, the Diameter of the Bung 22, and the interior Length 30 Inches; this Vessel will hold 27 *Septiers* of *Paris* Measure, and it's Diagonal Length, answering to the middle of the Bung-hole, will be 25 Inches, 9 Lines and a half, as is easy to find by Calculation: because in the right-angled Triangle A D C, the Side C D being 15 Inches, and D A 21, by adding their Squares together, you will have *(per Prop.* 47. *lib.* 1. *Eucl.)* the Square of the Hypothenuse A C; and by extracting the Square Root, A C will be had.

According to the same Proportions a Cask, whose Dimensions are one Third of the former ones, will contain one *Septier*, or eight Pints; that is, if the Diameter of the Head be 6 Inches, and 8 Lines; that of the Bung 7 Inches, 8 Lines; the Length 8 Inches, 8 Lines; and it's Diagonal 8 Inches, 7 Lines.

Another Cask, whose Dimensions are half of that before-mentioned, will contain one Pint; that is, if the Diameter of the Head be 3 Inches, 4 Lines; that of the Bung 3 Inches, 8 Lines; the interior Length of the Cask 5 Inches; and the Diagonal, answering to the middle of the Bung-hole, 4 Inches, 3 Lines and a half.

Now take a Rod about 3 or 4 Feet long, and chuse either of the three Measures, which you judge must proper: As, for Example; if you will make Divisions for *Septiers* upon the Rod, make a Point, in the middle of it's Breadth, distant from one of it's Ends, 8 Inches, 7 Lines, and there make the Division for one *Septier* upon it; double that Extent, and there make a Mark for 8 *Septiers*; triple the same Extent, and there make a Mark for 27 *Septiers*; quadruple

quadruple it, and there make a Mark for 64 Septiers; becaufe fimilar Solids are to each other, as the Cubes of their homologous Sides.

Again, to make Divifions upon it for the other Septiers, take between your Compaffes the Length of 8 Inches, 7 Lines; fet over this Diftance, upon each Line of Solids of your Sector, from the firft Solid to the firft. The Sector remaining thus opened, take the Diftance from the fecond Solid to the fecond, which mark upon the Rod for the Divifion of two Septiers.

Again; take the Diftance from the third Solid to the third, which mark upon the Rod for the Length of the Diagonal, agreeing to three Septiers, and fo on; by which means the Rod will be divided, for taking the Dimenfions of Veffels in Septiers. With the fame facility may the Divifions for Pints be made upon the Rod; for half of the Diftance of the Divifion of two Septiers, will give the Divifion for two Pints; half of the Diftance of the Divifion for three Septiers, will give the Divifion for three Pints; half of the Diftance of the Divifion for four Septiers, will give the Divifion for four Pints, and fo on.

If the Sector be not long enough to take the Diagonal Length anfwerable to one Septier, from the firft Solid to the firft, take the Diagonal Length anfwerable to one Pint; and having divided the Rod for any Number of Pints, the Diagonal Lengths of the fame Number of Septiers may be had, by doubling the Diagonal Lengths of the Pints. As, for Example; if the Diagonal Length for 6 Pints be doubled, that Diftance will be the Diagonal Length of a Veffel holding 6 Septiers: Alfo if the Diagonal Length of 7 Pints be doubled, the Length of the Diagonal of a Veffel, holding 7 Septiers, will be had; and fo of other Diagonal Lengths.

If the Diagonal Length is yet too long to be applied to the Diftance of the Divifion for the firft Solid, on each Leg of the Sector, it's half muft be applied to the fame; and the Sector remaining thus opened, take the Diftance of the Divifions for the fecond Solid on both Lines of Solids, and double it; then you will have the Diagonal Length of a Veffel holding two Pints. Having again taken the Diftance of the Divifion for the third Solid upon each Leg of the Sector, which Diftance being double, the Diagonal Length of a Veffel holding three Pints will be had, and may be marked upon your Rod; and fo of others.

The Divifions for Septiers go acrofs the whole Breadth of the Rod, upon which are their refpective Numbers graved; and the Divifions for Pints are fhorter than the others, for their better Diftinction.

In order for this Gauging-Rod to ferve to take the Quantity of Liquor contained in different diffimilar Veffels, other Divifions may be made upon it's Faces, according to the different Proportions of their Lengths and Diameters, and at the bottom of the Faces muft be writ the Diameters and Lengths by which the Divifions were made: For Example; at the bottom of the Face, upon which the precedent Divifions were made, there is wrote, the Diameter of the Head 20, the Diameter of the Bung 22, and the Length 30.

If, for dividing another Face, you ufe a Veffel, whofe Diameter of the Head is 21 Inches, that of the Bung 23, and the interior Length 27 $\frac{1}{2}$ Inches; this Veffel is fhorter than that before-named, but contains almoft the fame Quantity of Liquor, when full, *viz.* 27 Septiers, and the Length of it's Diagonal will be 26 Inches.

If another Veffel hath all it's Dimenfions $\frac{1}{3}$ of the precedent ones, this Veffel will hold one Septier, and it's Diagonal A C will be 8 Inches and 8 Lines in Length. Now by means of this Veffel, and it's Diagonal Length, you may divide the aforefaid Face in the Manner directed for dividing the firft Face, and at the Bottom of this Face you muft write, *Diameter reduced* 22, *Length* 27 $\frac{1}{2}$.

If the four Faces of the Rod are divided, as before-named, you will have four different Gauges for gauging four different kinds of Veffels; and by examining the Proportions of the Diameters of the Heads and Lengths, you muft make ufe of fuch a Face accordingly.

Inftead of ufing the Sector in dividing the before-mentioned Gauging-Rod, it is better ufing the Table of Solids.

For having found, by Calculation, that the Length of the Diagonal of a Veffel, holding 27 Septiers, is 6 Inches, it will be eafy to find the Diagonals of Veffels of any propofed Bigneffes, having the fame Proportions to the Diameters reduced, as 22 to 27 $\frac{1}{2}$, or as 4 to 5.

As, for Example; it is required to find the Diameter of a *Quarteau*, or *Firkin*, which holds 9 Septiers; feek, in the Table of Solids, the Number anfwering to the 9th Solid, which will be found 520; at the fame time find the correfpondent Number to the 27th Solid, which will be found 750: then ftate a Rule of Three, in the following manner; 750 : 520 : : 26 : 18; whence 18 Inches will be the Length of the Diagonal of a Veffel holding 9 Septiers. The Coopers about *Paris* make their Veffels almoft in the Proportion of 4 to 5; as is, for Example, a half *Muid*, having 19 Inches 2 Lines in Diameter reduced, and 24 Inches in Length; in which Cafe the Diagonal will be 22 Inches, 8 $\frac{1}{2}$ Lines, as you will eafily find by Calculation.

But, in general, as foon as the Proportions ufed in making Veffels are known, the Diagonal of fome one of thofe Veffels, holding a known quantity of Septiers being firft found (*per Prop.* 47. *lib.* 3. *Eucl.*) you may afterwards find the Lengths of the Diagonals of all Veffels made in the fame proportion, by means of the aforefaid Table of Solids.

SECTION VI.

Of the Construction and Use of other kinds of Gauging-Rods.

THE Gauging-Rod, of which we have already spoken, serves only to find the Quantity of Liquor contained in similar Vessels; but that which we are now going to mention, may be used in taking the Dimensions of dissimilar Vessels.

In order to construct the first Gauge of this kind, the Measure which you use must be determined, by comparing it with some regular Vessel, as a Concave Cylinder, in which a Quart or a Gallon of Water being poured, you must exactly note the Depth occupied by the Water.

As, for Example, if a Gauge is to be made for *Paris*, where a Pint is 48 Cubic Inches, or 61 Cylindrick Inches, you will find, by Calculation, that a Concave Cylinder, 3 Inches, 11 $\frac{1}{3}$ Lines in Diameter, and the like Number in Depth, contains one Pint of *Paris*; and a Cylinder, whose Dimensions are double the aforesaid ones, that is, 7 Inches, 10 $\frac{2}{3}$ Lines, will hold one Septier: for similar Solids are to each other, as the Cubes of their like Sides.

Fig. 14. This being supposed, lay off that Length of 3 Inches 11 $\frac{1}{3}$ Lines, upon one Face of the Rod, as often as the Length of the Rod will admit, and mark Points, whereon set 1, 2, 3, 4, 5, *&c.* each of these Parts may be subdivided into 4 or more. This Face, thus divided, is called the Face of equal Parts, and is used in measuring the Lengths of Vessels.

You must likewise mark, upon another Face of the Rod, the Diameter of the Cylinder of 3 Inches, 11 $\frac{1}{3}$ Lines, and then the Diameters of Circles double, triple, quadruple, *&c.* by any of the Methods before explained for dividing the Line of Planes on the Sector, the easiest and shortest of which is to make a right-angled Isosceles Triangle A B C; each of the Legs about the right Angle of which being 3 Inches, 11 $\frac{1}{3}$ Lines, the Hypothenuse B C will be the Diameter of a Circle double to that, whose Diameter is 3 Inches, 11 $\frac{1}{3}$ Lines: therefore having produced one of the Legs A B towards D, lay off the said Hypothenuse from A towards D, and at the Point whereon it terminates mark the Number 2; then take the Distance C 2, and having laid it off upon the Line A D, mark the Number 3 at the Point whreon it terminates. Again, take the Distance C 3, and having laid it off upon the Line A D, there mark the Number 4, *&c.*

Fig 15.

Note, A 4, which is the Diameter of a Circle quadruple the first, is double A C, or A B; because Circles are to each other as the Squares of their Diameters: whence since A B is 1, it's Square is also 1; and the Line A 4 being 2, it's Square must consequently be 4.

To use this Gauge, you must first apply the Face of equal Parts to the exterior Length of the Vessel, from which you must take the Depth of the two *Croes*, that thereby the true interior Length may be had.

This being done, apply the Face of Diameters to the Diameters of the Heads of the Vessel, and note the Number answering to them, and whether they are equal; for if there be any Difference between the Diameters of the Heads, you must add them together, and take half their Sum for the mean Head-Diameter.

Again; put the Rod downright in at the Bung-hole, in order to have the Diameter of the Bung, which add to the Head-Diameter, and take half the Sum for an arithmetical Mean; this being multiplied by the Length of the Vessel, will give the Number of Pints the Vessel holds.

As suppose the interior Length of a Vessel is 4 $\frac{3}{4}$ of the equal Parts of the Rod, the Diameter at the Head 15, and the Bung-Diameter 17; add 15 to 17, and their Sum is 32, half of which is 16; which multiplied by the Length 4 $\frac{3}{4}$, and the Product 76 will give the Number of Pints the Vessel holds.

Now to construct the second kind of Rods, it is found, by Experience, that a Cylinder, whose Height and Diameter is 3 Foot, 3 Inches, and 6 Lines, holds 1000 *Paris* Pints.

Then take upon a Ruler a Length of 3 Feet, 3 Inches, and 6 Lines, which divide into 10 Parts, each of which will be the Height and Diameter of a Cylinder holding one Pint, (because similar Cylinders are to each other as the Cubes of their Diameters). Again, divide each of these Parts into 10 more, which may easily be done by help of the Line of Lines on the Sector; then each of these last Parts will be the Height and Diameter of a Cylinder holding the 1000th part of a Pint: Every five of these small Parts being numbered, your

Fig. 16. Rod will be made. One of these Rods, of 4 or 5 Feet in Length, will serve to gauge great Vessels, as Pipes, *&c.*

To use this Rod, you must note how many of the small Divisions of the Rod the Diameters of the Head and Bung, as also the Length, contains.

But, *Note*, by the Length of a Vessel is understood the interior Length, which is the Distance between the Head and the Bottom; and by the Diameters is understood the interior Diameters included between the Staves.

Note, also, If the Diameters at Top and Bottom are unequal, compare one of them with the Bung-Diameter, and the middle between these two is called the mean Diameter of the Vessel.

But

But if the Diameters at Top and Bottom are unequal, add them together, and take half of their Sum, which is called the mean Diameter of the Head and Bottom; then compare this mean Diameter with the Diameter at the Bung, add them together, and take half their Sum for the mean Diameter of the Veſſel.

Then ſquare the mean Diameter of the Veſſel, and multiply the ſaid Square by the Length of the Veſſel; then the Product will give you the Quantity of Liquor in 1000th Parts the Veſſel holds; and by caſting away the laſt three Figures, you will have the Number of Pints contained in the Veſſel, when full.

Let, for Example, the Diameter at the Head be 58 Parts of the Guaging-Rod, and the Bung-Diameter 62; add theſe two Numbers together, and their Sum will be 120, whoſe half 60 is the mean Diameter of the Veſſel: then the Square of this mean Diameter will be 3600; and if this Square be multiplied by the Length of the Veſſel, which ſuppoſe 80, the Product will be 288000; and by taking away the three laſt Figures, the Number of *Paris* Pints the Veſſel holds will be 288.

This way of Gauging is exact enough for Practice, when there is but a ſmall Difference between the Bung and Head-Diameters, as are the Diameters of *Paris-Muids*; but when the Difference between the Bung and Head-Diameters is conſiderable, as in the Pipes of *Anjou*, whoſe Bung-Diameters are much greater than the Head-Diameters, Dimenſions taken in the before-directed manner will not give the Quantity of Liquor exact enough: But to render the Method more exact, divide the Difference of the Bung and Head-Diameters into 7 Parts, and add 4 of them to the Head-Diameter, and that will give you the mean Diameter: for Example; if the Diameter of the Head is 50, and the Bung-Diameter 57, the mean Diameter of the Veſſel will be 54; with which mean Diameter proceed as before.

Having found by the Rod how many *Paris* Pints a Veſſel holds, you may find how many other Meaſures the ſame Veſſel holds, in the following manner:

A *Paris* Pint of freſh Water weighs 1 Pound, 15 Ounces; therefore you need but weigh the ſought Meaſure full of Water, and by the Rule of Three you may have your Deſire.

As, for Example; a certain Meaſure of Water weighs 50 Ounces, and it is required to find how many of the ſame Meaſures is contained in a *Paris-Muid*, which holds 288 Pints: Say, by the Rule of Three, As 50 is to 31, So is 288 Pints to a fourth Number, which will be 178 $\frac{4}{5}$ of the ſaid Meaſures.

There may be marked Feet and Inches upon the vacant Faces of the aforeſaid Gauging-Rod, each of which Inches may be ſubdivided into four equal Parts, which will be a ſecond means to gauge Veſſels; the Feet are marked with Roman Characters, and the Inches with others.

We have already ſaid, that a *Paris* Pint contains 61 Cylindrick Inches; therefore having the Solidity of a Veſſel in Cylindrick Inches, it muſt be divided by 61, to have the Number of Pints the Veſſel holds. An Example or two will make this manifeſt.

Let the Length of a Veſſel be 36 Inches, the Head-Diameter 23, and the Bung-Diameter 25; add the two Diameters together, and their Sum will be 48, half of which is 24 for the mean Diameter. This Number 24 being ſquared, will be 576; and this Square being multiplied by the Length 36, gives 20736 Cylindrick Inches: which being divided by 61, the Quotient will give 339 Pints, and about $\frac{3}{4}$.

If the Diameters and Lengths of Veſſels are taken in fourth Parts of Inches, the laſt Product muſt be divided by 3904, to have the Number of Pints contained in a Veſſel, when full.

Let, for Example, the Length of a Veſſel be 35 $\frac{1}{4}$ Inches, the Head-Diameter 23 Inches, and the Diameter at the Bung 25 $\frac{1}{2}$ Inches; add the two Diameters together, and their Sum will be 48 , half of which will be 24 $\frac{1}{4}$; which, for eaſe of Calculation, reduce to 4ths: 97 is the Number to be ſquared, which will be 9409; which multiply by 141, and that Product again by 35 ', reduced to 4ths of Inches, will give this Product 1326669; which being divided by 3904, the Quotient will (as before) be 339 Pints, and about $\frac{3}{4}$.

The Construction and USE *of a new Gauging-Rod.*

Mr *Sauveur*, of the Academy of Sciences, has communicated to us a new Gauging-Rod of his Invention, by means of which may be found, by Addition only, the Quantity of Liquor that any Veſſel holds, when full; whereas hitherto Multiplication and Diviſion has been uſed in Gauging.

To make this Gauging-Rod, you muſt firſt chuſe a Piece of very dry Wood, as Sorbaple or Pear-tree, without Knots, about 5 Foot long, in Figure of a Parallelepipedon, and 6 or 7 Lines in Breadth; *Fig.* 17. ſhews it's four Faces. Fig. 17.

Now upon the firſt of the four Faces are made Diviſions for taking the Diameters of Veſſels.

The Diviſions of the ſecond Face ſerves to meaſure the Lengths of the Diameters.

The Diviſions upon the third Face are for finding the Contents of Veſſels.

And, Laſtly, upon the fourth Face, the Numbers of Septiers and Pints, which the Veſſel holds, are marked.

The aforeſaid Diviſions are made in the following manner:

Firſt,

First, divide the fourth Face into Inches, and each Inch into 10 equal Parts; those Divisions denote Pints, and are numbered 1, 2, 3, 4, 5, 6, *&c.* every 8 being Septiers, because 1 Septier is 8 Pints: On the End of this fourth Face is written *Pints* and *Septiers*.

The Divisions of the other three Faces are made by help of Logarithms, in manner following.

Note, The Divisions of the fourth Face serve as a Scale to the third, and ought to be contiguous to it.

To divide the third Face of the Rod.

If you have a mind to place any Number upon the third Face of your Rod; for Example, 240: seek in the Table of Logarithms for 240, or the nighest Number to it, which will be found against 251 in your Table; then place 240 upon the third Face, over against 251 Pints on the fourth Face, and, proceeding in this manner, you may divide the third Face.

But because, in the Table of Logarithms, 240 doth not stand against 251, but instead thereof there stands 2.39996, which nighly approaches it; therefore to make the Divisions as exact as possible, you must add 1 to the first Number of the Logarithm 2.40, and then seek for 3.40, over against which stands 2512; which shews, that the Logar. 240 must be placed not over against 251 of the Divisions of Pints, but against 251 and two Parts of the Divisions of a Pint, supposed to be divided into 10 Parts more. You must write *Contents* at one End of this third Face.

The Manner of dividing the second Face.

A Cylindrical Vessel, whose Length and Diameter is 3 Inches, 11 $\frac{1}{3}$ Lines, holds one *Paris* Pint; therefore the first part of the second Face, which is without Divisions, must be of that Length. This said Length must be laid off ten times, and more, if possible, upon the said Face, upon which make occult Marks; then one of these Parts must be divided into 100 more, upon a separate Ruler, serving as a Scale.

This being done, suppose any Number is to be placed upon the second Face; as, for Example, 60: Seek in the Table of Logarithms for 60, which will be found against 39 and 40, or rather against 3981, without having regard to the Numbers 1, 2, 3, that precede it, and which are called Characteristicks: therefore I take 98, or 981, by esteeming one Part divided into 10, upon the small Scale divided into 100, and I place this Distance next to the third occult Point, which denotes three Centesms, or three Thousandths. You must thus mark Divisions from 5 to 5, and every of these 5ths must again be subdivided into 5 equal Parts. Finally, upon the End of this Face, you must write *Lengths*.

The Manner of dividing the first Face.

The first Part of this Face, which is not divided, represents the Diameter of a Cylindrical Vessel holding one *Paris* Pint; therefore it's Length must be 3 Inches, 11 $\frac{1}{3}$ Lines.

And for dividing this Face, lay off upon it the Divisions of the second Face; but instead of writing 5, 10, 15, 20, 25, *&c.* write their Doubles 10, 20, 30, 40, 50, *&c.* and subdivide the Intervals into 10 Parts, and at the End of this Face write *Diameters*.

The USE *of the New Gauging-Rod.*

Measure the Length of the mean Diameter of the Vessel with the Face of *Diameters* of your Rod, which suppose to be 153.00. Likewise take the Length of the Vessel with the second Face of your Rod, which suppose to be 92.85; add these two Numbers together, then seek their Sum 245.85 upon the third Face, and over against it, on the fourth Face, you will have 36 Septiers, or 288 Pints.

$$\begin{array}{r} 153.00 \\ 92.85 \\ \hline 245.85 \end{array}$$

But to make the Use of this Rod general; suppose the Weight of a Pint of fresh Water of some Country be 50 Ounces Avoirdupoise; then seek 31, the Number of Ounces Avoirdupoise a *Paris* Pint of fresh Water weighs, upon the fourth Face of *Septiers* of the Rod, which will be found against 239.4 on the third Face.

Likewise, against 50 on the fourth Face, answers 260.2 on the third Face.

Then from 260.2	Again from 245.85 before found
Take 239	Take 20.80
And there remains 20.8	And there remains 225.05

Now against this Number 225.05, on the third Face, you will find, on the fourth Face, 22 Septiers 2 Pints, or 178 Pints, which is the Number of Pints of that Country a Vessel of the aforesaid Dimensions holds.

SECTION VII.

Of the USE *of the Line of Metals.*

USE I. *The Diameter of a Ball, of any one of the six Metals, being given; to find the Diameter of another Ball of any one of them, which shall have the same Weight.*

OPEN the Sector, and taking the given Diameter of the Ball between your Compasses, apply it's Extremes to the Characters upon each Line of Metals, expressing the Metal the Ball is is made of. The Sector remaining thus opened, take the Diſtance of the Characters of the Metal, the ſought Diameter is to be of, upon each Line of Metals, and this will be the Diameter ſought. As, for Example, let A B be the Diameter of a Ball of Lead, and it be required to find the Diameter of a Ball of Iron, having the ſame Weight. Open the Sector, Fig. 18. ſo that the Diſtance between the Points ♄ and ♄ be equal to the Line A B: The Sector remaining thus opened, take the Diſtance of the Points of ♂ on each Line of Metals, and that will give C D, the Length of the Diameter ſought. If, inſtead of Balls, ſimilar Solids of ſeveral Sides had been propoſed, make the ſame Operation, as before, for finding each of their homologous Sides, in order to have the Lengths, Breadths, and Thickneſſes of the Bodies to be made.

USE II. *To find the Proportion that each of the ſix Metals have to one another, as to their Weight.*

For Example; It is required to find what Proportion two ſimilar and equal Bodies, but of different Weights, have to one another.

Having taken the Diſtance from the Center of the Joint of your Sector, to the Point of the Character of that Metal of the two propoſed Bodies which is leaſt (and which is always more diſtant from the Center), apply the ſaid Diſtance acroſs to any two equal Diviſions on both the Lines of Solids. The Sector remaining thus opened, take the Diſtance on the Line of Metals, from the Center of the Joint to the Point, denoting the other Metal: and applying it to both Lines of Solids, ſee if it will fall upon ſome equal Number on each Line; if it will, that Number, and the other before, will, by permuting them, ſhew the Proportions of the Metals propoſed.

As, for Example: To find the Proportion of the Weight of a Wedge of Gold, to the Weight of a ſimilar and equal Wedge of Silver.

Now becauſe Silver weighs leſs than Gold, open the Sector, and having taken the Diſtance from the Center of the Joint to the Point ☽, apply it to the Numbers 50 and 50 on each Line of Solids. The Sector remaining thus opened, take the Diſtance from the Center to the Point ☉, and applying it on each Line of Solids, and you will find it to fall nearly upon the 27th Solid on each Line. Whence I conclude, the Weight of the Gold to the Weight of the Silver, Is as 50 to $27\frac{1}{6}$, or as 100 to $54\frac{1}{3}$; that is, if the Wedge of Gold weighs 100 Pounds, the Wedge of Silver will weigh $54\frac{1}{3}$ Pounds, and ſo of other Metals, whoſe Proportions are more exactly laid down by the Numbers of Pounds and Ounces that a cubick Foot of each of the Metals weighs, as is expreſſed in the Table adjoining to the Proof of the Line of Metals. If nevertheleſs their Proportions are required in leſſer Numbers, you will find, that if a Wedge of Gold weighs 100 Marks, a Wedge of Lead, of the ſame Bigneſs, will weigh about $60\frac{1}{2}$, one of Silver $54\frac{1}{2}$, one of Braſs $47\frac{1}{4}$, one of Iron $42\frac{1}{10}$, and one of Tin 39 Marks.

USE III. *Any Body of one of the ſix Metals being given; to find the Weight of any one of the five others, which is to be made ſimilar and equal to the propoſed one.*

For Example; Let a Ciſtern of Tin be propoſed, and it is required to make another of Silver equal and ſimilar to it. Firſt weigh the Tin-Ciſtern, which ſuppoſe 36 Pounds. This being done, open the Sector, and having taken the Diſtance from the Center of the Sector to the Point ☽ (which is the Metal the new Ciſtern is to be made of), apply that Diſtance to 36 and 36 on each Line of Solids. Then take the Diſtance, upon the Line of Metals, of the Point ♃, from the Center; and applying that Diſtance croſs-wiſe on each of the Lines of Solids, you will find it nearly fall upon 50 and 50 on each Line: Whence the Weight of a Silver Ciſtern muſt be $50\frac{1}{4}$ Pounds, to be equal in Bigneſs to the Tin-Ciſtern. The Proof of this Operation may be had by Calculation, *viz.* in multiplying the different Weights reciprocally by thoſe of a Cubic Foot of each of the Metals. As, in this Example; multiplying 720 *lib.* 12 Ounces, which is the Weight of a Cubick Foot of Silver, by 36 *lib.* which is the Weight of the Tin-Ciſtern; and again, multiplying 516 *lib.* 2 Ounces, which is the Weight of a Cubick Foot of Tin, by $50\frac{1}{4}$ Pounds, which is the Weight of the Silver Ciſtern, the two Products ought to be equal.

USE IV. *The Diameters, or Sides, of two ſimilar Bodies of different Metals, being given; to find the Ratio of their Weights.*

Let, for Example, the Diameter of a Ball of Tin be the right Line E F, and the Line Fig. 19. G H the Diameter of a Ball of Silver; it is required to find the Ratio of the Weights of theſe two Balls. Open the Sector, and taking the Diameter E F between your Compaſſes, apply it to the Points ♃ on each Line of Metals. The Sector remaining thus opened, take the Diſtance of the Points ☽ on each Leg of the Sector; which compare with the Diameter G H, in order to ſee whether it is equal to it: for if it be, the two Balls muſt be of the ſame Weight. But if the Diameter of the Ball of Silver be leſſer than the Diſtance of the Points ☽, on each Leg of the Sector, as here K L is, it is manifeſt that the Ball of Silver weighs leſs than the Ball of Tin; and to know how much, the Diameters G H and G L muſt be compared together. Wherefore apply the Diſtance of the Points ☽, which is G H, on each Leg of the Sector, to some equal Number on both the Lines of Solids; as, for Example, to the Numbers 60 and 60; then note upon what equal Number, on both Lines of Solids, the Diameter K L falls, which suppose 20: whence the Ball of Silver, whose Diameter is K L, weighs but $\frac{1}{3}$ of the Weight of the Ball of Tin, whose Diameter is E F.

USE V. *The Weight and Diameter of a Ball, or the Side of any other Body, of one of the six Metals, being given: to find the Diameter or homologous Side of another similar Body of one of the other five Metals, which shall have a given Weight.*

Fig. 20. Let, for Example, the right Line M N be the Diameter of a Ball of Brass, weighing 10 Pounds; and the Diameter of a Ball of Gold is required weighing 15 Pounds. You must first find the Diameter of a Ball of Gold, weighing as much as that of Silver, and then augment it by means of the Line of Solids. To do which, open the Sector, so that the Distance between the Points ♀, on each Line of Metals, be equal to M N. The Sector remaining thus opened, take the Distance of the Points ⊙ and ⊙ on each Line of Metals, which suppose to be the Diameter of the Ball of Gold O P; then open the Sector again, and apply this Distance to 10 and 10 of the Line of Solids, on each Leg of the Sector: The Sector remaining thus opened, take the Distance from 15 to 15, on each Line of Solids, and this last Extent Q R will be the Diameter of a Ball of Gold weighing 15 Pounds.

Of the Construction and Uses of the ENGLISH SECTOR.

THE principal Lines that are now generally put upon this Instrument, to be used Sector-wise, are the Line of Equal Parts, the Line of Chords, the Line of Sines, the Line of Tangents, the Line of Secants, and the Line of Polygons.

Plate 6. The Line of equal Parts, called also the Line of Lines, is a Line divided into 100 equal Parts; and if the Length of the Legs of the Sector is sufficient, it is again subdivided into Halfs and Quarters: they are placed on each Leg of the Sector, on the same Side, and are numbered by 1, 2, 3, *&c.* to 10, which is very near the End of each Leg. These Lines are denoted by the Letter L; it is divided into the same Number of equal Parts, as the same Line on the Sector described by our Author. And here note, that this 1 may be taken for 10, or for 100, 1000, 10000, *&c.* as Occasion requires; and then 2 will signify 20, 200, 2000, 20000, *&c.* and so of the rest.

The Line of Chords is a Line divided after the usual way of the Line of Chords, from a Circle, whose Radius is nearly equal in Length to the Legs of the Sector, beginning at the Center, and running towards the End thereof. It is numbered with 10, 20, *&c.* to 60, and to this Line, on each Leg, is placed the Letter C.

The Line of Sines is a Line of natural Sines, divided from a Circle of the same Radius, as the Line of Chords on the Sector was; these are also placed upon each Leg of the Sector, and numbered with the Figures 10, 20, 30, *&c.* to 90; at the End of which, on each Leg, is set the Letter S.

The Line of Tangents, is a Line of natural Tangents, divided from a Circle, and is placed upon each Leg of the Sector, and runs to 45 Degrees. It has the Numbers 10, 20, *&c.* to 45 placed upon it, with the Letter T for Tangent.

There is likewise another small Line of Tangents, divided from a Radius, of about two Inches, and is placed upon each Leg of the Sector; it begins at 45, which stands at the Length of the Radius from the Center, and runs to about 75 Degrees or farther, having the Numbers 45, 50, *&c.* to 75, with the Letter *t* set thereto. The Use of this Line (as hereafter shall be shewn) is to supply the Defect in the great Line.

The Line of Secants is only a Line of natural Secants, divided from a Circle of about two Inches Radius. These are placed upon each Leg of the Sector, beginning, not from the Center, but at two Inches Distance therefrom, and run to 75 Degrees. To these are set the Numbers 10, 20, *&c.* to 75, with the Letter S at the End thereof for Secants.

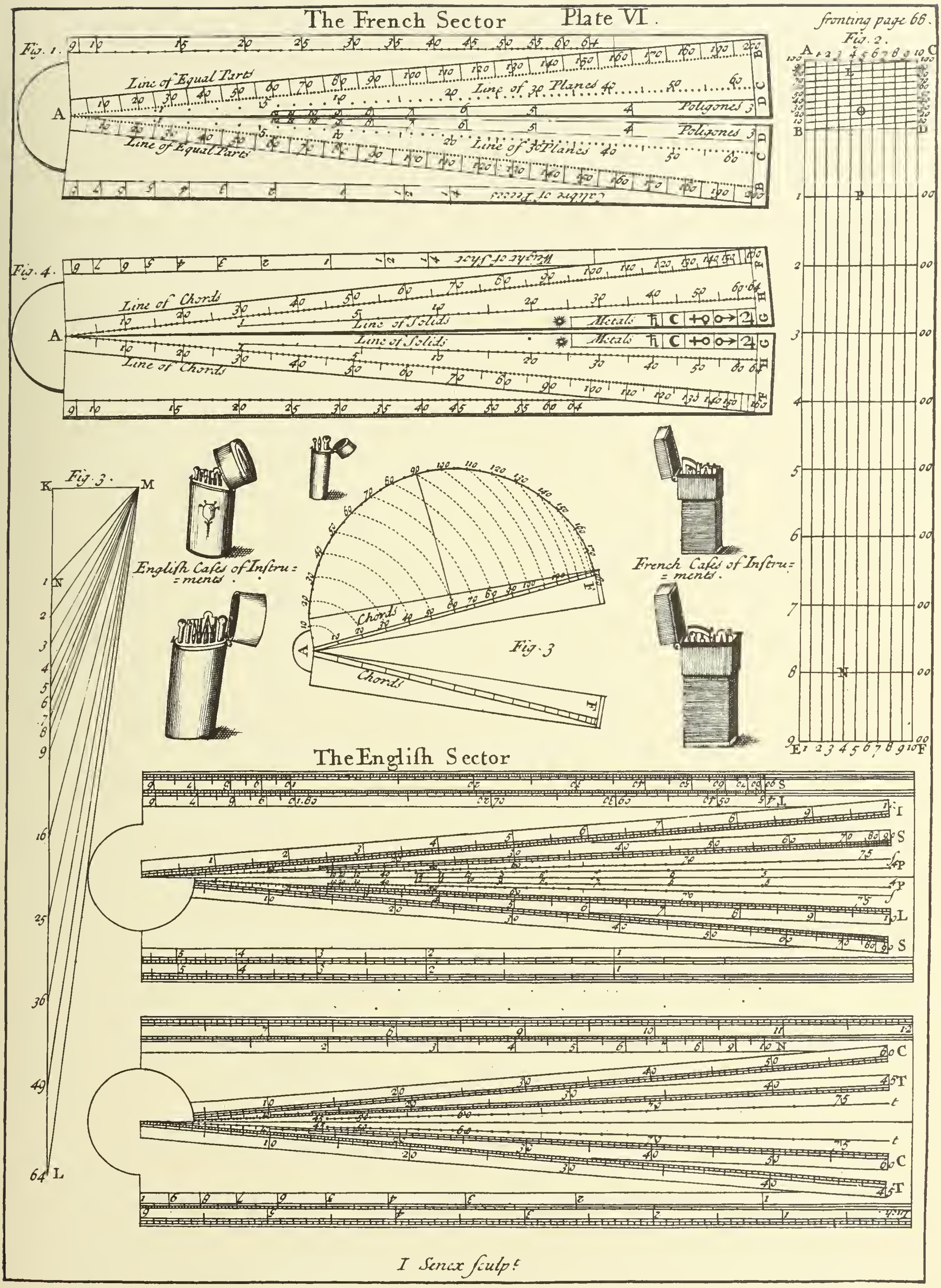
Finally, the Line of Polygons, denoted by the Letter P on each Leg of the Sector, is divided in the same manner as the Line of Polygons on the *French* Sector; only there the Number 3, for an equilateral Triangle, is the first Polygon, and here the Number 4 for a Square.

These are the principal Lines that are now put upon this Sector, to be used Sector-wise.

The other Lines, that are placed near and parallel to the outward Edges of the Sector, on both Faces thereof, and which are to be used, as on *Gunter*'s Scale, are,

1st, The Line of artificial Sines, numbered (as *per Fig.*) with 1, 2, 3, 4, 5, on one of the Legs, and with 6, 7, 8, *&c.* to 90, on the other Leg; which last Numbers, as they appear in the Figure, must be set backwards; to the End, that when the Sector is quite opened, they may become forwards. This Line is denoted by the Letter S, signifying Sines.

2dly, The



The French Sector
Plate VI.
fronting page 66.
Fig. 1.
Line of Equal Parts
Line of 30 Planes
Poligones
Calibre of Peeces
Fig. 2.
Fig. 4.
Line of Chords
Line of Solids
Metals
Weight of Shot
Fig. 3.
English Cases of Instru=
=ments.
Chords
Fig. 3
French Cases of Instru=
=ments.
The English Sector
I Senex sculpt.

2dly, The Line of artificial Tangents, placed next below the Line of artificial Sines, is numbered, on one Leg, 1, 2, 3, 4, 5, and on the other 6, 7, 8, *&c.* to 45. which laſt are likewiſe ſet backwards; but the Numbers 80, 70, 60, 50, placed at the Diviſions 10, 20, 30, 40, which ſignify their Complements, are ſet forwards.

3dly, Near the Edges, on the other Face of the Sector, is a Foot divided into 12 Inches, numbered 1, 2, 3, *&c.* and each Inch into 20 equal Parts. There is ſet to it *In.* ſignifying Inches.

4thly, and *Laſtly*, Next to that is placed *Gunter*'s Line of Numbers, denoted by the Letter N, as *per* Figure.

There are alſo ſome other Lines placed ſometimes upon the vacant Spaces of the Sector, as the Lines of Hours, Latitudes, and Inclinations of Meridians; which are no otherwiſe uſed than if they were placed upon common Scales.

All the aforeſaid Lines, except the ſmall Lines of Tangents, Secants, and the Line of Polygons, are furniſhed with Parallels, and the Diviſions marked by unequal Lines, that the Eye may the better diſtinguiſh them.

SECTION I.

Of the general USE *and Foundation of the Sector.*

The Excellency of this Inſtrument above the common Scales, or Rules, is, that it may be made to fit all Scales and Radius's; for by the Sector you may divide a Line, not exceeding it's Length when quite opened, into any Number of equal Parts; alſo from the Line of Chords, Sines, Tangents, *&c.* placed on the Sector, as before directed, you may have a Line of Chords, Sines, Tangents, *&c.* to any Radius, betwixt the Breadth and Length of the Sector when opened; by which Contrivance a Sector is made almoſt an univerſal Inſtrument. The Invention and Contrivance of this Inſtrument aroſe from a premeditate Conſideration of *Prop.* IV. *Lib.* 6. *Eucl.* where it is demonſtrated that ſimilar Triangles have their homologous Sides proportional.

For let the Lines A B, A C, repreſent the Legs of the Sector; and A D, A E, two equal Sections from the Center: then, I ſay, if the Points C, B, alſo D, E, are joined, the Lines C B, D E, *per Prop.* II. *Lib.* 6. *Eucl.* will be parallel; therefore becauſe the Lines D E, C B, are parallel, the Triangles A D E, A C B, *per Schol. Prop.* IV. *Lib.* 6. *Eucl.* will be ſimilar; and conſequently from the ſaid *Prop.* IV. the Sides A D, D E, A B, B C, are proportional: that is, As A D to D E, So is A B to B C; whence if A D be the half, or a third part of the Side A B, D E will be a half, or a third part of the Parallel C B; the like Reaſon holds of all other Sections: whence you ſee that if A D be the Chord, Sine, or Tangent of any Number of Degrees, to the Radius A B, D E will be the Chord, Sine, or Tangent of the ſame Number of Degrees to the Radius B C. *Plate 7. Fig. 21.*

Now the Lines found out by the Sector, are of two Sorts, *viz.* Lateral, or Parallel: Lateral are ſuch as are found upon the Sides of the Sector, as A B, A C: Parallel are the Lines that run from one Leg of the Sector to the other, in equal Diviſions from the Center, as D E, C B.

And here note, that the innermoſt of the Parallels, is the true divided Line, and therefore in uſing the Compaſſes, you muſt ſet them upon the innermoſt Line, both in lateral and parallel Entrance.

And further note, that the Lines are placed upon this Sector, different from thoſe that are placed upon Sectors formerly made; for inſtead of putting the ſame Lines at equal Diſtances from the inward Edges of both the Legs of the Sector, they are put at unequal Diſtances, as may be ſeen in the Figure: where, upon one Leg the Line of Chords is innermoſt, upon the other the Line of Tangents is innermoſt; that is, the innermoſt Line of Chords and Tangents are equally diſtant from the inward Edge, and ſo are the outermoſt Line of Chords and Tangents. The Benefit of the Contrivance is this, When you have ſet the Sector to a Radius for the Chords, it ſerves alſo for the Sines and Tangents without ſtirring it; for the Parallel betwixt 60 and 60 of the Chords, 90 and 90 of the Sines, alſo 45 and 45 of the Tangents, are all equal, which is the Reaſon that the Chords run but to 60 Degrees.

SECTION II.

Of the general USE *of the Lines of Chords, Sines, Tangents, and Secants, on the Sector.*

By diſpoſing and placing theſe Lines, as before directed, on this Inſtrument, we have Scales to ſeveral Radius's; that is, having a Length, or Radius given (not exceeding the Length of the Sector when opened), we can by the Sector find the Chord, Sine, *&c.* thereto; for which Property, this Inſtrument is often of great Uſe.

For Example; Suppoſe the Chord, Sine, or Tangent of 10 Degrees, to a Radius of three Inches, be required: take that three Inches, and make it a Parallel between 60 and 60 on the Line

Line of Chords, then, as I have already ſaid, the ſame Extent will reach from 45 to 45, on the Line of Tangents; alſo on the other Side of the Sector, the ſame Diſtance of three Inches, will reach from 90 to 90 on the Line of Sines: ſo that if the Lines of Chords be ſet to any Radius, the Lines of Sines and Tangents are alſo ſet to the ſame. Now the Sector being thus opened, if you take the parallel Diſtance between 10 and 10 on the Line of Chords, it will give the Chord of 10 Degrees. Alſo if you take the parallel Diſtance on the Line of Sines between 10 and 10, you will have the Sine of 10 Degrees. Laſtly, if you take the parallel Extent on the Line of Tangents, between 10 and 10, it will give you the Tangent of 10 Degrees.

If the Chord, or Tangent of 70 Degrees, had been required; then for the Chord you muſt take the parallel Diſtance of half the Arc propoſed, that is, the Chord of 35 Degrees, and repeat that Diſtance twice on the Arc you lay it down on, and you will have the Chord of 70 Degrees; and for finding the Tangent of 70 Degrees to the aforeſaid Radius, you muſt make uſe of the ſmall Line of Tangents: for the great one running but to 45 Degrees, the Parallel of 70 cannot be taken on that, therefore take the Radius of three Inches, and make it a Parallel between 45 and 45 on the ſmall Line of Tangents; and then the parallel Extent of 70 Degrees on the ſaid Line, is the Tangent of 70 Degrees to 3 Inches Radius.

If you would have the Secant of any Arc, then take the given Radius, and make it a Parallel between the Beginning of the Line of Secants, that is 0 and 0; ſo the parallel Diſtance between 10 and 10, or 70 and 70, on the ſaid ſecant Line, will give you the Secant of 10, or 70 Degrees, to the Radius of three Inches.

After this manner may the Chord, Sine, or Tangent of any Arc be found, provided the Radius can be made a Parallel between 60 and 60 on the Line of Chords, or between the ſmall Tangent of 45, or Secant of 0 Degrees. But if the Radius be ſo large, that it cannot be made a Parallel between 45 and 45 on the ſmall Line of Tangents, then there cannot be found a Tangent of any Arc above 45 Degrees, nor the Secant of no Arc at all to ſuch a Radius, becauſe all Secants are greater than the Radius, or Semi-diameter of a Circle.

If the Converſe of any of theſe things be required; that is, if the Radius be ſought, to which a given Line is the Chord, Sine, Tangent, or Secant of any Arc, ſuppoſe of 10 Degrees; then it is but making that Line (if it be a Chord) a Parallel on the Line of Chords between 10 and 10, and the Sector will ſtand at the Radius required; that is, the parallel Extent between 60 and 60, on the ſaid Chord-Line, is the Radius.

And ſo if it be a Sine, Tangent, or Secant, it is but making it a Parallel between the Sine, Tangent, or Secant of 10 Degrees, according as it is given; then will the Diſtance of 90 and 90 on the Sines, if it be a Tangent, the Extent from 45 to 45 on the Tangents, and if it be a Secant, the Extent or Diſtance between 0 and 0, be the Radius.

Hence, you ſee, it is very eaſy to find the Chord, Sine, Tangent, or Secant to any Radius.

SECTION III.

Of the USE *of the Sector in Trigonometry.*

USE I. *The Baſe* A C *of the right-lined right-angled Triangle* A B C *being given* 40 *Miles, and the Perpendicular* A B 30: *to find the Hypothenuſe* B C.

Fig. 22. Open the Sector, ſo that the two Lines of Lines may make a right Angle (by *Uſe* VI. of our Author's) then take, for the Baſe, A C, 40 equal Parts upon the Line of Lines on one Leg of the Sector; and for the Perpendicular A B, 30 equal Parts on the Line of Lines upon the other Leg of the Sector. Then the Extent from 40 on one Line, to 30 on the other, taken with your Compaſſes, will be the Length of the Hypothenuſe B C; and applying it on the Line of Lines, you will find it to be 50 Miles.

USE II. *The Perpendicular* A B *of the right-angled Triangle* A B C *being given* 30 *Miles, and the Angle* B C A 37 *Degrees*; *to find the Hypothenuſe* B C.

Fig. 22. Take the given Side A B, and ſet it over, as a Parallel, on the Sine of the given Angle A C B; then the parallel Radius will be the Length of the Hypothenuſe B C, which will be found 50 Miles, by applying it on the Line of Lines.

USE III. *The Hypothenuſe* B C *being given, and the Baſe* A C; *to find the Perpendicular* A B.

Fig. 22. Open the Sector, ſo that the two Lines of Lines may be at right Angles; then lay off the given Baſe A C on one of theſe Lines from the Center; take the Hypothenuſe B C in your Compaſſes, and ſetting one Foot in the Term of the given Baſe A C, cauſe the other to fall on the Line of Lines on the other Leg of the Sector, and the Diſtance from the Center to where the Point of the Compaſſes falls, will be the Length of the Perpendicular A B.

USE IV. *The Hypothenuſe* B C *being given, and the Angle* A C B; *to find the Perpendicular* A B.

Take the given Hypothenuſe B C, and make it a parallel Radius, and the parallel Sine of the Angle A C B will be the Length of the Side A B.

USE

USE V. *The Base* AC, *and Perpendicular* AB *being given, to find the Angle* BCA.

Lay off the Base AC on both Sides of the Sector from the Center, and note it's Extent; then take the Perpendicular AB, and to it open the Sector in the Terms of the Base AC: so the Parallel Radius will be the Tangent of BCA.

USE VI. *In any right-lined Triangle, as* ABC, *the Sides* AC, *and* BC, *being given, one* 20 *Miles, and the other* 30, *and the included Angle* ACB 110 *Degrees, to find the Base* AB.

Open the Sector, so that the two Lines of Lines may make an Angle equal to the given Angle ACB of 110 Degrees; then take out the Sides AC, CB, of the Triangle, and lay them off from the Center of the Sector on each of the Lines of Lines, and take in your Compasses the Extent between their Terms, or Ends, and that will be the Length of the sought Side AB, which will be found 41 $\frac{1}{2}$ Miles. Fig. 23.

USE VII. *The Angles* CAB, *and* ACB, *being given, and the Side* CB: *to find the Base* AB.

Take the given Side CB, and turn it into the parallel Sine of it's opposite Angle CAB, and the parallel Sine of the Angle ACB, will be the Length of the Base AB. Fig. 23.

USE VIII. *The three Angles of a Triangle, as* ABC, *being given, to find the Proportion of the Sides* AB, AC, BC.

Take the lateral Sines of the Angles ACB, CBA, CAB, and measure them in the Line of Lines, for the Numbers belonging to those Lines will give the Proportions of the Sides. Fig. 23.

USE IX. *The three Sides* AC, AB, CB, *being given, to find the Angle* ACB.

Lay the Sides AC, CB, on the Lines of Lines of the Sector from the Center, and let the Side AB be fitted over in their Terms; so shall the Sector be opened in those Lines, to the Quantity of the Angle ACB. Fig. 23.

USE X. *The Hypothenuse* AC, *of the right-angled Spherical Triangle* ABC, *being given, suppose* 43 *Degrees, and the Angle* CAB, 20 *Degrees, to find the Side* CB.

As Radius is to the Sine of the given Hypothenuse 43 Degrees, So is the Sine of the given Angle CAB 20 Degrees, to the Sine of the Perpendicular CB. Fig. 24.

Take either the lateral Sine of the given Angle CAB, 20 Degrees, and make it a parallel Radius; that is, take 20 Degrees from the Center on the Line of Sines, in your Compasses, and set that Extent from 90 to 90; then the parallel Sine of 43 Degrees, the given Hypothenuse, will, when measured from the Center on the Line of Sines, give 13 Deg. 30 Min. Or take the Sine of the given Hypothenuse AC, 43 Degrees, and make it a parallel Radius; and the parallel Sine of the given Angle CAB, taken and measured laterally on the Line of Sines, will give the Length of the Perpendicular CB, 13 Deg. 30 Min. as before.

USE XI. *The Perpendicular* BC *given, and the Hypothenuse* AC, *to find the Base* AB.

As the Sine Complement of the Perpendicular BC, is to Radius, So is the Sine Complement of the Hypothenuse AC, to the Sine Complement of the Base required. Fig. 24.

Make the Radius a parallel Sine of the given Perpendicular BC, *viz.* 76 Deg. 30 Min. and then the parallel Sine of the Complement of the given Hypothenuse, *viz.* 47 Degrees, measured laterally on the Line of Sines, will be found 49 Degrees, 25 Minutes: therefore the Complement of the required Base, will be 49 Degrees, 25 Minutes; and consequently the Base will be 40 Degrees, 35 Minutes.

The Use of the Sector in the Solution of the before-mentioned Cases of Trigonometry, being understood, it's Use in solving the other Cases, which I have omitted, will not be difficult.

Note, The several Uses of the Line of Lines, and Line of Polygons, on this Sector, are the same as the Uses of these Lines upon the *French* Sector, which see.

I now proceed to give some of the particular Uses of the Sector in Geometry, Projection of the Sphere, and Dialling.

SECTION IV.

USE I. *To make any regular Polygon, whose Area shall be of a given Magnitude.*

LET it be required to find the Length of one of the Sides of a regular Pentagon, whose superficial Area shall be 125 Feet, and from thence to make the Polygon.

Having extracted the square Root of $\frac{1}{5}$ Part of 125 (because the Figure is to have 5 Sides) which Root will be 5; make the Square AB, whose Side let be 5 Feet: then by means of the Line of Polygons (as directed by our Author in *USE* I. of the Line of Polygons) upon any right Line, as CD, make the Isosceles Triangle CGD so, that CG, being the Semi-diameter of a Circle, CD may be the Side of a regular Pentagon inscribed in it, and let fall the Fig. 25.

 Perpendicular

Perpendicular G E. Now continuing the Lines E G, and E C, make E F equal to the Side of the Square A B; and from the Point F, draw the right Line F H parallel to G C; then a mean Proportional between G E, and E F, will be equal to half the Side of the Polygon fought, which doubled, will give the whole Side. Now having found the Length of the whole Side, you muft, upon the Line expreffing it's Length, make a Pentagon (as directed by our Author in *USE* II. of the Line of Polygons), which will have the required Magnitude.

USE II. *A Circle being given, to find the Side of a Square equal to it.*

Fig. 26. Let E F be the Diameter of the given Circle, which divide into 14 equal Parts, by means of the Line of Lines (as directed by our Author in the Ufe of the Line of equal Parts), then E P, which is 12.4 of thofe Parts, will be the Side of the Square fought.

Note, 12.4 is the fquare Root of 11×14.

USE III. *A Square being given, to find the Diameter of a Circle equal to it.*

Fig. 27. Let A B be one Side of the given Square, which divide into 11 equal Parts, by means of the Line of Lines on the Sector; then continue the faid Side, fo that A G may be 12.4; that is, 1.4 of thofe Parts more, and the Line A G, will be the Diameter of a Circle, equal to the Square whofe Side is A B.

USE IV. *The tranfverfe and conjugate Diameters of an Ellipfis being given, to find the Side of a Square equal to it.*

Fig. 28. Let A B, and C D, be the tranfverfe and conjugate Diameters of an Ellipfis: firft, find a mean Proportional between the tranfverfe and conjugate Diameters, which let be the Line E F; then divide the faid Line E F, into 14 equal Parts, 12 and $\frac{4}{10}$ of which, will be E G, the Side of the Square equal to the aforefaid Ellipfis.

USE V. *To find the Magnitude of two right Lines which fhall be in a given Ratio; about which, an Ellipfis being defcribed, in taking them for the tranfverfe and conjugate Diameters, the Area of the faid Ellipfis, may be equal to a given Square.*

Fig. 29. Let the given Proportion that the tranfverfe and conjugate Diameters are to have, be as 2 to 1; then divide the Side A B of the given Square, into 11 equal Parts. Now as 2 is to 1 (the Terms of the given Proportion), So is $11 \times 14 = 154$ to a fourth Number; the fquare Root of which being extracted, will be a Number to which, if the Line A G is taken equal (fuppofing one of thofe 11 Parts the Side of the Square is divided into, to be Unity), the faid Line A G, will be the conjugate Diameter fought. Then to find the tranfverfe Diameter, fay, As 1 is to 2, So is the conjugate Diameter A G, to the tranfverfe Diameter fought. To work the firft of the faid Proportions by the Line of Lines on the Sector, fet 1 over as a Parallel on 2; then the parallel Extent of 154 taken, and laterally meafured on the Line of Lines, will give 77, the fourth Proportional fought. In the fame manner may the latter Proportion be worked.

USE VI. *To defcribe an Ellipfis, by having the tranfverfe and conjugate Diameters given.*

Fig. 30. Let A B, and E D, be the given Diameters: take the Extent A C, or C B, between your Compaffes, and to that Extent, open the Legs of the Sector fo, that the Diftance between 90 and 90 of the Line of Sines, may be equal to it: then may the Line A C be divided into a Line of Sines, by taking the parallel Extents of the Sine of each Degree, on the Legs of the Sector, between your Compaffes, and laying them off from the Center C; the Line A C being divided into a Line of Sines (I have only divided it into the Sine of every 10 Degrees) from every of them raife Perpendiculars both ways. Now to find Points in the faid Perpendiculars, through which the Ellipfis muft pafs, take the Extent of the femi-conjugate Diameter C E, between your Compaffes; and then open the Sector fo, that the Points of 90 and 90, on the Lines of Sines of the Sector, may be at that Diftance from each other. This being done, take the parallel Sines of each Degree, of the Lines of Sines of the Sector, and lay them off, on thofe Perpendiculars drawn thro' their Complements, in the Line of Sines A C, both ways from the faid Line A C, and you will have two Points in each of the Perpendiculars thro' which the Ellipfis muft pafs.

As for Example, the Sector always remaining at the fame Opening, take the Diftance from 80 to 80, on the Line of Sines, between your Compaffes, and fetting one Foot in the Point 10, on the Line A C; with the other make the Points *a* and *b*, in the Perpendicular paffing thro' that Point: then the Points *a* and *b*, will be the two Points in the faid Perpendicular, thro' which the Ellipfis muft pafs. All the other Points, in this manner, being found, if they are joined by an even Hand, there will be defcribed the Semi-Ellipfis D A E. In the fame manner may the other half of the Ellipfis be defcribed.

Fig. 31. USE VII. *The Bearings of three Towers, ftanding at* A B C, *to each other being given, that is, the Angles* A B C, B C A, *and* C A B; *and alfo the Diftances of each of them from a fourth Tower ftanding between them, as at* D, *being given; that is,* B D, D C, *and* A D *being*

given:

given: to find the Distances of the Towers at A B C *from each other; that is, to find the Lengths of the Sides* A B, B C, A C, *of the Triangle* A B C.

Having drawn the Triangle E F G similar to A B C, divide the Side E G in the Point H; so that E H may be to H G, as A D is to D C; which may be done by taking the Sum of the Lines A D and D C between your Compasses, and setting that Extent over as a Parallel on the Line of Lines of the Sector, upon the Side E G of the Triangle, laterally taken on the Line of Lines; for then the parallel Extent of A D will give the Length of E H, and consequently the Point H will be had. Fig. 32.

In like manner must the Side E F (or F G) be divided so in I, that E I may be to I F, as A D is to D B (or F G must be so divided, that the Segments must be as B D to D C).

Again, having continued out the Sides E G, E F, say, As E H — H G is to H G, So is E H + H G to G K; and as E I — I F is to I F, So let E I + I F be to F M, which Proportions may easily be worked by the Line of Lines on the Sector. This being done, bisect H K and I M, in the Points L N; and about the said Points, as Centers, and with the Distances L H and I N describe two Circles intersecting each other in the Point O; to which, from the Angles E F G, draw the right Lines E O, F O, and O G, which will have the same Proportion to each other, as the Lines A D, B D, D C. Now if the Lines E O, F O, and G O are equal to the given Lines A D, B D, D C, the Distances E F, F G, and E G, will be the Distances of the Towers sought. But if E O, O F, O G are lesser than A D, D B, D C, continue them out so, that P O, O R, and O Q be equal to them; then the Points P, Q, R being joined, the Distances P R, R Q, and P Q will be the Distances of the Towers sought. Lastly, If the Lines E O, O F, O G, are greater than A D, D B, D C, cut off from them Lines equal to A D, B D, D C, and join the Points of Section by three right Lines; then the Distances of the said three right Lines, will be the sought Distances of the three Towers.

Note, If E H be equal to H G, or E I to I F, the Centers L and N, of the Circles, will be infinitely distant from H and I; that is, in the Points H and I there must be two Perpendiculars raised to the Sides E F, E G, instead of two Circles, 'till they intersect each other: But if E H be lesser than H G, the Center L will fall on the other Side of the Base E G continued; understand the same of E I, I F.

USE VIII. *To project the Sphere Orthographically upon the Plane of the Meridian.*

Let the Radius of the Meridian Circle, upon which the Sphere is to be projected, be A E; then divide the Circumference of the said Circle into four equal Parts in E, P, Æ, S, and draw the Diameters E Æ, P S; the former of which will represent the Equator, and the latter P S, the Hour-Circle of 6, as also the Axis of the World; P being the North-Pole, and S the South-Pole. Then must each Quarter of the Meridian be divided into 90 Degrees, by making the Extent from 60 to 60 of the Line of Chords, on the Sector, equal to the Radius of the Meridian Circle; and taking the parallel Extent of every Degree, and laying them off from the Equator towards the Poles; in which if 23 Deg. 30 Min. be numbered, (*viz.* the Sun's greatest Declination) from E to ♋ Northwards, and from Æ to ♑ Southwards, the Line drawn from ♋ to ♑ will be the Ecliptick, and the Lines drawn Parallel to the Equator, through ♋ and ♑, will be the Tropicks. *Plate* 8. Fig. 1.

Now if each Semidiameter of the Ecliptick be divided into Lines of Sines (by making the Distance of the Points of 90 and 90, on the Line of Sines of the Sector, equal to either of the Semidiameters, and taking out the parallel Extent of each Degree, and laying them off both ways from the Center A), the first 30 Degrees, from A towards ♋, will stand for the Sign *Aries*; the 30 Degrees next following for *Taurus*; the rest for ♊, ♋, ♌, &c. in their Order.

If, again, A P, A S, are divided into Lines of Sines, and have the Numbers 10, 20, 30, &c. to 90 set to them, the Lines drawn thro' each of these Degrees, parallel to the Equator, will represent the Parallels of Latitude, and shew the Sun's Declination.

If, moreover, A E, A Æ are divided into Lines of Sines, and also the Parallels, and then there is a Line carefully drawn thro' each 15 Degrees; the Lines so drawn will be *Elliptical*, and will represent the Hour-Circles; the Meridian P E S the Hour of 12 at Noon; that next to it, drawn thro' 75 Degrees from the Center, the Hours of 11 and 1; that which is drawn thro' 60 Degrees from the Center, the Hours of 10 and 2, &c.

Then with respect to the Latitude, you may number it from E, Northwards, towards Z, and there place the Zenith (that is, make the Arc E Z 51 Deg. 32 Min. for *London*); thro' which, and the Center, the Line Z A N being drawn, will represent the vertical Circle passing thro' the Zenith and Nadir East and West; and the Line M A H, crossing it at right Angles, will represent the Horizon. These two being divided, like the Ecliptick and Equator, the Lines drawn thro' each Degree of the Radius A Z, parallel to the Horizon, will represent the Circles of Altitude, and the Divisions in the Horizon, and it's Parallels will give the Azimuths, which will be Ellipses.

Lastly, If thro' 18 Degrees in A N, be drawn a right Line I K, parallel to the Horizon, it will show the Time of Day-breaking, and the End of Twilight. For an Example of this Projection, let the Place of the Sun be the last Degree of ♉, the Parallel passing thro' this Place is L D, and therefore the Meridian Altitude will be M L; the Depression below the Horizon

Horizon at Midnight H D; the semidiurnal Arc L C; the seminocturnal Arc C D; the Declination A *b*; the ascensional Difference *b* C; the Amplitude of Ascension A C: The Difference between the End of Twilight, and the Break of Day, is very small; for the Sun's Parallel hardly crosses the Line of Twilight.

If the Sun's Altitude be given, let a Line be drawn for it parallel to the Horizon; so it shall cross the Parallel of the Sun, and there shew both the Azimuth and the Hour of the Day. As suppose the Place of the Sun being given, as before, the Altitude in the Morning was found, 20 Degrees, the Line F G, drawn parallel to the Horizon thro' 20 Degrees in A Z, would cross the Parallel of the Sun in ⊙; wherefore F ⊙ shews the Azimuth, and L ⊙ the Quantity of the Hour from the Meridian, which is about half an Hour past 6 in the Morning, and about half a Point from the East. The Distance of two Places may be also shewn by this Projection, in having their Latitudes and Difference of Longitude given.

For suppose a Place in the East of *Arabia* hath 20 Degrees of North Latitude, whose Difference of Longitude from *London*, by an Eclipse, is found to be five Hours and an half: Let Z be the Zenith of *London*, and the Parallel of Latitude for that other Place be L D, in which the Difference of Longitude is L ⊙; wherefore ⊙ representing the Position of that Place, draw thro' ⊙ a Parallel to the Horizon M H, crossig the vertical A Z about 70 Degrees from the Zenith; which multiplied by 69, the Number of Miles in a Degree, gives 4830 Miles, the Distance of that Place from *London*.

USE IX. *To project the Sphere Stereographically upon the Plane of the Horizon; suppose for the Latitude of* 51 *Degrees*, 32 *Minutes.*

Fig. 2. Draw a Circle of any Magnitude at pleasure, as N E, S W, representing the Horizon; in which draw the two Diameters, W E, N S, crossing one another at right Angles, which will be the Representations of two great Circles of the Sphere crossing each other at right Angles in the Zenith. Let N represent the North, E the East, S the South, and W the West Part of the Horizon.

Note, In all these Projections, the Eye is commonly supposed to be in the Under-pole of the primitive Circle, projecting that Hemisphere which is opposed to the Eye, which will all fall within the primitive Circle; but that Hemisphere in which the Eye is, will all fall without the primitive Circle, and will run out in an infinite annular Plane, in the Plane of the Projection, and consequently cannot all of it be projected by Scale and Compass.

I. But now let us begin with projecting the Equinoctial. And here we must first determine the Line of Measures, in which the Center of this Circle will be; and this will be done by determining in what Points a Plane, perpendicular to the primitive Circle, will cut the Horizon, whether in the North and South, East and West, or in what other intermediate Points such a Plane shall cut it. The Pole of the World, in this Projection, is elevated 51 Deg. 32 Min. and consequently the Equinoctial, on the Northern Part of the Horizon, will fall below the Horizon, and it is the Southern Part which here must be projected, or which will fall within the primitive Circle; that Plane, whose Intersection with the Horizon shall produce the Line of Measures, will be the Plane of a Meridian passing thro' the North and South Parts of the Horizon: wherefore N S will be the Line of Measures, in which the Center of the projected Equinoctial must fall; and since it is the Southern Part of the Equinoctial which we are to project, it's Center will be towards the North.

To find whereabouts in the Line of Measures the said Center will fall, you must first open the Legs of the Sector, so that the Distance from 45 Degrees to 45 Degrees, on the Lines of Tangents, is equal to the Radius of the primitive Circle; then take the parallel Extent of the Tangents of 38 Deg. 28 Min. the Height of the Equinoctial above the Plane of the Horizon, and lay it off from Z to *n*, and *n* will be the Center of the projected Equinoctial; and the Secant of the same, 38 Deg. 30 Min. will give it's Radius, with which the Circle W Q E must be described, which is the Representation of that part of the Equinoctial which is above our Horizon, for the Latitude of 51 Deg. 32 Min.

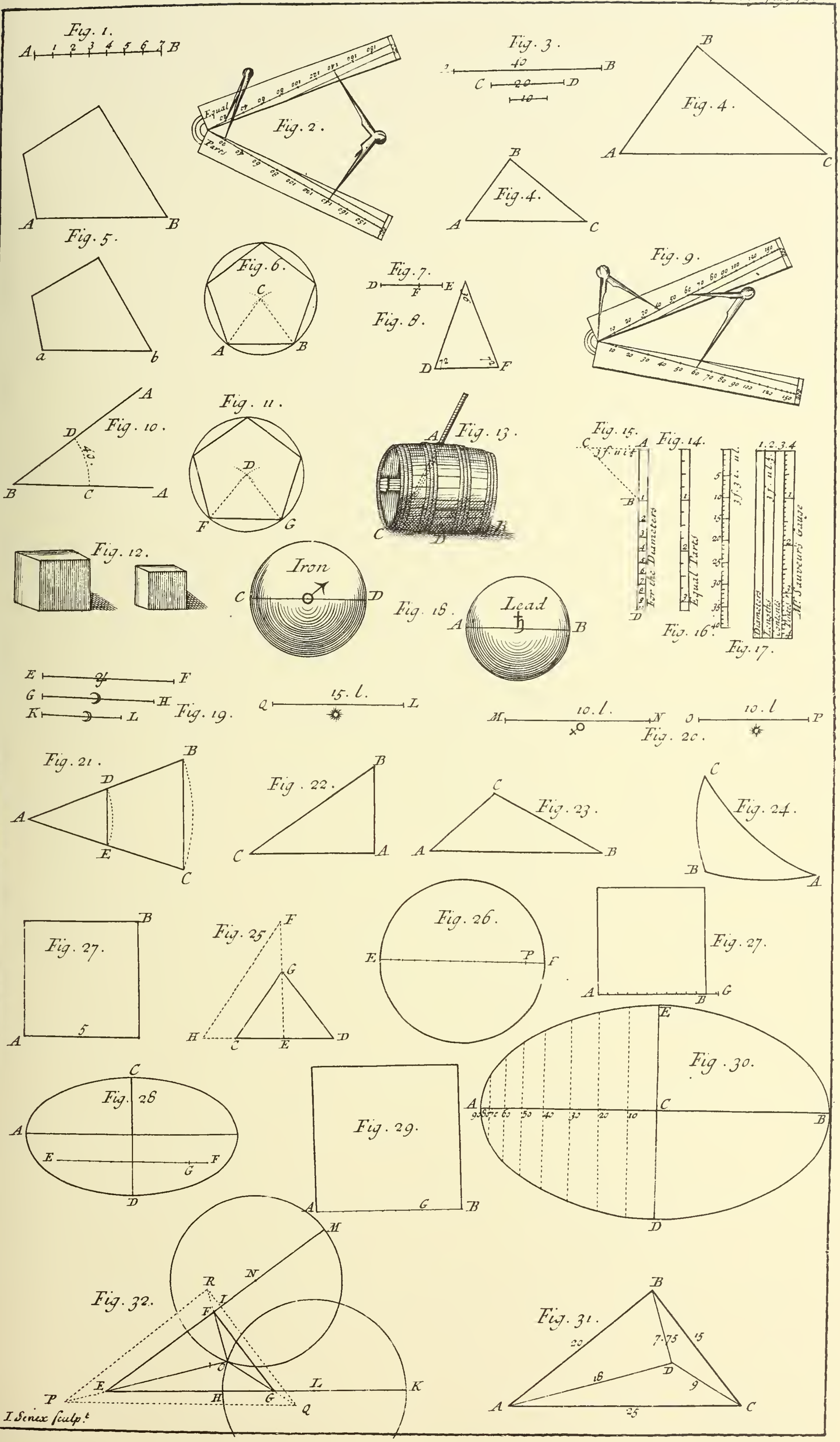
II. We will next project the Ecliptick, which being a great Circle of the Sphere, must cut the Equinoctial at a Diameter's Distance; that is, in E, W, the East and West Points of the Horizon, and consequently will have the same Line of Measures with that of the Equinoctial, *viz.* N S. Now let us consider whether the Center of the Ecliptick falls towards the North, or towards the South of the Horizon; and this will easily be determined, by considering that the Equinoctial is elevated above the Southern Part of the Horizon 38 Deg. 28 Min. and the Northern Part of the Ecliptick, or the Northern Signs, are elevated above the Equinoctial 23 Deg. 30 Min. which in all, make 62 Degrees, which is lesser than 90 Deg. So that it must fall towards the South, and consequently the Center must be Northwards, and will be found (the Sector remaining open as before), by setting off the Tangent of 62 Deg. from z to *b*, and the Secant of 62 Deg. will give it's Radius; with which the Circle W C E, the Representation of the Northern half of the Ecliptick, must be described.

The Southern Part of the Ecliptick is likewise, for the most part, projected on the horizontal Projection, and made to fall within the primitive Circle; but this cannot be, the Globe remaining fixed: for that part of the Ecliptick, which is below the Horizon, will be thrown out of the primitive Circle; so that it cannot be projected, unless the Globe be supposed to

be

Plate VII.

fronting page 72.



be turned round, and by that means the Southern Part of the Ecliptick to be brought above the Horizon; but such a Revolution of the Sphere, where it makes any Alteration, is scarce allowable: however, I shall shew how it is usually projected.

The same Line of Measures N S remains still, and the Circle must fall to the South, and consequently it's Center to the North of the Horizon; therefore nothing remains but to find it's Elevation above the Horizon. The Northern Part of the Ecliptick falls 23 Deg. 30 Min. nearer the Zenith than the Equinoctial does; therefore the Southern Part, being brought above the Horizon, must be 23 Deg. 30 Min. nearer the Horizon than the Equinoctial: So that 23 Deg. 30 Min. being taken from 38 Deg. 28 Min. there remains 15 Deg. for the Distance of that part of the Ecliptick above the Horizon. It will be represented by W *e* E, which is described by setting off the Tangent of 15 Degrees for the Center, and taking the Secant of the same for the Radius.

III. N S produced will also be the Line of Measures for all Parallels of Declination, and Parallels of Latitude: for the Poles of lesser Circles being the same as those of the great Circles, to which they are parallel, it is manifest that the same Plane, which is at right Angles to the Equinoctial and Horizon, will also be at right Angles to all lesser Circles parallel to the Equinoctial, and the same will hold as to Circles parallel to the Ecliptick: But N S is the Line of Measures of the Equinoctial and Ecliptick, and consequently must be the Line of Measures of all Circles parallel to either of them; therefore the Centers of such lesser Circles will be in N S produced, if there be Occasion. Now to project them, for Instance, the Tropick of *Cancer*; consider, in this Position of the Sphere, what will be it's nearest and greatest Distance from the Zenith, or the Pole of the primitive Circle, which you will find to be 28 Degrees; for the Equinoctial being elevated 38 Deg. 28 Min. above the Horizon, and the Tropick of *Cancer* being 23 Deg. 30 Min. from the Equinoctial, which, being added together, gives 62 Deg. which substracted from 90 Deg. leaves 28 Deg. it's Distance from the Zenith on the South-side of the Horizon; therefore the Half-Tangent of 28 Deg. or the Tangent of 14 Deg. set from Z to C, will give one Extremity of it's projected Diameter: Then the Distance from the Zenith to the Pole, being 38 Deg. 28 Min. and from the Pole to the Tropick of *Cancer* 66 Deg. 30 Min. the Sum of these, *viz.* 104 Deg. 58 Min. will be it's greatest Distance from the Zenith; the Half-Tangent of which, set from Z to *a*, will give the other Extremity of it's projected Diameter: therefore having got C *a* the Diameter, bisect it, and describe the Circle ♋ C ♋.

The Tropick of *Capricorn* may be described in the same manner: for the Distance of the Equinoctial and the Zenith being 51 Deg. 32 Min. if to this be added 23 Deg. 30 Min. you will have 55 Deg. 2 Min. equal to the nearest Distance of the Tropick of *Capricorn*, on the South-side of the Horizon; the Half-Tangent of which being set from Z to *e*, will give one Extremity of it's Diameter. Then the Distance between the Zenith and the Pole, *viz.* 38 Deg. 28 Min. and the Distance between the Pole and the Equinoctial, which is 90 Deg. and the Distance between the Equinoctial and the Tropick of *Capricorn*; which is 23 Deg. 30 Min. being all added together, will give the greatest Distance of the Tropick of *Capricorn*, from the Zenith, *viz* 152 Deg. 2 Min. the Semi-tangent of which being set from Z towards the North, will give the other Extremity of the Diameter. Bisect the Diameter found in *e*, and describe the Circle ♑ C ♑, which is the Representation of so much of the Tropick of *Capricorn*, as falls within the primitive Circle.

IV. The Polar Circle is 23 Deg. 30 Min. from the Pole; but the Pole being elevated, on the North-side the Horizon, 51 Deg. 32 Min. and 51 Deg. 32 Min. added to 23 Deg. 30 Min. whose Sum is 75 Deg. 2 Min. is lesser than 90 Deg. so that it does not pass beyond the Zenith; therefore 75 Deg. 2 Min. taken from 90 Deg. leaves 15 Deg. which is the nearest Distance of the Polar Circle from the Zenith: And the Half-Tangent of 15 Deg. set fron Z to *v*, will give one Extremity of it's projected Diameter; and then 15 Deg. added to 47 Deg. equal to 62 Deg. will be it's greatest Distance from the Zenith: the Half-Tangent of which Distance, set from Z to P, will give the other Extremity of it's projected Diameter; so that it's Diameter *v p* being found, it is but bisecting it, and the Circle may be described.

V. I shall now shew how to project the Hour-Circles. And, First, a Line of Measures must be determined, in which their Centers shall be, if possible; but you may easily discover it impossible for one Line of Measures to serve them all: for they are differently inclined to the Horizon, and so the Plane of no one great Circle can be at right Angles to the Horizon and all the Hour-Circles; therefore the Plane of a great Circle at right Angles to the Horizon, and one of them, must be found: which is possible, because the Hour-Circles being all at right Angles to the Plane of the Equinoctial, their Poles will be all found in this Circle; but the Poles of all great Circles, being 90 Degrees distant from their Planes, the Hour-Circle of 12, and the Hour-Circle of 6, must of necessity pass thro' each other's Poles, and so will be at right Angles to one another: But the Hour-Circle of 12 is at right Angles to the Horizon, and intersects it in N S; therefore the Line N S will be the Line of Measures, in which the Center of the Hour-Circle of 6 will be, and it's Center will be towards the South Parts of the Horizon, because all the Hour-Circles pass thro' the Pole which falls towards the North, the Elevation of this Circle above the Horizon being the same with that of the Pole, *viz.* 51 Deg. 32 Min. then take the Tangent of 51 Deg. 32 Min. and set it from Z to K; and upon the Center K, and with the Secant of the same Elevation, describe W P E, which is the Circle required.

The Point P, where N S, W E, intersect one another, is the Representation of the Pole of the World; for N S being the Representation of the Hour-Circle of 12, the projected Pole must be somewhere in this Line; but it must be somewhere in W E, which is likewise the Projection of an Hour-Circle: therefore it must be in that Point where these two projected Circles intersect one another, that is, in the Point P; P is the Point thro' which all the Hour-Circles must pass in the Projection.

In order to draw the rest of the Hour-Circles, we must have recourse to a *Secondary* Line of Measures, which may thus be determined: To P K, at the Point K, erect D B at right Angles, and produce the Circle W P E, 'till it meet the Line D B, in the Points D and B; and the Line D B will be the secondary Line of Measures in which the Centers of all the Hour-Circles will be found; for let the Hour-Circle of 6, D P B, be considered as the primitive Circle, in whose Under-Pole (which will be in the Equinoctial) K, let the Eye be placed; then D B will be the Representation of the Equinoctial, for it passing thro' the Eye will be projected into a right Line: but the Equinoctial is at right Angles to the Hour-Circles, both the primitive and all the rest; therefore it will be the secondary Line of Measures, upon this Supposition, upon which will be all their Centers. In order to find which, set the Sector to the Radius P K, then take off parallel-wise the Tangents of 15 Deg. 30 Deg. 45 Deg. the Elevations of the Hour-Circles above the Hour-Circle at 6, and set them both ways, from K to *r*, from K to *s*, from K to *r*, *&c.* then upon those Centers, and with the Secants of the same Elevations, describe the Circles P P, P Q, and P T, which will be the Hour-Circles; for they are all great Circles of the Sphere, passing thro' the Pole P, and make Angles with one another of 15 Deg. or are 15 Deg. distant from each other: and the Portions of those Circles which fall within the primitive Circle N E S W, as H P *h*, are the Representations of those Halves of the Hour-Circle, which are above our Horizon in our Latitude.

VI. In like manner the Circles of Longitude may be drawn, by determining the secondary Line of Measures R S, in which all their Centers will be; and this Line will be determined after the same manner with D B above, and the Circles of Longitude drawn as before the Meridians were drawn: for the Line N S will be the Line of Measures, with respect to one of them passing thro' E and W, the East and West Points of the Horizon. In order to draw this Circle, consider it's Elevation above the Horizon, which will be found by considering the Distance of the Pole of the Ecliptick, from the Pole of the World, which will be 28 Deg. 2 Min. the Elevation of this Circle above the Horizon. Set the Tangent of 28 Deg. 2 Min. from Z to Q, and with the Secant of the same Distance, describe the Circle W *p* E; to *p* Q, at the Point Q, erect R S at right Angles, which will be the secondary Line of Measures. In this Line from Q (the Sector being set to *p* Q), set off the Tangents of 24 Deg. 40 Deg. according to the Number of Circles you have a Desire to draw, from Q to *x*, from Q to *y*, *&c.* and with the Secants of 20 Deg. 40 Deg. *&c.* describe the Circles of Longitude, M P *m*, &c.

VII. The Representations of Azimuths, in this Projection, will be all right Lines, and any Number of them may be drawn, making any assigned Angles with one another, if the Limb be divided into it's Degrees by help of the Sector, and thro' these Degrees be drawn Diameters to the primitive Circle.

VIII. All Parallels of Altitude, in this Projection, will be Circles parallel to the primitive Circle, and may be easily drawn, by dividing a Radius of the primitive Circle, into Half-Tangents, and describing upon the Center Z, thro' the Points of Division, concentrick Circles. I shall omit drawing of them, lest the Scheme be too much perplexed.

USE X. *To project the Sphere Stereographically upon the Plane of the Solstitial Colure for the Horizon of 51 Deg. 32 Min.*

Fig. 3. Draw the Circle H B O C, representing the primitive Circle; and the Diameter H O, representing the Horizon: Set off the Chord of 51 Deg. 32 Min. from O to P, having first set the Sector to the Radius of the Circle, which will give the Polar Point, and draw the Diameter P *p*, representing the Hour-Circle of 6.

I. The Equinoctial may be represented, by drawing the Diameter E Q at right Angles to the Diameter P *p*.

II. Set off 23 Deg. 30 Min. from the Chords, from E to ♋, and from Q to ♑, which will represent the Ecliptick.

III. The Tropicks of *Cancer* and *Capricorn* may be drawn thus: Take the Secant of 66 Deg. 30 Min. the Distance of each of them from their respective Poles, and set it both ways, from the Center A in P *p* produced, which will give the two Points *e e* the Centers of the two Circles, and their Radii will be the Tangents of the same 66 Deg. 30 Min.

IV. The Polar Circles, as also all other Parallels of Declination, may be drawn in the same manner.

V. The Line of Measures for the Azimuths will be H O, and the Line of Measures for the Almacanters will be B C.

VI.

VI. ♋, ♑, or the Ecliptick, will be the Line of Meaſures for the Circles of Longitude, and the Line of Meaſures for the Circles of Latitude will be N S, all of which may be eaſily drawn from what is ſaid in the precedent Uſe.

VII. The Ecliptick may be divided into it's proper Signs in this Projection, by ſetting off the Tangents of 15 Deg. 30 Deg. 45 Deg. both ways from A.

USE XI. *To draw the Hour-Lines upon an erect direct South Plane, as alſo on an Horizontal Plane.*

Firſt, draw the indefinite right Line C C, for the Horizon and Equator, and croſs it at right Angles in the Point A, about the middle of the Line, with the indefinite right Line A B, ſerving for the Meridian, and the Hour Line of 12. then take out 15 Deg. from the Line of Tangents, on the Sector (the Sector being ſet to a parallel Radius leſſer than the Extent from 45 Deg. to 45 Deg. of the leſſer Lines of Tangents, when the Sector is quite opened), and lay them off in the Equator on both Sides from A, and one Point will ſerve for the Hour of 11, and the other for the Hour of 1. Again, Take out the Tangent of 30 Deg. (the Sector being opened to the ſame Radius), and lay it off on both Sides the Point A in the Equator, and one of theſe Points will ſerve one for the Hour of 10, and the other for the Hour of 2. In the ſame manner, lay off the Tangent of 45 Deg. for the Hours of 9 and 3, the Tangent of 60 Deg. for the Hours of 8 and 4, and the Tangent of 75 Deg. for the Hours of 7 and 5. But note, becauſe the greater Tangents on the Sector run but to 45 Deg. therefore you muſt ſet the parallel Radius of the leſſer Tangents, when you come above 45 Deg. to the Extent of the Radii of the greater Tangents. Fig. 4.

Now if you have a mind to ſet down the Parts of an Hour, you muſt allow 7 Deg. 30 Min. for every half Hour, and 3 Deg. 45 Min. for one quarter. This done, you muſt conſider the Latitude of the Place in which the Plane is, which ſuppoſe 51 Deg. 30 Min. then if you take the Secant of 51 Deg. 32 Min. off from the Sector, it remaining opened to the parallel Radius of the leſſer Tangents, and ſet it off from A to V, this Point V will be the Center of the Plane; and if you draw from V, right Lines to 11, 10, 9, *&c.* and the reſt of the Hour Points, they will be the required Hour Lines.

But if it happen, that ſome of theſe Hour Points fall out of the Plane, you may thus remedy yourſelf, by means of the larger Tangents.

At the Hour Points of 3 and 9, draw occult Lines parallel to the Meridian; then the Diſtances D C, between the Hour Line of 6, and the Hour Points of 3 and 9, will be equal to the Semi-diameter A V; and if they be divided in the ſame manner as the Line A C is divided, you will have the Points of 4, 5, 7, and 8, with their Halves and Quarters.

For take out the Semi-diameter A V, and make it a parallel Radius, by fitting it over in the Tangents of 45 and 45; then take the parallel Tangent of 15 Deg. and it will give the Diſtance from 6 to 5, and from 6 to 7. The Sector remaining thus opened, take out the parallel Tangent of 30 Deg. and it will give the Diſtance from 6 to 4, and from 6 to 8: the like may be done for Halves and Quarters of Hours.

The Hour Points may be otherwiſe denoted thus: Having drawn a right Line for the Equator, as before, and aſſumed the Point A for the Hour of 12, cut off two equal Lines A 10, and A 2, then upon the Diſtance between 10 and 2, make an equilateral Triangle, and you will have B for the Center of the Equator, and the Line A B, will give the Diſtance from A to 9, and from A to 3. This done, take out the Diſtance between 9 and 3, and this will give the Diſtance from B to 8, and from 8 to 7, and from 8 to 1: and again, from B to 4, and from 4 to 5, and from 4 to 11; ſo have you the Hour Points: and if you take out the Diſtances B 1, B 3, B 5, *&c.* the Points may be found not only for the Half-Hours, but for the Quarters.

In the ſame manner are the Hour Lines drawn on a Horizontal Plane, only with this Difference, that A H is the Secant of the Complement of the Latitude, and the Hour Lines of 4, 5, 7, 8, are continued thro' the Center.

USE XII. *To draw the Hour Lines upon a Polar Plane, as alſo on a Meridional Plane.*

In a Polar Plane, the Equator may be alſo the ſame with the Horizontal Plane, and the Hour Points may be denoted as before, in the laſt Uſe: but the Hour Lines muſt be drawn parallel to the Meridian. Fig. 5.

In a Meridional Plane, the Equator will make an Angle with the Horizontal Line, equal to the Complement of the Latitude of the Place; then may you aſſume the Point A, and there croſs the Equator with a right Line, which will ſerve for the Hour Line of 6: then the Tangent of 15 Deg. being laid off in the Equator on both Sides from 6, will give the Hour Points of 5 and 7; and the Tangent of 30 Deg. the Hour Points of 8 and 4; the Tangent of 45 Deg. the Hour Points of 3 and 9; the Tangent of 60 Deg. the Hour Points of 2 and 10: and laſtly, the Tangent of 75 Deg. will give the Hour Points of 1 and 11; and if right Lines are drawn thro' theſe Hour Points, croſſing the Equator at right Angles, theſe ſhall be the Hour Lines required.

USE

USE XIII. *To draw the Hour Lines upon a vertical declining Plane.*

Fig. 6. First draw A V the Meridian, and A E the Horizontal Line, crossing one another in the Point A; then take out A V, the Secant of the Latitude of the Place, which suppose 51 Deg. 32 Min. and prick it down on the Meridian from A to V. Now because the Plane declines, which suppose 40 Deg. Eastward, you must make an Angle of the Declination upon the Center A, below the Horizontal Line, on the left Side of the Meridian, because the Plane declines Eastwards; for if it had declined Westward, the said Angle must have been made on the right Side of the Meridian. This being done, take A H, the secant Complement of the Latitude, out of the Sector, and prick it down in the Line of Declination from A to H, as was done for the Semi-diameter in the Horizontal Plane: then draw an indefinite right Line thro' the Point A, perpendicular to A H, which will make an Angle, with the Horizontal Line, equal to the Plane's Declination, and will be as the Equator in the Horizontal Plane. Again, take the Hour Points out of the Tangents, as in the last Problem, and prick them down in this Equator on both Sides, from the Hour of 12 at A; then lay your Ruler, and draw right Lines thro' the Center H, and each of these Hour Points, and you will have all the Hour Lines of an Horizontal Plane, except the Hour of 6, which is drawn thro' H perpendicular to H A. Lastly, you must note the Interfections that these Hour Lines make with A E, the Horizontal Line of the Plane, and then if right Lines are drawn thro' the Center V, and each of these Interfections, they will be the Hour Lines required.

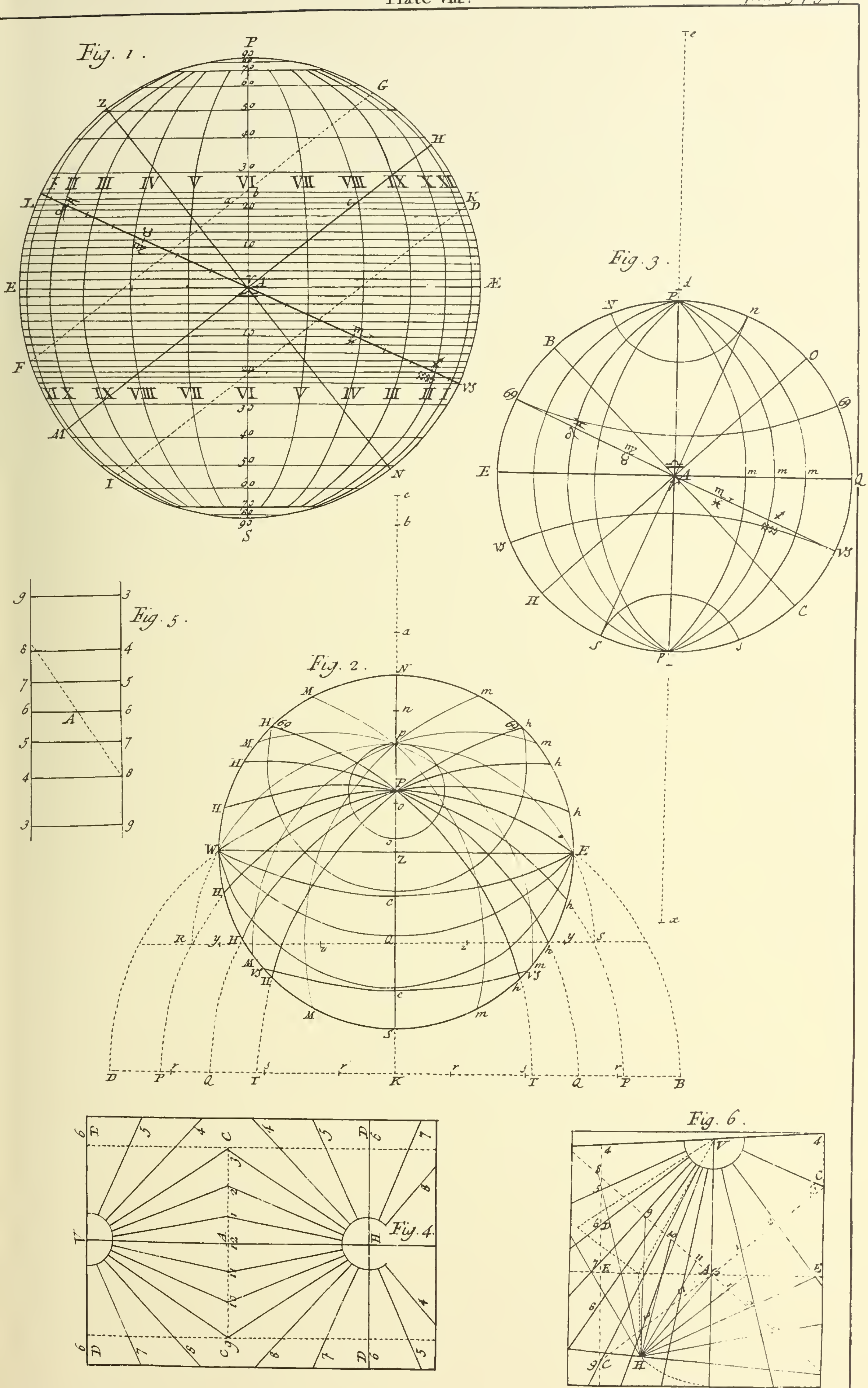
The Hour Points may be pricked down otherwise, thus: Take out the Secant of the Plane's Declination, and prick it down in the Horizontal Line from A to E, and thro' E draw right Lines parallel to the Meridian, which will cut the former Hour Lines of 3 or 9, in the Point C; then take out the Semi-diameter A V, and prick it down in those Parallels from C to D, and draw right Lines from A to C, and from V to D; the Line V D will be the Hour of 6: and if you divide those Lines A C, D C, in the same manner as D C is divided in the Horizontal Plane, the Hour Points required will be had.

Or you may find the Point D, in the Hour of 6, without knowing either H or C; for having pricked down A V in the Meridian Line, and A E in the Horizontal Line, and drawn Parallels to the Meridian thro' the Points at E, take the Tangent of the Latitude out of the Sector, and fit it over in the Sines of 90 Deg. and 90 Deg. and the parallel Sine of the Plane's Declination, measured in the same Tangent Line, will there shew the Complement of the Angle D V A, which the Hour Line of 6 makes with the Meridian: then having the Point D, take out the Semi-diameter V A, and prick it down in those Parallels from D to C; so shall you have the Lines D C, A C, to be divided, as before.

Thus have you the Use of the Sector applied in resolving several useful Problems. I might have laid down many more Problems in all the practical Parts of Mathematicks, wherein this Instrument is useful; but what I, and our Author have said of this Instrument, will, I believe, be sufficient to shew Persons skilled in the several practical Parts of Mathematicks, the Manner of using this Instrument therein.

For the Uses of the Lines of Numbers, Artificial Sines, and Tangents; as also the Lines of Latitude, Hours, and Inclination of Meridians; *See* USES *of* Gunter's *Scale.*

BOOK



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BOOK III.

Of the Conſtruction and Uſe of ſeveral different Sorts of Compaſſes, *and other Curious Inſtruments.*

CHAP. I.

Of the Conſtruction and Uſes of ſeveral Sorts of Compaſſes.

HAVING already treated of Common Compaſſes, uſually put into Caſes of Inſtruments, we proceed now to mention ſome others, ſometimes likewiſe placed in Caſes of different Bigneſſes.

The Conſtruction of Hair-Compaſſes.

Theſe Compaſſes are ſo called, becauſe of a Contrivance in the Body of them, by means of which an Extent may be taken to a hair's Breadth. We have before hinted, that the Goodneſs of Compaſſes conſiſts chiefly in having the Motion of their Head ſufficiently eaſy, and that they open and ſhut very equally; and that they may do ſo, the Joints ought to be well ſlit, and very equal in Thickneſs. *Plate* 9. Fig. A.

The Manner of conſtructing the Joints, is thus: We firſt, with a Steel-Saw, ſlit the Head in two Places, ſo that there remains a Middle-Piece, the Thickneſs of a Card; then we ſlit the other Leg of the Compaſſes, in the Middle of the Joint, to receive the Middle-Piece which was reſerved for that purpoſe; afterwards the Joints muſt be filed and ſtraightned, ſo that they may be well joined every where. This being done, we drill a round Hole thorow the Middle of the Head, in Bigneſs proportional to that of the Compaſſes, for the Rivet to go through; the Rivet ought to be very round, and exactly fill the aforeſaid Hole. When we have rivetted it, the Head of the Compaſſes muſt be warmed, and a little yellow Wax poured between the Joints, for leſſening the Friction of the Legs in opening and ſhutting. Laſtly, we generally put upon the Head two turned Cheeks, ſerving for Counter-Rivets, and to preſerve the Head.

The little Screw at the Bottom of the Body of theſe Hair-Compaſſes, is to move the Steel Point backwards or forwards, at pleaſure: this Point is faſtened to the Top of the Compaſſes by two Rivets, ſo that in turning the Screw it ſprings. The other Steel Point muſt be ſoldered to the other Leg, as all other Points of Compaſſes are that are fixed. Now to fit theſe Points for ſoldering, they muſt be filed ſo, as to go into two Slits made in the Bottom of the Body of the Compaſſes, that there they may be well joined, and the Solder ſtrongly hold them.

Note, Solder is commonly made with Silver and Thirds of Copper, that is, twice more Silver than Copper: For Example; with one Dram of Silver, we mix half a Dram of Copper, which muſt be firſt melted in a Crucible, and afterwards, when cold, hammered to about the Thickneſs of a Card, and cut into ſmall Pieces that it may the ſooner run, when there is uſe for it. Solder is likewiſe often made with Copper and Zink mixed together, *viz.*

X In

In melting $\frac{3}{4}$ of Copper, with $\frac{1}{4}$ of Zink: In soldering, we use Borax finely bruised, which makes the Solder better run and penetrate the Joints, or any thing else to be soldered.

Of the German *Compasses.*

Fig. B. The Legs of these Compasses are something bent, so that, when shut, the Points only touch each other. One Point of these Compasses may be taken off, and others put on, by means of a small square Hole made in the Bottom of the Body, for the Points to go in, and a Screw to keep them fast when in: but these Points ought very well to fit the aforesaid square Hole, that they may not shake.

The Points generally put on, are,

First, A Drawing-Pen Point, by means of which, Lines fine or coarse may be drawn with Ink, by help of a little Screw near the Point of the Drawing-Pen. This Drawing-Pen Point, as well as the other Points to be put on, has a small Joint, almost like the Head of a Pair of Compasses, by means of which it may be kept perpendicular to the Paper, according as the Compasses are more or less opened. This Point is represented by *Fig.* 3.

Secondly, A Pencil-carrying Point, represented by *Fig.* 2, for drawing Lines with a Pencil.

And *Lastly*, a Dotting-Wheel Point, (*Fig.* 1.) whose Use is to make dotted Lines. What we call a Dotting-Wheel, is a little Wheel of Brass, or other Metal, about 3 Lines in Diameter, round which is made little pointed Teeth. This Wheel is fastened between two little Pieces of Brass by a small Pin, so that it may freely turn round, almost like a Spur; but the said Teeth must not be too far distant from each other, because then the Dots the Wheel makes, will also be too far distant from each other.

The Construction of these Compasses as to their Joints, *&c.* being the same as those before spoken of, I shall only add, that since the Beauty of Compasses consists very much in their being well polished; for this effect, we first rub the Compasses with Slate-Stone dipped in Water; then we rub every part of the Compasses with a flat Stick of soft Wood, and a Mixture of Emery tempered with Oil, or fine Tripoly. And lastly, we wipe the Compasses clean with a Cloth or Piece of Shamoy.

Of the Spring-Compasses.

Fig. C. These Compasses are all made of tempered Steel, which are so hard every where, that a File cannot touch them; and the Head of these Compasses is rounded, that by it's Spring it opens and shuts itself: the Circular Screw fix'd to one of the Legs, serves to open or shut it, by means of a Nut. These Compasses are very fit to take small Lengths, and make small Divisions; yet they ought to be but short, and so tempered, that they may have a good Spring, and not break.

Of the Clock-makers Compasses.

Fig. D. These Compasses, which are strong and solid, serve to cut Past-board, Brass, and other the like things; the Quadrant crossing it, serves strongly to fix it to a proposed Opening, by help of a Screw pressing against it. The Nut at the End of the said Quadrant, is to open or shut the Compasses at pleasure, in turning the said Nut, which ought to be so riveted to the Leg of the Compasses, that it may make the other Leg move forwards or backwards. The Four Points ought to be made of well tempered Steel. That of *Fig.* 1. is filed slopewise, like a Graving-Tool, to cut Brass; that of *Fig.* 2. is like a pointed Button: and the two other Points are in figure of the fixed Points of common Compasses; but they must be very strong in proportion to the Compasses.

There are different ways of tempering the Points of Compasses, or other Pieces of Steel: For Example; the Points of small Compasses are tempered by means of a Lamp, and a small Brass Pipe: for blowing in the Pipe, causes a strong lively Flame, in which putting the Points, or other Things, to be hardened, and they will become almost instantly red hot, and when they are cold, they will be very hard. But the Points of great Compasses, and other Steel Tools, are tempered with a Charcoal Fire, by blowing thro' the aforesaid Pipe, and heating them to a Cherry Colour, and afterwards putting them into Water, and then they will be rendered very hard.

Of the Three-legged Compasses.

Fig. E. The Use of these Compasses is to take three Points at once, and so to form a Triangle, or to lay down three Positions of a Map to be copied at once, *&c.*

The Construction of these Compasses doth not much differ from the Construction of the others, excepting only that the third Leg has a Motion every way, by means of a turned Rivet, riveted by one End to the two other Legs; and at the other End there must be a turned Cheek, and a round Plate serving for a Joint to the third Leg: the little Figure 1 shows how the Rivet is made.

Of the Sea-Chart Compasses.

Fig. F. The Legs of these Compasses are crooked, and widened towards the Head, so that by pressing the two Legs with your Hand, you may open them. Their Construction sufficiently appears from the Figure, and their Use will be mentioned in the Instruments for Navigation.

Of

Of the simple Proportional Compasses.

These Compasses are used in dividing of Lines into 2, 3, 4, or 5 equal Parts, as also to reduce small Figures to greater ones, and contrariwise, *&c.* You must take care in making these Compasses, that the Head be drilled in a right Line with the Legs, and that the Points are not one forwarder than another. Now if you have a mind to make one of these Pair of Compasses to take the $\frac{1}{2}$ of a Line, the Distance from the Center of the Joint to the Ends of either of the longest Legs, must be twice the Length of either of the shortest Legs; and so in proportion for others. *Note*, The Compasses of Figure G, are for dividing of Lines into 3 equal Parts; whence the Distance from the Center marked 5, to the Points 2, 2, is three times the Distance from the said Center, to the Points 3 or 4: So that if the third Part of the Line 2, 2, be required, it's whole Length must first be taken between the longest Legs of the Compasses, which remaining thus opened, the Distance between the Points of the shortest Legs, will be $\frac{1}{3}$ of the given Line. Fig. G.

Of the Moveable-headed Proportional Compasses.

The Use of these proportional Compasses, is to divide a given Line into any Number of equal Parts, as also to divide the Circumference of a Circle, so that a regular Polygon may be inscribed therein. Fig. H.

These Compasses consist of two equal Legs, each of which is furnished with two Steel Points, and are hollowed in, for a Cursor to slip up and down; in the middle of which Cursor, there is a Screw serving to join the Legs, and to fasten them in divers Places by means of a Nut: but the Legs must be hollowed in exactly in the Middle of their Breadth, that so the Center of the Cursor may be in a right Line with the Points of the Legs, and the Cursor slide very exactly along the Legs: as also the Head-Screw must exactly fill the Hole in the Cursor, so that nothing may shake when the Legs are fastened with the Nut.

Figure 1, represents the Screw; Figure 2, the Nut; Figure 3, half the Cursor, which must be joined by a like half. You may see by that little Figure, that there is a Piece in the Middle left exactly to fit the Hollow of the Leg of the Compasses: the shadowed Spaces of the said Figure, are to contain the two Sides of the Leg; understand the same of the other half of the Cursor.

Figure 1, is one of the Legs separate, upon which are the Divisions for equal Parts: for upon one Side of one of the Legs, are the Divisions for dividing of Lines into equal Parts; and upon one Side of the other Leg, are denoted the Numbers shewing how to inscribe any regular Polygon in a proposed Circle.

Now to make the Divisions for dividing Lines into equal Parts, take a well divided Scale, or a Sector, which is better, because it is almost a universal Scale: then take the exact Length of one of the Legs of your proportional Compasses between your Compasses, and having opened the Sector, so that the Distance between 120 and 120 of the Line of Lines be equal to that Extent, take the Distance from 40 to 40, which lay off upon the Leg of your Compasses, and at the End thereof, set the Number 2, which will serve to divide any given Line into two equal Parts: The Sector still continuing opened to the same Angle, take the Distance from 30 to 30, on the Line of equal Parts, and lay off upon the aforesaid Leg of the Compasses, where set down the Number 3, and that will give the Division for taking $\frac{1}{3}$ of any given Line. Again; take 24 equal Parts, as before, from the Line of Lines, lay them off upon the Leg, and that will give the Division for dividing a Line into 4 equal Parts.

Moreover, take 20 equal Parts, and that will give you the Division upon the Leg of the Compasses, serving to divide a Line into 5 equal Parts: the same Opening of the Sector will still serve to divide a Line into 7, 9, and 11 equal Parts. But to avoid Fractions, the aforesaid Opening must be changed, to make the Division of 6, 8, 10, and 12, upon the Leg: but before the said Opening of the Sector be altered, take the Distance from 15 to 15, which will give the Divisions for dividing a Line into 7 equal Parts.

Again; Take 12, and that will give the Division for dividing a Line into 9 equal Parts; and lastly, the Distance from 10 to 10, will give the Division for dividing any Line into 11 equal Parts.

But to make the Division for dividing a Line into 6 equal Parts, take between your Compasses the Length of one of the Legs of the proportional Compasses, and open the Sector so, that the Distance between 140 and 140, on each Line of equal Parts, be equal to the aforesaid Length. The Sector remaining thus opened, take the Distance from 20 to 20, on each Line of equal Parts, and lay it off upon the Leg of the Compasses, and that will give the Division for dividing a Line into 6 equal Parts.

Again; having taken the Length of the Leg of your Compasses, open the Sector, so that the Distance from 180 to 180, of each Line of equal Parts be equal thereto. Then take the Extent from 20 to 20, and that laid off upon the Leg of the Compasses, will give the Division for dividing a Line into 8 equal Parts.

Moreover, open the Sector so, that the Distance from 110 to 110, be equal to the Length of the Leg of your Compasses. The Sector remaining thus opened, the Distance from 10 to 10, will give the Division for dividing a Line into ten equal Parts.

Lastly,

Laſtly, the Sector being opened, ſo that the Length of the Leg of your Compaſſes be equal to the Diſtance from 130 to 130; and then the Diſtance from 10 to 10 will give the Diviſion for dividing a Line into twelve equal Parts.

The Uſe of this Line is eaſy: for ſuppoſe a right Line is to be divided into three equal Parts; firſt puſh the Curſor, ſo that the Middle of the Screw may be juſt upon the Figure 3; and having firmly fixed it upon that Point, take the Length of the propoſed Line between the two longeſt Parts of the Legs; then the Diſtance between the two ſhorteſt Parts of the Legs will be $\frac{1}{3}$ of the given Line. Proceed thus for dividing a given Line into other equal Parts.

Now to make the Diviſions for regular Polygons, divide the Leg of your Compaſſes into two equal Parts; and having opened the Sector, let the Diſtance from 6 to 6, on the two Lines of Polygons, be equal to one of thoſe Parts. The Sector remaining thus opened, take the Diſtance from 3 to 3 for a Trigion, and lay it off from the End of the Leg of your proportional Compaſſes, where mark 3. Again, take the Diſtance from 4 to 4 for a Square, upon the Line of Polygons, and that will give the Diviſion for a Square. Moreover, take the Diſtance from 5 to 5, on the Lines of Polygons, and lay off upon the Leg of your Compaſſes, which will give the Diviſion for a Pentagon; proceed thus for the Heptagon, and the other Polygons, to the Dodecagon. It is needleſs to make the Diviſion for a Hexagon, becauſe the Semidiameter of any Circle will divide it's Circumference into ſix equal Parts.

The Uſe of this Line for the Inſcription of Polygons is very eaſy: for if, for Example, a Pentagon is to be inſcribed in a given Circle, puſh the Curſor ſo, that the Middle of the Screw may be againſt the Number 5 for a Pentagon; then with the ſhorteſt Parts of the Legs, take the Semidiameter of the Circle; and the Legs remaining thus opened, the Diſtance between the Points of the longeſt Parts of the Legs, will be the Side of a Pentagon inſcribed in the given Circle.

Again, ſuppoſe a Heptagon is to be inſcribed in a Circle; fix the Screw againſt the Number 7; then take the Semidiameter of the Circle between the longeſt Parts of the Legs of your Compaſſes, and the Diſtance between the ſhorteſt Parts of the Legs will be the Side of a Heptagon inſcribed in the ſaid Circle.

Of the Beam-Compaſs.

Fig. K. This Compaſs conſiſts of a very even ſquare Branch of Braſs or Steel, from 1 to 3 or 4 Feet in Length. There are two ſquare Braſs Boxes or Curſors exactly fitted to the ſaid Branch, upon each of which may be ſcrewed on Steel, Pencil, or Drawing-Pen Points, according as you have uſe for them. One of the Curſors is made to ſlide along the Branch, and may be made faſt to it by means of a Screw at the Top thereof, which preſſes againſt a little Spring; the other Curſor is fixed very near one End of the Branch, where there is a Nut ſo faſtened to it, that by turning it about the Screw, at the End of the Branch, the ſaid Curſor may be moved backwarde or forwards at pleaſure.

Theſe Compaſſes ſerve to take great Lengths, as alſo exactly to draw the Circumferences of great Circles, and exactly divide them.

Of the Elliptick-Compaſſes.

Fig. L. This Inſtrument, whoſe Uſe is to draw Ellipſes of any kinds, is made of a croſs Branch of Braſs, very ſtrait and equal, about a Foot long, on which are fitted three Boxes, or Curſors, to ſlide upon it. To one of the Curſors there may be ſcrewed on a Steel-Point, or elſe one to draw with Ink, and ſometimes a Porte-Craion. At the Bottom of the two other Boxes are joined two ſliding Dove-Tails (as the little Figure 1 ſhews), theſe ſliding Dove-Tails are adjuſted in two Dove-Tail Grooves, made in the Branches of the Croſs. The aforeſaid two ſliding Dove-Tails, which are affixed to the Bottoms of the Boxes by two round Rivets, and ſo have a Motion every way, by turning about the long Branch, move backwards and forwards along the Croſs; that is, when the long Branch has gone half way about, one of the ſliding Dove-Tails will have moved the whole Length of one of the Branches of the Croſs; and then, when the long Branch is got quite round, the ſame Dove-Tail will go back the whole Length of the Branch: underſtand the ſame of the other ſliding Dove-Tail.

Note, The Diſtance between the two ſliding Dove-Tails, is the Diſtance between the two Foci of the Ellipſis; for by changing that Diſtance, the Ellipſis will more or leſs ſwell.

Underneath the Ends of the Branches of the Croſs, there is placed four Steel-Points, to keep it faſt upon the Paper. The Uſe of this Compaſs is eaſy; for by turning round the long Branch, the Ink, or Pencil-Point, will draw an Oval, or Ellipſis, required. It's Figure is enough to ſhew the Conſtruction and Uſe thereof.

Of Cylindrick and Spherick Compaſſes.

Fig. M. Figure M is a Pair of Compaſſes uſed in taking the Thickneſſes of certain Bodies, as Cannon, Pipes, and the like things, which cannot be well done with Compaſſes of but two Points. Theſe Compaſſes are made of two Pieces of Braſs, or other Metal, having two circular Points, and two flat ones, a little bent at the Ends. When you uſe them, one of the

flat

flat Points must be put into the Cannon, and the other without; then the two opposite Points will shew the Thickness of the Cannon.

Note, The Head of these Compasses ought to be well drilled in the Center; that is, if a Line be drawn from one Point to the opposite one, the said Line must exactly pass thro' the Center; and when the Compasses are shut, all the Points ought to touch one another.

The Figure N is a Pair of Spherick Compasses, which differs in nothing from the Construction of Common Compasses, except only that the Legs are rounded, to take the Diameters of round Bodies, as Bullets, Globes, &c. Fig N.

Lastly, the Figure O is another Cylindrical Pair of Compasses, whose Legs are equal: The Figure is enough to shew their Construction and Use.

Additions *to* CHAP. I.

Of the Turn-up Compasses, and the Proportional Compasses, with the Sector Lines upon them.

Of the Turn-up Compasses.

THE Body of these Compasses, is much like the Body of common Compasses, nigh the Bottom of which, and on the outward Faces, are adjusted two Steel Points, one of them having a Drawing-Pen Point at the End, and the other a Porte-Craion at it's End, so that they may turn round. Nigh the Middle of the outward Faces, are two little Steel Spring Catches, to hinder the Points giving way when using. The Benefit of this Contrivance, is, that when you want to use a Drawing-Pen Point, or a Pencil, you have no more to do, but turn the Drawing-Pen Point, or the Porte-Craion, until the Steel Points come to the Catch: whereas, in a common Pair of Compasses, you have the trouble of taking off a Steel Point, in order to put either of the aforesaid Points in it's place. The Figure of these Compasses is sufficient to show their Construction and Use. Fig. 1.

Of the Proportional Compasses, with the Sector Lines upon them.

These Compasses are made of two equal Pieces of Brass or Silver, of any Length, the Breadth and Thickness of which must be proportionable. Along the greatest Part of their Length are two equal Dove-tail Slits made, in each of which go two Sliding Dove-tails of the same Length, each having a Hole drilled in the Middle, thro' which passes a Rivet, with a turned Cheek fixed at one End (which turned Cheek is fastened to one of the Sliding-Dove-Tails), and a Nut at the other. There is another equal turned Cheek, fastened to the other Dove-tail; so that the two Sliding Dove-tails, together with the two turned Cheeks and Rivet, make a Cursor to slip up and down the Slits, and likewise serve as a moveable Joint for the Branches of the Compasses to turn about. Fig. 2.

At the Ends of the aforesaid Pieces of Brass, or Silver, are fixed four equal Steel-Points, the Lengths of each of which must be such, that when the Cursor is slid as far as it can go, to either of the Ends of the Slits, the Center of the Rivet may be exactly $\frac{2}{3}$ Parts of the Distance from one Point to the other.

At a small Distance from the four Ends of the two Sliding Dove-tails, are drawn across four Lines, or Marks; and when the Center of the Rivet is in the Middle between the Points, the Divisions of the Lines on the Broad-Faces, begin from those Lines, and end at them: But the Divisions on the Side-Faces, begin and end against the Center of the Rivet, when it is in the Middle between the Points

The Lines on the first broad Face of these Compasses, are, 1*st*, the Line of Lines, divided into 100 unequal Parts; every 10th of which are numbered, at the Top of which is writ *Lines*. 2*dly*, A Line of Chords to 60 Degrees, at the Top of which is writ *Chords*. On the other broad Face, are, 1*st*, A Line of Sines to 90 Degrees, at the Top of which is writ *Sines*. 2*dly*, A Line of Tangents to 45 Degrees, at the Top of which is writ *Tangents*.

On the first Side-Face, are the Tangents from 45 Deg. to 71 Deg. 34 Min. to which is writ *Tang.* and on the second, are the Secants from 0 Deg. to 70 Deg. 30 Min. to which is writ *Sec.*

Construction of the Line of Lines on these Compasses.

Draw the Lines A D, C B, of the same Length that you design to have the Branches of the Compasses, crossing each other in the Middle G; with one Foot of your Compasses in A, and the Distance A D, describe the Arc E D; and with the same Distance in the Point B, describe the Arc C E: thro' the Points E, G, draw the right Line E M, which will bisect the Line drawn from C to D, in the Point F; also bisect F D in H, and raise the Perpendicular H R. Now if from the Point R, a right Line be drawn to A, it will cut the Line E M in Fig. 3.

the Point *k*; and if with one Foot of your Compasses in A, and the Distance A *k*, you describe an Arc cutting the Side A D in the Point 50; the said Point 50, on the Side A D, will be the Division for 50 and 50 of the Line of Lines, if the Center of the Cursor was to be slid to the Divisions, when the Compass is using. But because the Lines drawn across near the Ends of the Sliding Dove-tail, are to be slipped to the Divisions, when the Compasses are to be used, the Division for 50 must be as far beyond the Point 50, as the aforesaid Line on the Sliding Dove-tail, is distant from the Center of the Cursor; which Distance suppose to be G Q, or G L it's Equal. Understand the same for all other Divisions, which are found in the manner that I am now going to shew.

Divide D H into 50 equal Parts, and from every of which raise Perpendiculars to cut the Arc E D (I have only drawn every 10). Now if from the Point A, to all the Points wherein the Perpendiculars cut the Arc E D, right Lines be drawn, cutting the Line E M; and if the Distances of these Sections from the Point A, are laid off from the same Point on the Line A D, the Divisions from 0 to 50, for the Line of Lines, will be had; and likewise from 50 to 100, which are at the same Distance from the Center G; in observing to place each of them, found out as directed, so much further from the Center G, as the Line G Q is distant from it.

The Divisions for the Line of Lines being found, as before directed, they must each of them be transferred to the Face of your Compasses, and be numbered as *per* Figure.

Construction of the Line of Chords, Sines, Tangents, and Secants.

Fig. 4. Having taken half of the Line of Lines, and divided the Spaces from 0 to 10, 10 to 20, 20 to 30, 30 to 40, and 40 to 50, into 100 Parts, by means of Diagonals; that half so divided, will serve as a Scale whereby the Tables of Natural Sines, Tangents, and Secants, and the Divisions of all the other Lines on these Compasses may be easily made.

Now having slid the Center of the Cursor to the Middle of the Compasses, the Beginning and Ending of the Line of Chords must be (as in all the other Lines drawn upon these Compasses, two broad Faces) where the Line drawn across the Sliding Dove-tail cuts the Sides of the Slit: then to find where the Division of any Number of Degrees, or half Degrees, suppose 10, must be, look in the Table of Natural Sines for the Sine of 5 Degrees, which is half 10, and you will find it 871.557; which doubled, will give the Chord of 10 Degrees, *viz.* 1743.114: but because the Radius to the Table of Natural Sines, Tangents, and Secants, is 10000, and from the aforesaid Semi-Line of Lines made into a Diagonal Scale, can be taken but 5.00 Parts; therefore reject the last Figure to the right-hand, together with the Decimals, and you will have 174 for the Chord of 10 Degrees, when the Radius is but 1000, or the Length of the Line of Lines. Now take 174 Parts from the Diagonal Scale, and lay them off from 0, on the Parallels drawn to contain the Divisions of the Line of Chords, and you will have the Division for 10 Degrees. Again, to find the Division for 20 Degrees, look for the Natural Sine of 10 Degrees, and it will be found 1736.482; which doubled, will give the Chord of 20 Degrees, *viz.* 3472.964, and rejecting the last Figure to the right-hand, and the Decimals, you will have 347, which being taken from your Diagonal Scale, and laid off from 0 0 on the Parallels, you will have the Division for the Chord of 20 Degrees. In this manner proceed for finding the Divisions for the Chords of any Number of Degrees, or half Degrees. But note, when you come to the Chord of 29 Degrees, you are got to the furthest Division from the Center; because, from the Table of Sines, the Chord of 29 Deg. is half Radius (or at least near enough half for this Use), or 500, and consequently the Length of your whole Scale: therefore you must, for the Divisions of the Chords of any Number of Degrees above 29, lay off the Parts above 500, taken on the Diagonal Scale, from the Division of 29 Degrees, back again towards the Center, on the other Side the Slit, to 60. As for Example; to find the Division for the Chord of 40 Degrees; the Chord is 684, from which 500 being substracted, you must take the Remainder 184 from your Diagonal Scale, and lay it off towards the Center, on the Parallels drawn on the other Side of the Slit, from a Point over-against the Division for the Chord of 29 Degrees; and so for any other.

The Lines of Sines, or Tangents, on the other broad Face of these Compasses, are made in the same manner as the Line of Chords is: As, for Example, to make the Division for the Sine of any Number of Degrees, suppose 10; you will find from the Table of Natural Sines, that the Sine of 10 Degrees is 173; whence lay off 173 Parts, taken on the Diagonal Scale, from the Beginning of the Lines drawn to contain the Divisions, and you will have the Point for the Sine of 10 Degrees. Again; to find the Division for the Sine of 25 Degrees, you will find from the Table, that 422 is the Sine of 25 Degrees; therefore take on your Scale 422 Parts, and lay them off from 0, and you will have the Division for the Sine of 25 Degrees. Thus proceed for the Divisions of any other Number of Degrees, until you come to 30, whose Sine is equal to Half-Radius, and from 30 back again to 90, in observing the Directions aforegiven about the Chords, when they return towards the Center.

The Divisions for the Tangent of any Number of Degrees, suppose 10, are likewise thus found; for the Tangent of 10 Degrees, by the Table, is 176; wherefore taking 176 Parts from your Scale, and laying them off from 0 0 on the Parallels drawn to contain the Divisions, the Division for the Tangent of 10 Degrees will be had. Again; to find the Division for the Tangent

Tangent of 25 Degrees; by the Table of Tangents, the Tangent of 25 Degrees will be found 466; whence taking 466 Parts from your Scale, and laying them off from o o, you will have the Diviſion for the Tangent of 25 Degrees. Thus proceed for the Diviſions of the Tangents of any other Number of Degrees, until you come to the Diviſion of the Tangent of 26 Deg. 30 Min. which is half the Radius; and from 26 Deg. 30 Min. back again to 45 Deg. whoſe Tangent is equal to Radius, in obſerving the Directions afore-given about the Line of Chords, when they return.

The Conſtruction of the Tangents to a ſecond and third Radius, on the ſide Face of theſe Compaſſes, is thus: Let the Beginning of the ſecond Radius, which is at the Tangent of 45 Degrees, be in the Middle between the Points of the Compaſſes; becauſe when the Compaſſes is uſing, a little Notch in the Side of the turned Cheek, which is directly againſt the Center of the Curſor, is ſlid to the Diviſions: then to make the Diviſions for the Tangents of the Degrees, and every 15 Minutes, from the Tangent of 45, to the Tangent of 56 Degrees, and about 20 Minutes, which is half a ſecond Radius, you muſt look for the reſpective Tangents in the Table of Natural Tangents; and having caſt away the laſt Figure to the right-hand, and the Decimals (which always do), ſubſtract 1000 from each of them, becauſe that is equal to one of our Radius's, and the Remainders take from your Scale, and lay off from 45; ſo ſhall you have the Diviſions to the Tangent of 56 Deg. and about 20 Min. Then again, to have the Diviſions from 56 Deg. 20 Min. to 63 Deg. and 27 Min. the Tangent of which is equal to 2000, or two of our Radius's, you muſt ſubſtract 1500, which is 2 and a half of our Radius's, from every of the reſpective Tangents, found and ordered as before directed; and then take each of the Remainders from the Scale, and lay them off from 56 Deg. 20 Min. on the Top, and you will have the Diviſions of the Tangents of the Degrees, and every 15 Min. from 56 Deg. 20 Min. to 63 Deg. 27 Min. which will fall againſt 45 Deg. on the Side of the other Branch. Again; to find the Diviſions of the Tangents of the Degrees, and every 15 Minutes, from 63 Deg. 27 Min. to 68 Deg. 12 Min. which makes two Radius's and a half, or 2500, you muſt ſubſtract 2000 from each of the Tangents, found and ordered as aforeſaid, and the Remainders muſt be taken off your Scale, and laid off from 63 Deg. 27 Min. and you will have the Diviſions for the Tangents of the Degrees, and every 15 Min. from 63 Deg. 27 Min. to 68 Deg. 12 Min. *Laſtly*, To have the Diviſions from 68 Deg. 12 Min. to 71 Deg. 34 Min. which ends at 45 Deg. and makes up the third Radius, or 3000: you muſt ſubſtract 2500 from each of the Tangents found in the Table, and ordered as before directed; and take off the Remainders from your Scale, which laid off upwards from 68 Deg. 12 Min. will give the Diviſions for the Tangents of the Degrees, and every 15 Minutes, between 68 Deg. 12 Min. and 71 Deg. 34 Min.

The Diviſions for the Secants, on the other narrow Face of the Compaſſes, which run from o Degrees, in the Middle between the two Points of the Compaſſes, to 70 Degrees, 32 Minutes, that is, which are the Secants to a ſecond and third Radius (like as the Tangents laſt mentioned) are made exactly in the ſame manner, from the Table of Natural Secants, as thoſe Tangents to a ſecond and third Radius are made.

USE *of theſe Proportional Compaſſes.*

USE I. *To divide a given right Line into any Number of equal Parts, leſs than* 100.

Divide 100 by the Number of equal Parts the Line is to be divided into, and ſlip the Curſor ſo, that the Line drawn, upon the ſliding Dove-Tail, may be againſt the Quotient on the Line of Lines: then taking the whole Extent of the Line between the two Points of the Compaſſes, that are furtheſt diſtant from the Center of the Curſor, and afterwards applying one of the two oppoſite Points to the Beginning or End of the given Line, and the other oppoſite Point will cut off from it one of the equal Parts that the Line is to be divided into.

As, for Example; To divide the Line A B into two equal Parts: 100, divided by 2, gives 50 for the Quotient; therefore ſlip the Line on the Dove-Tail to the Diviſion 50 on the Line of Lines, and taking the whole Extent of the Line A B between the Points furtheſt from the Center; then one of the oppoſite Points ſet in A or B, and the other will fall on the Point D, which will divide the Line A B in two equal Parts. Fig. 5.

Again; to divide a right Line into three equal Parts, divide 100 by 3, and the Quotient will be 33.3; therefore ſlip the Line of the Dove-Tail to the Diviſion 33, and for the three Tenths conceive the Diviſion between 33 and 34 to be divided into 10 equal Parts, and reaſonably eſtimate 3 of them. Proceed as before, and you will have a third Part of the ſaid Line, and therefore it may eaſily be divided into 3 equal Parts. Moreover, to divide a given Line into 50 equal Parts, divide 100 by 50, and the Quotient will be 2; therefore ſlip the Line, on the ſliding Dove-Tail, to the Diviſion 2 on the Line of Lines. Proceed as at firſt, and you will have a 50th Part of the Line propoſed; whence it will be eaſy to divide it into 50 equal Parts.

Note, If each of the Subdiviſions, on the Line of Lines, be ſuppoſed to be divided into 100 equal Parts; then a Line may, by means of the Line of Lines on theſe Compaſſes, be divided into any Number of equal Parts leſs than 1000. As, for Example; to divide a Line into 500 equal Parts: Divide 1000 by 500, and the Quotient will be 2; therefore ſlip the Line,

on

on the Dove-Tail, to 2 Tenths of one of the Subdivisions of 100, and proceed, as at first directed, and you will have the 500th Part of the Line given, which afterwards may easily be divided into 500 equal Parts. Again; To divide a Line into 200 equal Parts: divide 1000 by 200, and the Quotient will be 50; therefore slip the Line, on the Dove-Tail, to 5 of the Subdivisions of 100, on the Line of Lines, which will now represent 50; proceed as at first, and you will have the 200th Part of the Line given: therefore it will be easy to divide it into 200 equal Parts. Moreover, to divide a given Line into 150 equal Parts, divide 1000 by 150, and the Quotient will be 6.6; wherefore reasonably estimate 6 of the 10 equal Parts that the first of the Subdivisions of 100 is supposed to be divided into, and slip the Line, on the sliding Dove-Tail, to the 6th; then proceeding as at first, and the Line may be divided into 150 equal Parts. If a Line be so long, that it cannot be taken between the Points of your Compasses, you must take the half, third, or fourth Part, &c. and proceed with that as before directed; then one of the Parts found being doubled, trebled, &c. will be the correspondent Part of the whole Line.

USE II. *A right Line being given, and supposed to be divided into* 100 *equal Parts: to take any Number of those Parts.*

Slip the Line, on the sliding Dove-Tail, to the Number of Parts to be taken, as 10; then the Extent of the whole Line being taken between the Points of the Compasses, furthest distant from the Cursor, if one of the opposite Points be set in either Extreme of the given Line, the other will cut off the Part required.

USE III. *The Radius being given; to find the Chord of any Arc under* 60 *Degrees.*

Slip the Line, on the sliding Dove-Tail, to the Degrees sought on the Line of Chords; then take the Radius between the Points of the Compasses, furthest distant from the Center of the Cursor, and the Extent, between the two opposite Points, will be the Chord sought, if the given Number of Degrees be greater than 29, whose Chord is nearly Half-Radius; but if the Number of Degrees be less than 29, then the Distance of the two opposite Points, taken from Radius, will be the Chord of the Degrees required.

If the Chord of a Number of Degrees under 60 is given, and the Radius to it be required; you must slip the Line, on the sliding Dove-Tail, to the Degrees given on the Line of Chords; and taking the Length of the given Chord between the two Points of your Compasses, that are nighest the Cursor, the Extent of the two other opposite Points will be the Radius required.

Fig. 6. Example, for the first Part of this Use: Suppose the Length of the Radius be the Line A B, and the Chord of 35 Degrees be required; Slip the Line, on the sliding Dove-Tail, to 35 Degrees on the Line of Chords; take the whole Extent of the Line A B between the Points of the Compasses, furthest distant from the Cursor; and placing one of the opposite Points in the Point A, the other Point will give the Extent A D for the Chord of 35 Degrees. Again; To find the Chord of 9 Degrees: Slip the Line, on the sliding Dove-Tail, to 9 Degrees on the Line of Chords; then take the Extent of the Radius, which suppose A B, between the two Points of the Compasses, furthest distant from the Center; and placing one of the opposite Points in the Point A, the other will fall on the Point C, and the Difference between A B and A C, *viz.* C B, will be the Chord of 9 Degrees.

USE IV. *The Radius being given, suppose the Line* A B; *to find the Sine of any Number of Degrees, as* 50.

Fig. 7. Slip the Line, on the sliding Dove-Tail, to 50 Degrees on the Line of Sines; then if the Extent A B be taken between the two Points of the Compasses, furthest from the Cursor, and one of the opposite Points be set in the Point A, the other will give A C for the Sine of 50 Degrees; but if the Sine sought be lesser than the Sine of 30 Degrees, which is equal to Half-Radius, the Difference, between the Extents of the opposite Points, will be the Sine of the Angle required.

USE V. *The Radius being given; to find the Tangent of any Number of Degrees, not above* 71.

If the Tangent of the Degrees, under 26 and 30 Minutes, whose Tangent is equal to Half-Radius, be sought: You must slip the Line, on the sliding Dove-Tail, to the Degrees proposed on the Line of Tangents; and then take the Radius between the Points of the Compasses, furthest distant from the Cursor, and the Difference between the opposite Points will be the Tangent of the Number of Degrees proposed.

If the Tangent of any Number of Degrees above 26 and 30 Minutes, and under 45, be sought; then you must slip the Line, on the sliding Dove-Tail, to the Number of Degrees given on the Tangent-Line, and take the Radius between the Points of the Compasses furthest from the Cursor; then the Distance, between the two opposite Points, will be the Tangent of the Degrees required.

If the Tangent required be greater than 45 Degrees, but less than 56 Degrees, and about 20 Minutes; you must slip the Notch, on the Side of the turned Cheek, to the Degrees of the

the Tangents upon the Side of the Compaffes, and take the Radius, between the Points of the Compaffes, furtheft diftant from the Curfor; the Difference between the oppofite Points, added to Radius, will be the Tangent of the Degrees fought.

If the Tangent required be greater than that of 56 Degrees, 20 Minutes, but lefs than 63 Degrees, 27 Minutes, you muft flip the Notch to the Degrees propofed, and take the Radius, as before, between the Points of the Compaffes; then the Extent, between the two oppofite Points, added to Radius, will be the Tangent required.

If the Tangent required be greater than 63 Degrees, 27 Minutes, but lefs than 68 Degrees; you muft flip the Notch, on the Side of the turned Cheek, to the Degrees propofed, and take the Radius between the Points of the Compaffes, as before; then the Difference between the oppofite Points, added to twice Radius, will be the Tangent of the Degrees propofed.

Laftly, If the Tangent be greater than 68 Degrees, but lefs than 71, you muft add the Diftance between the oppofite Points of the Compaffes, to two Radius's, and the Sum will be the Tangent of the Degrees fought.

The Secant of any Number of Degrees, under 70, by having Radius given, in obferving the aforefaid Directions about the Tangents, may be eafily found.

CHAP. II.

Of the Conftruction of divers Mathematical Inftruments.

Of the Sliding Porte-Craion, *or* Pencil-Holder.

THIS Inftrument is commonly about four or five Inches long, the Outfide of which is filed into eight Faces, and the Infide perfectly round, in which a Porte-Craion is put, which may be flid up and down by means of a Spring and Button, of which we fhall fpeak hereafter. The Compaffes of the Figure B is made to fcrew into one End of this Inftrument. *Plate* 10. Fig. A.

There are commonly drawn, upon the Faces of this Porte-Craion, the Sector-Lines, whofe manner of drawing is the fame, as thofe on the Sector; and their Ufe is the fame as the Ufe of thofe on the Sector, excepting only that they are not fo general. For Example; If you have a mind to make an Angle of 40 Degrees upon a given Line; take the Extent of 60 Degrees of the Line of Chords, and therewith defcribe an Arc upon the given Line: then take the Extent of 40 Degrees, and lay off upon that Arc, and from it's Center draw a Line, which will make an Angle of 40 Degrees with the given Line.

Note, There are alfo round Inftruments of this kind, whofe Outfides are divided into Inches, and each Inch into Lines.

This is another Porte-Craion made of Brafs, round within, and commonly fo without, having the Porte-Craion of Figure D made to flip up and down in it. In the Ends of the faid Porte-Craion are put Pencils, which are made faft by two Rings; and in the Middle is placed a well-hammered Brafs or Steel Spring, having a Female Screw made in it at 1, in order to receive the Male Screw at the End of the Button E, which goes thro' a Slit made in the Body of the Inftrument. The Figure, and what I have faid, is enough to fhew the Nature of this Porte-Craion. Fig. C.

Of the Fountain-Pen.

This Inftrument is compofed of different Pieces of Brafs, Silver, *&c.* and when the Pieces F G H are put together, they are about five Inches long, and it's Diameter is about three Lines. The middle Piece F carries the Pen, which ought to be well flit, and cut, and fcrewed into the Infide of a little Pipe, which is foldered to another Pipe of the fame Bignefs, as the Lid G; in which Lid is foldered a Male Screw, for fcrewing on the Cover: as likewife for ftopping a little Hole at the Place 1, and fo hindering the Ink from running through it. At the other End of the Piece F, there is a little Pipe, on the Outfide of which the Top-Cover H may be fcrewed on. In this Top-Cover there goes a Porte-Craion, that is to fcrew into the laft mentioned little Pipe, and fo ftop the End of the Pipe at which the Ink is poured in, by means of a Funnel. Fig. F.

When the aforementioned Pen is to be ufed, the Cover G muft be taken off, and the Pen a little fhaken, in order to make the Ink run freely. *Note*, If the Porte-Craion does not ftop the Mouth of the Piece F, the Air, by it's Preffure, will caufe the Ink all to run out at once. *Note*, alfo, That fome of thefe Pens have Seals foldered at their Ends.

Of Pincers for holding Papers together.

This little Inftrument is made of two well-hammered thin Pieces of Brafs, faftened together at top, and having a Brafs Spring between them, and a Ferril, that flides up and down, in order to draw them together. The whole Piece is about two Inches long, and it's Figure is enough to fhew the Conftruction and Ufe thereof. Fig. I.

Of the Pentograph, or Parallelogram.

Fig. K. This Instrument, called a *Pentograph*, as serving to copy any manner of Designs, is composed of four Brass, or very hard Wooden Rulers, very equal in Breadth and Thickness; two of them being from 15 to 18 Inches in Length, and the other two but half of their Length, and their Thickness is usually 2 or 3 Lines, and Breadth 5 or 6.

The Exactness of this Instrument very much depends upon having the Holes made at the Ends, and in the Middle of the longest Rulers, at an equal Distance from the Holes at the Ends of the shortest Rulers; for this Reason, That being put together, they may always make a Parallelogram: and when the Instrument is to be used, there are six small Pieces of Brass put on it.

The Piece 1 is a little turned Brass Pillar, at one End of which is a Screw and Nut, serving to join and fasten the two long Rulers together; and at the other is a little Knob for the Instrument to slide upon. The Piece 2 is a turned-headed Rivet, with a Screw and Nut at the End; two of which there must be for joining the two Ends of the two short Rulers to the Middle of the long ones, at the Places 2, 2. The Piece 3 is a Brass Pillar, one End of it being hollowed into a Screw, having a Nut to fit it; and at the other End is a Worm to screw into the Table, when the Instrument is to be used. This Piece holds the two Ends of the short Rulers together, at *Fig.* 3. *Fig.* 4. is a Porte-Craion, or Pen, which may be screwed into the Pillar 4, which is fixed on at the Place 4, to the End of the great Ruler. Lastly, *Fig.* 5. is a Brass Point, something blunt, screwed into a Pillar like one of the former ones, which is screwed on to the End of the other long Ruler. This Instrument being put together, and disposed, as *per* Figure, the next thing will be to shew it's Use.

Now when a Design, of the same Bigness as the Original, is to be copied, the Instrument must be disposed, as in Figure K; that is, you must screw the Worm into the Table at the Place 3, and lay the Paper under the Pencil 4, and the Design under the Point 5; then there is no more to do but move the Point 5 over every part of the Design 5, and at the same time the Pencil, at Figure 4, will mark the said Design upon your Paper. But if the Design is to be reduced, or made less by half, the Worm must be placed at one End of the long Ruler, the Paper and the Pencil in the Middle, and then you must make the Brass Point pass over all the Tracts of the Design, and the Pencil at the same time will also have described all those Tracts; but they will be of but half the Length of the Tracts of the Design: for this reason; because the Pencil, placed in the manner aforesaid, moves but half the Length, in the same time, as the Brass Point does. And, for the contrary Reason, if a Design is to be augmented, for Example, twice the Original, the Brass Point and the Design must be placed in the Middle, at Figure 3, the Pencil and Paper at the End of one of the long Rulers, and the Worm at the End of the other long Ruler; by this means a Design twice the Original may be drawn.

But to augment or diminish Designs in other Proportions, there are drilled Holes at equal Distances upon each Ruler, *viz.* all along the short ones, and half-way the great ones, in order to place the Pieces carrying the Brass-Point, the Pencil, and the Worm in a right Line in them; that is, if the Piece carrying the Brass-Point be put into the third Hole, the two other Pieces must be likewise each put into the third Hole.

Note, If the Point and the Design be placed at any one of the Holes of one of these great Rulers, and the Pencil with the Paper under one of the Holes of the short Ruler, which forms the Angle, and joins to the Middle of the said long Ruler, that then the Copy will be less than half the Original: But if the Pencil and Paper be placed under one of the Holes of that short Ruler, which is parallel to the long Ruler, then the Copy will be greater than half the Original. In a word, all these different Proportions will be easily found by Experience.

Construction of Sizes: To know the Weight of Pearls.

Fig. M. This little Instrument, whose Use is to find the Weight of very fine and round Pearls, is made of five thin Pieces, or Leaves, of Brass, or other Metal, about two Inches long, and six or seven Lines broad. The said Leaves have several round Holes drilled in them of different Diameters; the Holes in the first Leaf serve for weighing Pearls from half a Grain to 7 Grains; those in the second Leaf are for Pearls from 8 Grains, which is 2 Carats, to 5 Carats; those in the third for Pearls weighing from 2 $\frac{1}{2}$ Carats to 5 $\frac{1}{2}$ Carats; the fourth for Pearls weighing from 6 Carats to 8; and the fifth for Pearls weighing from 6 $\frac{1}{2}$ Carats to 8 $\frac{1}{2}$.

Now the Diameters of the greatest and least Holes of each Leaf being found, by weighing of Pearls in nice fine Scales, the Diameters of all the other Holes from thence, by proportion, may be found.

The Hole, shewing the Weight of a Pearl of one Grain, is 2 $\frac{1}{3}$ Lines in Diameter; that shewing the Weight of a Pearl of 2 Carats, is 2 $\frac{1}{7}$ Lines; that shewing the Weight of a Pearl of 5 Carats, is 4 Lines; that shewing 2 $\frac{1}{2}$ Carats, is 2 $\frac{1}{4}$; that of 5 $\frac{1}{2}$ Carats, is 4 $\frac{1}{2}$ Lines; that of 6 Carats, is 4 $\frac{1}{4}$ Lines; that of 8 Carats, is 4 $\frac{1}{2}$ Lines; and, Lastly, the Diameter of that Hole for Pearls weighing 8 $\frac{1}{2}$ Carats, is 4 $\frac{1}{4}$ Lines.

The

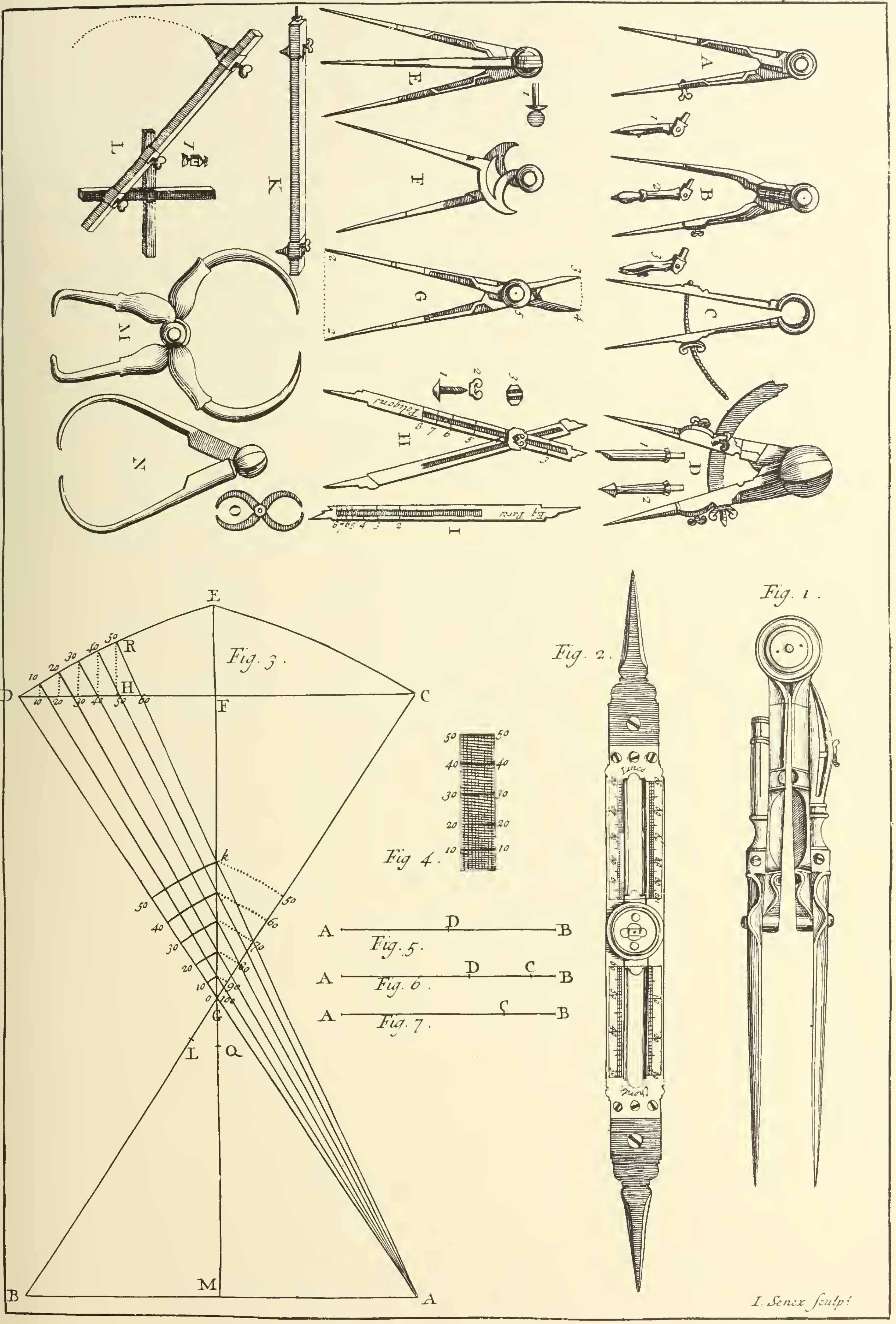
Fig. 1.
Fig. 2.
Lines
Chord
Fig. 3.
E
R
H
D
F
C
k
G
L
Q
B
M
A
Fig 4.
A D B
Fig. 5.
A D C B
Fig. 6.
A C B
Fig. 7.
I. Senex sculp.

The Leaves are faſtened together at one End by a Rivet, about which they are moveable, and included between two thin Pieces of Braſs, ſerving as a Caſe for them.

Jewellers likewiſe uſe little Scales, and very ſmall Weights, which they call Carats, to weigh Diamonds, and other precious Stones, as alſo Pearls that be not round. A Carat is 4 Grains, and is divided into $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$, and $\frac{1}{4}$ of a Carat: the word *Carat* is alſo uſed for the Degrees of Perfection of Gold; as a Carat of fine Gold is the 24th part of an Ounce of pure Gold, which is ſo ſoft, that it cannot be worked; for which Reaſon the Goldſmiths of *Paris* uſe Gold of 22 Carats, that is, 22 Parts of fine Gold, and two Parts of Braſs; by which Mixture it is rendered harder and fitter to work.

Of the Fixed Square.

This Inſtrument is called a Fixed Square, becauſe it's Sides do not open or ſhut; all it's Exactneſs conſiſts in being very ſtrait, and that both the inward and outward Faces of the two Sides be at right Angles; which that they may be, it is neceſſary for them to be parallel to each other. Fig. L.

The Figure N is another Square, which opens or ſhuts. Theſe Squares principal Uſes are to know whether any Line or Plane be at right Angles to another.

Of the Foot-Level.

This Inſtrument is compoſed of two Branches of Braſs, or other Matter, about half an Inch broad, and opens and ſhuts like a Two-foot Rule; half-way the Inſide of both theſe Branches are hollowed in, to receive a kind of Tongue, or thin Piece of Braſs (which is faſtened to one of the Branches) that ſo the two Branches may be ſhut cloſe together. The Uſe of this Tongue is ſuch, that when the End of it is placed in the Branch it is not faſtened to, where there is a Pin that holds it, the two Branches of the Level will be fixed at right Angles, as *per* Figure. There is likewiſe a thin ſquare Piece of Braſs adjuſted to the Head of this Inſtrument, that ſo it may ſerve for a Square, and at the Bottom of the Angle of the ſaid Piece of Braſs is a little Hole made, in which is faſtened a Silken Line, with a Plummet at the End thereof; which falling upon a perpendicular Line, drawn on the Middle of the Tongue, ſhews whether any thing the Inſtrument is applied to be level or not. *Note*, The inward Angles of the Branches are cut away, that ſo the Inſtrument may better ſtand upon a Plane to be levelled. *Note* alſo, That this Inſtrument ſerves for a Level, a Square, and a Foot-Rule. Fig. O.

Of the Paris *Foot-Rule, and the Compariſon of it's Length with that of other Countries.*

The Conſtruction of the Body of this Inſtrument does not differ from that of the Sector before ſpoken of; and when the *Paris* Foot is only put thereon, each Leg is but about five Lines in Breadth; but when the Foot of other Countries, compared with the *Paris* Foot, is put thereon, it is made broader. I ſhall here lay down the Compariſon between the Foot of moſt chief Towns in *Europe*, compared with that of *Paris*. Fig. P.

A Point is [illegible] of an ordinary Grain of Barley; a Line is 12 Points, or the Thickneſs of one Grain of Barley; an Inch is 12 Lines, and a Foot is 12 Inches. The Foot Royal of *Paris* is 12 of the aforeſaid Inches, but ſometimes it is divided into 720, or 1440 equal Parts, for better expreſſing it's Relations to the Meaſures of other Countries. The Foot of *Lyons* and *Grenoble* is ſomething bigger than that of *Paris*; for it contains 12 Inches, 7 Lines. The Foot of *Dijon* is leſſer, and contains but 11 Inches, 7 Lines; that of *Beſançon* 11 Inches, 5 Lines; that of *Maçon* 12 Inches, 4 Lines; and the Foot of *Rouen* is equal to that of *Paris*.

	Inches.	*Lines.*
A Foot of *Sedan* is	12	3
A Foot of *Lorrain*	10	9
A Foot of *Bruſſels*	10	9
A Foot of *Amſterdam*	10	5
A Foot of the *Rhine*	11	7
A Foot of *London*	11	3
A Foot of *Dantzick*	10	7
A Foot of *Sweden*	12	1
A Foot of *Denmark*	10	9
A Foot of *Rome*	10	10
A Foot of *Bologne*	14	1
A Foot of *Venice*	11	11

The great Foot of *Milan* is 1 Foot, 10 Inches.
And the ſmall one, 1 Foot, 2 Inches, 8 Lines.
A Foot of *Turin* is 1 Foot, 6 Inches, 11 Lines.
A Foot of *Savoy* 10 Inches.
A Foot of *Geneva* is 18 Inches.
A Foot of *Vienna* is 11 Inches, 8 Lines.
A Foot of *Conſtantinople* is 2 Feet, 2 Inches, 2 Lines.

Some

Some other Measures compared with the Paris *Foot.*

A *Roman* Palme is 8 Inches, 2 Lines; that of *Genoa* is 9 Inches, 1 Line; that of *Naples* is 9 Inches, 9 Lines; and that of *Portugal*, is 8 Inches, 2 Lines.

A Pan, which is a Measure used in many Places of *Italy*, is 8 or 9 Inches.

The Ell of *Paris*, is 3 Feet 8 Inches; that of *Provence*, *Montpelier*, and *Avignon*, is $\frac{2}{3}$ of that of *Paris*, and the Ell of *Flanders* and *Germany*, is $\frac{7}{12}$ of that of *Paris*.

The Fathom of *Milan*, used by Mercers, is 1 Foot, 7 Inches, 4 Lines; and that of Linen-Drapers, is 2 Foot, 11 Inches.

A Fathom of *Florence* is 1 Foot, 9 Inches, 6 Lines.

The Ras of *Piedmont* and *Lucca* is 22 Inches.

The Yard of *Seville* is 30 Inches, 11 Lines.

The Varre of *Madrid* and *Portugal* is 3 Foot, 9 Lines.

The Varre of *Spain* in general is 5 Foot, 5 Inches, 6 Lines.

The Cane of *Toulouse* is of the same Length.

The Cane of *Rome* is 6 Feet, 11 Inches, 7 Lines.

The Cane of *Naples* is 6 Feet, 10 Inches, 2 Lines.

The Pic of *Constantinople* is 2 Foot, 2 Inches, 2 Lines.

The Geuse of *India* and *Persia* is 2 Foot, 10 Inches, 11 Lines.

Construction of Parallel Rules.

Fig R. These Instruments are commonly made of Brass, or hard Wood, from 6 to 18 Inches in Length, and about two Lines in Thickness: the two parallel Pieces ought to be very straight every way, and parallel, that is, very equal in Breadth from one End to the other; for this is the chief thing upon which the Exactness of these Instruments depends.

The two parallel Pieces of this Instrument are joined together by two Brass Blades, from about 2 to 3 $\frac{1}{2}$ Inches long, and 6 Lines broad, filed and fashioned, as *per* Figure, near the Ends of which are round Holes very equally drilled thro' them, which ought to be done by laying them one upon the other. Then the parallel Pieces must be divided Length-wise into two equal Parts, and afterwards one of the Halves of each into 3 equal Parts, and at the first of these Parts from the Middle, a Hole must be made in each parallel Piece, in the Middle of their Breadth, in which must be placed two turned-headed Rivets, for joining one End of each Blade to the said parallel Piece. Likewised, near, and equally distant from the two opposite Ends of each Piece, must two more Holes be made, in which must be put two more Rivets, for joining the other two Ends of the two Blades, to the parallel Pieces. The Pieces being thus joined, if you move them backwards and forwards, to the right-hand and the left, and the inward Edges of the said Pieces do exactly meet each other, it is a sign the Rule is well made.

Fig Q. The Figure Q, is another kind of parallel Rule, the two parallel Pieces of which, are joined together by two others something shorter, which are joined to each other in the Middle, and make a kind of Cross, which opening or shutting, cause the two parallel Pieces to recede parallelly from, or accede to each other. In the Middle of each parallel Piece of both these Instruments, is fixed a Brass Button, for more easier managing them.

The principal Use of these Instruments, is to draw parallel Lines, by opening or shutting the parallel Pieces, and are of excellent Use in Architecture and Fortification, wherein a great Number of parallel Lines are to be drawn.

Construction of the Pedometer or Waywiser.

Fig. S. This Instrument is about two Inches in Diameter, commonly about 7 Lines in Thickness, and hath all it's Parts joined together a Case, almost like that of a Watch.

The Plate T, is placed in the Bottom of the Case, upon which are fastened several Pieces, as they appear *per* Figure. The Piece 1, is a little Steel Catch with it's two Springs; this Catch is held by a round Tenon going into a Hole in the said Plate, so that by pulling the Piece F, which is fastened to one End of the Catch, the said Catch turns round the Steel Star 2, having 6 Points, and carrying a Pinion of Six Teeth of the same Height as the two Wheels, of which we are going to speak. The Spring 4, is for hindering the Star from going back; and that marked 5, is to lift up the End of the Catch, when it hath made the Star move one Point forwards.

The Plate V is like the Plate T, only it hath upon it two equal Wheels placed on each other; the upper Wheel hath 100 Teeth, and the under one 101, which are both put in Motion by the Pinion upon the Star; so that when the upper Wheel hath gone round once, and run 100 equal Parts, with it's Hand upon the greater Dial-Plate S; the Wheel which hath 101 Teeth, wants one of going round, and makes the lesser Hand move the $\frac{1}{100}$ Part of the Circumference of the lesser Dial-Plate the contrary way; whence the greater Hand must go round 100 times, before the little Hand hath gone round once the contrary way; and consequently, the Piece F must be pulled 10000 times, before the little Hand will go round once: there are 3 Tenons fixed to the under Plate, by means of which, the upper Plate is fastened to it with little Pins.

The

The whole Machine is incloſed in it's Caſe, covered with a Glaſs, and having on one Side of it two Rings, thro' which a String is put for hanging the Inſtrument to any thing; and at the other Side of the Caſe, is an Opening left for the Piece F to come out thro', which Piece receives a String faſtened to one's Garter.

The Uſe of this Inſtrument is ſuch, that being hung to a Perſon's Belt, at each Tenſion of the Knee, that is, every time he ſteps forwards, the String pulls the Piece F, and this the Catch, which cauſes the greater Hand to move one Diviſion forwards. When any Perſon hath a mind to know how many Paces he hath moved, he muſt look upon the Dial-Plate, and that will inform him. *Note*, A Pace is nearly 2 Foot, and a Perſon in walking may ſo accuſtom himſelf, as to take his Steps of that Length; but when Ground is not level, Paces are not equal, for in deſcending they are longer, and in aſcending ſhorter, which muſt be regarded, and corrected by Experience.

There are alſo theſe kinds of Inſtruments made, and fitted to Wheels of known Circumferences; for Example, a Fathom round: ſo that every time the Wheel comes to a certain Point, where there is a Tenon which pulls the Piece F, the Catch cauſes the larger Hand to move one Diviſion forwards; and by this means you may know how many Fathom you have gone.

Pedometers are likewiſe adjuſted behind Coaches, ſo that when one of the great Wheels of a Coach comes to a certain Point, it cauſes the Catch to move the Hand one Diviſion forwards, ſo that in knowing the Circumference of the ſaid Wheel, the Length the Coach hath moved may be known.

Note, ' The leſſer Dial-Plate muſt be carried round by the upper Wheel of 100 Teeth, or ' elſe it will not at any time be eaſy to tell, how many Paces you have gone by the ſaid leſſer ' Plate, but muſt ſtay 'till the Hand of the greater Plate hath made one Revolution.'

The Conſtruction of a Machine for cutting and dividing the Wheels, and Pinions of Clocks, or Watches.

The Machine A, is for cutting and dividing the Wheels and Pinions of Clocks and Watches, and is very commodious, and extremely ſhortens the Time of doing them.

The Plate A is made of Braſs, very even, about 8 Inches Diameter, and one Line in Thickneſs, having ſeveral Concentrick Circles drawn upon it, whoſe Peripheries are divided into ſeveral even or uneven Numbers of equal Parts, the greater of which are always more diſtant from the Center. Fig. 1.

As for Example; to divide the Periphery of one of the Circles into 120 equal Parts, you muſt firſt divide the ſaid Periphery into 2 equal Parts, each of which will be 60, which again ſubdivide by 2, and each Part will be 30; which again divided by 2, and each Part will be 15, which being divided by 3, produces 5. Laſtly, dividing each of theſe laſt Parts by 5, the whole Periphery will be found divided in 120 equal Parts.

But if one of the Circles is to be divided into an odd Number of equal Parts, for Example, into 81, you muſt firſt divide it into 3 equal Parts, each of which will be 27, which being divided by 3, will produce 9; each of which being divided by 3, will produce 3; each of which being again divided by 3, will produce 1: wherefore the Periphery of the Circle will be divided into 81 equal Parts.

The like may be done for any other Number, in taking the moſt proper aliquot Parts thereof, to make a propoſed Diviſion.

The Circles of the Plate being divided, there ought to be made, at every Diviſion, ſmall round Holes, with a fine Steel Point.

Now when a Clock Wheel is to be ſimply divided by means of theſe Concentrick Circles, in order to cut it with the Hand, the ſaid Wheel muſt be placed upon the Arbre in the Center of the Plate, and having fixed it faſt, you muſt divide it with a fine Steel Ruler, one End of it being placed in the Center: then by moving the ſaid Ruler round from Diviſion to Diviſion, upon the Circumference of one of the Concentrick Circles, anſwerable to the Number of Teeth the Wheel is to have, the Wheel may be divided; which being done, the Teeth muſt be made with a very fine File, obſerving to leave as much Space between them, as you file away.

But when this Machine is uſed to very expeditiouſly cut Clock Wheels, it is compoſed in the following Manner.

Fig. 1. repreſents the Plan of the whole Machine put together, and fit for Uſe.

The Piece 1, is a Steel Saw-Wheel, the Breadth of the Interval between the Teeth of a Wheel to be cut by it: this Saw-Wheel is placed upon a ſquare Arbre, as likewiſe a little Pully, to turn it between two Steel Points. The Place 2, is the Porte-Touret, having a Motion at the two Ends thereof, like the Head of a Pair of Compaſſes, that ſo the file Wheel may be raiſed, or lowered, at pleaſure.

At the Place 1, of Figure 2, is the Saw-Wheel put upon it's Arbre, as likewiſe the Pully between the two Steel Points, that are faſtened by 2 headed-Screws 7, 7. The two Ends of the Porte-Touret, are repreſented by 2, 2. The Screws 9, 9, are for fixing the Part of the Machine carrying the Saw-Wheel, upon the Square Iron Ruler 3, which is put thro' a ſquare Hole, between the Screws 9, 9. There are two of the ſaid Iron Rulers, that is, there is one above the circular Brafs Plate, and another underneath it, both of them being of a convenient Bignefs, and are fo faftened together at the Ends by ftrong Screws, that there is room enough left between them for the circular Brafs Plate, and alfo for the Touret, or Frame, and a kind of Spring, which carries the Point (of which we fhall fpeak prefently), to flide freely along the fquare Iron Ruler 3.

Figure 3, reprefents the Side-Draught of the whole Machine put together, whereof the Piece 1, is the Touret, or Frame, placed near the Wheel to be cut, which is reprefented by Number 6: this Wheel is placed in the Center of the Brafs Plate, and is faftened by the Arbre Screw. The Piece 3, is the Iron Ruler along which the Touret of Figure 2 flides, as alfo the Spring carrying the Point 4: and Number 5 is a Piece of Iron, by means of which the Machine may be faftened in a Vice, when it is to be ufed.

Figure 4, is a very fine and well-tempered Steel-Point, fcrewed into the End of a kind of Spring, having a circular Motion, that thereby the faid Steel-Point may be put into any of the Holes of the Circumferences of either of the concentrick Circles upon the Plate. There is likewife another Piece joined to the Spring, is order to keep, by means of a Screw, the Point upon any propofed Divifion of the Circumference of any of the concentrick Circles, while one Tooth of a Wheel is fawing.

Laftly, Figure 5, is the Arbre placed in the Center of the Machine, and upon which is put the Wheels to be cut, which are firmly fixed thereon, by means of Screws at the Top and Bottom. There are commonly feveral Arbres of different Bigneffes, in proportion to the Holes in the Centers of Wheels to be cut.

The Ufe of this Machine is eafy, for you have no more to do but fix a Wheel to be cut into Teeth, in the Center (at Number 6), and then fit the Spring (reprefented by *Fig.* 4.) fo that it's Point may exactly fall upon the Divifions of that concentrick Circle, which is divided into the fame Number of equal Parts you defign your Wheel to have Teeth; and then you muft move the Touret, with it's Saw-Wheel, to cut the Wheel, by means of a Male-Screw (one End of which goes into a round Hole 8, in the Bottom of the Touret, and is there faftened with a Pin), and a Female-Screw to fit it, at the End of the Iron Ruler, denoted by Number 5; fo that by turning the faid Male-Screw, the Touret may be moved backwards and forwards at pleafure. The Saw-Wheel being thus placed, you muft turn it 4 or 5 times about, by means of a Bow, whofe String is put about the Pully, and then one Side of a Tooth will be cut; and having moved the Steel-Point 4, to the next Divifion in the Circumference of that concentrick Circle upon the Plate, whofe Divifions are the fame in Number you defign your Wheel to have Teeth, give 4 or 5 Strokes with the Bow, and the other Side of the Tooth will be cut: and in this manner may all the Teeth be cut; Pinions are alfo thus cut.

Note, There are Saw-Wheels of divers Thickneffes, conformable to the Space there ought to be left between the Teeth of different Wheels.

The Conftruction of Armour for Load-Stones, as alfo how to cut the faid Stones, in order to arm them.

The Figures 6, 7, reprefent two armed Load-Stones; the firft in the Form of a Parallelepipedon, and the fecond in the Form of a Sphere: But before we fhew the beft way of arming them, we will enumerate fome of the Properties and Virtues of Load-Stones.

The Load-Stone is a very hard and heavy Stone, found in Iron Mines, and is almoft the Colour of Iron, for which reafon it is reckoned among the Metallick Kind: it hath two wonderful Properties, one whereof is to attract Iron, and the other to direct itfelf towards the Poles of the World.

The Load-Stone attracts Iron, and reciprocally Iron attracts the Load-Stone, notwithftanding any other Body's Interpofition between them. This Stone likewife communicates to Iron a Faculty of attracting Iron: For Example, an Iron Ring that hath been touched with a good Load-Stone, will lift up another Iron Ring by only touching it, and this fecond a third, *&c.* but the firft Ring muft have a greater Degree of Attraction, than the fecond, and the fecond than the third, *&c.*

The Blade of a Knife that hath been touched with a Load-Stone, will likewife lift up Needles, and fmall Pieces of Iron: alfo feveral Sewing-Needles being laid upon a Table in a Row, and a Load-Stone being brought near the firft, by which receiving the Magnetick Virtue, the faid firft Needle will attract the fecond, the fecond the third, *&c.* 'till they all come together.

That Iron reciprocally attracts the Load-Stone, when it can move freely, may be thus fhewn: For if you put a Load-Stone into a hollow Piece of Cork, and fet it floating upon the Surface of a Bafon of Water, and bring a Piece of Iron at a convenient Diftance to it, the Piece of Cork, together with the Stone, will accede to the Iron.

That Property of the Load-Stone which is always to refpect the Poles of the World, may be fhewn by the following Experiment: For having put a Load-Stone into a hollow Piece of Cork, and fet them both a floating upon the Surface of ftill Water (there being no Iron, or other Obftacle near), the Load-Stone will always fo difpofe itfelf, that one certain Point thereof will regard the North, and the oppofite Point the South.

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But you must note, that the Load-Stone doth not exactly respect the North, it having at different Times, and in different Places of the Earth's Superficies, different Declinations, or Variations therefrom, and at this time at *Paris*, varies 12 Deg. 15 Min. Westward: so that the South Pole of the Load-Stone varies above 12 Degrees from that of the World, and it's Opposite so likewise. The Poles of a Load-Stone, are those two Places thereof, that respect the two Magnetick Poles of the World; and the principal Axis, is a right Line drawn from one Pole to the other, about which, the greatest Force of the Load-Stone manifests itself, and at the two Poles is greatest. Spherical Load-Stones have also ficted Equators, and Meridians, *&c.* from whence they are called Magnetick Spheres.

Now, in order to find the Poles of a Load-Stone, you must cut a Hole in a Card of the Figure of the Stone, in which the Stone must be put, so that it's principal Axis may be found in the Plane of the Card. This being done, Iron or Steel Filings must be strewed upon it: after which strike the Card softly with a little Stick, so that by putting the Filings in Motion, the Magnetick Matter may let them take a Circuit conformable to the way which that Matter takes in moving from a North Pore to another South one, and you will perceive the Filings ranged in the Figure of several Semi-Circumferences, whose opposite Ends are the Poles of the Load-Stone.

The Poles of a Load-Stone may otherwise be found, in plunging it into Iron or Steel Filings, or into very little Bits of Steel Wire; for then they will make different Configurations round the Stone, some of them lying flat on it, others half bent; and finally, others quite upright on it: and those Places of the Stone where the little Bits of Steel are perpendicular to it, are the Poles; and where they lie along, is the Equator.

Having thus found the Poles of a Load-Stone; which is the North or South Pole, may be known in laying the Stone in a hollow Piece of Cork, swimming on Water, or by suspending it with a Thread, so that it's Axis be parallel to the Horizon; for then that Pole of the Stone turning towards the North Pole of the World, will be the South Pole of the Stone, and the opposite Point the North Pole.

The Poles of a Load-Stone may likewise be found by means of a Compass; for bringing a touched Needle to the Stone, the End that was touched, will immediately turn toward that Pole of the Stone agreeing therewith, and the other End of the Needle will likewise turn towards the other Pole of the Stone.

The Poles of the Stone being found, the next thing will be to cut, and give it a regular Figure, in taking away the Superfluities either with a Saw, and Powder of Emery, or else with a Knife-Grinder's Grind-stone, preserving it's Axis as long as possible, and giving a like Figure to it's Poles.

Now to make a great many Experiments, it is necessary to give to a Load-Stone the most regular Figure possible, which is determined by the Likeness it hath to that of the irregular Mass it is composed of: the Cube, the Parallelepipedon, the Oval, and the Round are to be preferred, on account of having the principal Axis of the Stone as long as may be. If a Load-Stone is to be made in Form of a Sphere, it will not be difficult to find it's Poles and Axis; you need only figure it with Powder of Emery in a round Iron Concave, and afterwards finish it with find Sand, in a round Brass Concave.

A Load-Stone in Figure of a Sphere, is very fit for many Experiments, and it's Poles may be found in manner aforesaid: but it is necessary, before any pains be taken in cutting and figuring of a Load-Stone, to be assured of it's Goodness, in observing whether it strongly attracts Filings, or little Bits of Steel; and whether there be not other Matter passing thro' it's Pores, which hinders the Magnetical Matter from circulating and passing from one Pole to the other.

The Goodness of a Load-Stone consists in two essential Things; which are, first, That it be homogeneous, having a great Number of Pores filled with Magnetick Matter, which passing thro' them form about the Stone, as it were, a very extensive Whirlwind. In the second place, it's Figure very much contributes to it's Force (as we have already said), for it is certain, that of all Load-Stones of a like Goodness, that which hath the best Poles, it's Axis longest, and whose Poles meet exactly in the Extremes, will be most vigorous.

Two Load-Stones placed in two hollow Pieces of Cork, which are both set floating upon the Surface of the Water, having their Poles of contrary Denominations turned to each other, will accede to each other; but if the Poles of the same Denomination be turned towards each other, then the Load-Stones will mutually recede from one another.

If a Load-Stone be cut into two Pieces, parallel to it's Axis, the Sides of the Pieces that were together before the Division, will mutually recede from each other.

But if a Load-Stone be cut into two Pieces, according to it's Equator, the Sides of the Pieces that were together before they were cut, will be found to have Poles of a contrary Denomination, and will accede to each other.

A strong Load-Stone touching a weak one, will attract it with it's Pole of the same Denomination, *&c.*

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The Description of the Armour, or Capping for Load-stones.

Fig. 6. The Armour for a Load-stone, cut into the Form of a right-angled Parallelepipedon, is composed of two square Pieces of very smooth Iron or Steel; but tempered Steel is better than Iron, because it's Pores are closer, and there are a greater Number of them. Care must be taken, that the Armour well encompasses, and exactly touches the Poles of the Load-stone, and that the Armour is in Thickness proportionable to the Goodness of the Stone: for if strong Armour be put upon a weak Stone, it will produce no Effect, because the magnetick Matter will not have force enough to pass thro' it; and, on the contrary, if the Armour of a strong Stone be too thin, it will not contain all the magnetick Matter it ought, and consequently the Stone will not produce so great an Effect, as when the Armour is thicker.

Now, to fit the Armour exactly, you must file it thinner by Degrees; and when you find the Effect of the Stone to be augmented as much as possible, the Armour will be in it's just Proportion, and will have it's convenient Thickness; after which it must be smoothed within Side, and polished without.

The Heads of the Armour (whereon is writ *North* and *South*) must be thicker than the other Parts, and cover about $\frac{2}{3}$ of the Length of the Axis.

The Breadth and Length of the Armour, best fitting a Stone, may also be found by filing it by little and little; but, above all, Care must be taken that the two Heads are equal in Thickness, and that their Bases very exactly meet in the same Plane. Number 5 is a Brass or Silver Girdle fitted about the Stone, serving to fasten and hold the Armour, by means of two Screws 1, 1; and at 6 and 6 are two Screws fastening a round Brass Plate, carrying the Pendant 4, and it's Ring, to the Top of the Armour.

Fig. 7. The Armour of a spherical Load-stone is composed of two Steel Shells, fastened to the Piece 8 by two Joints 6, 6; of a Girdle 5, 5; of a Pendant and Ring 4; and of a Piece (or *Porte-Poid*) 2, to hold the Hook 3 Great Care must be taken that the Shells very exactly join the Superficies of the Stone, and that they well encompass the Poles of the Stone, and cover the greatest part of the Convexity thereof. The convenient Breadth and Thickness of this Armour may be found by Trials, as before-mentioned.

It is very wonderful, that two little Pieces of Steel, composing the Armour of a Load-stone, should give it such a Property, that a good Stone, after it is armed, will attract above 150 times more than before it was armed.

There are indifferent good Stones, which, unarmed, weigh about three Ounces, and will lift up but half an Ounce of Iron; but being armed, will lift up more than seven Pounds.

To preserve a Load-stone, you must keep it in a dry Place among little Bits of Steel-Wire; for Filings, which are always full of Dust, make it rusty.

We sometimes suspend Load-stones, so that having the liberty to move, they may conform themselves to the Poles of the World; and if, in this Situation, the Piece carrying the Hook, or *Porte-Poid*, be put on, and the Weight the Stone commonly carries be hung on, and from time to time there be hung to it some small Weight more, you will find that, when the Stone has continued suspended some Days, that it will lift up a much greater Weight than it did before it was hung up.

Several common Experiments made with the Load-stone.

The first and usefullest Experiment made with the Load-stone, is that of touching the Needles of Sea-Compasses; for rightly doing of which, you must draw the Needle softly over one of the Poles of the Load-stone, from it's Middle to it's End, and then it will receive it's Virtue. But, *Note*, that that End of the Needle, which hath been touched with one of the Poles of a Load-stone, will turn towards the opposite Part of the World, to that which that Pole regards; therefore if the End of a Needle is to turn towards the North, it must be touched with that Pole of the Stone respecting the South. *Note*, The longer Needles are, the less will they vibrate.

This admirable Direction of the Load-stone and Touched Needle hath not been known in *Europe* much above two hundred Years, by means of which, Navigation hath been almost infinitely advanced. But there is one Inconveniency, which is, that a Touched Needle doth not exactly respect the Poles of the World, but declines or varies therefrom towards the East or West, at different Times, and in different Places, variously. In the Year 1610, it varied at *Paris* 8 Degrees North-Easterly; in 1658, it had no Variation; and in the Year 1716, it varied about 12 Deg. 15 Min. Westward.

Moreover, the Needle hath also an Inclination as well as a Declination; that is, the Needle of a Sea-Compass being *in Equilibrio* upon it's Pivot, will, when touched, lose that Equilibrium, and the End that turns North, on this side the Equator, will drip or incline towards the Earth, as if it was heavier on that Side; for which reason the North Side of a Needle must be made lighter, before the Needle be touched, than the South Side, and going towards the Poles, this Inclination grows greater; but in going towards the Equator, it grows lesser: so that under the Equator, the Inclination will be nothing; and in passing the Line, the other End of the Needle, respecting the South, will begin to incline; so that Pilots are obliged to stick as much Wax to the End of the Needle, as will make it *in Equilibrio*. *Note*,

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The greater Force that Load-ſtones, which touch Needles, have, the more will the Needles incline.

There are Needles purpoſely made to obſerve this Inclination, which at *Paris* is about 70 Degrees.

If a long thin Piece of Steel be drawn over one of the Poles of an armed Load-ſtone (in the ſame manner as was ſaid before of the Needles), this Piece of Steel will in an inſtant acquire the magnetick Virtue, and will not loſe it but by degrees after ſeveral Months, unleſs it be put in the Fire. *Note*, A Piece of Steel, touched by a good Stone, will lift up 14 Ounces.

The two Ends of a Steel Blade thus touched will become North and South Poles; that End whoſe Contact ends on the South Pole of the Stone, being the North, and the other the South Pole: for if this Piece of Steel be made light enough to ſwim, one End thereof will turn to the North, and the other to the South.

Again; that End of the Steel Blade where the Contact ended, will attract much ſtronger than the other End; and if the ſaid Blade be once drawn over the Stone the contrary way, it will quite loſe it's Virtue, and attract no more. Underſtand the ſame of the Needle of a Compaſs, the Blade of a Knife, &c. two touched Steel Blades will avoid each other, and approach like two Load-ſtones.

A Piece of Steel, in a hollow Piece of Cork ſwimming on the Water, may be any ways moved, by bringing the Pole of a Load-ſtone towards it, or another touched Piece of Steel.

A fine Sewing-Needle, ſuſpended by a Thread, will ſhew what is meant by Sympathy and Antipathy; for this Needle will be repelled by one Pole of a Load-ſtone, and attracted by the other.

A Needle may be kept upright, without it's touching a Load-ſtone; ſo that there may be put between it and the Stone, a Piece of Silver, or other Matter, provided it be not Iron.

If, about a Load-ſtone, ſuſpended by a String, be circularly placed ſeveral little touched Needles of a Compaſs, upon their Pivots, and the Load-ſtone be moved any how, you will likewiſe ſee all the Needles move in a pleaſant manner; and when the Stone ceaſes moving, the Needles will alſo ceaſe.

What we have already ſpoken about ſtrewing of Filings about a Load-ſtone, may be ſaid alſo of ſtrewing them about a Piece of touched Steel.

If Filings be ſtrewed upon a Piece of Paſteboard, and a Load-ſtone be moved under it, the Filings will erect themſelves, and then lie along on that Side from whence the Stone came.

If, inſtead of Filings, you lay upon a Piece of Paſteboard ſeveral Bits of the Ends of broken Needles; by bringing one Pole of a Load-ſtone towards them, they will erect themſelves upon one of their Ends; and by bringing the other Pole, they will fall, and riſe upon their other Ends.

It is eaſy to ſeparate a black Powder mixed with white Sand, and propoſing it to a Perſon, not knowing the Secret, he will think it impoſſible; for if Iron Filings be mixed with fine Sand, they may be ſeparated from it by a Load-ſtone, or Piece of touched Steel: for either of them being put into the Mixture, at divers times, you may get all the Filings from among the Sand.

A Load-ſtone will lift up a Whirlegig in Motion, whoſe Axis is Steel; and if it be ſomething heavy, it will turn a longer Time in the Air than upon a Table, where the Friction ſoon ſtops it's Motion; and if the Stone be a good one, this Whirlegig may lift up another, and both of them will turn contrary ways. Another diverting Experiment may yet be made, by putting little Steel Fiſhes, or Swans, into a flat Baſon of Water; for by moving a good Load-ſtone under the Baſon, you will ſee them prettily ſwimming about; and moving the Stone different ways, they will likewiſe have different Motions; if the Stone be turned round, the Fiſhes will alſo turn round; if the Pole of the Stone be turned towards them, they will plunge themſelves, as it were, to join themſelves to the Stone. You may likewiſe put little Steel Soldiers into the Baſon, which may be made to approach to, or recede from, each other in form of a Battel; and by bringing the Equator of the Stone towards them, they will fall down.

It is pleaſant enough to ſee a Sewing-Needle threaded, or a little Arrow, faſtened by a Hair to the Arc of a *Cupid*'s Bow, remain ſuſpended in the Air eight or ten Lines diſtant from a good Load-ſtone.

There are ſeveral other Experiments made with the Load-ſtone, but mentioning them here would take up too much time.

The Conſtruction of an Artificial Magnet.

This Inſtrument, invented by Mr *Joblot*, is compoſed of ſeveral very ſtrait Steel Blades laid upon one another; and to make it paſſably good, there ought to be at leaſt 20 of them (according to the force of the Magnet to be made), each about 10 Inches long, 1 Inch broad, and half a Line in Thickneſs. It is uſeleſs to make them thicker, becauſe the magnetick Virtue will not penetrate further into the Steel Blades. Fig. 8.

Now theſe Blades being firſt touched with a good Stone, are afterwards laid one upon another, having their Poles, of the ſame Denomination, turned the ſame way, forming a Parallelepipedon; then they are preſſed together with four Braſs Stirrups, and as many little Wedges 3, 3, 3, 3, of the same Metal, and encompassed with Iron Armour of a proper Length, Breadth, and Thickness. This Armour is held by a Brass Girdle, and fastened with the Screws 2, 2. At the Top is placed a Brass Plate, to which is fastened the Pendant 4, and it's Ring; and at the Bottom is the *Port-Poids* 5. But, *Note*, That the Base of the *Porte-Poids* must make the perfectest Contact possible with the Heads *a*, *b*, of the Armour. When artificial Magnets are well made, and touched with good Stones, they will have as much Virtue in them as good natural ones, and may be used for the same Experiments.

The Construction of the Spring Steel-yard.

Fig 9. This Machine, which is portable, and serves to weigh any thing from one Pound to about forty, is composed of a Brass Tube or Pipe, open at the Ends, about 4 or 5 Inches long, and 7 or 8 Lines broad, one End whereof is marked 3; the rest being open for shewing the Inside, which is a Spring (2) of tempered Steel-Wire, made like a Worm. *Number* 6. is a little Feril screwed upon the Top of the square Brass Rod 1, which the Spring crosses. Upon this Rod are the Divisions of Pounds, and Parts of a Pound, which are made in successively hanging on the Hook (4.) 1, 2, 3, *&c.* Pounds: for the Spring being fastened by a Screw to the Bottom of the square Rod, the greater the Weight is, that is hung on the Hook, the more will the Spring be contracted; and consequently a greater part of the Rod will come out of the Tube, thro' the square Hole C: therefore if you have a mind to mark the Division for any Number of Pounds upon the Rod, suppose 10, hang 10 Pounds upon the Hook, and where the Edge of the square Hole C, at the Top of the Tube, cuts the Rod, make a Mark upon the Rod for 10 Pounds, and so for any other.

The Use of this Instrument is very easy; for having screwed the Feril 6 on the Top of the Rod, if you hold the Instrument in your Hand by the Hook 5, and hang any thing to be weighed upon the Hook 4; then where the Edge C of the square Hole cuts the Rod, will be the Weight of the thing required.

The chief Goodness of this Instrument consists in having a well-tempered Spring; so that it may fold according to the Force of the Weight it is to carry, and also in having a Bigness proportionable.

The Construction of the Beam Steel-yard.

Fig. 10. This Instrument, which is a kind of Steel-yard, or Balance of Mr *Cassini*'s Invention, consists of a Rod suspended by a Beam, in it's Point of Equilibrium 5, which divides the said Rod into two Arms (like the two Arms of a common Balance) each of which are lengthwise divided into equal Parts, beginning from the Point of Suspension or Equilibrium.

The Use of this Balance is to find both the Weight and Price of Goods at the same time. If you use it for weighing any thing, the Counter-Weight 4 of one Pound, or one Ounce, must be hung to one of the Arms (according as Goods are to be weighed by Pounds or Ounces), so that it may slide along the Arm, like as in *Roman* Balances; and on the other Arm must be hung on a silken Line, for sustaining things to be weighed. Then to weigh any thing, you must place the silken Line, to which the thing is hung, upon the first Division of the Arm, nighest the Point of Equilibrium; and moving the Counter-weight upon the other Arm, 'till it makes an Equilibrium, the Point whereon it falls will show the Weight sought.

To know the Weight of Goods, according to any Price; for Example, at seven Pence an Ounce or Pound; place the Line, sustaining the Goods, upon the Division 7 of the Arm; then placing the Line, carrying the Counter-weight upon the other Arm, so that it be *in Equilibrio*, and the Number of Divisions, from the Point of Suspension to the Line sustaining the Counter-weight, will give the Value of the Goods weighed.

But for Goods that cannot be weighed, unless in a Scale, take a Scale of a known Weight, and having hung it upon a Hook to the Arm, proceed as before, and substract the Weight of the Scale.

A *Paris* Pound is 16 Ounces, and is divided into 2 Marks, each of which is 8 Ounces; an Ounce is subdivided into 8 Drams, a Dram into 72 Grains, and a Grain, which is nighly the Weight of a Grain of Wheat, is the least Weight used.

A Quintal weighs 100 Pounds.

The Paris *Pound compared with those of other Countries.*

The Pound of *Avignon*, *Lyons*, *Montpelier*, and *Thoulouse* is 13 Ounces.
The Pound of *Marseilles* and *Rochelle* is 19 Ounces.
The Pound of *Rouen*, *Besançon*, *Strasburgh*, and *Amsterdam* is 16 Ounces, like that of *Paris*.
The Pound of *Milan*, *Naples*, and *Venice*, is 9 Ounces.
The Pound of *Messina* and *Genoa* is 9 $\frac{1}{4}$ Ounces.
The Pound of *Florence*, *Leghorne*, *Pisa*, *Sarragossa* and *Valence* is 10 Ounces.
The Pound of *Turin* and *Modena* is 10 $\frac{1}{2}$ Ounces.
The Pound of *London*, *Antwerp* and *Flanders* is 14 Ounces.
The Pound of *Basil*, *Berne*, *Franckfort* and *Nuremburgh* is 16 Ounces and 14 Grains.
That of *Geneva* is 17 Ounces.

Construction

Conftruction of an Inftrument for raifing of Weights.

The Inftrument of *Fig.* 11. confifts of two Sheaves, each of which carries eight Pullies, hollowed in to receive a Rope, which is faftened at one End to the upper Sheave; and after having put it round all the Pullies, the other End of it muft be joined to the Power reprefented by the Hand. Four of the Pullies are carried upon one Axel-Tree, and four upon another, as well in the upper Sheave as in the lower one. At the Top of the upper Sheave is a Ring to hang the Machine in a fixed Place, and at the Bottom of the other, there is another Ring to hang the Weights to. Fig. 11.

The Ufe of this Machine is to lift up or draw great Burdens, by multiplying the Force of the Power, which augments, in the Ratio of Unity, to double the Number of the Pullies in the lower Sheave; fo that in this Inftrument, where the lower Sheave carries eight Pullies, if the Weight (4) weighs 16 Pounds, the Power need be but a little above one Pound to make an Equilibrium; I fay, a little above, becaufe of the Friction of the Ropes and Axes. The Pullies of the upper Sheave do not at all contribute to the Augmentation of the Force, but only to facilitate the Motion in taking away the Friction of the Rope, becaufe being as Leavers of the firft kind, whofe fixed Point is in the Middle, the Power will be equal to the Weight; but the Pullies below are as Leavers of the fecond Kind, whofe fixed Point is at one of the Ends: for their Diameter is, as it were, fixed at one End, and lifted up at the other; by which each of the Pullies double their Force, fince the way moved through by the Power, is double to that moved through by the Weight.

The Conftruction of the Wind-Cane.

This Inftrument is about three Foot long, and twelve or fifteen Lines in Thicknefs. The Tube 3 is made of Brafs, very round, and well foldered, from 4 to 6 Lines in Diameter, ftopped at one End *a*. At the Place 1 is likewife another larger Tube, fo difpofed about the former one, that there remains a Space 4, wherein the Air may be clofely included. Thefe two Tubes ought to be joined together at one End by a circular Plate *c c*, exactly foldered to them both, for hindering the Air's getting out of the Space 4. The Piece 8 is a Valve ftopping a Hole, permitting the Air to pafs from 2 towards 1, but not to return from 1 towards 2. There are, moreover, two Holes near the ftopped End of the Tube 3; thro' one of thefe Holes, which is marked 6, the Air would come out of the Space 4 into the Tube 3, if it was not hindered by a Spring-Valve opening outwardly. The other Hole is marked 5, thro' which there is a Communication with the outward Air, and the Air in the Cavity of the Tube 3; but yet fo, that the Air, inclofed in the Space 4, cannot come out thro' the Hole 5, it being hindered by a little fhort Tube foldered to the Tubes 1 and 3. Laftly, the Tube 2 reprefents the Body of a Syringe, by which as much Air as poffible may be intruded into the Space 4; after which having put a Bullet into the Cavity of the Tube 3, near the little Tube 5, the Cane will be charged. Now, to difcharge it, you muft pufh up the Spring-Valve 6, by means of a little Pin exactly filling the Cavity of the little Tube 5; then the compreffed Air, in the Cavity 4, will dilate itfelf; and paffing thro' the Hole 6, into the Cavity of the Tube 3, will pufh the Bullet out with a great force, even to it's penetrating thro' a Board of an ordinary Thicknefs. Fig. 12.

Note, At Number 7 this Cane may be taken into two Pieces, by unfcrewing of it; and the Handle 12 may be taken out, and inftead thereof the Head of a Cane put thereon.

The Conftruction of the Æolipile.

This Inftrument is made of hammered Copper, in form of a Ball, or hollow Pear, having a Neck foldered to it, and a very little Hole drilled at the End of this Neck. Fig. 13.

The Air in the Ball is firft rarefied, by bringing it to the Fire; and afterwards plunging it into cold Water, will condenfe the Air in it, and the Water will pafs thro' the little Hole into the Cavity of the Inftrument.

Now having let about as much Water, as will fill $\frac{1}{3}$ of the Æolipile, get into it, if it be fet upon a good Fire, in the fame Situation as in the Figure, the Water, as it grows hot, will dilate itfelf by little and little, and throw up Vapours into the Space of Air contained between the Surface of the Water, and the little Hole at the End of the Neck, which, together with the Air, will very fwiftly crowd thro' the little Hole, and produce a Wind and violent Hiffing, continuing 'till all the Water be evaporated, or the Heat extinguifhed. *Note*, This Wind has all the Properties of the natural Wind blowing upon the Surface of the Earth.

The Conftruction of four different Microfcopes.

This is a Microfcope for viewing very minute Objects and Animals that are in Liquors. It is compofed of two Plates of Brafs, or other Metal, about 3 Inches long, and 8 Lines broad, faftened together, nigh the Ends, by two Screws, 2, 2, which likewife ferve to fix the Plates at fuch a Diftance from each other, that a Wheel may turn which has fix round Holes, in every of which are flat Pieces of Glafs to put different Objects upon, marked 3, 4, 5, &c. Next to the Eye there is a concave Piece of Brafs 1, having a Hole in the Middle, in which is put a very fmall *Lens*, or Ball of Glafs. This Ball ought to be very convex, and well polifhed, Fig. 14.

in

in order to diſtinguiſh minute Objects. The End of the Machine is filed in manner of a Handle to hold it.

The Uſe of this Inſtrument is very eaſy; if the Objects are tranſparent, as the Feet of a Flea, or of Flies, their Wings, the Mites in Cheeſe, or other minute Animals; as likewiſe Hairs of the Head, their Roots, *&c.* they are put upon the Glaſs Plates on the Wheel, and are held faſt with a little Gum-water: and to ſee the little Animals in ſtale Urine, Vinegar, in Water where there has been infuſed Pepper, Coriander, Straw, Hay, or almoſt any kind of Herbs; little Drops thereof muſt be taken up with the End of a little Glaſs Pipe, and laid upon the aforeſaid Glaſſes: then the Wheel muſt be turned and raiſed, or depreſſed by means of the Screws 2, 2, and a Spring between the Plates, which ſerves to keep the Wheel in any Situation required, in ſuch manner that a little Drop may be exactly under the Lens. Things being thus ordered, take the Microſcope in your Hand, and having placed your Eye to the Concave 1, over the Lens, look ſteadily at the Drop in broad Day-light, or at Night by the Light of a Wax-Candle; at the ſame time turn the Screw at the End by little and little, to bring the Drop nigher, or make it further from the Lens, until the Point be found where the Object will be tranſparent, or the Animals ſwimming in the Drop of Liquor, appear very large and diſtinct.

Conſtruction of another Microſcope.

Fig. 15. This Microſcope is compoſed of a Braſs Plate about three Inches high, and ½ an Inch broad, cut in Form of a Parallelogram, at the Bottom of which there is a Handle to hold it. The Place marked 1, is a little Groove drilled thro' the Middle, in the Hole of which is placed a Lens faſtened in a little Frame; there may be put into it Lenſes of diverſe Foci, according to the different Objects to be obſerved. *Note*, That the Focus of a Glaſs, is it's Diſtance from the Object, and that Lenſes are uſed in theſe Microſcopes, whoſe Foci are from half a Line to four Lines.

On the Backſide of the aforeſaid Plate (at the Place 2.), is fixed a little ſquare Branch of Braſs or Steel, carrying another Plate that ſlides upon it by means of a little Box, a Spring, and a Screw, turned by help of a Wheel, cut into Teeth, which ſerves to bring the ſaid Plate nigher to, or more diſtant from that which carries the Lens. Towards the Top of the ſecond Plate; which has a Hole drilled in it, is alſo a Groove, in which is placed little Pieces of plain Glaſs, and round Concaves to put Liquors on. There may be different Glaſſes put in that Groove for viewing different Objects. Laſtly, Obſerve that all the Objects anſwer to the Center of the Lens, and that there muſt be adjuſted on the other Side of the Plate a little Tube (marked 3.) of Braſs, about an Inch Diameter, and one or two long, whoſe Center muſt very exactly anſwer to the Center of the Lenſes. It has been found that with ſuch a Tube, theſe Microſcopes will have much more effect upon tranſparent Objects, than without it. The Circulation of the Blood may pretty diſtinctly be obſerved in the Tails of little Fiſhes by this Microſcope, which is, in my Opinion, the moſt commodious of any.

The Uſe of this Inſtrument is very eaſy; for having placed the Object over-againſt the Center of the Lens, move it backwards and forwards by means of the Screw, 'till it be ſeen very diſtinctly.

Conſtruction of a ſingle Glaſs Microſcope.

Fig. 16. The little Inſtrument of *Fig.* 16. is a Microſcope commodious enough, compoſed of a Branch of Braſs, or other Metal, having a Motion towards the Top, for putting it into the Situation as *per Fig.* The Piece, at the End, carries a very convex Lens, magnifying the Object very much: this Branch is ſcrewed into a little Box 5, bored through the Bottom. The Piece 4, is two Springs faſtened to one another in the Middle with a Rivet, to give it a Motion deſired. The Branch which carries the Lens, is put through one of the Springs; and through the other there is put a little Branch, carrying at one End the Piece 2, which is white on one Side, and black on the other, for different Objects. The other End 3, is a little kind of Pincer, which opens by preſſing two little Buttons; it ſerves to hold little Animals, or other Bodies. The Foot 5, is about 1 ½ Inch in Diameter, the Branch ſcrews into it, in order to take to pieces the Inſtrument.

The Uſe of it is very eaſy, for the Objects being placed upon the little round Piece, or at the End of the Pincer, you muſt bring the Lens towards them, by ſliding the Spring along the Branch, 'till the Objects be ſeen very diſtinct.

There may likewiſe be diſcovered with this Microſcope, the Animals which are in Liquors, by putting a flat Glaſs in the Place of the little round Piece 2, which unſcrews.

Conſtruction of a Three-Glaſs Microſcope.

Fig. 17. This Inſtrument is compoſed of three Glaſſes, *viz.* the Eye Glaſs 3, the Middle Glaſs 4, and the Object Lens 5. There is a Cover ſcrewed on at the Top to preſerve the Eye Glaſs from Duſt: theſe three Glaſſes are ſet in wooden Circles, and ſcrewed into their Places, for eaſier taking them out to cleanſe. The Eye Glaſs, and the Middle one, are placed at the Ends of a Tube of Parchment, exactly entering into the outward Tube, in order to lengthen the Microſcope, and place it at it's exact Point, according to a Line drawn round about the

afore-

aforeſaid Tube. To have this Inſtrument of a reaſonable Bigneſs, the focal Diſtance of the Eye Glaſs ought to be about 20 Lines, that of the middle Glaſs about 3 Inches, and placed about 3 Inches 3 Lines diſtant from one another.

The Object Lens is placed at the End of a wooden Tail-piece, glued to the End of the outward Tube, and is encloſed in a little Box, bored through the Bottom, which unſcrews in order to change the Object Lenſes, and put in others of different focal Diſtances, which are commonly 2, 3, 4, and 5 Lines in Diameter, and are more or leſs convex. The Goodneſs of theſe Glaſſes conſiſts in having the concave Braſs Baſons they are ground in, turned in a juſt Proportion to the Glaſſes to be worked; as alſo in the Motion of the Hand, and the Goodneſs of the Matter uſed to conſtruct them, and above all in well poliſhing them. Brown Free-ſtone is firſt uſed to faſhion them in the Baſon, then fine Sand to ſmooth them, and Tripoli to poliſh them. I ſhall ſay no more of the Conſtruction of theſe Glaſſes, M. *Cherubin* having ſufficiently ſpoken thereof.

The Foot 1, which ought to be pretty heavy to keep the Miſcroſcope from falling, is made of Braſs 4 or 5 Inches in Diameter, having a Cavity in the Middle, wherein is put a little Piece, white on one Side, and black on the other: black Objects are placed upon the white Side, and white Objects upon the black Side.

The round Braſs Branch is faſtened at the Edge of the Foot, upon which the Microſcope may ſlide up or down, and turn round by means of the Support or double Square 2: there is a Circle, or Ring, ſtrongly faſtened to the Support, and which very exactly encompaſſes the outward Tube. There is alſo a Steel Spring which bears againſt the Branch, and keeps the Inſtrument in a required Situation.

Number 6, is a little Braſs Frame, having in it a Piece of flat Glaſs to lay tranſparent Objects upon. This Frame may ſlide up and down the Branch underneath the Microſcope, and is ſupported by a double Square.

Laſtly, Number 7 is a convex Glaſs converging the Rays of Light, coming from a Candle under it; and throwing them ſtrongly under the tranſparent Object on the Glaſs, makes it be ſeen more diſtinctly. The aforeſaid Glaſs is ſet in a Braſs Circle, and riſes, falls, and turns by means of a little Arm carrying it, as the Figure ſhews.

USE *of the aforeſaid Microſcope.*

To uſe this Inſtrument, for Example, to obſerve the Circulation of the Blood in ſome Animal; a live Fiſh muſt be placed upon the Glaſs 6, ſo that one part of the Fins of the Tail be exactly oppoſite to the Object Glaſs, and over the Ray of the Convex-Glaſs in broad Daylight, or the Spot of the Candle, in the Night; then place the Microſcope exactly to ſuch a Point, and you will ſee the Blood riſe, deſcend, or circulate.

Number 9, is a little Piece of Lead hollowed, to keep the Fiſh from any how ſtirring to hinder the Experiment.

Liquors may alſo by this Microſcope be very well examined; for if you put a little Drop of Vinegar upon the Glaſs juſt over the bright Spot; the little Animals in it will very diſtinctly be obſerved. The ſame may be obſerved of Water in which Pepper or Barley has been infuſed, *&c.* as alſo the Eels and other little Animals obſerved in ſtanding Water.

A Drop of Blood may be obſerved by putting it hot over the Speck of the Candle, upon the Glaſs; after which it's Seroſity, and little Globules of a reddiſh Colour, may be diſcovered therein.

The beſt way to get a Drop of Blood is to tie a Thread about one's Thumb, and then prick it with a Needle.

The beſt way to put Liquors upon the Glaſs, is by taking a Drop of them up with the ſmall End of a little Glaſs Tube; and then blowing ſoftly at the other End, will make the Liquor deſcend and drop upon the Glaſs.

To get a great Number of little Eels in a ſmall Quantity of Liquor; the Liquor muſt be put into a very narrow-necked Bottle, and always kept full; for by this means, the Animals coming to the Top to get Air, may be ſucked into a little Tube in greater Numbers, than if the Neck of the Bottle was wider.

The Eyes of Flies, Ants, Lice, Fleas, and Mites, are put in the Middle of the Foot of the Microſcope, as alſo Sand, Salt, *&c.* to examine their Colours and Qualities; always obſerving to lay black Objects upon the white, and white Objects upon the black Side.

I ſuppoſe here that the Microſcope Glaſſes are well worked, and placed in their Foci. Note alſo, that the ſhorter the Focus of an Object Glaſs is, the greater will the Object appear, but not altogether ſo diſtinct.

BOOK IV.

Of the Construction and Uses of Mathematical Instruments for measuring and laying out of Land, taking of Plots, Heights, and Distances; the most usual of which, are Staffs, Lines, the Toise or Fathom, the Chain, Surveying-Crosses or Squares, Recipient-Angles or Measure-Angles, Theodolites, the Quadrant, the Semicircle, and the Compass.

CHAP. I.

Containing the Description and Uses of Staffs, Lines, the Fathom or Toise, and the Chain.

Plate 11. Fig. A. STAFFS are made of hard Wood, 2 or 3 Foot long, cut pecked at one End, upon which are put pointed Caps of Iron, to make them go easier into the Ground. There are sometimes longer ones made, in order to be seen at a great distance.

Fig. B. Lines ought to be of good Packthread, or Whipcord, well twisted, and of a convenient Thickness, that they may not easily stretch.

Fig. C. The Toise, or Fathom, is a round Staff 6 Foot long, divided into Feet by little Rings, or Brass Pins; the last Foot being divided into 12 Inches, likewise distinguished by little Brass Pins.

Fig. D. There are Toises that may be taken into 2, 3, or 4 Pieces, by means of Ferils and Brass Screws at the End of each Piece.

There are also two Brass or Steel Ferils, put upon each End of the Toise, to preserve it's Length.

Fig. E. The Chain is composed of several Pieces of thick Iron or Brass Wire, bent at the Ends, each of which is a Foot long, and are joined together with little Rings.

Chains are commonly a Perch, or else 4 or 5 Toises in Length, distinguished by a great Ring from Toise to Toise. These sort of Chains are very commodious, because they will not entangle themselves, as those will that are made with little Iron Rings.

In the Year 1668, there was placed a new Toise for a Standard, at the Foot of the Stairs of the *Grand Chatelet* at *Paris*, for having recourse to in case of Need.

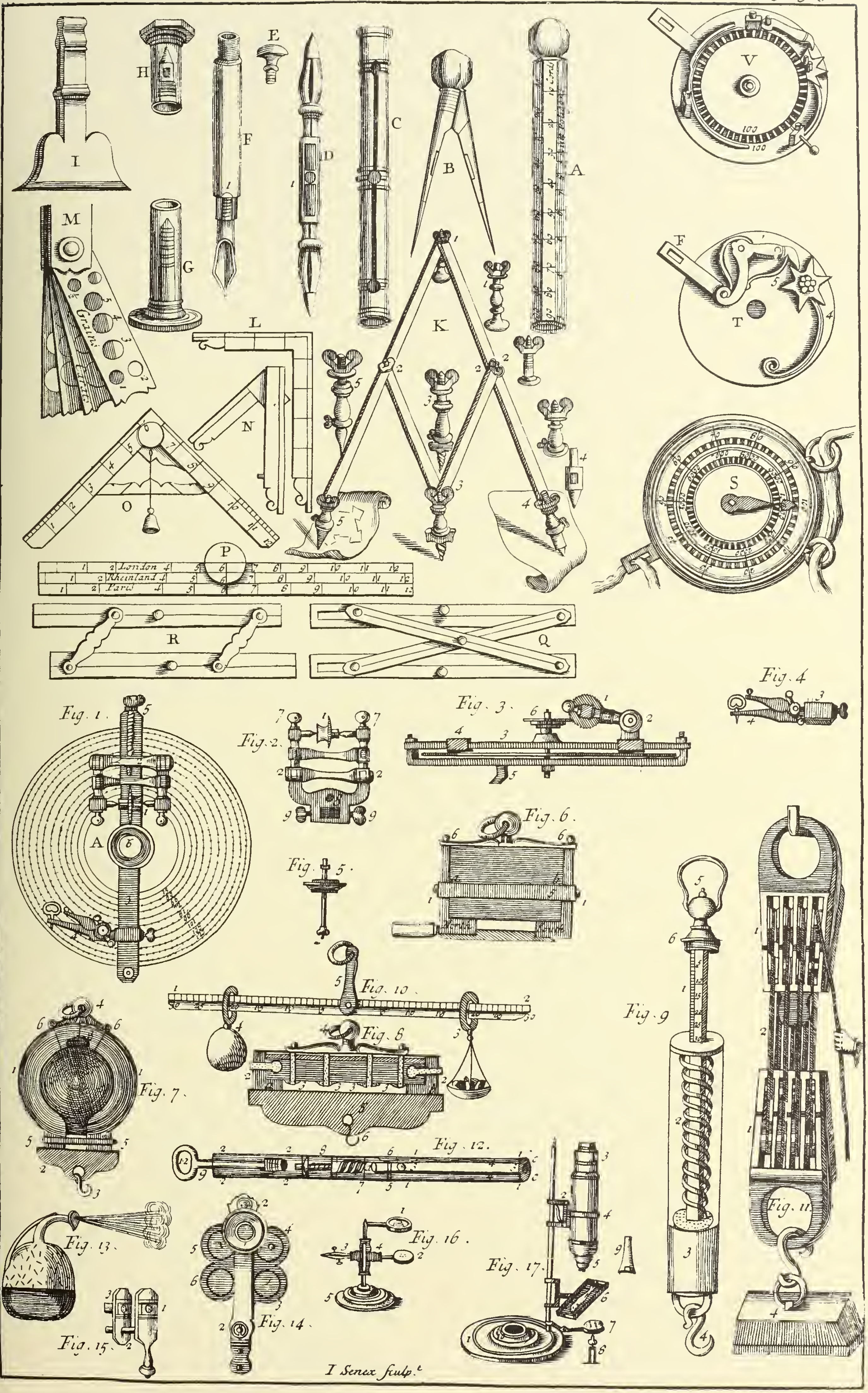
We have said that a Toise in Length contains 6 Feet, and each Foot 12 Inches.

A square Toise contains 36 square Feet, and a square Foot 144 Inches; because 6 times 6 is 36, and 12 times 12 is 144.

A Cubick

Plate X

fronting page 98.



A Cubick Toise contains 216 Cubick Feet, and a Cubick Foot 1728 Cubick Inches; becaufe the Cube of 6 is 216, and the Cube of 12 is 1728.

The Length of a Perch is not determined.

That of *Paris* is 3 Toifes, or 18 Feet; in other Countries it is 20, 22, and 24 Feet.

The Perch, ufed in *France*, to meafure Waters and Forefts, according to the laft Regulation, is 22 Feet long, and confequently a fquare Perch is 484 fquare Feet.

The Arpent is a fuperficial Meafure, ufed to meafure Ground or Woods.

The Arpent of *Paris*, and the adjacent Parts, contains 100 fquare Perches, or 900 Toifes: the Side of which muft confequently be 10 Perches, or 30 Toifes.

A League is a Meafure for High-ways, or great Diftances; it's Length is not determined, being different in different Countries.

It is reckoned from the Gate of *Paris*, nigh the *Grand Chatelet*, to the Gate of the Church of *St Dennis*, 2 Leagues, each of which is 2200 Toifes.

The Gentlemen of the Academy of Sciences have found, that a Degree of a great Circle of the Earth contains 57060 Toifes; and giving 25 Leagues to a Degree, each League will contain 2282 Toifes.

A Sea-League is greater, for there goes but 20 to make a Degree; therefore it contains about 3000 Toifes.

The *Italians* reckon by Miles, each of which contains 1000 Geometrical Paces.

A Geometrical Pace is five of the antient Feet, one of which the antient *Roman* Palm is three quarters, which may be efteemed about 11 of our Inches; and confequently an *Italian* Mile contains about 769 of our Toifes.

The *Germans* alfo reckon by the Mile, but they are much greater than the *Italian* Miles; for one of them contains 3626 Toifes.

They count by Leagues in *Spain*, one of which contains 2863 Toifes, 20 of which exactly make one Terreftrial Degree,

The fame may be faid of the *Englifh* and *Dutch* Leagues.

USE I. *To draw a right Line thro' two Points given upon the Ground, and produce it to any required Length.*

Plant a Staff upon each of the given Points, very upright, and having ftrained a Line from one Staff to the other; by that Line, as a Guide, draw a Line upon the Ground.

That right Line may be continued by planting a third Staff, fo that by placing the Eye to the Edge of the firft, the Edges of the two others may be but juft feen; and again, the Line may be continued, by taking that Staff, which was the firft, and placing it as a third, *&c.*

USE II. *To meafure a right Line upon the Ground.*

When a long Line upon the Ground is to be meafured, Precaution muft be ufed that we do not miftake, and be obliged to begin again. To do which, two Men muft each of them have a Toife; the firft having laid down his, muft not lift it up, 'till the fecond has placed his at the End of the firft Man's Toife. The firft Man having lifted up his Toife, muft loudly count 1; and when he has again laid his down to the End of the fecond Man's, the fecond Man muft lift up his, and count 2. In thus continuing on to the End, and in order to lay the Toifes in a right Line, there muft be placed two Staffs, at a Diftance before them, to look at; for if there is but one, the Toifes cannot be fo truly laid in a right Line by help of it.

To fpare Time and Pains, you ought to have a Chain of 30 Feet, or 5 Toifes long, with a Ring at each End, carried by two Men, the firft of which carries feveral Staffs. When the Chain is well extended on the right Line to be meafured, the foremoft Man muft place a Staff at the End of 5 Toifes, to the End that the hinder Man may know where the Chain ended; for the whole Matter confifts in well counting, and exactly meafuring.

USE III. *From a Point given in a right Line, to raife a Perpendicular.*

Let the given Line be A B, and the given Point C.

Plant a Staff in the Point C, and two others, as E, D, in the fame Line, equally diftant from the Point C; then faften the two Ends of a Line to the two Staves E, D, and fold the Line into two equal Parts in F; afterwards ftretch the Line tight, and at the Point F plant a Staff, and the Line F C will be perpendicular to A B. Fig. 1.

Otherwife; meafure 4 Feet, or 4 Toifes, from the Point C, on the Line A B, and plant there the Staff G; take a Line containing 8 Feet, or 8 Toifes (according as the former are Feet or Toifes) faften one End of the Line to the Staff C, and the other to the Staff G; then ftretch the Line, fo that 3 of thofe Parts be next to the Point C, and 5 next to G; plant a Staff in H, and the Line H C will be perpendicular to A B. Fig. 2.

USE IV. *From a given Point without a Line, to draw a Perpendicular.*

Let the given Line be A B, and the Point F.

Fold your Line into two equal Parts, and fix the Middle to the Staff F; ftretch the two Halves (which I fuppofe long enough) to the Line A B; then plant two Staffs, namely, one Fig. 3.

to each End of your Line, and divide their Diſtance into two equal Parts, which may be done by folding a Line as long as the Diſtance A B; plant a Staff in the middle C, and the Line CF will be perpendicular to the Line AB.

USE V. *To draw a Line parallel to another, at a given Diſtance from it.*

Fig. 4. Let the given Line be A B, and it is required to draw a Line parallel to it at the Diſtance of 4 Toiſes.

Raiſe (by *Uſe* 3.) two Perpendiculars, each of 4 Toiſes, upon the Points A, B; and upon the Points C, D plant two Staffs; by which draw the Line C D, which will be parallel to A B.

USE VI. *To make an Angle on the Ground, at the End of a Line, equal to an Angle given.*

Fig. 5. Let A B C be the given Angle (which ſuppoſe is drawn upon Paper).

About the Point B, as a Center, deſcribe upon the Paper the Arc A C, and draw the right Line A C, which will be the Chord of the ſaid Arc. Meaſure with a Scale, or the Line of equal Parts of the Sector, the Length of one of the equal Legs A B, or B C of the ſaid Angle; likewiſe meaſure, with the ſame Scale, the Length of the Chord A C; which, for Example, ſuppoſe 36 of thoſe equal Parts, whereof the Leg A B contains 30.

Now let there be upon the Ground a right Line, as B C, to which it is required to draw another Line F B, making an Angle with B C equal to the propoſed one. Plant a Staff in the Point B, and having meaſured 30 Feet, or 5 Toiſes, on the Line B C, there plant a Staff, as D; then take two Lines, one of 30 Feet long, which faſten to the Staff B, and the other 36 Feet, which likewiſe faſten to the Staff D: Draw the Lines tight, and make their Ends meet in the Point F, where again plant a Staff, from which draw the Line F B; which will form, at the Point B, the Angle F B C equal to the propoſed one A B C.

USE VII. *To draw upon Paper an Angle, equal to a given one upon the Ground.*

Fig. 5. This Problem is the Converſe of the former.

Let the given Angle upon the Ground be F B C; meaſure 30 Feet, or 5 Toiſes, from B towards C, at the End of which plant the Staff D; meaſure likewiſe 30 Feet from B towards F, and there plant another Staff; meaſure alſo the Diſtance of the Staffs F, D, which ſuppoſe will be 36 Feet (as in *Uſe* VI.).

Now let B C be a Line upon the Paper; then about the Point B, as a Center, and with a Length of 30 equal Parts (taken from a Scale) deſcribe the Arc A C; and take 36 of the ſame Parts, and lay them off from the Point C, upon the Arc C A, and a Line drawn from B to A will make, with the Line B C, the Angle required.

If, moreover, the Quantity of the aforeſaid Angle be deſired, it will be found, by the Protractor, ſomething leſs than 64 Degrees.

The Quantity of Angles (whoſe Chords are known), in Degrees and Minutes, may more exactly be known by the following Table, which is calculated for Angles, always contained under equal Sides of 30 Feet each.

The Uſe of the ſaid Table is very eaſy for finding the Quantity of any Plane Angles upon the Ground: for meaſure 30 Feet upon each of the Lines forming an Angle, and plant a Staff at the End of 30 Feet upon each Line; then meaſure the Diſtance between the two Staffs, which ſuppoſe to be 36 Feet (as in the preceding Example), look in the Table in the Column of Baſes of 36 Feet, and you will find over againſt it, in the Column of Angles, 63 Degrees, 44 Minutes, the Quantity of the ſaid Angle.

A TABLE *of Plane Angles, contained under Sides of 30 Feet.*

Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.
2	0	19	2	6	3	2	11	48	2	17	34	2	23	24	2	29	17	2	35	15	2	41	19	2	47	30	2	53	51
4	0	38	4	6	22	4	12	8	4	17	54	4	23	44	4	29	37	4	35	35	4	41	40	4	47	51	4	54	12
6	0	57	6	6	41	6	12	27	6	18	13	6	24	3	6	29	56	6	35	55	6	41	0	6	48	12	6	54	34
8	1	8	8	7	0	8	12	46	8	18	32	8	24	23	8	30	16	8	36	15	8	42	20	8	48	33	8	54	55
10	1	36	10	7	20	10	13	5	10	18	52	10	24	42	10	30	36	10	36	35	10	44	40	10	48	54	10	55	18
1	1	55	4	7	39	7	13	24	10	19	11	13	25	1	16	30	56	19	36	55	22	43	1	25	49	15	28	55	38
2	2	14	2	7	58	2	13	43	2	19	30	2	25	21	2	31	16	2	37	15	2	43	22	2	49	36	2	56	0
4	2	33	4	8	17	4	14	2	4	19	50	4	25	41	4	31	36	4	37	36	4	43	42	4	49	57	4	56	22
6	2	52	6	8	36	6	14	22	6	20	19	6	26	1	6	31	56	6	37	56	6	44	3	6	50	18	6	56	43
8	3	11	8	8	55	8	14	41	8	20	29	8	26	20	8	32	16	8	38	16	8	44	24	8	50	39	8	57	5
10	3	30	10	9	14	10	15	0	10	20	48	10	26	40	10	32	35	10	38	36	10	44	44	10	51	0	10	57	26
2	3	49	5	9	34	8	15	20	11	21	8	14	26	53	17	32	55	20	38	56	23	45	5	26	51	21	29	57	48
2	4	8	2	9	53	2	15	39	2	21	27	2	27	18	2	33	15	2	39	17	2	45	26	2	51	42	2	58	10
4	4	28	4	10	12	4	15	58	4	21	46	4	27	38	4	33	35	4	39	38	4	45	46	4	52	3	4	58	32
6	4	47	6	10	31	6	16	18	6	22	16	6	27	58	6	33	55	6	39	58	6	46	7	6	52	24	6	58	54
8	5	6	8	10	50	8	16	37	8	22	25	8	28	18	8	34	15	8	40	18	8	46	28	8	52	46	8	59	16
10	5	25	10	11	9	10	16	56	10	22	45	10	28	38	10	34	35	10	40	38	10	46	48	10	53	8	10	59	38
3	5	44	6	11	29	9	17	15	12	23	6	15	28	57	18	34	55	21	40	59	24	47	9	27	53	29	30	60	0

Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.	Bases.	Angles. D.	M.
2	60	22	2	67	7	2	74	8	2	81	30	2	89	18	2	97	40	2	106	48	2	117	2	2	129	3	2	144	39
4	60	44	4	67	30	4	74	32	4	81	55	4	89	45	4	98	9	4	107	20	4	117	39	4	129	48	4	145	43
6	61	6	6	67	53	6	74	56	6	82	20	6	90	12	6	98	38	6	107	52	6	118	16	6	130	33	6	146	48
8	61	28	8	68	16	8	75	20	8	82	46	8	90	39	8	99	8	8	108	25	8	118	53	8	131	19	8	147	57
10	61	50	10	68	39	10	75	44	10	83	12	10	91	6	10	99	37	10	108	57	10	119	31	10	132	6	10	149	8
31	62	13	34	69	2	37	76	9	40	83	37	43	91	33	46	100	6	49	109	30	52	120	9	55	132	53	58	150	20
2	62	35	2	69	25	2	76	33	2	84	3	2	92	1	2	100	36	2	110	4	2	120	47	2	133	44	2	151	36
4	62	58	4	69	48	4	76	57	4	84	29	4	92	29	4	101	6	4	110	37	4	121	26	4	134	30	4	152	55
6	63	20	6	70	12	6	77	22	6	84	54	6	92	56	6	101	36	6	111	11	6	122	6	6	135	20	6	154	19
8	63	43	8	70	35	8	77	46	8	85	20	8	93	24	8	102	7	8	111	44	8	122	45	8	136	11	8	155	48
10	64	5	10	70	59	10	78	9	10	85	46	10	93	52	10	102	37	10	112	18	10	123	25	10	137	3	10	157	22
32	64	28	35	71	22	38	78	35	41	86	13	44	94	20	47	103	8	50	112	53	53	124	6	56	137	57	59	159	3
2	64	50	2	71	46	2	79	0	2	86	39	2	94	48	2	103	39	2	113	28	2	124	47	2	138	49	2	160	53
4	65	13	4	72	10	4	79	25	4	87	5	4	95	16	4	104	10	4	114	3	4	125	28	4	139	44	4	162	54
6	65	35	6	72	33	6	79	50	6	87	32	6	95	20	6	104	41	6	114	38	6	126	10	6	140	40	6	165	12
8	65	58	8	72	56	8	80	15	8	87	58	8	96	13	8	105	12	8	115	14	8	126	52	8	141	38	8	167	48
10	66	21	10	73	20	10	80	40	10	88	25	10	96	42	10	105	44	10	115	49	10	127	35	10	142	36	10	171	28
33	66	44	36	73	44	39	81	5	42	88	51	45	97	11	48	106	16	51	116	26	54	128	19	57	143	36	60	180	0

Note, That in the Columns of Bafes are only fet down every 2 Inches, and the Feet from 1 to 60. By means of this Table may be eafily and exactly found the Opening and Quantity of any Angle; for fuppofe your Bafe be in Length 50 Feet, 3 Inches, and the other 2 Sides each 30 Feet, which they muft always be. Seek 50 Feet, 2 Inches, in the Column of Bafes; and againft it you will find, in the Column of Angles, 113 Deg. 28 Min. whence by making due Proportion with the Inches and Minutes, the Quantity of the Angle fought will be 113 Deg. 44 Min. This Table, together with a well divided Brafs Scale, may be ufed in meafuring or laying off Angles upon Paper, with as much Exactnefs as Lines will do them upon the Ground; becaufe the Sides of equi-angled Triangles are proportional to each other.

This Method of meafuring plane Angles, may likewife ferve to make Defigns of Fortification, both regular and irregular, to find the Quantities of Angles, as well of Baftions as of the Polygon, formed by the Concourfe of the Lines of the Bafes, or exterior Sides, either upon Paper or the Ground.

To draw Angles by this Table, feek for the Degrees and Minutes you defign an Angle to confift of, which, for Example, fuppofe 54 Deg. 34 Min. and againft them, in the Column of Bafes, is the Number of Feet and Inches correfponding thereto, *viz.* 27 Feet, 6 Inches; which is the Length of the Bafe of the Angle, each of the other Sides of which is 30 Feet, and fo of others.

USE VIII. *To take the Plane, or Plot of a Place within it.*

Fig. 6. Let the Place whose Plan is required, be A B C D E.

First, make a Figure upon your Paper, something like the Plan to be taken, and after having measured with a Toise the Sides A B, B C, C D, D E, and E A, write the Lengths found upon each of their corresponding Lines on the Paper; then instead of measuring the Angles made by the Sides, measure the Diagonals A D, B D, which write down in your Book, and the Figure will be reduced into three Triangles, whose Sides are all known, because they have been actually measured. Then the Figure must be drawn neat in your Book by means of a Scale of equal Parts.

Note, Of all the Ways to take the Planes of Places, that of taking it within is the best.

USE IX. *To take the Plot of any Place (as a Wood, or marshy Ground) by measuring round about it.*

Fig. 7. First draw a rough Sketch of the Figure in your Field-Book: if it takes not too much time in going round the Place; then measure with a Toise, or Chain, all the Sides encompassing the Figure proposed, and set the Numbers found upon each correspondent Line, in your Book; but for the Angles, you must measure them as follows.

To measure, for Example, the Angle E F G, produce the Side E F, 5 Toises, and plant a Staff at the End K; produce also the Side G F, the Length of 5 Toises, and plant a Staff at the End L. Measure the Distance L K, and supposing it 6 Toises, 4 Feet, that is 40 Feet, set it down upon the Line L K in your Book, by which means the three Sides of the Isosceles Triangle L F K will be had; and consequently the Angle L F K, may be known by the aforementioned Table, or otherwise. Now the aforesaid Angle is equal to it's opposite one E F G, and if you seek 40 Feet in the Column of Bases, the Angle will be found 83 Deg. 37 Min.

In the same manner may the Angle F G H, or any other of the proposed Figure, be measured: or else thus, Produce the Side H G, the Length of 5 Toises, to N, where plant a Staff; make likewise G M, 5 Toises. Measure the Distance M N, which suppose, for Example, 6 Toises, 2 Feet, or 38 Feet, which write upon the Line M N in your Book.

This Number sought in the Column of Bases, corresponds to 78 Deg. 35 Min. for the exterior Angle M G N, whose Complement 101 Deg. 25 Min. is the Quantity of the Angle F G H.

Then the Figure in your Field-Book must be drawn neat by means of a Scale of equal Parts, as well to denote the Lengths of the Sides, as the Bases of all the Angles, which may exactly be had without the Trouble of taking their Quantities in Degrees and Minutes.

USE X. *To draw any regular Polygon upon a given Line on the Ground.*

Fig. 8. Let, for Example, the given Line be A B, upon which it is required to make an equilateral Triangle.

Measure 30 Feet upon the Line A B, from A to D, where plant a Staff: then take 2 Lines, each 30 Feet long, one of which fasten to the Staff D, and the other to the Staff A, and stretch them 'till their Ends join in the Point C, where plant another Staff.

Make the same Operation at the other End of the given Line, and produce the Lines A C, and B F, 'till they meet in the Point E, and form the equilateral Triangle A E B required.

Fig. 9. If a Square be to be made upon the given Line A B, raise upon each End A and B, a Perpendicular (by *USE* III.).

Then make each of those Perpendiculars equal to the Line given, plant Staffs at their Ends C and D, and draw the Line C D, which will compleat the Square proposed.

Fig. 10. If a Pentagon is required to be drawn upon the given Line A B:

You will find that the Angles formed by the Sides of a Pentagon, are each 108 Degrees; (as before has been said, in USE 3. of the Protractor, and in the third Section, concerning the Line of Polygons of the Sector); therefore seek for, in the Table of Plane Angles, the Number that answers to 108 Degrees, or nighly approaches it, and you will find 48 Feet, and something above 6 Inches: for that Number answers to 107 Deg. 52 Min. which is lesser by 8 Min. than 108 Degrees; whence 48 Feet, 6 $\frac{1}{2}$ Inches, may be taken for the aforesaid Base.

Now measure upon the given Line, from the Point A towards B, 30 Feet, and plant a Staff in the Point C, where the said Length terminates: then take 2 Lines, one 30 Feet, the End of which fasten to the Staff A; and the other 48 Feet, 6 $\frac{1}{2}$ Inches, which likewise fasten to the Staff C; strain the Lines equally, 'till they join in the Point E, where plant a Staff, and by that means will be had an Angle of 108 Degrees: then produce the Line A E, 'till it be equal to A B; make the same Operation at the End B of the given Line, by which means three Sides A B, A G, B D, of the required Pentagon will be had, which afterwards may be compleated by the same Method.

If the Pentagon be not too big, it may be compleated by means of 2 Lines, each equal to the given Side, one fastened to the Staff D, and the other to the Staff G; for if they are equally strained, they will form the two other Sides of the Polygon, by meeting in the Point H.

Any

Any other regular or irregular Polygon, by the fame Method, may be made upon the Ground, by feeking in the before-mentioned Table, the Number of Feet and Inches anfwering to the Angle of the Polygon to be drawn.

USE XI. *To find the Diftance of two Objects, inacceffible in refpect of each other.*

The Diftance, for Example, from the Tower A, to the Windmill B, is required. Fig. 11.
Plant the Staff C in fome Place from whence it may be eafy to meafure the Diftance in a right Line from it to the Places A and B.

Meafure thefe Diftances exactly, as for Example, from C to A, which fuppofe 54 Toifes; then produce the Line A C to D, likewife 54 Toifes: meafure alfo the Line B C, which fuppofe 37 Toifes, and produce it to E, fo that C E may be 37 Toifes likewife; by which means the Triangle C D E, will be formed equal and fimilar to the Triangle A B C, and confequently the Diftance D E will be equal to the propofed inacceffible Diftance from B to A.

USE XII. *To find the Diftance of two Objects, one of which is inacceffible.*

Let it be propofed, for Example, to find the Breadth A B of a River: being at one of it's Sides A, plant there a Staff A C, 4 or 5 Feet high, and very upright; make a Slit towards the Top of the Staff, in which put a very ftraight Piece of Steel or Brafs (that may flide up and down), about 3 Inches long, which muft be flipped up or down, 'till the Point B, on the other Side of the River be feen along it; afterwards turn the Staff, and look along (keeping the aforefaid Piece of Brafs in the fame Pofition) the Side of the River upon level Ground, 'till you fee the Point D, where the vifual Rays terminate. The Diftance A D meafured with a Chain, will give the Breadth of the River, to which it is equal. Fig. 12.

This Propofition, as fimple as it is, may ferve to know what Length Timber muft be of, to make Bridges over Ditches or Rivers.

USE XIII. *To draw upon the Ground a right Line from the Point* A, *to the Point* B, *between which there is a Building, or other Obftacle, that hinders the continuing of it.*

Find, upon very level Ground, a third Point, as C, from which you may fee Staffs planted in the Points A and B; then meafure exactly the Diftance from C to A, and from C to B: this being done, take the Half, Third, or any other Part of each of thofe Lines, whereat plant Staffs, as in D bifecting C B, and in E bifecting C A; then draw a right Line from D to E, which produce as is neceffary, and draw a Parallel to it paffing by the Points A and B, by means of Staffs planted betweent the Point A and the Houfe, as alfo between the Houfe and the Point B, which will fhew the Direction from A to B. Fig. 13.

USE XIV. *It is required to cut a Paffage thro' a Hill from the Point* A *to* B.

Draw on one Side of the Hill a right Line, as D C, and on the other Side another right Line, as E F, parallel to C D; then let fall from the Point A, to the Line C D, the Perpendicular A G; and in fome other Point beyond the Hill, draw another Perpendicular, as C H, equal to A G. Fig. 14.

Again; from the Point B, let fall upon the Line E F the Perpendicular B I; and from fome other Point beyond the Hill, draw another Perpendicular to the fame Line, as L M, equal to B I, fo that the Diftance I L, may be equal to C G; then draw a right Line from the Staff H, to the Staff M (and produce it as far as is neceffary), which will be parallel to the Paffage to be made from A to B; therefore any Number of Staffs may be planted at an equal Diftance to that Parallel H M on both Sides the Hill, as O, P, Q, which will ferve as a Guide to pierce the Hill thro' from A to B.

I fhall again mention the Ufe of the aforefaid Inftruments, in the little Treatife of Fortification, hereafter laid down.

CHAP. II.

Of the Defcription and Ufe of the Surveying-Crofs.

THE Surveying-Crofs is a Brafs Circle of a good Thicknefs, and 4, 5, or 6 Inches Diameter. It is divided into 4 equal Parts, by two Lines cutting one another at right Angles in the Center. At the four Ends of thefe Lines, and in the Middle of the Limb, there are fixed four ftrong Sights well riveted in fquare Holes, and very perpendicularly flit over the aforefaid Lines, having Holes below each Slit, for better difcovering of diftant Objects: the Circle is hollowed to render it more light. Fig. 15.

Under-

Fig. 16. Underneath, and at the Center of the Instrument, there ought to be screwed on a Ferrel, serving to sustain the Cross upon it's Staff of 4 or 5 Feet long, according to the Height of the Observer's Eye. This Staff must be furnished with an Iron Point, to go into the Ground the better.

All the Exactness of this Instrument consists in having it's Sights well slit at right Angles, which may be known by looking at an Object thro' two Sights, and another Object thro' two other Sights: then the Cross must be exactly turned upon it's Staff, and you must look at the same Objects through the opposite Sights; if they are very exactly in the Direction of the Slits, it is a sign the Instrument is very just.

To avoid breaking or damaging the Cross, the Staff must first be put in the Ground, and when it is well fixed, the Cross must be screwed upon it.

These kinds of Crosses sometimes are made with eight Sights, in the same manner as the aforesaid one, and serve to take Angles of 45 Degrees; as also for Gardeners to plant Rows of Trees by.

USE I. *To take the Plot and Area of a Field within it.*

Fig. 17. Let the Field proposed be A B C D E, and having placed at all the Angles Staffs, or Poles very upright, exactly measure the Line A C (in the manner we have already laid down, or any other at pleasure) then make a Memorial, or rough Draught, somewhat representing the Field proposed, on which write all the Dimensions of the Parts of the Line A C, and of Perpendiculars drawn from the Angles to the Line A C. If, for Example, you begin from the Staff A, find the Point F in the Line A C, upon which the Perpendicular E F falls: then measure the Lines A F and E F, and set down their Lengths upon their correspondent Lines in your Memorial.

Now to find the Point F, plant several Staffs at pleasure in the Line A C; as also the Foot of your Cross in the same Line, in such a manner that you may discover thro' two opposite Sights, two of those Staffs, and thro' the other two Sights (which make right Angles with the two first ones), you may see the Staff E. But if in this Station the Staff E cannot be seen, remove the Instrument backwards or forwards, 'till the Lines A F, E F, make a right Angle in the Point F, by which means the Plot of the Triangle A F E will be had.

In the same manner may the Point H be found, where the Perpendicular D H falls, whose Length, together with that of G F, must be set down in your Memorial, in order to have the Plot of the Trapezium E F H D. Again, Measure H C making a right Angle with H D, and the Plot of the Triangle D H C will be had.

Having likewise measured the whole Line A C, there is no more to do but find the Point G, where the Perpendicular B G falls; and proceeding as before, the Plot of the Triangle A B C may be had, and consequently the Plot of the whole Field A B C D E. The Area of the Field will likewise be had, by adding the Triangles and Trapeziums together, which may easily be done by the Rules of Planometry, in the following manner:

Suppose, for Example, A F to be 7 Toises, and the Perpendicular E F 10; multiply 7 by 10, and the Product is 70, half of which is 35, the Area of the Triangle A F E.

If moreover the Line F H be 14 Toises, and the Perpendicular H D 12, add 12 to 10 (which is the Perpendicular F E), the Sum will be 22, half of which being 11, multiplied by 14, will give 154 square Toises, for the Area of the Trapezium E F H D; and if the Line H C is 8 Toises, multiplying 8 by 12, the Product is 96, whose half 48, will be the Area of the Triangle C H D.

The whole Line A C is 29 Toises, and the Perpendicular B G 10; whence the Product is 290, whose half 145, is the Area of the Triangle A B C. Finally, adding together 35, 154, 48, and 145, the Sum 382, will be the Number of square Toises contained in the Field A B C D E.

USE II. *To take the Plane of a Wood, Morass, &c. in which it is not easy to enter.*

Fig. 18. Let the Morass E F G H I be proposed: Set up Staves at all the Angles, so made as to include the Morass within a Rectangle, which measure; then substract the Triangles and Trapezia included between the Sides of the Morass, and the Sides of the Rectangle, from the said Rectangle, and the Area of the proposed Morass will be had.

If, for Example, you begin at the Staff E, produce by help of the Cross the Line E F, as far as is necessary, to which, from the Point G, let fall the Perpendicular G K; set up a Staff at K, and produce K G to L, to which, from the Point H, draw the Perpendicular L H, which likewise produce as far as is necessary: afterwards draw from the Staff E, to the Line H L, produced, the Perpendicular E M: whence the Rectangle E M L K will be had, whose Sides must be measured with a Chain or Toise.

Suppose, for Example, the Line E K, or it's Parallel M L (which ought to be equal to it) is 35 Toises, and the Line E M, or it's Parallel, 10 Toises; multiplying these two Numbers by one another, there will arise 350 square Toises for the Area of the Rectangle E M L K: but if F K be 5 Toises, and G K 4, by multiplying 4 by 5, the Product is 20, whose half 10 Toises, is the Area of the Triangle F K G. The Line G L, being 6 Toises, and H L 4, the Product of 4 by 6 is 24, whose half 12 is the Area of the Triangle G L H.

After-

Afterwards a Point must be found in the Line HM, where a Perpendicular drawn from the Staff 1 falls, which forms a Triangle and a Trapezium; so that if the Distance HN be 24 Toises, and the Perpendicular NI 4 Toises, 24 by 4 gives 96, whose half 48, is the Area of the Triangle HNI. Lastly, NM being 7 Toises, ME 10, and it's Parallel NI 4 Toises, adding 10 to 4, the Sum will be 14, whose half 7, multiplied by 7, produces 49 for the Area of the Trapezium EMNI.

Therefore adding together the Areas of the three Triangles, and that of the Trapezium, there will be had 119 Toises, which taken from 350, the Area of the Rectangle, and there remains 231, the Area of the proposed Morass. The same may be done with any other Figure. These two Uses are enough to shew how Surveyors use their Instruments for measuring and taking the Plot of any Piece of Ground.

CHAP. III.

Of the Construction and Uses of divers Instruments to take Angles.

THERE are several Sorts of Instruments to take Angles, but the best and most in use, are those whose Description we are now going to give.

The Instrument A, is composed of two Rules very equal in breadth, for the Insides of them must be parallel to their Outsides; their Breadth is about an Inch, and their Length a Foot or more. Those two Rulers are equally rounded at the Top, and fastened to one another by means of a Rivet artificially turned, so that the Instrument may easily open and shut. When an Angle is taken with it, the Center of a Protractor must be put to the Place where the two Rulers join each other, and the Degrees cut by the Edge, will show the Quantity of the Angle; or else the Angle which the two Rulers make, is drawn upon Paper, and then it is measured with a Protractor. Fig. A.

The Instrument B, is made like the precedent one, only there are two Steel Points at the Ends, in order for it to serve as a Pair of Compasses. Fig. B.

The Instrument C, is different from the others, because it shews the Quantities of Angles without a Protractor. Fig. C.

It is composed of 2 Brass Rulers of equal Breadth and parallel, about 2 Feet long, and 2 or 3 Inches thick, joined together by a very round Rivet: it has besides a Circle divided into 360 Degrees at the End of one of the Rulers, and a little Index fixed to the Rivet, which shews the Number of Degrees the 2 Rulers contain between them. I shall not here shew how to divide the Circle, having sufficiently spoken of it in the Construction of the Protractor; only note, that the Degrees are always reckoned from the Middle of the Rule, where the Center is.

There are these Sorts of Instruments made by dividing a Circle upon the under Ruler, and filing the upper one like the Head of a Sector, that thereby the Degrees of the Opening of the Legs may be known, by means of the two Shoulders of the upper Leg.

To measure a saliant Angle with any one of the three Recipient-Angles, apply the Insides of the two Rulers, to the Lines forming the Angle; and to measure a rentrant Angle, apply the Outsides of the same Rulers to the Lines forming the Angle.

The Instrument D, is made of 4 Brass Rules, equal in Breadth, joined together by 4 round Rivets, forming an equilateral Parallelogram. Fig. D.

At the End of one of the Rules there is a Semi-circle, divided into 180 Degrees. The other Branch passing upon the Semi-circle, is continued to the Divisions of the Semi-circle, in order to shew the Quantities of Angles.

The said Rules are made one or two Feet long, 8 or 10 Lines broad, and of a convenient Thickness; they ought to be drilled very equal in Length, namely, that where the Center of the Semi-circle is (marked 2.) and at the other End in the Point 1. That which serves for an Index, ought to be drilled in the Points 2 and 3. And lastly, The two other Rules in the Point 4. The Rule serving for an Index, must be fastened to the Center of the Semi-circle; and the two other Rules, which are of equal Length, must be fastened underneath the two others, all of them so as their Motion may be very uniform.

When a saliant Angle is to be measured with this Instrument, the 2 equal Rules must be put underneath the 2 others, so that the End 4 be underneath 2, and thereby the 4 Rules make but 2 to encompass the Angle: but when a rentrant Angle is to be measured, the two Rules must be drawn out (as *per* Figure), and applied to the Corner of the Angle; and since in every Parallelogram the opposite Angles are equal, the Degrees of the Angle may be known by the Semi-circle.

USE I. *Of the Recipient-Angle.*

To take the Plan of a Bastion; as, for Example; ABCDE, make a Memorial, and then measure, with the Instrument, the rentrant Angle E, made by the Courtine of the Place, Fig. 19.

and the flanquant Angle of the proposed Bastion, by applying it horizontally, in such manner that one of the Rules may be in the Direction of the said Courtine, and the other in the Direction of the Flank; and having found the Quantity of it in Degrees, set it down upon a little Arc in your Memorial; then measure the Flank E D, which set down upon the Line *e d* in your Memorial. Again, apply the Rules of your Instrument to the saliant Angle D, and set down it's Quantity upon a little Arc; measure the Length of the left Face C D, take the Quantity of the flanquant Angle C, and of all the other Angles of the Bastion, as likewise the Length of the Faces and Flanks; after which, by help of a Scale, the Plan of the Bastion may be drawn neat.

But since it often happens that these Angles, which are commonly made of Free-Stone, are not well cut, by the Negligence of Workmen, who make them either too acute or obtuse; to remedy this, there must be a long Rule horizontally applied to each Wall, whose Direction is good, tho' the Angles are not; and putting the Legs of the Instrument level upon those two Rules, the Angle to be measured may be more exactly had.

USE II. *To take the Plot of a Piece of Ground encompassed by right Lines.*

Fig. 20. Let the Piece of Ground proposed be A B C D E F G; measure exactly the Length of all the Sides, and set them down upon the relative Lines of your Memorial; then take, with any recipient Angle, the Quantity of each Angle, as, for Example, the Angle A G F, and set down the Quantity of it upon the relative Angle *a g f*, in the Memorial; measure also the Angle F E D, by applying the Instrument to it (as *per* Figure), and set down the Quantity thereof upon the relative Angle of the Memorial, and so of all the other Angles, whose Quantities being noted in Degrees, as likewise the Lengths of all the Lines, the Plot *a b c d e f g* may be neat drawn, and similar to A B C D E F G.

In this Plate may be seen the Plane of a Pentagon fortified, with the Names of the Parts of it's Fortification.

CHAP. IV.

Of the Construction and Use of the Theodolite.

Plate 12. *Fig.* A. THIS Instrument is made of Wood, Brass, or any other solid Matter, commonly circular, and about one Foot in Diameter. In the Center of this Instrument is set upright a little Brass Cylinder, or Pivot, about which an Index turns, furnished with two Sights, or a Telescope, having a right Line, called *The Fiducial Line*, exactly answering to the Center of the aforesaid little Cylinder, whose Top ought to be cut into a Screw, for receiving a Nut to fasten the Index, upon which is fixed a small Compass for finding the Meridian Line.

The Limb of the Theodolite is a Circle of such a Thickness, as to contain about six round Pieces of Pasteboard within it (of which we are going to speak), and of such a Breadth as to receive the Divisions of 360 Degrees, and sometimes of every fifth Minute.

There are several round Pieces of Pasteboard, of the Bigness of the Theodolite, pierced thro' the Middle with a round Hole, exactly to fit the Pivot; so that the Pivot may be put thro' each of the aforesaid Holes in the Pieces of Pasteboard, and the upper Pasteboard may have the Index moving upon it. This upper Pasteboard may be fixed at pleasure, by means of a little Point fastened to the Limb of the Instrument, and entering a little way into the Pasteboard. There is commonly drawn with Ink, upon each of these Pasteboards, a Radius or Semidiameter, serving for a Station-Line.

Underneath the Theodolite is fastened a Ball and Socket, represented by the Figure D, which is a Brass Ball enclosed between two Shells of the same Metal, that may be more or less opened by means of a Screw, and a Socket G, in which goes the Head of a three-legged Staff, of which more by and by.

Fig. A, represents the Instrument put together. We now proceed to shew the Construction of the Pieces composing it, in beginning with the Division of it's Limb.

First, Draw upon the Limb two or three concentrick Circles, to contain the Degrees, and the Numbers set at every tenth Degree; then divide one of these Circumferences into four very equal Parts, each of which will be 90 Degrees; and dividing each of these four Parts into 9 more, the Circumference will be divided into every tenth Degree. Again, each of these last Parts being divided by 2, and each of those arising into 5 equal Parts, the whole Circumference will be divided into 360 Degrees. This being done, you must draw the Lines of these Divisions upon their convenient Arcs, by means of a Ruler moving about the Center. Afterwards Numbers must be set to every tenth Degree, beginning from the Fiducial Line, which is that whereon the two fixed Sights or Telescope is fastened.

A Theodolite thus divided is of much greater Use than those whose Limbs are not divided; for it may serve exactly to take the Plots of Places, and measure inaccessible Distances by Trigonometry.

The

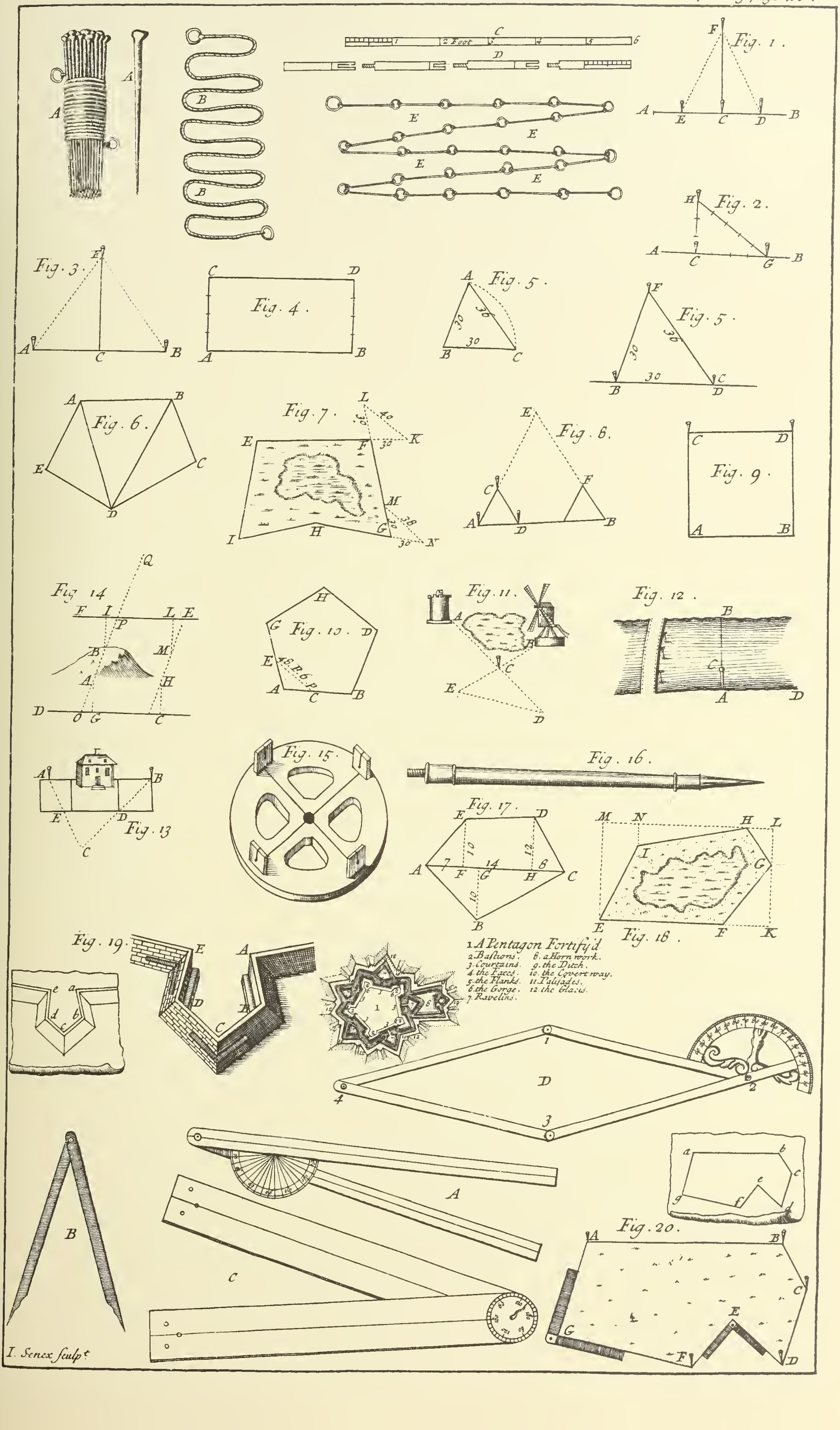


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 14
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
2 Foot
1 A Pentagon Fortify'd
2 Bastions. 8. a Horn work.
3. Courtains. 9. the Ditch.
4. the Faces. 10. the Covert way.
5. the Flanks. 11. Palisades.
6 the Gorge. 12. the Glacis.
7. Ravelins.
Fig. 20.
I. Senex sculp.

The Figures B reprefent the Sights which are placed upon different Inftruments; that to which is placed the Eye, hath a long ftrait Slit, which ought to be very perpendicular, made with a fine Saw; and that which is turned towards the Object, hath a fquare Hole, fo large, that the adjacent Parts of a diftant Object may be perceived thro' it: And along the Middle of this Hole is ftrained a very fine Gut, in order to vertically cut Objects, when they are perceived thro' the Slit of the other Sight. But that the Eye may be indifferently placed at any one of the two Sights at pleafure, fo that Objects may be as well perceived thro' the Sights on one Side the Inftrument, on which they are placed, as on the other; there is made in each Sight a fquare Hole and a Slit, the Hole in one Sight being below the Slit, and in the other Sight above it, as the little Figures fhew. Thefe Sights ought to be exactly placed on the Extremes, and in the fiducial Line, as well of Inftruments as Indexes, and are faftened in little fquare Holes with Nuts underneath, or elfe by means of Screws, according as the Place they are faftened on requires.

The little Figure C reprefents the aforefaid Cylinder, or Pivot, with it's Nut, for joining the Index to the Theodolite; thofe of Semicircles, and other Inftruments, are made in the fame manner, only they are rivetted underneath.

The Figure D reprefents the Ball and Socket for fupporting the Inftrument, and is compofed of a Brafs Ball inclofed between two Shells of the fame Metal, which are made very round, with Balls of tempered Steel cut in manner of a File. Thefe Shells are locked more or lefs by means of a Screw, that fo they may prefs the Ball inclofed between them according to neceffity. One of thefe Shells is foldered to the Socket G, which is a turned Brafs Ferrel, in which the Foot of the Inftrument is put. Balls and Sockets are made of different Bigneffes, according to the Bigneffes of Inftruments, and are faftened to the Inftruments with Screws, in a Plate riveted to the Top of the Ball.

Conftruction of the Feet for fupporting of Inftruments.

We have already mentioned the fimple Feet for fupporting Surveying-Croffes, which are to be forced into the Ground; but thofe whofe Defcription we are now going to give, are not to be forced into the Ground, but are opened or fhut according as the Inequality of the Ground, the Inftrument is to be ufed upon, requires.

The Foot E is a triangular Plate, in whofe Middle is a Piece *b*, which is to go into the Socket G.

Underneath the aforefaid Plate are faftened three Ferrels, or Sockets, moveable by means of Joints, for receiving three round Staves of fuch a Length, that the Obferver's Eye, when the Inftrument is ufing, may commodioufly view Objects thro' the Telefcope, or Sights. The Extremities of thefe Staves are furnifhed with Ferrels and Iron Points, in order to keep the Inftrument firm when it is ufing.

The Foot F confifts of four Staves, about two Foot long, whereof that in the Middle, called the Shank, hath it's Top rounded, that fo it may go into the Socket; the reft of this Staff is cut in Figure of a Triangle, that fo the three Faces thereof may receive upon them three other Staves, faftened by means of three Screws (all of a piece) and fo many Nuts. Thefe three Staves are furnifhed with Ferrels and Iron Points, being flat within fide, and have three Faces without.

When we have a mind to carry this Foot, we re-unite all the Staves together, fo that they make, as it were, but one, and by this means are fhorter by about the half, than when the Foot is ufing.

We generally hang to the Middle of each of thefe Feet a Thread and Plummet, in order to know the Station-Point.

USE *of the Theodolite.*

To take the Map of a Country by this Inftrument, chufe two high Places, for Example, the *Obfervatory*, and the *Salt-Petre Houfe*, from whence the Country nigh *Paris*, a Map of which is to be made, may be feen; then mark round the Center of the upper Pafteboard the Name of the Place chofen for the firft Station, and having fixed it by means of the Point on the Limb of the Theodolite, put the Index upon it, which fufficiently fcrew down by means of the Nut and Screw. Fig. 1.

Now having placed the Theodolite upon it's Foot, planted at the *Obfervatory*, and given it a Situation nearly horizontal, fo that it may remain fteddy while the Index is moving, obferve thro' the Sights the Steeple of the Salt-Petre Houfe, and along the fiducial Line of the Index from the Center draw the Station-Line.

Then turn the Index, and obferve fome remarkable Object thro' the Sights, as the Steeple of *Vaugirard*, towards which a Line muft be drawn upon the Pafteboard, from the Center, along the fiducial Line of the Index, and along this Line write the Name of the Place viewed thro' the Sights.

Again, direct the Index towards fome other Object (as *Mont-rouge)* and draw a Line towards it from the Center, along the fiducial Line, and upon this Line write the Name of the

Place obferved. Proceed in the fame Manner with all the confiderable Places that can be feen from the Obfervatory.

Now having removed the Theodolite from it's firft Station, having well obferved it's Place, and tranfported it to fome other defigned Place, as to the Salt-Petre Houfe; meafure the exact Diftance between the two Stations upon level Ground, the Number of Toifes of which muft be fet down upon your Pafteboard, which muft now be turned, or taken from under the Index, that fo at every different Station, the upper Face of the Pafteboard, upon which the Index is, may be clean: then fet down about the Center of this new Pafteboard, the Name of the Place of your fecond Station, and upon the Bafe Line the Number of Toifes meafured, that fo you may remember this Line is the fame as that on the precedent Pafteboard. The Theodolite being placed here, difpofe it fo, that placing the fiducial Line of the Index upon the Station Line, you may difcover thro' the Sights, the *Obfervatory*, which was your firft Station.

The Inftrument remaining firm in this Situation, turn the Index, and fucceffively view thro' the Sights the former Objects obferved from the *Obfervatory*, and draw Lines, as before, upon the Pafteboard, along the Index, from the Center towards the Places viewed, and upon each Line write the correfpondent Name of the Place.

If all the Places you have a mind to fet down in your Map, cannot be feen from the two precedent Stations, you muft chufe a third Place from whence they may be obferved, and make as many new Stations, as are neceffary for perceiving each remarkable Object, from two Places fufficiently diftant from each other.

Now to reprefent this Map upon a Sheet of Paper, firft draw a right Line at pleafure upon it, for a common Bafe, which divide into the fame Number of equal Parts, as you have meafured Toifes upon the Ground. About one End of this Line, as a Center, defcribe circular Arcs equal to thofe drawn upon the firft Pafteboard, and upon the other Extreme, Arcs equal to thofe drawn upon the fecond Pafteboard, and produce the Lines forming the Arcs 'till they meet each other; then the Points of Concourfe, will be the Points of Pofition of the Places obferved.

The aforefaid Places may be laid down upon the Paper eafier, by placing the Centers of the Pafteboards upon the Extremities of the common Bafe, and noting upon the Paper the Ends of the Lines drawn upon the Pafteboard, and then drawing Lines from the Stations thro' thofe Points 'till they interfect.

By means of this Theodolite may be had in Degrees, or Parts, all the Angles that the Places viewed thro' the Sights or Telefcopes, make, with the Places whereat the Inftrument is placed.

What we have faid, is fufficient to fhew the Manner of ufing the Theodolite in taking the Pofition of Places, and making of Maps, becaufe the Operations are the fame for all different Places; but for it's Ufes, with regard to Trigonometry, they are the fame as thofe of the Semi-circle and Quadrant, of which we are going to treat.

CHAP. V.

Of the Construction and Uses of the Quadrant, and Geometrick Quadrat.

Fig. G. THE Figure G, reprefents a Quadrant and Geometrick Square, with it's Index and Sights.

It is commonly made of Brafs, or other folid Matter, 12 or 15 Inches Radius, and an anfwerable Thicknefs. It's Circumference is firft divided into 90 Degrees, and every Degree into as many equal Parts as poffible, without Confufion, and in fuch manner, that the Divifions and Subdivifions may be juft, and very diftinctly marked upon the Limb of the Inftrument.

To do which, there muft firft be 2 Arcs drawn nigh the Edge of the Quadrant, about 8 or 9 Lines diftant from each other; and after having divided them into Degrees, draw Diagonal Lines between them, from the firft Degree to the fecond, from the fecond to the third, and fo on to the laft.

After which, if you have a mind to fubdivide every Degree into 10 Minutes, there muft 5 other concentrick Arcs be defcribed from the Center of the Inftrument, cutting all the aforefaid Diagonals; but if every Degree is to be fubdivided into Minutes, there muft be 9 concentrick Arcs defcribed between thofe two firft drawn.

The Diftances between all thefe Arcs, muft not be all equal, becaufe the Extent of a Degree taken in the Breadth of the Limb, forms a kind of Trapezium, broader towards the outward Arc, and narrower towards the inward one; whence a mean Arc dividing every Degree

gree into 2 equal Parts, muft be nigher the inward Arc than the outward one, and the others in proportion.

To make thefe Subdivifions exactly, the Diagonals muft be Curve Lines, as B D C, defcribed in making the Portion of a circular Arc pafs thro' the Center B, the beginning of the 1ft Degree marked D, upon the inward Arc, and the End C of the fame Degree, on the outwark Arc: which is eafy to do by *Ufe* 18. *Lib.* 1. which fhews how to make a Circle pafs thro' 3 Points given, by which means the Point F, the Center of the Diagonal Curve, paffing thro' the firft Degree, will be found. Fig H.

Afterwards one of thefe Diagonal Curves muft be divided into equal Parts, and from the Center of the Inftrument, there muft be drawn as many concentrick Arcs, as each Degree is to have equal Parts.

The Reafon of this Operation is, that the Diagonal Curve being divided into equal Parts, if from the Center of the Inftrument there are drawn right Lines thro' all the Points of Divifion of that Arc, there will be had *(per Prop.* 27. *Lib.* 3. *Eucl.)* as many equal Angles in the Center, becaufe they will be all in the Circumference of the fame Circle, and ftand upon equal Arcs.

But fince it is troublefome to find the Centers of 90 Arcs, each paffing thro' 3 Points; and fince it is manifeft, that all the Centers of thefe Arcs ought to be placed in the Circumference of a Circle whofe Center is the Point B; there is no more to do but draw a Circle from the Center B, with the Diftance B F, and divide it's Circumference into 360 equal Parts; upon every of which, fetting one Foot of your Compaffes, you may defcribe with the fame Extent F B, all the Arcs between the Circles A C, D E, and then the circular Arcs, which are Diagonals, will likewife divide the Circumferences, upon the Limb of the Inftrument, into Degrees. *Note*, Becaufe the Figure is too little, it is divided but into every 5th Degree.

Diagonal Curves may alfo be drawn without transferring the Foot of your Compaffes from one Degree to another, upon the aforefaid Arc, in fixing the Foot of your Compaffes in only one Point, as F, and letting the Inftrument be gradually turned about the Center of a large Circle, whofe Limb is already divided into Degrees, by means of a Rule ftrongly faftened upon the Inftrument, and reaching to the Divifions of the large Circle.

Ingenious Workmen may fhorten their Work by adjufting a fine Steel Ruler, according to the Curvature of the firft Diagonal, which being drawn, by this means they may draw all the others. If Diagonal right Lines are to be drawn from one Degree to the other, the Lengths of the Radii of each of the Circumferences cutting the Diagonals, may be found by Trigonometry, an Example of which is as follows:

Suppofe a Quadrant be 6 Inches Radius, which is the fmalleft accuftomed to be divided by Diagonals. Suppofe alfo you have a Scale of 1000 equal Parts, and that the Diftance from the inward Arc to the outward one, is 9 Lines, anfwering to 125 of fuch Parts, whereof the Radius is 1000; whence, by Calculation, I find that the right-lined Diagonal, drawn from one Degree to that which follows it, is 126 of the fame Parts; and that the Radius of the inward Arc, which is 5 Inches, 3 Lines, contains 875 of them.

The obtufe Angle made by the Radius and the Diagonal, is 172 Deg. 2 Min. and afterwards calculating the Lengths of the Radii of the Circumferences cutting the Diagonals, and dividing them into every 10 Minutes, I find that the Radius of 10 Min. is 894 of the fame equal Parts, inftead of 896 which it would have contained, if the Diftance between the inward and outward Arc had been divided into 6 equal Parts. The Radius of 20 Minutes ought to contain 913 of them, inftead of 917; the Radius of 30 Minutes ought to contain 933 of them, inftead of 938; the Radius of 40 Minutes ought to contain 954 of them, inftead of 959. Laftly, the Radius of 50 Minutes ought to contain 977, inftead of 980, which it muft, if the aforefaid Diftance be divided into 6 equal Parts.

The greateft Error, which is about 5 Parts, anfwers to about $\frac{1}{3}$ of a Line, which may caufe an Error of 2 Minutes; but this Error diminifhes in proportion as the Radius of the Quadrant augments in refpect of the Diagonals, fo that the Error will be lefs by half, if the Radius of the Quadrant be one Foot, and the Diftance of the inward and outward Arcs is but 9 Lines.

What we have faid as to the Divifions of the Quadrant, may likewife be applied to Theodolites, Circles, Semi-circles, or any other Portions of Circles to be divided into Minutes.

As to the Geometrick Square, each Side of it is divided into 100 equal Parts, beginning at the Ends, that fo the Number 100 may end at the Angle of 45 Degrees. Thefe Divifions are diftinguifhed by little Lines from 5 to 5, and by Numbers from 10 to 10; all thofe Divifions being produced from a kind of Lattice, both ways containing 10000 fmall and equal Squares.

This Quadrant is furnifhed with two immoveable Sights, faftened to one of it's Semi-diameters, and with a Thread and Plummet fixed to the Center, as likewife a moveable Index, with two other Sights, faftened to the Center, with a Headed-Rivet. The Sights are nearly like thofe belonging to the Theodolite.

Inftead of immoveable Sights, there is fometimes faftened to one of the Radius's of the Quadrant a Telefcope, and then the 1ft Point of Divifion of the Circumference may be

 found

found in the manner as is explained hereafter in the Astronomical Quadrant: for this Quadrant is designed only to take the Heights and Distances of Places on Earth.

Upon the under Surface of this Quadrant, is a Ball and Socket fastened with 3 Screws, by means of which it may be put into any Position fit for Use.

This Instrument may be put in Use in different Situations; for first, it may be so disposed that it's Plane may be at right Angles with the Horizon, for observing Heights and Depths, which may yet be done two different ways, *viz.* in using the fixed Sights, and the Thread and Plummet, and then neither of it's Sides will be found parallel to the Horizon; or else by keeping the Sights fastened to the Index moveable, and then one of the Semi-diameters of the Quadrant will always be parallel to the Horizon, and the other perpendicular: which may be done by means of a Plummet suspended in the Center, and then the fixed Sights are useless.

Finally, the Quadrant may be placed so as it's Plane may be parallel to the Horizon, for observing horizontal Distances with the Index and immoveable Sights, and then the Thread, with it's Plummet, is not in use.

Uses of the Quadrant, with two fixed Sights and a Plummet.

USE I. *To take the Height or Depth of any Object in Degrees.*

As suppose the Height of a Star or Tower is to be taken in Degrees; place the Quadrant vertically, then place your Eye under that fixed Sight next the Circumference of the Quadrant, and direct it so, that the visual Rays passing through the Holes of the Sights, may tend to the Point of the Object proposed (as to the Sun, it is sufficient that it's Rays pass thro' the aforesaid Holes): then the Arc of the Circumference contained between the Thread and it's Plummet, and the Semi-diameter on which the Sights are fastened, will show the Complement of the Star's Height above the Horizon, or it's Distance from the Zenith: Whence the Arc contained between the Thread, and the other Semi-diameter towards the Object, shews it's Height above the Horizon. The same Arc likewise determines the Quantity of the Angle made by the visual Ray, and a horizontal Line, parallel to the Base of the Tower.

But to observe Depths, as those of Wells or Ditches, the Eye must be placed over that Sight, which is next the Center of the Quadrant.

The whole Operation consists in calculating Triangles by the Rule of Three, formed in the the Proportion of the Sines of Angles, to the Sines of their opposite Sides, according to the Rules of right-lined Trigonometry, of which we are now going to give some Examples.

USE II. *Let it be required to find the Height of the Tower* AB, *whose Base is accessible.*

Fig. 2. Having planted the Foot of your Instrument in the Point C, look at the Top of the Tower thro' the fixed Sights; then the Thread of the Plummet freely playing, will fix itself upon the Number of Degrees, determining the Quantity of the Angle made at the Center of the Quadrant, by the visual Ray, and the horizontal Line, parallel to the Base of the Tower, accounting the Degrees contained between the Thread and the Semi-diameter next to the Tower.

Now suppose the Thread fixes upon 35 Deg. 35 Min. and having exactly measured the level Distance from the Foot of the Tower, with a Chain, to the Place of Observation, you will find it 47 Feet; then there will be 3 things given, to wit, the Side B C, and the Angles of the Triangle A B C: for since Walls are always supposed to be built upright, the Angle B is a right Angle, or 90 Deg. and consequently the 2 acute Angles A and C, are together equal to 90 Degrees, because the three Angles of any right-lined Triangle, are equal to 180 Degrees, or 2 right Angles.

Now the Angle observed, is 35 Deg. 35 Min. whence the Angle A is 54 Deg. 25 Min. therefore you may form this Analogy, As the Sine of 54 Deg. 25 Min. is to 47 Feet, So is the Sine of 35 Deg. 35 Min. to a fourth Term, which will be found 33 $\frac{1}{2}$ Feet; to which adding 5 Feet, the Height of the Observer's Eye, and the Height of the proposed Tower will be found 38 $\frac{1}{2}$ Feet.

USE III. *Let it be required to find the Height of the inaccessible Tower* D E.

Fig. 3. In this Case two Observations must be made, as follow:

Place the Foot of your Quadrant in the Point F, and look thro' the two immoveable Sights to the Top of the Tower D; then see on what Degree the Thread of the Plummet fixes, which suppose on the 34th. This being done, remove the Instrument, planting a Staff in it's Place, and set it up in some other Place level to the Place it was in before, as in the Point G, in the same right Line, and look thro' the afore-mentioned Sights, at the Point D of the Tower. Note the Point in the Limb of the Quadrant that the Thread cuts, which suppose 20 Degrees. Measure likewise very exactly, the Distance between the two Stations, which suppose 9 Toises, or 54 Feet.

This being done, all the Angles of the Triangle D F G will be known, as also the Side F G measured; by which means it will be easy to find the Side D F, and afterwards the Side E D, by making the following Analogies.

The

The Angle E F D being found 34 Deg. it's Complement D F G to 180 Deg. will be 146 Deg. and the Angle G having been found 20 Deg. it follows that the Angle F D G is 14 Deg. therefore ſay, As the Sine of 14 Deg. is to 54 Feet, So is the Sine of 20 Deg. to a fourth Term, which will be 76 Feet, and about $\frac{1}{3}$, for the Side D F: then ſay, As Radius is to the Hypothenuſe F D, So is the Sine of the Angle D F E, to the Side E D, which will be found 42 $\frac{2}{3}$ Feet; to which adding 5 Feet, the Height of the Center of the Inſtrument above the Ground, and there will be had 47 $\frac{2}{3}$ Feet, for the Height of the Tower propoſed.

Theſe Calculations are much better made with Logarithms, than by common Numbers, becauſe they may be done by only the help of Addition and Subſtraction, as is more fully explained in Books of Trigonometry.

Theſe Propoſitions, and others the like, may be alſo geometrically ſolved, by making Triangles ſimilar to thoſe formed upon the Ground.

As to ſolve the preſent Queſtion, make a Scale of 10 Toiſes, that is, draw the right Line A B ſo long, that the Diviſion of it may be exact; and then divide it into 10 equal Parts, and ſubdivide one of theſe Parts into 6 more, to have a Toiſe divided into Feet.

Then draw the indeterminate Line E G, and make with a Line of Chords, or Protractor, an Angle at the Point G of 20 Degrees, and draw the indeterminate Line G D. Lay off 9 Toiſes, or 54 Feet, from G to F; then make at the Point F an Angle of 34 Degrees, and draw the Line F D, cutting the Line G D in ſome Point as D, from which let fall the Perpendicular D E, which will repreſent the Height of the propoſed Tower, and meaſuring it with the Scale, you will find it to contain 47 Feet, 8 Inches. All the other Sides of theſe Triangles may likewiſe be meaſured with the ſame Scale.

USE IV. *To find the Breadth of a Ditch, or Well, whoſe Depth may be meaſured.*

Let it be propoſed to meaſure the Breadth of the Ditch C D, which may be approached. Fig. 4.

Place the Quadrant upon the Brink in the Point A, ſo that you may ſee thro' the Sights the Bottom of the Ditch, at the Point D; then find the Angle made by the Thread upon the Limb, which ſuppoſe is 63 Degrees, and meaſure the Depth A C, from the Center of the Quadrant, which ſuppoſe 25 Feet; then make a ſimilar right-angled Triangle, one of whoſe acute Angles is 63 Degrees (and conſequently the other will be 27 Degrees), and the leaſt Side is 25 Parts of ſome Scale. Laſtly, meaſure with the ſame Scale the Side C D, which will be about 49; therefore the Breadth of the Ditch is 49 Feet.

USE *of the Geometrick Quadrat.*

The Quadrant being vertically placed, and the Sights directed towards the Top of the Tower propoſed to be meaſured; if the Thread of the Plummet cuts the Side of the Quadrat, whereon is writ *right Shadows*, the Diſtance from the Baſe of the Tower, to the Point of Station, is leſs than the Tower's Height: if the Thread falls upon the Diagonal of the Square, the Diſtance is equal to the Height; but if the Thread falls upon the Side of the Square, whereon is writ *verſed Shadows*, the Diſtance of the Tower from you, is greater than it's Height. Fig. G.

Now having meaſured the Diſtance from the Foot of the Tower, it's Height may be found by the Rule of Three, in having 3 Terms known, but their Diſpoſition is not always the ſame; for when the Thread cuts the Side, denoted *right Shadow*, the firſt Term of the Rule of Three, ought to be that part of the Side cut by the Thread, the ſecond Term will be the whole Side of the Square, and the third, the Diſtance meaſured.

But when the Thread cuts the other Side of the Square, the firſt Term of the Rule of Three, muſt be the whole Side of the Square; the ſecond Term, the Parts of that Side cut by the Thread; and the third, the Diſtance meaſured.

Suppoſe, for Example, that looking to the Top of a Tower, the Thread of the Plummet cuts the Side of *right Shadows* in the Point 40, and that the Diſtance meaſured is 20 Toiſes: I order the Rule of Three in the following manner; [40. 100. 20.

Multiplying 20 by 100, and dividing the Product 2000 by 40, there will be found the fourth Term 50, which ſhews the Height of the Tower to be 50 Toiſes.

But if the Thread of the Plummet falls on the other Side of the Square, as, for Example, upon the Point 60, and the Diſtance meaſured is 35 Toiſes; diſpoſe the three firſt Terms of the Rule of Three thus, [100. 60. 35.

Multiply 35 by 60, and the Product 2100 being divided by 100, will give 21 for the Height of the Tower.

USE *of the Quadrat without Calculation.*

All the aforeſaid Operations, with many others, may be made without Calculation, as we ſhall make manifeſt by ſome Examples.

USE I. Let us ſuppoſe (as we have already done) that the Thread falls upon 40 on the Side of right Shadows, and that the Diſtance meaſured is 20 Toiſes; ſeek amongſt the little Squares for that Perpendicular to the Side, which is 20 Parts from the Thread, and that Perpendicular will cut the Side of the Square next to the Center in the Point 50, which will be the Height of the propoſed Tower in Toiſes. Fig. G.

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USE II. But if the Thread cuts the Side of versed Shadows in the Point 60, and the Distance is 35 Toises, count upon the Side of the Quadrant, from the Center, 35 Parts; count also the Divisions of the Perpendicular from that Point 35 to the Thread, which will be 21, the Height of the proposed Tower in Toises.

Note, In all Cases the Height of the Center of the Instrument above the Ground, must be added.

USE III. *To take an inaccessible Height with the Quadrat.*

To do which, there must be made two Stations, whose Distance must be measured, and then there will be three Cases.

CASE I. *When the right Shadow is cut in both Stations by the Thread.*

Let us suppose, for Example, that at the first Observation the Side of right Shadows is cut in the Point 30, and the Instrument being removed 20 Toises to a second Station, the Side of right Shadows is cut in the Point 70; then note the Position of the Thread in these two Stations, by drawing a Line upon the Lattice with a Pencil, from the Center to the aforesaid Point 30, and another to the Point 70. Seek between these two Lines a Portion of a Parallel, which may have as many Parts as the Distance measured has Toises, which in this Example must be 20: then the said Parallel being continued, will meet the Number 50, counting from the Center, whence the Height of the Tower observed, will be 50 Toises. You will likewise by the same means find that the Distance from the Base of the Tower, to the first Station, is 15 Toises, because there is 15 Parts contained upon the Parallel between the Number 50, and the Line drawn with the Pencil to the Number 30.

Instead of drawing Lines with a Pencil, two Threads fastened to the Center will do, one of which may be the Thread of the Plummet.

CASE II. *When the Side of versed Shadows is cut at both Stations by the Thread.*

Suppose, in the first Station, that the Thread cuts the Side of versed Shadows in the Point 80, and that being removed 15 Toises to another Station, the Thread falls upon the Number 50 on the same Side. Mark with a Pencil upon the Lattice, the two different Positions of the Thread in both Stations, and find between these two Lines, a Portion of a Parallel containing as many Parts as the Distance measured contains Toises, which, in this Example, is 15 Toises: to these 15 Parts add 25, which is the Continuation of the same Parallel to the Side of the Square next to the Center, and the Sum makes 40; whence the Distance of the Tower, from the second Station, is 40 Toises: and to find it's Height, seek the Number 40 upon the Side of the Square next the Center, and count from that Number to the first Line drawn on the Lattice with the Pencil, the Parts of the Parallel, which in this Example will be found 20; therefore the Height of the Tower is 20 Toises, by always adding the Height of the Quadrant.

CASE III. If in one Station the Thread falls upon the Diagonal of the Square, and in the other it cuts the Side of right Shadows, you must proceed in the same manner as when the Thread at both Stations falls upon the Side of right Shadows.

But when the Thread falls along the Diagonal in one Station, and upon the Side of versed Shadows in the other, you must proceed in the same manner, as when the Thread cuts, at both Stations, the Side of versed Shadows.

The Reason of all this is, because there is always made upon the Lattice a little Triangle similar to a great one, made upon the Ground, altho' diversly posited. The Line made by the Thread and Plummet always represents the Visual Ray; the two other Sides of the little Triangle, which make a right Angle, represent the Height of the Tower and it's Distance; and when the Thread cuts the Side of right Shadows, the Height is represented by the Divisions of the Sides of the Lattice, which is perpendicular to the Side of the Quadrat; but when the Thread cuts the Side of versed Shadows, the Distance is represented by the Divisions of the Side distant from the Center, and the Height by the Perpendicular answering to the Number of Divisions of the same Side.

USE IV. *To find the Depth of a Ditch or Well.*

The Breadth of the Ditch (or Well) must first be measured, and afterwards you must place the Quadrant upon the Brink, and look thro' the two Sights, 'till you see the opposite Point, where the Surface of the Water touches the Side of the Ditch; then the Thread will cut the Parallel, answering to the Feet or Toises of the Ditch's Breadth; and that Perpendicular, at which the Parallel ends, will determine the Depth, from which must be substracted the Height of the Instrument above the Brink of the Ditch.

USE *of the Quadrant in taking of Heights and Distances, by means of an Index and it's Sights.*

Place the Quadrat so that it's Plane may be at right Angles with the Plane of the Horizon, and one of it's Sides parallel thereto, which will be done when the Plummet, freely hanging, falls along the other Side of the Quadrant.

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In this Situation the two fixed Sights are of no Ufe, unlefs they are ufed to obferve the Diftance between two Stars, and then the Quadrant muft be inclined, by directing the immoveable Sights towards one Star, and the moveable ones towards the other; and the Number of Degrees, comprehended between them, will be the Diftance of the Stars in Degrees.

If it is ufed to obferve an Height, the Center of the Inftrument muft be above the Eye; but if a Depth is to be obferved, the Eye muft be above the Center of the Inftrument.

USE I. *To take an Height, as that of a Tower, whofe Bafe is acceffible.*

Having placed the Quadrant, as already fhewn, turn the Index, fo that you may fee the Top of the Tower thro' the two Sights; and the Arc of the Limb of the Quadrant, between that Side of it parallel to the Horizon, and the Index, will be the Height of the Tower in Degrees. If afterwards the Diftance from the Foot of the Tower, to the Place where the Inftrument ftands, be exactly meafured, there will be three things given in the Triangle to be meafured; namely, the Bafe, and the two Angles made at it's Ends, one of which will be always a right Angle, becaufe the Tower is fuppofed to be built upright, and the other the Angle before obferved; whence the other Sides of the Triangle may be found by the Rules of right-lined Trigonometry, or elfe without Calculation, by drawing a little Triangle fimilar to the great one, whofe Bafe is the Ground, and Perpendicular the Height of the Tower; or otherwife by the Geometrick Square, in obferving, that in *that* Pofition of the Quadrant, the Side of right Shadows ought always to be parallel to the Horizon, and the Side of verfed Shadows perpendicular thereto.

USE II. *To find the Height of a Tower, whether acceffible or inacceffible, by means of the Quadrat.*

In the aforementioned Pofition of the Quadrant, there are always formed, in the Quadrat, little fimilar Triangles, whofe homologous Sides are parallel and fimilarly pofited to thofe of the great ones formed upon the Ground; by which means the Operations are rendered more fimple and eafy than in the other Situation of the Quadrant; as we come now to explain, by making three different Suppofitions, according to the different Cafes that may happen.

CASE I. Let us fuppofe, for Example, that having obferved the Height of a Tower, whofe Bafe is acceffible, thro' the Sights of the Index, the Index cuts the Side of right Shadows in the Point 40, and the Diftance to the Bafe of the Tower is 20 Toifes; feek among the Parallels to the Horizon, from that which paffes thro' the Center to the Index, the Parallel of 20 (becaufe 20 Toifes is the Diftance fuppofed), and you will find that it terminates at the Number 50, on the perpendicular Side of the Square, reckoning from the Center; whence the Height of the Tower is 50 Toifes above the Center of the Inftrument.

CASE II. Suppofe, in another Obfervation, that the Index cuts the Side of verfed Shadows in the Point 60, and the Diftance meafured is 35 Toifes; count from the Center of the Quadrant upon the Side parallel to the Horizon 35, and from this Point, reckoning the Parts of the Perpendicular, to the Interfection of the Index, and you will find 21; whence the Height of the Tower is 21 Toifes.

CASE III. *Laftly*, Suppofe the Bafe of the Tower to be inacceffible, and that there muft be made two Stations (as we have faid before); the Height of it may be found without any Diftinction of right or verfed Shadows: for having meafured the Diftance between the two Stations, and drawn two Lines in the Quadrat, fhewing the Situation of the Index in thofe two Stations, find between thofe two Lines a Portion of a Parallel to the Horizon, which fhall have as many Parts, as the Diftance meafured contains Toifes: then if you continue that to the perpendicular Side of the Square diftant from the Center, you will there find a Number expreffing the Height of the Tower, and the Continuation of that Parallel to this Number, will fhew the Diftance to the Bafe of the Tower.

Note, In this Situation of the Quadrant, horizontal Diftances are always reprefented in the Quadrat by Lines parallel to the Horizon, and Heights are always reprefented by Lines perpendicular to the Horizon, which renders (as we have already faid) Operations more eafy.

It does not happen fo in that other vertical Pofition of the Quadrant, when the fixed Sights are ufed; for if in obferving the Height of an inacceffible Tower, the Thread of the Plummet in one Station falls upon the Side of right Shadows, and in the other Station, on the Side of verfed Shadows, the Diftance between the two Lines drawn with a Pencil on the Lattice, croffes the Squares of the Lattice by their Diagonals, which will not have common Meafures with the Sides; whence it cannot be ufed to find the Height of the propofed Tower.

USE *of the Quadrant in meafuring of Horizontal Diftances.*

Altho' a Quadrant is not fo proper to meafure horizontal Diftances, as a Semi-circle or whole Circle, becaufe by it obtufe Angles cannot well be taken, yet we fhall here give fome Ufes of it by means of the Quadrat. Place the Quadrant upon it's Foot nighly parallel to the Horizon; for there is no Neceffity of it's Plane being perfectly level, becaufe fometimes it muft be inclined to perceive Objects thro' the Sights.

Then put the Foot of the Inftrument in the Line to be meafured, and make two Obfervations in the following manner, not ufing the Plummet, but the four Sights.

Fig. 5. Suppose, for Example, the perpendicular Distance A B is to be measured; plant several Staffs in the Line A C D, and the Quadrant in the Point A, in such manner that the two fixed Sights may be in the Line A C, and the Point B may be seen thro' the two moveable Sights, placed at right Angles with the Line A C: then remove the Quadrant, planting a Staff in it's place, and measure from A towards C, any Length; as, for Example, 18 Toises: at the End of which, having placed the Instrument, so that the two fixed Sights may be in the Line A C, move the Index 'till you see the Point B thro' it's Sights, and you will have upon the Lattice a little Triangle, similar to the great one made upon the Ground; therefore seek amongst the Parallels cut by the Index, that which contains as many Parts as the Distance measured does Toises; that is, in this Example, 18, which will terminate on the Side of the Quadrant, at a Number containing as many Parts as there are Toises in the Line A B proposed to be measured.

The Distance A B may yet otherwise be found, whether perpendicular or not, without making a Station at right Angles with the Point A.

Suppose, for Example, that the first Station is made in the Point C, and the second in the Point D; draw upon the Lattice two right Lines with a Pencil, or otherwise, shewing the two different Positions of the Index in both the Stations; and having measured the Distance of the Points C and D, which suppose 20 Toises, seek between the two Lines drawn with a Pencil, a Portion of a Parallel which is 20 Parts, and that will correspond, upon the Semidiameter of the Geometrick Quadrat, to a Number, which, reckoned from the Center, will contain as many Parts as the right Line A B does Toises.

You will likewise find the Lengths of the Distances C B and D B, by the Divisions of the Index; for there is upon the Lattice a little oblique-angled Triangle similar to the great one C D B upon the Ground.

CHAP. VI.

Of the Construction and Uses of the Semi-Circle.

Fig. I. & K. THESE Instruments which are also called Graphometers, are made of beaten or cast Brass, from 7 Inches Diameter to 15; the Divisions of them are made in the same manner as those of the Theodolite and Quadrant, before explained. The simplest of these Instruments, is that of *Fig.* K; at the Ends of it's Diameter, and in little square Holes made upon the fiducial Line, there is adjusted two fixed Sights, fastened with Nuts underneath, and upon it's Center there is a moveable Index furnished with two other Sights, made in the same manner as those before-mentioned for the Theodolite, and which is fastened with a Screw. There is a Compass placed in the Middle of it's Surface, for finding the North Sides of Planes. There is also fixed underneath to it's Center, a Ball and Socket, like that mentioned in the Construction of the Theodolite, and for the same Use.

Note, These Instruments ought to be well straightned with hammering; then they must be fashioned with a rough File, and afterwards smoothed with a Bastard-File, and a fine one. When they are filed enough, you must see whether they are not bent in filing; if they are, they ought to be well straightned upon a Stone, or very plain Piece of Marble; then they must be rubbed over with Pumice-Stone and Water, to take away the Tracts of the File. To polish Semi-Circles well, as also any other Instruments, you must use *German*-Slate Stone, and very fine Charcoal, so that it does not scratch the Work: afterwards, to brighten them, you must lay a little Tripoli, tempered in Oil, upon a Piece of Shamoy, and rub it over them.

The Semi-Circle I, carries Telescopes for seeing Objects at a good Distance, and has the Degrees of it's Limb divided into Minutes, by right-lined or curved Diagonals, as in the Quadrant before-mentioned.

There is one Telescope placed underneath along the Diameter of the Semi-Circle, whose Ends are B B; and another Telescope adjusted to the Index of the Semi-Circle. When the fiducial Line cuts the Middle of the Index, the Telescope fastened to it must be a little shorter than the Index, to the End that the Degrees cut by the fiducial Line may be seen; but the best way is for the Telescopes to be of equal Length, and then the fiducial Line must be drawn from the End C, passing thro' the Center of the Semi-Circle, and terminating in the opposite End D. The two Ends of the Index are cut so as to agree with the Degrees upon the Limb, as may be seen at the Places C F, G D, in such manner that the Line C F E G D, may be the fiducial Line of the Semi-Circle.

Note, The Degrees on this Semi-Circle do not begin and end at the Diameter, as in others; but at the Lines C F, G D, when the Telescopes are so placed over each other, that the visual Rays agree. To make which, the little Frame carrying the cross Hairs, must be moved backwards or forwards by means of Screws. The Breadth from the Middle of the Telescope,

ſcope, to the Points F, G, is commonly about 5 Degrees; and this is the Reaſon why the Diviſions begin further from the Diameter than they end, as may be ſeen *per* Figure.

Theſe Teleſcopes have two or four Glaſſes, and have a very fine Hair ſtrained in the Focus of the Object-Glaſs, ſerving for a Sight.

Teleſcopes with four Glaſſes ſhew Objects in their true Situation, but thoſe with two Glaſſes invert them; ſo that *that* which is on the right Hand appears on the left, and that which is above appears below: but this does not at all hinder the Truth of Operations, becauſe they always give the Point of Direction.

Theſe Teleſcopes are made with Braſs Tubes ſoldered, and turned in a Cylindrick Form, as may be ſeen by the Figure L, which repreſents a Teleſcope taken to pieces; the Eye-Glaſs, being that to which the Eye is applied to look at Objects, is at the End 1. It is put in another little Tube apart (likewiſe marked 1) which is drawn out, or ſlid into the Teleſcope, according to different Sights. This little Tube alſo ſometimes carries the Hair in the Focus of the Glaſs, ſerving as a Sight; but it is better for the Hair to be faſtened to a little Piece of Braſs (ſeen apart) on which there is very exactly drawn a ſquare Tract 2, upon which the Hairs are placed. The ſaid Piece is placed in a Groove made in a little Braſs Frame, ſoldered to the Tube of the Teleſcope at the Place 2; the ſmall Screw 5 is to move forwards or backwards, the little Piece carrying the Hairs; the Object-Glaſs is placed at the other End of the Teleſcope, next to the Object to be ſeen. It is alſo placed in the little Tube 3, which being put into the Tube of the Teleſcope, muſt be binded pretty much by it, that it may not eaſily change it's Place when the Teleſcope is adjuſted. The Glaſſes are convex, which renders their Middle thicker than their Edges; but the Eye-Glaſs muſt have more Convexity than the Object-Glaſs, to the end that Objects may appear greater than by the naked Eye.

The Focus of a Convex Glaſs is that Place where the Rays, coming from a luminous or coloured Object, unite, after having paſſed thro' the Glaſs; whence the Picture of Objects, oppoſite to the Glaſs, are there very diſtinctly repreſented. For example, the Point R, at the End of the Cone of the Figure H, is the Focus of the Glaſs S, becauſe it is the Point where the Rays, entering at the other End N of the Tube, unite, after having paſſed thro' the Glaſs S.

The Teleſcopes moſt in Uſe (for Semi-Circles) are thoſe with two Glaſſes, which are ſo placed, that their Foci are common, and unite in the ſame Point in the Tube of the Teleſcope, in which Point the Hairs are placed; if the focal Length of the Object-Glaſs is ſeven or eight times greater than that of the Eye-Glaſs, the Object will appear ſeven or eight times greater than when the Foci of the two Glaſſes are equal.

The Focus of the Eye-Glaſs being common with that of the Object-Glaſs, the coloured Rays, which falling upon the Surface of the Object-Glaſs, and uniting in the Focus of the Glaſs, afterwards continue their way diverging to the Eye-Glaſs, and paſs thro' it; ſo that placing the Eye behind it, Objects may be perceived, whoſe Pictures are repreſented in the Focus: for it is the Object that ſends forth it's Species to the Eye, as may be yet very manifeſtly proved by the following Experiment.

Darken a Room, by ſhutting the Window-Shutters, and make a round Hole in ſome Shutter, whoſe Window is expoſed to a Place on which the Sun ſhines: in which Hole place a Convex Glaſs, and alſo a white Piece of Paper or Sheet in the Room, oppoſite to the Hole, and at the Glaſs's focal Diſtance from it; then a very diſtinct Repreſentation of all outward Objects, oppoſite to the Hole in the Shutter, will be painted upon the Paper in the Room in an inverted Situation; and this Picture is made by Rays of Light coming from the Objects without. The focal Diſtance of the Glaſs may be found, by moving the Paper backwards and forwards, 'till the Repreſentation of the Objects are diſtinctly perceived.

There is a Ball and Socket belonging to this Semi-Circle, which, being well made, in the aforeſaid manner, is the moſt perfect that can be made.

The Inſtrument M is a Protractor about 8 or 10 Inches Diameter, with it's moveable Index; we make them ſometimes as large as Graphometers, and uſe them both in taking Angles in the Field to a Minute, and alſo plotting them upon Paper.

The Index of this Protractor turns about a circular Cavity, in the Middle of which is a little Point, ſhewing the Center of the Protractor. The Diviſions of the Limb of this Protractor are made in the ſame manner as thoſe on the Limb of the Semi-Circle, and by the Method before explained.

USE I. To take the Plot of a propoſed Field, as A B C D E; plant a Staff very upright, at each Angle of the Field, and meaſure exactly, with a Toiſe, one of it's Sides, as A B, which ſuppoſe 50 Toiſes, 2 Feet; then make a Memorial, on which draw a Figure ſomething like the Field propoſed: This being done, place the Semi-Circle, with it's Foot, in the Place of the Staff A; ſo that looking thro' the fixed Sights of the Diameter, you may ſee the Staff B. Afterwards, the Semi-Circle remaining fixed in this Poſition, turn the Index, ſo that you may ſee thro' the Sights the Staff C. Note the Angle made by the fiducial Line with the Side A B, and write down, in your Memorial, the Quantity of the Angle B A C; afterwards turn the Index ſo, that you may ſee the Staff D thro' the Sights, and write down Fig. 6.

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in your Memorial the Quantity of the Angle B A D: Again, turn the Index so that you may see thro' the Sights the Staff E, and set down the Quantity of the Angle B A E; but every time you look thro' the Sights, Care must be taken that the Staff B is in a right Line with the Sights of the Diameter.

This being done, remove the Semi-Circle with it's Foot, and having replanted the Staff A, place the Semi-Circle, with it's Foot, in the Place of the Staff B, in such manner, that by looking thro' the fixed Sights of the Diameter, you may see the Staff A; and the Semi-circle remaining fixed in this Situation, turn, as you have already done, the Index so that you may successively see the Staffs C, D, E, and write down in the Memorial the Quantities of the Angles A B C, A B D, A B E.

Finally, Plot the Field exactly with a Semi-Circle or Protractor, by laying down all the Angles, whose Quantities are marked at the Ends of the Line A B, from whence may be drawn as many right Lines, and from their Intersections other Lines, which will form the Plot of the Field proposed. The Lengths of all those Sides which have not been measured, may be found by a Scale of equal Parts, of which the Line A B is 50 $\frac{1}{3}$, and the Area of the Field may be found by finding the Area of all the Triangles it may be reduced into.

Note, It is proper to measure one of the longest Sides of the Field, for using it as a common Base, and making at both it's Ends all the Observations necessary for there forming the Angles of the Triangles required to be made; for if one of the shortest Lines be taken for a common Base to all the Triangles, the Angles formed by the Intersections of the visual Rays in looking at the Staffs, will be too acute, and so their Intersections very uncertain.

The Meridian Line of Plans may be known by help of the Compass, whose Meridian is generally parallel to the Diameter of the Semi-Circle: for since the common Base of all the Triangles observed, is parallel to the said Diameter, you need but note the Angle which it makes with the Needle of the Compass, and this may be easily done by directing the fiducial Line parallel to the Needle; after which you may draw upon the Plot a little Card in it's true Position.

USE II. *To find the Distance from the Steeple* A, *to the Tower* C, *they being supposed inaccessible.*

Fig. 7. Having chosen 2 Stations, from which the Steeple and Tower may be seen, and measured their Distance serving as a Base, place the Semi-Circle at one of them, as D, and the Staff in the other, as in the Point E, and turn it so, that thro' the fixed Sights of it's Diameter, or thro' the Telescope, you may espy the Staff E: then move the Index so, that thro' it's Sights you may see the Steeple A; and the Degrees of the Semi-Circle between the Diameter and the Index, will give the Quantity of the Angle B D E, being in this Example 3 Deg. which note in your Memorial. Again; turn the Index 'till you see the Tower C thro' the Sights or Telescope, always keeping the Diameter in the Line D E; then the Degrees between the Diameter and Index, will shew the Quantity of the Angle C D E, 123 Deg. which likewise note in the Memorial. Now having removed the Semi-Circle from the Station D, and placed a Staff in it's Place, measure the Distance from the Staff D to the Staff E, which suppose 32 Toises, writing it in the Memorial: then put the Semi-Circle in the Place of the Staff E, so that the fixed Sights of the Diameter, or Telescope, may be in the Line E D; and turn the Index, that the Tower C may be seen thro' it's Sights, then the Degrees contained between the Diameter, and the Index, will give the Angle C E D, which in this Example is 26 Degrees. Finally, Turn the Index 'till you see the Steeple A thro' the Sights, and the Angle A E D will be 125 Degrees, which set down in the Memorial, and by help of a Scale and Protractor, the Distance A C may be known.

To solve the same Problem trigonometrically; first, We have found by Observation in the Triangle D A E, that the Angle A D E is 32 Degrees, and the Angle D E A 125 Degrees, whence the Angle D A E is 23 Degrees (because the three Angles of any right-lined Triangle, are equal to 2 right Angles), and to find the Side A E, make this Analogy: As the Sine of 23 Degrees is to 32 Toises, So is the Sine of 32 Degrees to the Line A E, about 43 Toises. Likewise you will find by Observation in the Triangle C D E, that the Angle C D E is 26 Degrees, and the Angle E D C 123 Degrees, whence the Angle D C E is 31 Degrees; and to find the Side C E, make this second Analogy: As the Sine of 31 Degrees is to 32 Toises, So is the Sine of 123 Degrees, or it's Complement 57, which is the same, to C E 52 Toises. Now to find the Distance C A, examine the Triangle C A E, whose two Sides C E, A E, with the included Angle A E C of 99 Degrees, are known, and consequently the Sum of the two unknown Angles are equal to 81 Degrees; and to find either of them, make again this Analogy: As the Sum of the two known Sides 95 Toises, is to their Difference 9, So is the Tangent of 40 Deg. 30 Min. half the Sum of the opposite Angles, to the Tangent of half their Distance, which answers to 4 Deg. 37 Min. and being added to 40 Deg. 30 Min. will give the greatest of the unknown Angles C A E, 45 Deg. 7 Min. and consequently the other Angle A C E, will be 35 Deg. 53 Min. Lastly, to find the Length C A, say, As the Sine of 35 Deg. 53 Min. is to 43 Toises, So is the Sine of 99 Deg. to the Distance A C, 72 Toises, 2 Feet.

USE

USE III. To find the Height of the Tower A B, whofe Bafe cannot be approached becaufe of a Rivulet paffing by it's Foot; chufe two Stations fomewhere upon level Ground, as in C and D, and place the Semi-Circle vertically in the Point D, fo that it's Diameter may be parallel to the Horizon, which you may do by means of a Thread and Plummet, hung on the Top of a Perpendicular drawn on the backfide of the Semi-Circle: then turn the Index, in order to fee the Top of the Tower B thro' the Sights, and take the Quantity of the Angle B D A, which fuppofe 42 Degrees, noting it down in your Memorial. Now having removed the Semi-Circle, and placed it at the other Station C, meafure the Diftance D C, which fuppofe 12 Toifes; and after having adjufted the Semi-Circle, fo that it's Diameter may be parallel to the Horizon, turn the Index 'till you fee the Top of the Tower B, and fet down the Quantity of the Angle B C D, which fuppofe 22 Degrees, in the Memorial; then make a fimilar Figure by means of a Scale and Protractor, and the Height of the Tower A B will be found; which may likewife be found by Calculation in the following manner: The Angle B D A of 42 Degrees, gives the Angle B D C of 138 Degrees; and fince the Angle C of 22 Degrees has been meafured, the third Angle of the Triangle C B D will be 20 Degrees. Now fay, As the Sine of 20 Degrees is to 12 Toifes, So is the Sine of 22 Degrees, to the Line B D, about 13 Toifes; but B D is the Hypothenufe of the right-angled Triangle B D A, all the Angles of which are known: therefore fay by a fecond Rule of Three, As Radius is to about 13 Toifes, So is the Sine of 42 Degrees to the Height A B, 8 Toifes, and one Foot. Fig. 8.

USE IV. *To take the Map of a Country.*

Firft, chufe 2 high Places, from whence a great Part of the Country may be feen, which let be fo remote from each other, as that their Diftance may ferve as a common Bafe to feveral Triangles that muft be obferved for making of the Map; then meafure with a Chain the Diftance of thefe two Places. Thefe two Places being fuppofed A and B, diftant from each other 200 Toifes, place the Plane of the Semi-Circle horizontally, with it's Foot in the Point A, in fuch manner, that you may difcover the Point B thro' the fixed Sights or Telefcope: the Inftrument remaining fixed in this Situation, turn the Index, and fucceffively difcover Towers, Steeples, Mills, Trees, and other remarkable Things defired to be placed in the Map: examine the Angles which every of them make with the common Bafe, and fet them down together with their proper Names in the Memorial: As, for Example, the Angle B A I 14 Degrees, B A G 47, B A H 53, B A F 68, B A E 83, B A D 107; and, laftly, the Angle B A C 130 Degrees: which being done, and the Diftance of the two Stations A B fet down, place the Semi-Circle in the Point B, for a fecond Station. Fig. 9.

The Inftrument being fo placed that it's Diameter may be in the Line B A, turn the Index, and obferve the Angles made by the Objects before feen from the Point A; as for Example, the Angle A B C 20 Degrees, A B F 37, A B D 44, A B E 56, A B G 83, A B H 96, and the Angle A B I 133 Degrees, which note down in the Memorial.

If any Object viewed from the Point A, cannot be feen from the Point B, the Bafe muft be changed, and another Point fought, from whence it may be difcovered; for it is abfolutely neceffary for the fame Object to be feen at both Stations, becaufe it's Pofition cannot be had but by the Interfection of two Lines drawn from the Ends of the Bafe, with which they form a Triangle.

Note, The Bafe muft be pretty long, in proportion to the Triangles for which it ferves, and moreover very ftraight and level.

To make the Map, reduce all thofe Triangles obferved, to their juft Proportion, by means of a Scale and Protractor, in the manner as we have already given Directions, in the Ufe of the Theodolite.

CHAP. VII.

Of the Conftruction and Ufe of the Compafs.

THIS Inftrument is made of Brafs, Ivory, Wood, or any other folid Matter, from 2 to 6 Inches in Diameter, being in figure of a Parallelopipedon, in the Middle of which is a round Box, at the Bottom of which is defcribed a Card (of which more in the Conftruction of the Sea-Compafs) whofe Circumference is divided into 360 Degrees. In the Center of this Card is fixed a well-pointed Brafs or Steel Pivot, whofe Ufe is to carry the touched Needle placed upon it, *in Equilibrio*, fo that it may freely turn. This Box is covered with a round Glafs, for hindring left the Air fhould any wife agitate the Needle. Fig. O.

One of the Ends of the Needle always turns towards the North Part of the World, but not exactly, it declining therefrom, and the other towards the South.

According to Obfervations made in *October*, in the Year 1715, in the Royal Obfervatory, the Needle declined 2 Deg. 5 Min. Weftwardly.

Needles are made of Pieces of Steel, the Length of the Diameter of the Box, having little Braſs Caps ſoldered to their Middle, hollowed into a conical Figure ſo, that the Needle being put upon the Pivot, may move very freely upon it, and not fall off; they are nicely filed into different Figures, thoſe which are large being like a Dart, and ſmall ones have Rings towards one End, for knowing that End which reſpects the North, as may be ſeen in the little Figures nigh the Compaſs.

To touch a Needle well, having firſt got a good Stone, begin your Touch near the Middle of the Needle, and preſſing it pretty hard hard upon the Pole of the Stone, draw it ſlowly along to the End of the Needle, and lifting your Hand a good Diſtance from the Stone, while you put the Needle forward again, begin a ſecond Touch in the ſame manner, and after that a third, which is enough, only take Care not to rub the Needle to and fro on the Stone, whereby the backward Rubs take away what Virtue the forward ones gave; but lift it out of the Sphere of the Stone's Virtue, when you carry it forward again to begin a new Touch.

This admirable Property, by help of which great Sea-Voyages were firſt undertaken, and vaſt Nations both in the Eaſt and Weſt diſcovered, was not known in *Europe* 'till about the Year 1260.

A Man by means of this Inſtrument, and a Map, may likewiſe go to any propoſed Place, at Land, without enquiring of any body the way; for he need but ſet the Center of the Compaſs, upon the Place of Departure, on the Map, and afterwards cauſe the Needle to agree with the Meridian of this Place upon the Map: then if he notes the Angle that the Line leading to the Place makes with the Meridian, he need but in travelling keep that Angle with the Meridian, and that will direct him to the Place deſired.

This Inſtrument is alſo very uſeful to People working in Quarries, and Mines under Ground; for having noted upon the Ground the Point directly over that you have a mind to go to, you muſt place the Compaſs at the Entrance into the Quarry or Mine, and obſerve the Angle made by the Needle with the Line of Direction: then when you are under Ground, you muſt make a Trench, making an Angle with the Needle equal to the aforeſaid Angle; by means of which you may come to the propoſed Place under Ground. There are ſeveral other Uſes of this Inſtrument, the principal of which we are now going to ſpeak.

USE I. *To take the Declination of a Wall with the Compaſs.*

You muſt remember that there are 4 Points, called Cardinal ones, *viz.* North, South, Eaſt, and Weſt, dividing the Horizon into 4 equal Parts, and when one of theſe Points are found, all the others may likewiſe: for if you have North before you, South will be behind, Eaſt on the right-hand, and Weſt on the left.

A Wall built upon a Line tending from North to South, will be in the Plane of the Meridian; ſo that one Side thereof will face the Eaſt, and the other the Weſt.

Another Wall, at right Angles with the former, that is, one built upon the Line of Eaſt and Weſt, will be parallel to the Prime Vertical, and will not decline at all, and one of it's Sides will be directly South, and the other North.

Fig. 10. But if a Wall is ſuppoſed to be built upon the Line D E, it is ſaid to decline as many Degrees as is contained in the Arc F; therefore if, for Example, that Arc be 40 Degrees, the Side of the Wall facing towards the South, declines from the South towards the Eaſt 40 Degrees, and the oppoſite Side of the Wall will decline from the North towards the Weſt 40 Degrees: ſo that the Declination of a Wall, is no more than the Angle made by the Wall and the Prime Vertical. Another Wall parallel to the Line G H, will decline as many Degrees as is contained in the Arc C; therefore if that Arc be 30 Degrees, the Side of the Wall reſpecting the South, will decline 30 Degrees from the South towards the Weſt, and the other Side will decline 30 Degrees from the North to the Eaſt.

In all Operations made with a Compaſs, you muſt take care of bringing it nigh Iron or Steel, and that there be none concealed; for Iron or Steel entirely changes the Direction of the Needle.

I ſuppoſe here that the Pivot, upon which the Cap of the Needle is put, is in the Center of a Circle divided into 360 Degrees, or four Nineties, whoſe firſt Degree begins from the Meridian Line, and alſo that the Compaſs be ſquare, as that which is repreſented in the Figure.

Apply the Side of the Compaſs where the North is marked, to the Side of the Wall; then the Number of Degrees over which the Needle fixes, will be the Wall's Declination, and on that Side. If, for Example, the North Point of the Needle tends towards the Wall, it is a ſign that *that* Side of the Wall may be ſhone on by the Sun at Noon; and if the Needle fixes over 30 Degrees, counting from the North towards the Eaſt, the Declination is ſo many Degrees from South towards the Eaſt. If it fixes over 30 Degrees from the North towards the Weſt, the Declination of the Needle will be ſo many Degrees from the South towards the Weſt.

But ſince the Declination of the Needle is at *Paris* 12 Deg. 15 Min. N. W. for correcting that Defect, 12 Deg. 15 Min. muſt always be added to the Degrees ſhown by the Needle, when the Declination of the Wall is towards the Eaſt; and, on the contrary, when the Declination is towards the Weſt, the Declination of the Needle muſt be ſubſtracted.

As

As fuppofing, as we have already done, that the Needle fixes over the 30th Degree towards the Eaft, the Declination of the Wall will be 42 Deg. 15 Min. from the South towards the Eaft; but if the Needle fixes on the Weft-fide of the Wall, over the 30th Degree, the Declination will be 17 Deg. 45 Min. from the South towards the Weft.

If the South Point of the Needle tends towards the Wall, it is a Sign that the South is on the other Side of the Wall, and confequently that Side of the Wall, whofe Declination is to be found, will not be fhone upon by the Sun at Noon; whence it's Declination will be from the North towards the Eaft or Weft, according as it faces towards thofe Parts of the World. This will be more fully explained in the Treatife of Dialling.

USE II. *To take an Angle with the Compafs.*

Let the Angle D A E be propofed to be meafured; apply that Side of the Compafs, where the North is marked, to one of the Lines forming the Angle, as A D: fo that the Needle may freely turn upon it's Pivot, and when it refts, obferve what Number the North Point of the Needle ftands over; and finding it, for Example, 80 Degrees, the Declination of the faid Line will be fo many Degrees. Afterwards take, in the fame manner, the Declination of the Line A E, which fuppofe 215 Degrees: fubftract 80 Degrees from 215 Degrees, there will remain 135, which fubftract from 180, and there will remain 45 Degrees, the Quantity of the Angle propofed to be meafured. Fig. 11.

But if the Declination of the Line A D had been, for example, but 30 Degrees, and the Line A E 265 Degrees, the Difference of thofe two Declinations, which would be 235 Degrees, would be too great to fubftract from 180 Degrees; whence in this Cafe 180 Degrees muft be taken from 235 Degrees, and the Remainder 55 Degrees, will be the Angle propofed.

When Angles are meafured with the Compafs, there need not any regard be had to the Variation of the Needle, becaufe the Variation will always be the fame in all the different Pofitions of the Needle, provided at all times there be no Iron near it: and when the Compafs cannot be put nigh the Plane, by means of fome Impediment, it is fufficient to place it parallel, as the Figure fhews, and the Effect will be the fame.

USE III. *To take the Plot of a Foreft, or Morafs.*

Let it be required to take the Plot of the Morafs A B C D E, in which one may enter. To make thefe kinds of Operations, there muft be faftened two Sights to the Meridian Line of the Compafs; now plant long Staffs upright, fo that they may be in Lines parallel to the Sides encompaffing the Morafs, and place the Compafs upon it's Foot in a horizontal Pofition: then look at two of the Staffs thro' the Sights, putting always the Eye to that which is on the South Side of the Compafs; and having drawn a Figure upon Paper fomething reprefenting the Plot of the Morafs, write upon the correfpondent Line the Number of Degrees which the Needle, when fixed fhews. At the fame time meafure the Length of each Side of the Morafs, and fet down their Lengths upon the correfpondent Lines of your Memorial. When you have gone round the Morafs, the Degrees denoted by the Needle, will ferve to form the Angles of the Figure, and the Length of each Line will determine the Plot of the Morafs propofed. Fig. 12.

Let us fuppofe, for Example, that having placed the Compafs along the Side A B, or which is all one, along a Line parallel to that Side, and placing the Eye next to the South Sight of the two Sights, two Staffs fet up in that Line are efpied. If the Needle fixes on the 30th Degree towards the Eaft, fet down the Number 30 upon the Line A B in the Memorial, and alfo 50 Toifes, the Length of the Side A B: afterwards fet the Compafs, with it's Foot, along the Side B C, or in the Direction of the Staffs, putting always the Eye next the South Sight. If the Needle fixes on the 100th Degree, I write that Number on the Line B C, and at the fame time 70 Toifes, the Length of the Side B C: doing thus quite round the Morafs, you may fet down upon each correfpondent Line of the Memorial, the Numbers of Degrees and Toifes; by means of which, the Plot may be drawn in the following manner, by help of a Scale and Protractor.

Angles obferved	*Remaining Angles.*
30 *Degrees.*	
100 - - - - -	70
130 - - - - -	30
240 - - - - -	110
300 - - - - -	60

Set down, one after the other, all the Angles obferved with the Compafs, and fubftract the leaft from it's next greater, as in this Table.

Draw the Indefinite Line A B, of 50 equal Parts, reprefenting the 50 Toifes meafured; make the exterior Angle at the Point B 70 Degrees, and draw the indefinite Line B C, on which lay off 70 Toifes from B to C. Make at the Point C an exterior Angle of 30 Degrees, and draw the indefinite Line C D, whofe Length let be 65 Toifes, conformable to the Length meafured. Make likewife at the Point D an exterior Angle of 110 Degrees, and draw the Line D E of 70 Toises. Lastly, Make an exterior Angle of 60 Degrees at the Point E, draw the Line A E of 94 Toises, and the Plot will be compleated.

Note, All the Angles of the Figure taken together, ought to make twice as many right Angles, wanting 2, as the Figure has Sides: As, for Example, the Figure of this Use, having 5 Sides, all the Angles added together make 540 Degrees, or 6 times 90, which may serve to prove Operations.

This Manner of taking Plots is expeditious enough, but it is very difficult to make Operations exact with a Compass, because there may be Iron concealed nigh the Places whereat a body is obliged to place the Instrument.

CHAP. VIII.

The Uses of the aforesaid Instruments, applied to the Fortification of Places.

Plate 13. Fortification is the Art of putting a Place into such a State, that a small Body of Troops therein may advantageously resist a considerable Army.

The Maxims serving as a Foundation to the Art of Fortification, are certain general Rules established by Engineers, founded upon Reason and Experience.

The chief Engineer having examined the Extent and Situation of the Place to be fortified, communicates his Design in a Plane and Profil, as may be seen in Plate 13. to which he commonly adds a Discourse, orderly explaining the Materials employed by the Undertakers: and having searched the Ground in several Parts of the Place proposed, makes a Computation of each Toise of Work, by means of which the Engineer may nighly estimate the Charge of the whole Work, the Number of Workmen necessary to perfect it, and also the Time it will be done in.

The Plane of a Fortification represents, by several Lines drawn horizontally, the Inclosure of a Place.

This Design contains several Lines drawn parallel to one another; but the first and principal Track, which ought to be marked by a Line more apparent than the others, represents the chief Inclosure of the Body of the Place between the Rampart and the Ditch; so that by the Plan and it's Scale, the Lengths and Breadths of all the Works composing the Fortification may be known. (*Fig.* 1.)

The Profil represents the principal Tracks appearing upon a plane Surface vertically cutting and separating all the Works thro' the Middle. There is commonly a larger Scale to draw it, than to draw a Plan, for better distinguishing their Breadths, Heights, or Depths (as appears in *Fig.* 3.).

The Names of the chief Lines, and principal Angles, forming the Plane.

Fig. 1. The Line A B, is called the exterior Side of the Polygon, and L M the interior Side thereof.

L G the Demi-gorge of the Bastion, of which E G is the Flank, A E the Face, and A L the Capital.

G H is the Courtain, and A H the Line of Defence *Razant*.

The Figure A L G E represents a Demi-Bastion.

The Angle A N B is the Angle of the Center.

The Angle K A B is the Angle of the Polygon.

The Angle I A E, made by the two Faces, is the flanquant Angle, or Angle of the Bastion.

The Angle A E G made by the Face and the Flank, is called the Angle *of the Shoulder*.

The Angle E G H, made by the Flank and the Courtain, is called the Angle of the Flank.

The Angle E G B, made by the Flank and the Line of Defence, is called the interior flanquant Angle.

The Angle E D F, made by the two *Razant* Lines interfecting one another towards the Middle of the Courtain, is called the exterior flanquant Angle, or Angle of the *Tenaille*.

The Angle E H G, made by the Courtain and Line of Defence *Razant*, is called the diminished Angle, which is always equal to that made by the Face of the Bastion and the Base, or exterior Side.

Fundamental Maxims of Fortification.

The principal Maxims may be reduced to six.

I. Every Side round about a Place, must be flanked or defended with Flanks; for if there be any Side about a Place not seen or defended by the Besieged, the Enemy may there lodge themselves, and become Masters of the Place in a short time.

It

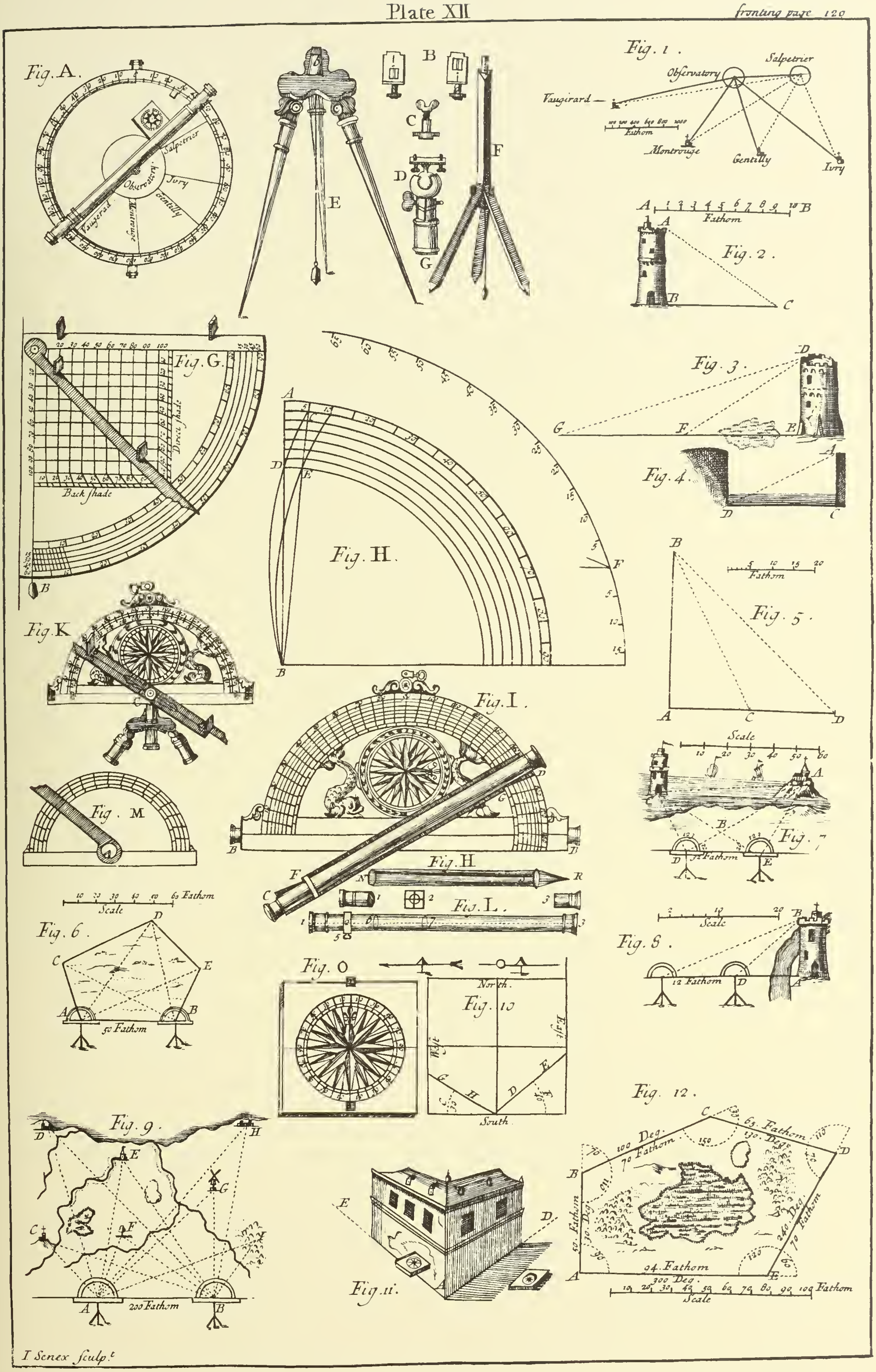


Fig. A.
Salpetrier
Observatory
Jury
Gentilly
Vaugerad
Fig. 1.
Observatory
Salpetrier
Vaugirard
Fathom
Montrouge
Gentilly
Ivry
Fathom
Fig. 2.
Fig. G.
Direct shade
Back shade
Fig. 3.
Fig. 4.
Fig. H.
Fathom
Fig. 5.
Fig. K
Fig. I.
Scale
Fig. 7
Fathom
Fig. M
Fig. H
Fig. L.
60 Fathom
Scale
Fig. 6.
50 Fathom
Scale
Fig. 8.
12 Fathom
Fig. O
North
Fig. 10
East
West
South
Fig. 12.
63 Fathom
100 Deg.
70 Fathom
70 Fathom
94 Fathom
300 Deg.
100 Fathom
Scale
Fig. 9.
200 Fathom
Fig. 11.
I Senex sculp.

It follows from this Maxim, that the flanquant Angle, or the Angle made by the Faces of the Bastion, being too acute, is defective, because it's Point may easily be blunted or broken by the Cannon of the Besiegers, and afterwards Miners may there work safe in widening of the Breach.

It is also a like Fault to round the Points of Bastions, for the same Reason.

II. The Force, as much as possible, must be equally distributed every where, for if there be any Side weaker than the rest, that will be it which the Enemy will attack; therefore if from the Nature of the Ground, one Side be weaker than the others, some Work must be there added to augment it's Force, in multiplying it's Defence.

III. The flanquant Parts must be no further remote from those which flank them, than a Musquet-shot will do Execution; therefore the Line of Defence, or the Distance from the Point of a Bastion to it's neighbouring Bastions, ought not much to exceed 125 Toises, which is the Distance that a Musquet, well charged, will do Execution.

IV. The Flanks of Bastions must be large enough to contain at least 30 Soldiers in Front, and 4 or 5 Pieces of Cannon mounted on their Carriages, in order to defend well the Face of the Bastion attacked by the Enemy; and since the principal Defence arises from Flanks, it is more proper for them to be perpendicular to the Line of Defence, than to have any other Situation. This Method was assigned by Count *Pagan*, and has been followed by the ablest Engineers since his Time, and particularly by Monsieur *Vauban*, who, by his singular Services, merited the Esteem of all warlike Nations, and able Engineers of his Time.

V. The Fortress must not be commanded by any Side out of the reach of Fire-Arms, which are Musquets and Cannon; but, on the contrary, it ought to command all Places round about.

VI. The Works nighest the Center, must be highest, and command those Places more distant, so that when the Enemy endeavour to make themselves Masters of some Outwork, they may be repulsed by those in the Body of the Place.

To draw upon Paper a fortified Plane, according to the Method of Count Pagan.

Let it be, for Example, an Hexagon: first draw the Line A B 180 Toises, for the exterior Side of the Hexagon, and raise the Perpendicular C D from the Point C of 30 Toises; then draw the Lines A D H, B D G, interfecting each other in the Point D, and take 55 Toises from your Scale, to determine the Length of the Faces A E, B F: from the Point E draw the Flank E G, making a right Angle in the Point G, at the End of the Line of Defence B G, and likewise the other Flank F H at right Angles to A H: finally, draw the Courtain G H, and you will have one Side of the Hexagon fortified. The other Sides are fortified in the same manner. About this Side of the Polygon thus fortified, you must draw a Ditch, represented by the Lines A C, C B, parallel to the Faces of the Bastions, meeting each other towards the Middle of the Courtain in the Point C. This Ditch ought to be 20 Toises in Breadth, and 3 Toises deep. The Ground taken out in making of the Ditch, serves to form the Rampart with it's Parapet, and the Glacis of the Covered Way, preserving the finest for the Parapet of the Body of the Place, and the Covered Way; for if the Ground be stony, Cannon-Balls, coming from the Besiegers against Parapets made with it, will make the Stones fly about, and annoy the Soldiers defending the Body of the Place. On the contrary, when the Ground is fine, the Bullets will but make Holes, and enter therein, provided Parapets have Breadth enough to deaden them: by Experience it is found, that Parapets must consist of well-rammed Earth at least 20 Foot thick, to be Proof against Cannon. Fig. 1.

The Parapet is made upon the Rampart 24 Feet broad, containing the Banquette, or little Bank, made parallel to the Faces, Flanks, and Courtains, forming the Inclosure of the Place.

The Base of the Rampart is 15 Toises broad, and is made parallel to the Courtains only, to the End that the Bastions may be full, and that there may be there found Earth in case of need, to make an Intrenchment.

When any Bastion is left open, a Mine must be made therein well arched, Bomb proof, and covered with Ground well rammed, and it must be endeavoured to be made so that the Rain-Water cannot get into it, to the End that Provisions put therein, may be preserved from time to time.

The Covered Way is made parallel without the Ditch, about 5 Toises broad, and upon it there is a Parapet made 6 Foot high, and a Banquette, at the Foot of the said Parapet, 3 Foot broad, and a Foot and a half high, so that Soldiers may commodiously use their Arms on the Top of the Parapet, whose Top must be sloped, that is, having a Descent or Slope going down 20 or 30 Toises into the Country.

There must be no hollow Places about this Slope, for the Enemy to cover themselves in; therefore when an Engineer visits the Fortification of a Place, it is requisite for him to examine the adjacent Parts, and have the hollow Places filled up, at least within the reach of a Musquet-shot from the Covered Way; and also to have all Places too high levelled, that so those which defend the Place, may discover all the adjacent Parts.

To draw the Profil of a Fortified Place upon Paper.

Fig. 2. Draw the indefinite Line O N, representing the Level of the Country, and take 15 Toises, which lay off from O to Q, for denoting the Base of the Rampart; then lay off 20 Toises from Q to R, for the Breadth of the Ditch, over-against one of the Faces of the Bastion, for it is wider over-against the Courtain: lay off 5 Toises from R to P, for the Breadth of the Covered Way; and lastly, 20 or 30 Toises from P to N, for the Base of the *Glacis*. *Note*, The longer the Base of the *Glacis* is made, the better will it be.

After having determined the Breadths or Thicknesses; the Heights above the Level of the Country, and Depths below, must be as follows.

Take 3 Toises from your Scale, and raise from the Points O, Q, Perpendiculars of that Height, for raising above the Level of the Country the Platform of the Rampart, whereof O S is the interior Talud, or Slope, going up from the City to the Platform of the Rampart S T; which Platform ought to be 6 or 7 Toises broad, that so Cannon may be commodiously used thereon, as also the other necessary Munitions for the Defence of the Place.

Note, The Rising of the Rampart ought to be very easy over-against the Gorge of the Bastions, for Coaches to go easily there up and down it.

The Base of the Talud O Z, is made with new-dug Earth, equal to the Height all along the Courtains; as if the Height be 3 Toises, the Base of the Slope must be also 3 Toises.

But at the Entry of the Bastions, the Base must be at least twice the Height; that is, if the Height of the Slope be 3 Toises, the Base of it must be at least 6 or 8 Toises, for Coaches to go up it.

When the Rampart is formed, and the Earth sufficiently raised upon it, which cannot be done but with Time and Precaution, in well ramming it every 2 Feet in Height, and laying Fascines to keep it together; a Parapet is made upon the Earth of the Rampart, 6 Feet of interior Height, and 4 Feet of exterior Height (for the Top of the Earth to have a Declivity), to discover any thing beyond the Ditch, and being mounted upon the Banquette, the Covered Way may be seen, and defended in case of Need.

The Base of the Parapet X Y, ought to be about 4 Toises broad, to the End that the Top thereof may be at least 20 Feet broad. At the Bottom of the interior Slope of the Parapet, there is made a little Bank 3 Feet wide, and a Foot and a half high, so that the Parapet will be 4 $\frac{1}{2}$ Feet above the Bank, which is sufficient for Soldiers to use their Fire-Arms on the Top thereof.

Care must be taken to lay Beds of Fascines every Foot in height, between the Earth of the Parapet; and in order to keep the Earth of the said Parapet from crumbling, it is covered with Grass-Turfs, cut with a Turfing-Iron, from some neighbouring Common, about 15 Inches long, and 10 broad.

Now to lay these Turfs, you must place the first Bed, or Row of them, very level all along the Distance of several Toises, and then lay the Turfs of the second Bed so, that the Joints of the first may be covered with them, and the Joints of the second likewise covered with the Joints of the third, *&c.* that so they may all make a good joining.

It is sufficient to give 2 Inches of Declivity to one Foot in height, for the interior Slope; and about 4 Inches to one Foot in height, for the exterior Slope of the Parapet. *Note*, There ought to be Gardiners to cut and lay the Turfs.

At the Foot of the exterior Slope of the Parapet and the Rampart, there is left a little Berm (marked Q), about 4 Feet wide, for retaining the loose Ground falling down from the Slope.

Q B represents the inward Slope of the Ditch, which is 3 Toises deep, and B K is the exterior Slope. If the Ground be brittle, they must have more Slope given them, for hindering it's falling to the Bottom of the Ditch. The Line K P represents the Platform of the Covered Way, which must be 5 Toises broad. P A represents the Parapet of the Covered Way, with it's Banquette at the Foot thereof. The whole must be 6 Feet high, for covering those which are on the Covered Way.

The superior Slope of the Slope A N, ought to be made of fine Earth, the Stones in which, if there be any, must be taken away with an Iron Rake, and buried at the Foot of the Slope, so that Cannon Balls shot from the Enemy upon the Covered Way, may enter therein, without making the broken Pieces of the Stones fly about upon the Covered Way.

To lay off the Plan of a Fortification upon the Ground.

Let, for Example, the Plan of the first Figure be proposed to be drawn upon the Ground.

Instead of a Scale and Compasses, there must be used Staffs, the Toise, and Lines; therefore, after having well examined the Ground, and considered where the Gates and Bastions must be made, which are commonly in the Middle of the Courtains, long Staffs must first be placed, where the flanquant Angles of the Bastions are intended to be.

Now having planted a long Staff upright, in the Place fixed on for the Point of the Bastion (marked A), measure very exactly, with a Toise, or Chain, 90 Toises; at the End of which plant a Staff (marked C): from the Point C continue that Line 90 Toises more; at the End of which plant another Staff, which will be the Point of the Bastion B. In the mean

mean time you are meaſuring with Chains or Lines, ſome Workmen muſt follow, and make a little Trench from Staff to Staff, before the Lines are taken away.

After which, a Perpendicular muſt be drawn from the Staff C, to the Track A C B.

To draw the ſaid Perpendicular, meaſure two or three Toiſes from C to A, where plant a Staff; meaſure likewiſe from C towards B an equal Number of Toiſes; at the End of which plant a ſecond Staff: Take two Lines very equal, and having made Loops in the two Ends of each of them, put thoſe Loops about each of the Staffs, and holding the two other Ends of the Lines in your Hands, ſtretch them 'till they join upon the Ground, and in their point of Junction plant a third Staff. Laſtly, Faſten a Line tight to the Point C, and that third Staff, by which make a Track, which will be perpendicular to the Line A C B.

Meaſure 30 Toiſes from the Point C along the Track, at the End of which plant another Staff very upright, which will ſhew the Point D of the Plan. Return to the Staff A, from which to the Staff D make a Track; along which from the Point A meaſure 55 Toiſes towards D, for the Face of the Baſtion A E; plant a Staff in the Point E, for denoting the Angle *de l'Epaule.*

Go to the Point B, and there make the ſame Operations for drawing the Face B F, and plant a Staff at the Angle *de l'Epaule* F.

Produce B F from D, towards G; and alſo A E from D towards H; then meaſure with the Scale of the Plane the Lines D G, D H, and lay off their Lengths on the Ground from D to G, and from G to H, where plant Staffs: After which it will be eaſy to draw the Flanks E G, F H, and the Courtain G H.

By this means you will have one Front of a fortified Place, drawn on the Ground; the others may be drawn in the ſame manner by Staffs and Lines.

Note, It will not be improper to examine with a Semi-Circle, or other ſuch Inſtrument, whether the Angles drawn upon the Ground are equal to thoſe taken off of the Plane, and to rectify them before the Works are begun.

Care muſt likewiſe from time to time be taken, that the Tracks are followed; for without theſe Precautions there will ſometimes happen great Deformities.

Of the Conſtruction of the Outworks.

The Outworks of a Fortification, are thoſe Works made without the Ditch of a fortified Place, to cover it and augment it's Defence.

The moſt ordinary kinds of theſe Works, are the Ravelins or Half-Moons, which are formed between the two Baſtions upon the Flanquant Angle of the Counterſcarp, and before the Courtain, for covering the Gates and Bridges commonly made in the Middle of the Courtains, as the Figures P P ſhow.

The Ravelins are compoſed of two Faces furniſhed with one or two little Banks, and a good Parapet raiſed on the Side next the Country; and two Demigorges, without a Parapet, on the Side next to the Place, with an Entrance and Slope for mounting the great Ditch on the Platform of the Ravelin.

In each Ravelin there is built a Guard-Houſe, to ſhelter the Soldiers neceſſary for it's Defence, from the Injuries of Weather; but it is proper for the Guard-Houſe to be built in form of a Redoubt, with Battlements all round, for the Soldiers, in caſe of being attacked, to retire in, and obtain ſome Capitulation, before they lay down their Arms.

To draw a Ravelin before a Courtain, open your Compaſſes the Length of the interior Side of the Polygon, and having fixed one of the Points in one of the Ends of the Line, with the other Point deſcribe an Arc without the Counterſcarp; likewiſe ſet one Foot of the Compaſſes in the other End of the interior Side, and with the other Point deſcribe a ſecond Arc, cutting the firſt in a Point, which will be the Point or Flanquant Angle of the Ravelin: then lay a Ruler on the aforeſaid Interſection, and upon each of the Ends of the interior Side of the Polygon, for drawing the Faces of the Ravelin, which will terminate to the Right and Left upon the Edge of the Counterſcarp. The two Demigorges are drawn from the End of each Face, to the Rentrant Angle of the Counterſcarp.

But that the Flanquant Angle may not be too acute, it's Capital R S muſt be but about 40 Toiſes; and proceed with the reſt, as before.

Sometimes a ſimilar Work is made before the Point of a Baſtion; and ſince it's Gorge is built upon the Edge of the Counterſcarp, which is commonly rounded over-againſt the Point of the Baſtions, this Work is called a Half-Moon (becauſe it's Gorge is in the Form of an Arc): They are very often confounded, and the greateſt Part of the Soldiers give, without diſtinction, the Name of Half-Moons to Ravelins made before the Courtains.

The Defect of this Work is, that it is too diſtant from the Flanks of the Baſtions, for being ſufficiently defended by them; therefore a Half-Moon muſt not be made before the Point of a Baſtion, unleſs at the ſame time there are made other Out-Works to the Right and Left before the adjacent Courtains, to defend it.

It is proper for theſe Works to be lined with Walls, as well as the Body of the Place; for when they are not, the Ground muſt have ſo great a Slope, that it will be eaſy to mount the Works.

In

In the mean time the new-dug Earth the Works are made with, must settle at least a Year or two before the Walls are built, to the End that the Walls may not be thrown down by it after they are built.

Construction of the Hornworks.

Fig. 3. These kind of Works are commonly made before the Courtains, and because the Expence in making them is greater than the Expence in making the Ravelins, they are not made without absolute necessity; they serve to cover some Side of the Place, weaker than the others; they likewise serve to occupy an Height, which cannot be done by Persons inclosed in the Body of the Place.

Now to draw a Hornwork, first raise the Indefinite Perpendicular 1, 2, on the Middle of the Courtain; and to this Line draw two Parallels 3, 4, and 5, 6, from the Angles of the Shoulders. These two Parallels, which are called the Wings of the Hornwork, ought to draw their defence from the Faces of the Bastions; whence their Length ought not much to exceed 120 Toises, counting from the Shoulders. Thro' the Ends of the Wings draw the Line 4, 6, which will be the exterior Side of the Hornwork, and is divided into two equal Parts in the Point 7, by the Perpendicular 1, 2; then take half that exterior Side in your Compasses, and lay it off upon the Sides, from 4 to 8, and from 6 to 9; draw the Lines 4, 9, and 6, 8, which intersecting one another in the Point 10, will form the Angle of the *Tenaille*, that represents a Work called the Simple *Tenaille*, which is common enough made before the Courtains, with a little Ravelin without the Ditch, between the two Saliant Angles, and over-against the Middle of the Rentrant Angle.

But to strengthen this Work, there is added thereto two Demi-bastions, and a Courtain between them; which is better than two simple Rentrant Angles.

To draw the Demi-bastions, bisect the Line 4, 10, in the Point 11; and likewise the Line 10, 6, in the Point 12; then from the Points 11 and 12, draw to the Middle of the Courtain of the Place, as at the Point 1, the occult Lines 121, 111, by which means will be had the little Courtain 1314 of the Hornwork, the two Flanks 1113, 1214, and the two Faces 114, 126.

The Sides of these Works, which are next to the Country (as the Demi-bastions, the Courtain, and the Wings of the Hornwork are), ought to be furnished with a good Parapet of fine Earth well rammed, 18 or 20 Feet thick, and 6 Feet high before, containing a *Banquette*, like that in the Body of a Place; observing at all times, that the Parapets of the Works nigher the Center of the Place, must be higher above the Level of the Country, than those Works more distant; to the End that when the Besiegers have made themselves Masters of some Outwork, the Besieged, defending the Body of the Place, seeing them altogether uncovered, may dislodge them therefrom.

These Parapets ought to be sustained by a Rampart, whose Platform having a *Banquette*, is three or four Toises wide; but when Earth is wanting, we must be content to make several little Banks upon one another eighteen Inches high, and three or four Feet broad; and the Parapet ought to be about $4\frac{1}{2}$ Feet above the highest Bank, for covering the Soldiers: the Top of the Parapet must be sloped, gradually descending towards the Country, so that the Besieged may see the Enemy.

The parts of those Works, which are next the Place, must be without a Parapet, and only inclosed with a single Wall, or a Row of Palisadoes, to avoid the Surprizes of the Enemy. It is on this side that a Gate must be (for a Communication from the Works to the Body of the Place); as also a Guard-House, for covering the Soldiers designed for it's defence.

All these Works ought to be environed with a Ditch 10 or 12 Toises broad, communicating with the Ditch of the Body of the Place, and also as deep.

On the outside of that Ditch is made a Covered Way five or six Toises broad, with a Parapet, and it's Bank, commonly furnished with an enclosure of strong Palisadoes, drove 4 or 5 Feet into the Ground. The Top of that Parapet must be sloped next to the Country, and if it can be produced 20 or 30 Toises it will be better: for a Slope (or Glacis) cannot be too long; because, by means thereof, the Enemy cannot approach the Body of the Place, without being discovered.

The Outworks of which we have spoken, are the most common ones: There are many other sorts of them, which we shall not mention, it requiring a great Volume.

How to measure the Works of Fortifications.

The Ground of which the Ramparts and Parapets are formed, is generally taken out of the Ditches made about the Place; to know the Quantity of which, measure the Cavity of the Ditches, and reduce it to Cubic Toises. As, for example, If the Ditch over-against the Face of a Bastion, be 50 Toises long, 20 broad, and 4 deep; multiply the Length by the Breadth, and the Product will be 1000 square Toises, which multiplied by 4 the Depth, and there will arise 4000 Cubic Toises.

Note, That since there is a Necessity to give the Ground a great Slope, to keep it from crumbling to the Bottom, the Ditch will be wider at the Top than at the Bottom; whence, if if a Ditch be 20 Feet broad in the Middle of it's Depth, at the Top it must at least be 22 Toises broad, and 18 Toises at the Bottom: Those 22 Toises added to 18, make 40, whose half 20, is the mean Breadth to be used.

The Stone, or Brick-work, keeping together the Earth, ought to have thickness proportionable to it's height, and also about a Foot in Talud or Slope, the Height of every Toise.

If, for Example, a Wall be built to sustain the Earth of the Rampart of a Place, and it is 6 Toises high, the least thickness that can be given to that height, at the Top, must be 3 Feet, and at the Bottom, just above the Foundation, 9 Feet, because of it's Talud of 1 Foot every Toise in height: Now these two thicknesses, 9 and 3 make 12, whose half 6 Feet is the mean thickness of the Wall; and, consequently, to line the Face of a Bastion, 50 Toises long, 6 Toises high, and one Toise of mean thickness, there must be 300 Cubic Toises of Walling, excluding the Foundation, which cannot be determined without knowing the Ground. Besides this, there are commonly made Counter-Forts for sustaining the Earth, and hindering it's pressing too much against the Walls. These Counter-Forts ought to be sunk in firm Ground, and enter in the dug Earth, at least a Toise; they are 7 or 8 Feet broad at the Root, that is, on the Side where they are fastened to the Wall, and 4 or 5 Feet at the End, going into the Earth of the Rampart, which amounts to one Toise of Surface, in supposing (as we have already) that the Root is 7 Feet, and the End going into the Earth of the Rampart 5 Feet, which makes 12 Feet, half of which being 6, is the mean thickness; and supposing them 4 Toises in height, one with another, each will be 4 Cubic Toises: and since there ought to be 10 in the Extent of 50 Toises, the Stone, or Brick-Work of 10 Counter-Forts will be 40 Cubic Toises: So that there will be about 1000 Cubic Toises to wall the two Faces, and the Flanks of a Bastion, and to wall a Courtain, 80 Toises in length, there must be about 600 Cubic Toises of Stone, or Brick-Work; whence the Walling for the whole Place may be easily computed.

Note, It is better to make an Estimation too great, than too little.

It remains that we say something of the Carpenters Toise, required to construct Bridges and Gates, and other Works of the like Nature.

In measuring of Timber, we reduce it to Solives.

A Solive is a Piece of Timber 12 Feet long, and 36 Inches in surface; that is, 6 Inches broad, and 6 thick, which makes 3 Cubic Feet of Timber, being the seventy second Part of a Cubic Toise.

We shall give here two Ways of Calculation, to the End that the one may prove the other.

The first is, to reduce the Bigness of the Piece of Timber into Inches, that is, the Inches of it's Breadth and Thickness, and after having multiplied these two Quantities by one another, the Product must be multiplied by the Toises, Feet and Inches of it's Length, which last Product being divided by 72, the Quotient will give the Number of Solives contained in the Piece of Timber.

The Reason of this is, because 72 Pieces, 1 Inch Base, and a Toise long, make a Solive.

Suppose, for Example, a great Piece of Timber is to be reduced to Solives, whose length is 2 Toises, 4 Feet, 6 Inches, and 12 by 15 Inches Base; multiply 15 by 12, the Product is 180 square Inches, which again multiplied by 2 Toises, 4 Feet, 6 Inches, and the Product 495, divided by 12, will give 6 $\frac{7}{8}$ Solives.

The second Method is founded upon this, that a Solive contains 3 Cubic Feet.

As, for Example, If a Piece of Timber (the same as before) be 2 Toises, 4 Feet, 6 Inches long, and Base be 12 by 15 Inches; multiplying 12 by 15, the Product will be 180 square Inches; the 12*th* Part of that Number, which is 15, being considered as Feet, makes 2 Toises 3 Feet, which, multiplied by the Length 2 Toises, 4 Feet, 6 Inches, make 6 Solives, 5 Feet, and 3 Inches: So that there wants but 9 Inches, or the eighth Part of a Toise, to make 7 Solives, as in the Calculation of the first Method.

ADDITIONS of English Instruments.

Of the Theodolite, Plain-Table, Circumferentor, and Surveying-Wheel.

CHAP. I.

Of the Theodolite.

Fig. B. THIS Theodolite consists of a Brass Circle, cut in form of the Figure B, usually about 12 or 14 Inches in Diameter, whose Limb is divided into 360 Degrees, and each Degree into as many Minutes, either Diagonally, or otherwise, as the largeness of the Instrument will admit.

Underneath, at the Places *c c* of this Circle, are fixed two little Pillars *d d*, for supporting an Axis, upon which is fixed a Telescope, with a square Brass Tube, having two Glasses therein, for better perceiving Objects at a great distance; whence this Telescope may be raised or lowered, according as Objects be Horizontal or not. The Ends of the aforesaid Pillars are joined by the Piece *g g*, upon the Middle of which is soldered a Socket with it's Screw, for receiving the Top of the Ball and Socket E. Upon and about the Center of the Fig. C. Circle B, must the Index C move, which is a Circular Brass Plate, having upon the Middle thereof a Box and Needle, or Compass, whose Meridian Line answers to the Fiducial Line *a a*. At the Places *b b* of the Index are fixed two little Pillars for supporting an Axis, carrying a Telescope in the Middle thereof, whose Line of Collimation must be answerable to the Fiducial Line *a a* of the Index. This Telescope hath a square Brass Tube, and two Glasses therein, and may be raised or lowered, like that beforementioned. At each End of one of the perpendicular Sides of each Tube of the Telescopes, are fixed four small Sights for viewing nigh Objects thorough them.

The Ends of the Index *a a* are cut Circular, so as to fit the Divisions upon the Limb of the Circle B, and when the said Limb B is Diagonally divided, the Fiducial Line at one End of the Index shews the Degrees and Minutes upon the Limb. But when the Limb is only divided into Degrees, and every 30th Minute, we have a much better Contrivance for finding the Degrees, and every 2 Minutes upon the Limb, which is thus: Let the half Arc *p a* of one End of the Index contain exactly 8 Degrees of the Limb; then divide the said half Arc into 15 equal Parts, at every five of which set the Numbers 10, 20, 30, beginning from the Fiducial Line or middle of the Index. Now each of these equal Parts will be 32 Minutes: Therefore if you have a mind to set the Fiducial Line of the Index to any Number of Degrees, and every 2 Minutes upon the Limb; for Example, to 40 Degrees 10 Minutes; move the Index so, the Fiducial Line being between the 40th Degree, and the 40th Degree and 30 Minutes, that the Line of Division, numbered 10 upon the Index, may exactly fall upon some Line of Division of the Limb; and then the Fiducial Line will shew 40 Degrees, 10 Minutes.

Again: Suppose the Fiducial Line being between the 50th Degree and 30 Minutes, and the 51st, then that Line of Division, of equal Parts on the Index, exactly falling upon some Line of the Divisions of the Limb, will give the even Minutes above 50 Degrees 30 Minutes the Fiducial Line stands at. As suppose the 4th Line of Division of the Index stands exactly against some Line of Division of the Limb; then the Minutes above 40 Degrees 30 Minutes will be 8, that the Fiducial Line stands at: Understand the same of others.

Fig. D. is the Brass Ball and Socket, in which goes the Head of the three-legged Staff E, for supporting the Instrument when using: These three Legs are moveable by means of Joints, and may be taken shorter by half at the Places *a a a*, by means of Screws, for better conveniency of Carriage.

Thus

Thus have you the beſt Theodolite, as now made in *England*, briefly deſcribed.

The Uſe thereof will be ſufficiently underſtood by what our Author ſays of the Uſe of the Semi-Circle (which is but half a Theodolite), and I in the Uſe of the Plain-Table, and Circumferentor.

Note, There are ſome Theodolites that have no Teleſcopes, but only 4 Perpendicular Sights; two being faſtened upon the Limb, and two upon the Ends of the Index. Note likewiſe, That the Index, and Box and Needle, or Compaſs of the Theodolite, will ſerve for a Circumferentor.

CHAP. II.

Of the Conſtruction and Uſe of the Plain-Table, and Circumferentor.

THE Table itſelf is a Parallelogram of Oak, or other Wood, about 15 Inches long, and 12 broad, conſiſting of two ſeveral Boards, round which are Ledges of the ſame Wood; the two oppoſite of which being taken off, and the Spangle unſkrewed from the Bottom, the aforeſaid two Boards may be taken aſunder for eaſe and conveniency of Carriage. For the binding of the two Boards and Ledges faſt, when the Table is ſet together, there is a Box Jointed-Frame, about $\frac{3}{4}$ of an Inch broad, and of the ſame thickneſs as the Boards, which may be folded together in 6 Pieces. This Frame is ſo contrived, that it may be taken off and put on the Table at pleaſure, and may go eaſily on the Table, either ſide being upwards. This Frame alſo is to faſten a Sheet of Paper upon the Table, by forcing down the Frame, and ſqueezing in all the Edges of the Paper; ſo that it lies firm and even upon the Table, that thereby the Plot of a Field, or other Incloſure, may conveniently be drawn upon it. Fig. F.

On both ſides this Frame, near the inward Edge, are Scales of Inches ſubdivided into 10 equal Parts, having their proper Figures ſet to them. The Uſes of theſe Scales of Inches, are for ready drawing of Parallel Lines upon the Paper; and alſo for ſhifting your Paper, when one Sheet will not hold the whole Work.

Upon one ſide of the ſaid Box Frame, are projected the 360 Degrees of a Circle from a Braſs Center-Hole in the Middle of the Table. Each of theſe Degrees are ſubdivided into 30 Minutes; to every 10th Degree is ſet two Numbers, one expreſſing the proper Number of Degrees, and the other the Complement of that Number of Degrees to 360. This is done to avoid the trouble of Subſtraction in taking of Angles.

On the other Side of this Frame, are projected the 180 Degrees of a Semi-Circle from a Braſs Center-Hole, in the Middle of the Table's length, and about a fourth Part of it's breadth. Each of theſe Degrees are ſubdivided into 30 Minutes; to every 10th Degree is ſet likewiſe, as on the other ſide, two Numbers; one expreſſing the proper Number of Degrees, and the other the Complement of that Number of Degrees to 180, for the ſame Reaſon, as before.

The manner of projecting the Degrees on the aforeſaid Frame, is, by having a large Circle divided into Degrees, and every 30 Minutes: For then placing either of the Braſs Center-Holes on the Table, in the Center of that Circle ſo divided, and laying a Ruler from that Center to the Degrees on the Limb of the Circle; where the Edge of the Ruler cuts the Frame, make Marks for the Correſpondent Degrees on the Frame.

The Degrees thus inſerted on the Frame, are of excellent uſe in wet or ſtormy Weather, when you cannot keep a Sheet of Paper upon the Table. Alſo theſe Degrees will make the Plain-Table a Theodolite, or a Semi-Circle, according as what ſide of the Frame is uppermoſt.

There is a Box, with a Needle and Card, covered with a Glaſs, fixed to one of the long Sides of the Table, by means of a Screw, that thereby it may be taken off. This Box and Needle is very uſeful for placing the Inſtrument in the ſame Poſition upon every remove.

There belongs to this Inſtrument a Braſs Socket and Spangle, ſcrewed with three Screws to the Bottom of the Table, into which muſt be put the Head of the three-legged Staff, which may be ſcrewed faſt, by means of a Screw in the Side of the Socket.

There is alſo an Index belonging to the Table, which is a large Braſs Ruler, at leaſt 16 Inches long, and 2 Inches broad, and ſo thick as to make it ſtrong and firm, having a ſloped Edge, called the Fiducial Edge, and two Sights ſcrewed perpendicularly on it, of the ſame Height. They muſt be ſet on the Ruler perfectly at the ſame Diſtance from the Fiducial Edge. Upon this Index it is uſual to have many Scales of equal Parts, as alſo Diagonals, and Lines of Chords.

SECTION

SECTION I.

Of the Construction of the Circumferentor.

Fig. G. THIS Instrument consists of a Brass Index and Circle, all of a Piece; the Index is commonly made about 14 Inches long, an Inch and half broad, and of a convenient Thickness. The Diameter of the aforenamed Circle is about 7 Inches. On this Circle is made a Card, whose Meridian Line answers to the Middle of the Breadth of the Index: That Card is divided into 360 Degrees. There is a Brass Ring soldered on the Circumference of the Circle, on which screws another Ring with a flat Glass in it; so that they make a kind of Box to contain the Needle suspended upon the Pivot placed in the Center of the Circle.

There are also two Sights to screw on, or slide up and down the Index, like those beforenamed, belonging to the Index of the Plain-Table; as likewise a Spangle and Socket screwed on to the back-side of the Circle, for putting the Head of the Staff in.

SECTION II.

Of the Use of the Plain-Table and Circumferentor.

BUT first, it is necessary to know how to set the Parts of the Plain-Table together, to make it fit for use.

When you would make your Table fit for use, lay the two Boards together, and also the Ledges at the Ends in their due Places, according as they are marked. Then lay a Sheet of white Paper all over the Table, which must be stretched over the Boards, by putting on the Box Frame, which binds both the Paper to the Boards, and the Boards to one another: Then screw the Socket on the back-side the Table, and also the Box and Needle in it's due Place, the Meridian Line of the Card lying parallel to the Meridian or Diameter of the Table; which Diameter is a Right Line drawn upon the Table, from the Beginning of the Degrees thro' the Center, and so to the End of the Degrees. Then put the Socket upon the Head of the Staff, and there screw it: Also put the Sights upon the Index, and lay the Index on the Table. So is your Instrument prepared for use, as a Plain-Table, Theodolite, or Semi-Circle.

But *Note*, It is either a Theodolite, or Semi-Circle, according as the Theodolite or Semi-Circular Side of the Frame is upwards; for when you use your Instrument as a Plain-Table, you may place your Center in any part of the Table, which you judge most proper for bringing on the Work you intend. But if you use your Instrument as a Theodolite, the Index must be turned about upon the Brass Center-Hole in the Middle of the Table; and if for a Semi-Circle, upon the other Brass Center-Hole, by means of a Pin or Needle placed therein.

If you have a mind to use this Instrument, as a Circumferentor, you need only screw the Box and Needle to the Index, and both of them to the Head of the Staff, with a Brass Screw-Pin fitted for that purpose: So that the Staff being fixed in any Place, the Index and Sights may turn about at pleasure, without moving the Staff.

USE I. *How to measure the Quantity of any Angle in the Field, by the Plain-Table, considered as a Theodolite, Semi-Circle, and Circumferentor.*

I. *How to observe an Angle in the Field by the Plain-Table.*

Plate 14. Fig. 1. Suppose E K, K G, to be two Hedges, or two Sides of a Field, including the Angle E K G, and it is required to draw upon the Table an Angle equal thereto: First place your Instrument as near the Angular Point K as conveniency will permit, turning it about, 'till the North End of the Needle hangs directly over the Meridian Line in the Card, and then screw the Table fast. Then upon your Table, with your Protracting-Pin (which is a fine Needle put into a Piece of Box or Ivory, neatly turned) or Compass-Point, assign any Point at pleasure upon the Table, and to that Point apply the Edge of the Index, turning the Index about upon that Point, 'till thro' the Sights thereof you see a Mark set up at E, or parallel to the Line E K: And then with your Protracting-Pin, Compass-Point, or Pencil, draw a Line by the Side of the Index to the assigned Point upon the Table. Then (the Table remaining immoveable) turn the Index about upon the forementioned Point, and direct the Sights to the Mark set up at G, or parallel thereto, that is, so far distant from G, as your Instrument is placed from K; and then by the Side of the Index draw another Line to the assigned Point. Thus will there be drawn upon the Table two Lines representing the Hedges E K, and K G; and which include an Angle equal to the Angle E K G. And tho' you know not the Quantity of this Angle, yet you may find it, if required: For in working by this Instrument, it is sufficient only to give the Proportions of Angles, and not their Quantities in Degrees, as in working by the Theodolite, Semi-Circle, or Circumferentor. Also in working by the Plain-Table, there needs no Protraction at all, for you will have upon your Table the true

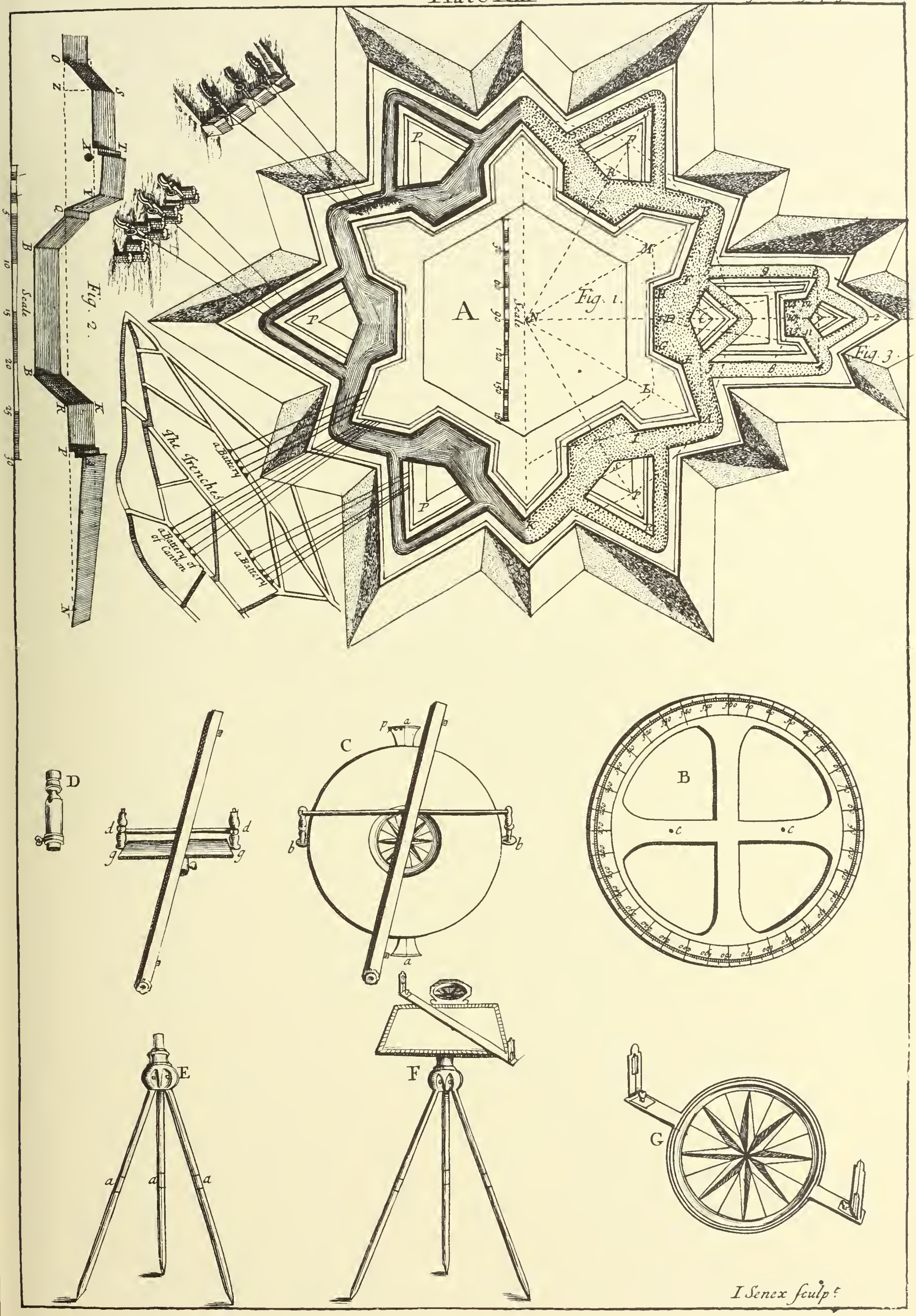


Fig. 1.
Fig. 2.
Fig. 3.
A
B
C
D
E
F
G
Scale
The Trenches
a Battery
a Battery of Cannon
I Senex sculp.

Figure of any Angle or Angles that you obſerve in the Field, in their true Poſitions, without any further trouble.

II. *How to find the Quantity of an Angle in the Field, by the Plain-Table, conſidered as a Theodolite, or Semi-Circle.*

Let it firſt be required to find the Quantity of the Angle E K G by the Plain-Table, as a Theodolite: Place your Inſtrument at K, with the Theodolite ſide of the Frame upwards, laying the Index upon the Diameter thereof; then turn the whole Inſtrument about (the Index ſtill reſting upon the Diameter) 'till thro' the Sights you eſpy the Mark at E: Then ſcrewing the Inſtrument faſt there, turn the Index about upon the Theodolite Center-Hole in the Middle of the Table, 'till thro' the Sights you eſpy the Mark at G. Then note what Degrees on the Frame of the Table are cut by the Index, and thoſe will be the Quantity of the Angle E K G ſought. Fig. 1.

You muſt proceed in the ſame manner for finding the Quantity of an Angle by the Plain-Table as a Semi-Circle; only put the Semi-Circle ſide of the Frame upwards, and move the Index upon the other Center-Hole.

III. *How to obſerve the Quantity of an Angle by the Circumferentor.*

If it be required to find the Quantity of the former Angle E K G by the Circumferentor, Firſt, place your Inſtrument (as before) at K, with the *Flower-de-luce* in the Card towards you. Then direct your Sights to E, and obſerve what Degrees are cut by the South-End of the Needle, which let be 296; then turning the Inſtrument about (the *Flower-de-luce* always towards you), direct the Sights to G, noting then alſo, what Degrees are cut by the South-End of the Needle, which ſuppoſe 182. This done (always) ſubſtract the leſſer from the greater, as in this Example 182 from 296, and the Remainder is 114 Degrees; which is the true Quantity of the Angle E K G. Fig. 1.

Again; The Inſtrument ſtanding at K, and the Sights being directed to E, as before, ſuppoſe the South-End of the Needle had cut 79 Degrees; and then directing the Sights to G, the ſame End of the Needle had cut 325 Degrees. Now, if from 325 you ſubſtract 79, the Remainder is 246. But becauſe this Remainder 246 is greater than 180, you muſt therefore ſubſtract 246 from 360, and there will remain 114, the true Quantity of the Angle ſought.

This adding and ſubſtracting for finding of Angles may ſeem tedious to ſome. But here note, That for quick diſpatch the Circumferentor is as good an Inſtrument as any, for in going round a Field, or in ſurveying a whole Manor, you are not to take notice of the Quantity of any Angle; but only to obſerve what Degrees the Needle cuts: as hereafter will be manifeſt.

USE II. *How by the Plain-Table, to take the Plot of a Field at one Station within the ſame, from whence all the Angles of the ſame Field may be ſeen.*

Having entered upon the Field to ſurvey, your firſt work muſt be to ſet up ſome viſible Mark at each Angle thereof; which being done, make choice of ſome convenient Place about the Middle of the Field, from whence all the Marks may be ſeen, and there place your Table covered with a Sheet of Paper, with the Needle hanging directly over the Meridian Line of the Card (which you muſt always have regard to, eſpecially when you are to ſurvey many Fields together). Then make a Mark about the Middle of the Paper, to repreſent that part of the Field where the Table ſtands; and laying the Index upon this Point, direct your Sights to the ſeveral Angles where you before placed Marks, and draw Lines by the Side of the Index upon the Paper. Then meaſure the Diſtance of every of theſe Marks from your Table, and by your Scale ſet the ſame Diſtances upon the Lines drawn upon the Table, making ſmall Marks with your Protracting-Pin, or Compaſs-Point, at the End of every of them. Then Lines being drawn from the one to the other of theſe Points, will give you the exact Plot of the Field; all the Lines and Angles upon the Table being proportional to thoſe of the Field. Fig. 2.

Example; Suppoſe the Plot of the Field A B C D E F was to be taken. Having placed Marks in the ſeveral Angles thereof, make choice of ſome proper Place about the Middle of the Field, as at L, from whence you may behold all the Marks before placed in the ſeveral Angles; and there place your Table. Then turn your Inſtrument about, 'till the Needle hangs over the Meridian Line of the Card, denoted by the Line N S.

Your Table being thus placed with a Sheet of Paper thereon, make a Mark about the Middle of your Table, which ſhall repreſent the Place where your Table ſtands. Then, applying your Index to this Point, direct the Sights to the firſt Mark at A, and the Index reſting there, draw a Line by the Side thereof to the Point L. Then with your Chain meaſure the Diſtance from L, the Place where your Table ſtands, to A, the firſt Mark, which ſuppoſe 8 Chains, 10 Links. Then take 8 Chains 10 Links from any Scale, and ſet that Diſtance upon the Line from L to A

Then directing the Sights to B, draw a Line by the Side of the Index, as before, and meaſure the Diſtance from your Table at L, to the Mark at B, which ſuppoſe 8 Chains 75

Links. This diſtance taken from your Scale, and applied to your Table from L to B, will give the Point B, repreſenting the ſecond Mark.

Then direct the Sights to the third Mark C, and draw a Line by the Side of the Index, meaſuring the Diſtance from L to C, which ſuppoſe 10 Chains 65 Links. This Diſtance being taken from your Scale, and applied to your Table from L to C, will give you the Point C, repreſenting the third Mark.

In this manner you muſt deal with the reſt of the Marks at D, E, and F, and more, if the Field had conſiſted of more Sides and Angles.

Laſtly; When you have made Obſervations of all the Marks round the Field, and found the Points A B C D E and F upon your Table, you muſt draw Lines from one Point to another, 'till you conclude where you firſt begun. As, draw a Line from A to B, from B to C, from C to D, from D to E, from E to F, and from F to A, where you begun; then will A B C D E F, be the exact Figure of your Field, and the Line N S the Meridian.

Note, Our Chains are commonly 4 Poles in Length, and are divided into one hundred equal Parts, called Links, at every tenth of which are Braſs Diſtinctions numbering them.

USE III. *To take the Plot of a Wood, Park, or other large Champain Plain, by the Plain-Table, in meaſuring round about the ſame.*

Suppoſe A B C D E F G to be a large Wood, whoſe Plot you deſire to take upon the Plain-Table.

Fig. 3. I. Having put a Sheet of Paper upon the Table, place your Inſtrument at the Angle A, and direct your Sights to the next Angle at B, and by the Side thereof draw a Line upon your Table, as the Line A B. Then meaſure by the Hedge-Side from the Angle A to the Angle B, which ſuppoſe 12 Chains 5 Links. Then from your Scale take 12 Chains 5 Links, and lay off upon your Table from A to B. Then turn the Index about, and direct the Sights to G, and draw the Line A G upon the Table. But at preſent you need not meaſure the Diſtance.

II. Remove your Inſtrument from A, and ſet up a Mark where it laſt ſtood, and place your Inſtrument at the ſecond Angle B. Then laying the Index upon the Line A B, turn the whole Inſtrument about, 'till thro' the Sights you ſee the Mark ſet up at A, and there ſcrew the Inſtrument. Then laying the Index upon the Point B, direct your Sights to the Angle C, and draw the Line B C upon your Table. Then meaſuring the Diſtance B C 4 Chains 45 Links, take that Diſtance from your Scale, and ſet it upon your Table from B to C.

III. Remove your Inſtrument from B, and ſet up a Mark in the room of it, and place your Inſtrument at C, laying the Index upon the Line C B; and turn the whole Inſtrument about, 'till thro' the Sights you eſpy the Mark ſet up at B, and there faſten the Inſtrument. Then laying the Index on the Point C, direct the Sights to D, and draw upon the Table the Line C D. Then meaſure from C to D 8 Chains 85 Links, and ſet that Diſtance upon your Table from C to D.

IV. Remove the Inſtrument to D (placing a Mark at C, where it laſt ſtood), and lay the Index upon the Line D C, turning the whole Inſtrument about, 'till thro' the Sights you ſee the Mark at C, and there faſten the Inſtrument. Then lay the Index on the Point D, and direct the Sights to E, and draw the Line D E. Then with your Chain meaſure the Diſtance D E 13 Chains 4 Links, which lay off on the Table from D to E.

V. Remove your Inſtrument to E (placing a Mark at D, where it laſt ſtood), and laying the Index upon the Line D E, turn the whole Inſtrument about, 'till thro' the Sights you ſee the Mark at D, and there faſten the Inſtrument. Then lay the Index on the Point E, and direct the Sights to F, and draw the Line E F. Then meaſure the Diſtance E F 7 Chains 70 Links, which take from your Scale, and lay off from E to F.

VI. Remove your Inſtrument to F, placing a Mark at E (where it laſt ſtood), and lay the Index upon the Line E F, turning the Inſtrument about, 'till you ſee the Mark ſet up at E, and there faſten the Inſtrument. Then laying the Index on the Point F, direct the Sights to G, and draw the Line F G upon the Table, which Line F G will cut the Line A G in the Point G. Then meaſure the Diſtance F G 5 Chains 67 Links, and lay it off from F to G.

VII. Remove your Inſtrument to G (ſetting a Mark where it laſt ſtood), and lay the Index upon the Line F G, turning the whole Inſtrument about, 'till thro' the Sights you ſee the Mark at F, and there faſten the Inſtrument. Then laying the Index upon the Point G, direct the Sights to A (your firſt Mark), and draw the Line G A, which, if you have truly wrought, will paſs directly thro' the Point A, where you firſt began.

In this manner may you take the Plot of any Champain Plain, be it never ſo large. And here note, That very often Hedges are of ſuch a Thickneſs, that you cannot come near the Sides or Angles of the Field, either to place your Inſtrument, or meaſure the Lines. Therefore in ſuch Caſes you muſt place your Inſtrument, and meaſure your Lines parallel to the Side thereof; and then your Work will be the ſame as if you meaſured the Hedge itſelf.

NOTE alſo, That in thus going about a Field, you may much help yourſelf by the Needle. For looking what Degree of the Card the Needle cuts at one Station, if you remove your Inſtrument to the next Station, and with your Sights look to the Mark where the Inſtrument laſt ſtood, you will find the Needle to cut the ſame Degree again, which will give you no ſmall Satisfaction in the proſecution of your Work. And tho' there be a hundred or more Sides, the Needle will ſtill cut the ſame Degree at all of them, except you have committed ſome former Error: therefore at every Station have an Eye to the Needle.

Of Shifting of Paper.

In taking the Plot of a Field by the Plain-Table, and going about the ſame, as before directed, it may ſo fall out, if the Field be very large, and when you are to take many Incloſures together, that the Sheet of Paper upon the Table will not hold all the Work. But you muſt be forced to take off that Sheet, and put another clean Sheet in the room thereof: and, in Plotting of a Manor or Lordſhip, many Sheets may be thus changed, which we call Shifting of Paper. The Manner of performing thereof is as follows. Fig. 4.

Suppoſe in going about to take the Plot A B C D E F G, as before directed, that you having made choice of the Angle at A for the Place of the Beginning, and proceeded from thence to B, and ftom B to C, and from C to D, when you come to the Angle at D, and are to draw D E, you want room to draw the ſame upon the Table. Do thus:

Firſt, thro' the Point D draw the Line D O, which is almoſt ſo much of the Line D E, as the Table will contain. Then near the Edge of the Table H M, draw a Line parallel to H M, by means of the Inches and Subdiviſions on the oppoſite Sides of the Frame, as P Q, and another Line at Right Angles to that thro' the Point O, as O N. This being done, mark this Sheet of Paper with the Figure (1) about the Middle thereof, for the firſt Sheet. Then taking this Sheet off your Table, put another clean Sheet thereon, and draw upon it a Line parallel to the contrary Edge of the Table, as the Line R S. Then taking your firſt Sheet of Paper, lay it upon the Table ſo, that the Line P Q may exactly lie upon the Line R S, to the beſt advantage, as at the Point O (*Fig.* 5.). Then with the Point of your Compaſſes draw ſo much of the Line O D upon the clean Sheet of Paper as the Table will hold. Having thus done, proceed with your Work upon the new Sheet, beginning at the Point O; and ſo going forward with your Work, as in all Reſpects has before been directed; as from O to E, from E to F, from F to G, and from G to A (by this direction), ſhifting your Paper as often as you have occaſion. Fig. 5.

USE IV. *How to take the Plot of any Wood, Park, &c. by going about the ſame, and making Obſervations at every Angle thereof, by the Circumferentor.*

Suppoſe A B C D E F G H K is a large Field, or other Incloſure, to be Plotted by the Circumferentor. Fig. 6.

1. Placing your Inſtrument at A (the *Flower-de-luce* being towards you), direct the Sights to B, the South-End of the Needle cutting 191 Degrees, and the Ditch, Wall, or Hedge, containing 10 Chains 75 Links. The Degrees cut, and the Line meaſured, muſt be noted down in your Field-Book.
2. Place your Inſtrument at B, and direct the Sights to C, the South-End of the Needle cutting 279 Degrees, and the Line B C containing 6 Chains 83 Links; which note down in your Field-Book.
3. Place the Inſtrument at C, and direct the Sights to D, the Needle cutting 216 Deg. 30 Min. and the Line C D containing 7 Chains 82 Links.
4. Place the Inſtrument at D, and direct the Sights to E, the Needle cutting 327 Degrees, and the Line D E containing 9 Chains 96 Links.
5. Place the Inſtrument at E, and direct the Sights to F, the Needle cutting 12 Deg. 30 Min. and the Line F E 9 Chains 71 Links.
6. Place the Inſtrument at F, and direct the Sights to G, the Needle cutting 342 Deg. 30 Min. and the Line F G being 7 Chains 54 Links.
7. Place the Inſtrument at G, and direct the Sights to H, the Needle cutting 98 Deg. 30 Min. and the Line G H containing 7 Chains 52 Links.
8. Place the Inſtrument at H, and direct the Sights to K, the Needle cutting 71 Deg. and the Line H K containing 7 Chains 78 Links.
9. Place the Inſtrument at K, and direct the Sights to A (where you began), the Needle cutting 161 Deg. 30 Min. and the Line K A containing 8 Chains 22 Links.

Having gone round the Field in this manner, and collected the Degrees cut, and the Lines meaſured, in the Field-Book, you will find them to ſtand as follows, by which you may protract and draw your Field, as preſently I ſhall ſhew.

Degrees

	Degrees.	Minutes.	Chains.	Links.
A	191	00	10	75
B	279	00	6	83
C	216	30	7	82
D	325	00	6	96
E	12	30	9	71
F	324	30	7	54
G	98	30	7	54
H	71	00	7	78
K	161	30	8	22

In going about a Field in this manner, you may perceive a wonderful quick Dispatch; for you are only to take notice of the Degrees cut once at every Angle, and not to use any Back-Sights, that is, to look thro' the Sights to the Station you last went from. But to use Back-Sights with the Circumferentor, is best to confirm your Work: For when you stand at any Angle of a Field, and direct your Sights to the next, and observe what Degrees the South-End of the Needle cuts; if you remove your Instrument from this Angle to the next, and look to the Mark or Angle where it last stood, the Needle will there also cut the same Degrees as before.

So the Instrument being placed at A, if you direct the Sights to B, you will find the Needle to cut 191 Degrees; then removing your Instrument to B, if you direct the Sights to A, the Needle will then also cut 191 Degrees.

Notwithstanding the quick Dispatch this Instrument makes, one half of the Work will almost be saved; if, instead of placing the Instrument at every Angle, you place it but at every other Angle. An Instance of which take in the aforegoing Example.

1. Placing the Instrument at A, and directing the Sights to B, you find the Needle to cut 191 Degrees. Then,

2. Placing the Instrument at B, directing the Sights to C, you find the Needle to cut 279 Degrees. And,

3. Placing the Instrument at C, and directing the Sights to D, you find the Needle to cut 216 Degrees.

Now, having placed your Instrument at A, and noted down the Degrees cut by the Needle, which was 191, you need not go to the Angle B at all, but go next to the Angle C, and there place your Instrument; and directing your Sights backwards to B, you will find the Needle to cut 279 Degrees, which are the same as were before cut when the Instrument was placed at B: so that the Labour of placing the Instrument at B is wholly saved. Then (the Instrument still standing at C) direct the Sights to D, and the Needle will cut 216 Degrees, as before, which note in your Field-Book. This done, remove your Instrument to E, and observe according to the last Directions, and you will find the Work to be the same as before. Then remove the Instrument from E to G, from G to K, and so to every second Angle.

Fig. 7. I now proceed, to shew the Manner of Protracting the former Observations.

According to the largeness of your Plot provide a Sheet of Paper, as L M N O, upon which draw the Line L M, and parallel thereto draw divers other Lines quite thro' the whole Paper, as the pricked Lines, in the Figure, drawn between L M and N O. These Parallels thus drawn, represent Meridians. Upon one or other of these Lines, or parallel to one of them, must the Diameter of your Protractor be always laid.

1. Your Paper being thus prepared, assign any Point upon any of the Meridians, as A, upon which place the Center of the Protractor, laying the Diameter thereof upon the Meridian Line drawn upon the Paper. Then look in your Field-Book what Degrees the Needle cuts at A, which was 191 Degrees. Now, because the Degrees were above 180, you must therefore lay the Semi-Circle of the Protractor downwards, and keeping it there, make a Mark with the Protracting-Pin against 191 Degrees; thro' which Point, from A, draw the Line A B, containing 10 Chains 75 Links.

2. Lay the Center of the Protractor on the Point B, with the Diameter in the same Position as before directed (which always observe). And because the Degrees cut at B were more than 180, *viz.* 279, therefore the Semi-Circle of the Protractor must lie downwards; and so holding it, make a Mark against the 279 Degrees, and thro' it draw the Line B C, containing 6 Chains 83 Links.

3. Place the Center of the Protractor on the Point C. Then the Degrees cut by the Needle at the Observation in C, being above 180, namely, 216 Degrees 30 Minutes, the Semi-Circle of the Protractor must lie downwards. Then making a Mark against 216 Deg. 30 Min. thro' it draw the Line C D, containing 7 Chains 82 Links.

4. Lay the Center of the Protractor upon the Point D; the Degrees cut by the Needle at that Angle being 325: which being above 180, lay the Semi-Circle downward; and against 325 Degrees make a Mark, thro' which Point, and the Angle D, draw the Line D E, containing 6 Chains 96 Links.

5. Remove

5. Remove your Protractor to E. And becaufe the Degrees cut by the Needle at this Angle were lefs than 180, namely, 12 Degrees 30 Min. therefore lay the Semi-Circle of the Protractor upwards, and make a Mark againft 12 Degrees 30 Minutes, thro' which draw the Line E F, containing 9 Chains 71 Links.

6. Lay the Center of the Protractor upon the Point F; and becaufe the Degrees to be protracted are above 180, *viz.* 342 Degrees 30 Minutes, lay the Semi-Circle of the Protractor downwards, and make a Mark againft 342 Degrees 30 Minutes, drawing the Line F G, containing 7 Chains 54 Links.

And in this Manner muft you protract all the other Angles, G, H, and K, and more, if the Field had confifted of more Angles.

CHAP. III.

Of the Conftruction and Ufe of the Surveying-Wheel.

THIS Inftrument confifts of a wooden Wheel, fhod with Iron, to prevent it's wearing, exactly two Feet feven Inches and a half in Diameter, that fo it's Circumference may be eight Feet three Inches, or half a Pole. Fig. 8.

At the End of the Axle-Tree of this Wheel, on the left Side thereof, is, at Right Angles to the Axle-Tree, a little Star, about three fourths of an Inch Diameter, having eight Teeth. Now the Ufe of this Star is fuch, that when the Wheel moves round, the faid Star's Teeth, by falling at Right Angles into the Teeth of another Star of eight Teeth, fixed at one End of an Iron Rod (Q) caufes the Iron Rod to move once round in the fame Time the Wheel hath moved once round. Therefore every time you have drove the Wheel half a Pole, the Iron Rod goes once round.

This Iron Rod, lying along a Groove in the Side of the Body of the Inftrument, hath on the other End a fquare Hole, in which goes the fquare End *b* of the little Cylinder P. This Cylinder is faftened underneath the upper Plate H, of a Movement, covered with a Glafs, placed in the Body of the Inftrument at B, yet fo, that it may be moveable about it's Axis, having the End *a* cut into a fingle threaded perpetual Screw, which falling into the Teeth of the Wheel A, being thirty-two in Number, when you drive the Inftrument forwards, caufes the Wheel A to go once round at the End of each 16th Pole. The Pinion B hath fix Teeth, which falling into the Teeth of the Wheel C, whofe Number is fixty, caufes that to move once round at the End of each 160th Pole, or half Mile. This Wheel carries round a Hand, once in 160 Poles, over the Divifions of an annular Plate, fixed upon the Plate H, whofe outmoft Limb is divided into 160 equal Parts, each tenth of which is numbered, and fhews how many Poles the Inftrument is drove. Fig. 9.

Again; The Pinion D, which is fixed to the fame Arbre as the Wheel C is, hath twenty Teeth, which by their falling into the Teeth of the Wheel E, which hath forty Teeth, caufes the faid Wheel E to go round once in 320 Poles, or one Mile; and the Pinion F, of twelve Teeth, falling into the Teeth of the Wheel G, whofe Number is 72, caufes the Wheel G to go once round in 12 Miles. This Wheel G carries another leffer Hand once round in 12 Miles, over the Divifions of the innermoft Limb of the aforefaid annular Plate, which is divided into twelve equal Parts for Miles, and each Mile fubdivided into Halves and Quarters (that is, into eight equal Parts, for Furlongs), with Roman Characters numbering the Miles.

The Ufe of this Inftrument is fuch, that by driving the Wheel before you, the Number of Miles, Poles, or both, you have gone, is eafily fhewn by the two Hands. And fo this Inftrument, together with a Theodolite or Circumferentor, for taking of Bearings, is of excellent Ufe in Plotting of Roads, Rivers, *&c.* For having placed your Wheel and Circumferentor at the Beginning of the Road you defign to plot, which call your firft Station, caufe fome Perfon to go as far along the Road as you find it ftraight; and then take a Bearing to him, which fet down. This being done, drive the Wheel before you to the Place where the Man ftands, which call the fecond Station, and note, by the Hands of the Dial-Plate, the Diftance from the firft Station to the fecond, which fet down. Again, Having placed your Circumferentor at the fecond Station, caufe the Man to go along the Road 'till he comes to another Bend therein. And from the fecond Station take a Bearing to the Man at the third, which fet down. Then drive the Wheel from the fecond Station to the third, and note the Diftance, which fet down. And in this Manner proceed 'till you come to your Journey's End. Then in Plotting the Road, you muft obferve the fame Directions, as are given in Plotting the Example of Ufe IV. of the laft Chapter.

BOOK V.

Of the Construction and Uses of Levels, for conducting of Water; as also of Instruments for Gunnery.

CHAP. I.

Of the Construction and Uses of different Levels.

Construction of a Water-Level.

Plate 15. Fig. A. THE first of these Instruments is a Water-Level, composed of a round Tube of Brass, or other solid Matter, about 3 Feet long, and 12 or 15 Lines Diameter, whose Ends are turned up at Right Angles, for receiving two Glass Tubes, 3 or 4 Inches long, fastened on them with Wax or Mastick. At the Middle and underneath this Tube, is fixed a Ferrel, for placing it upon it's Foot.

There is as much common or coloured Water poured into one End of it, as that it may appear in the Glass Tubes.

This Level, altho' very simple, is very commodious for levelling small Distances.

It is founded upon this, that Water always naturally places itself level; and therefore the Height of the Water in the two Glass Tubes will be always the same, in respect to the Center of the Earth.

Fig. B. The Air-Level B, is a very straight Glass Tube, every where of the same Thickness, of an indetermined Length, and Thickness in proportion; being filled to a drop with Spirit of Wine, or other Liquor, not subject to freeze. The Ends of the Tube are hermetically sealed, that is, the End through which the Spirit of Wine is poured must afterwards be closed, by heating it with the Flame of a Lamp, blown thro' a little Brass Tube, to make the heat the greater; and then when the Glass is become soft, the End must be closed up.

When this Instrument is perfectly level, the Bubble of Air will fix itself just in the Middle, and when it is not level, the Bubble of Air will rise to the top.

Construction of an Air-Level.

Fig. C. This Instrument is composed of an Air-Level 1, about 8 Inches long, and 7 or 8 Lines in Diameter, set in a Brass Tube 2; which is left open in the Middle for seeing the Bubble of Air at the Top.

It is carried upon a very strong straight Rule, about a Foot long, at the Ends of which are placed two Sights exactly of the same Height, and like that of Number 3, which has a square Hole therein, having two Fillets of Brass very finely filed, crossing one another at Right Angles, in the Middle of which Fillets is drilled a little Hole. There is fastened a little thin Piece of Brass to this Sight, with a small Headed-Rivet, to stop the said Square opening, when

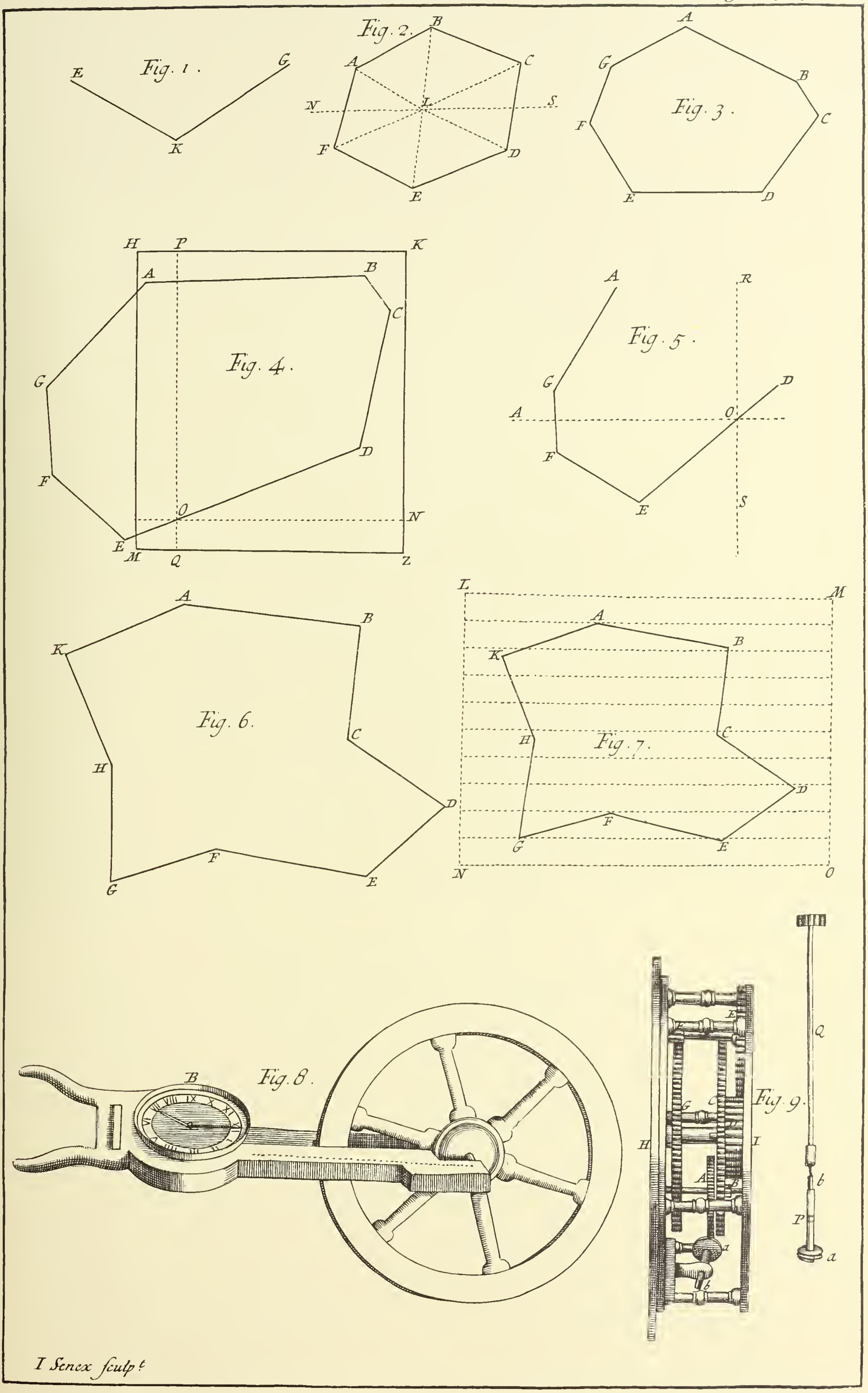


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
I Senex sculp^t

when there is occaſion, and having a little Hole drilled thro' it, anſwering to that which is in the Middle of the Fillets. The Braſs Tube is faſtened upon the Rule, by means of two Screws, one of which marked 4, ſerves to raiſe or depreſs the Tube at pleaſure, for placing it level, and making it agree with the Sights.

The Top of the Ball and Socket is riveted to a little Rule, that ſprings, one of whoſe Ends is faſtened with two Screws to the great Rule, and at the other End there is a Screw 5, ſerving to raiſe or depreſs the whole Inſtrument when it is nearly level.

The Manner of adjuſting this Level is eaſy, for you need but place it upon it's Foot, ſo that the Bubble of Air may be exactly in the Middle of the Tube; then ſhutting the Sight next to the Eye, and opening the other, the Point of the Object which is cut by the horizontal Fillet is level with the Eye; and to know whether the Air-Level agrees well with the Sights, you muſt turn the Inſtrument quite about, and ſhut the Sight which before was opened, and open the other. Then looking through the little Hole, if the ſame Point of the Object before obſerved be cut by the horizontal Fillet, it is a ſign the Level is juſt; but if there be found any difference, the Tube muſt be raiſed or depreſſed by means of the Screw 4, 'till the Sights agree with the Level; that is, that looking at an Object, the Bubble of Air being in the Middle, and afterwards turning the Inſtrument about, the ſame Object may be ſeen.

The Level D is a little Glaſs Tube incloſed within a Braſs Tube, faſtened upon a Rule perfectly equal in Thickneſs, and ſerves to know whether a Plane be level, or not. Fig. D.

Conſtruction of a Telescope Air-Level.

This Level is like the Level C, but inſtead of Sights, it carries a Teleſcope to diſcover Objects at a good Diſtance. This Teleſcope is in a little Braſs Tube, about 15 Inches long, faſtened upon the ſame Rule as the Level, which ought to be of a good Thickneſs, and very ſtraight. Fig. E.

At the End of the Tube of the Teleſcope, marked 1, enters the little Tube 1, carrying the Eye-Glaſs, and a human Hair horizontally placed in the Focus of the Object-Glaſs 2. This little Tube may be drawn out or puſhed into the great one, for adjuſting the Teleſcope to different Sights.

At the other End of the Teleſcope is placed the Object-Glaſs, whoſe Conſtruction is the ſame as that before-mentioned, belonging to the Semi-Circle.

The whole Body of the Teleſcope is faſtened to the Rule, as well as the Level, with Screws, upon two little ſquare Plates, ſoldered towards the Ends of each Tube, which ought to be perfectly equal in Thickneſs.

The Screw 3, is for raiſing or lowering, the little Fork carrying the human Hair, and making it agree with the Bubble of Air, when the Inſtrument is level; and the Screw 4, is for making the Bubble of Air agree with the Teleſcope.

Underneath the Rule there is a Braſs Plate with Springs, having a Ball and Socket faſtened thereto.

The Level F, is in form of a Square, having it's two Branches of equal length; at the Junction of which there is made a little Hole, from which hangs a Thread and Plummet, playing upon a Perpendicular Line, in the Middle of the Quadrant, often divided into 90 Degrees. It's Uſe is very eaſy, for the Ends of the Branches being placed upon a Plane, we may know that the Plane is level when the Thread plays upon the Perpendicular in the Middle of the Quadrant. Fig. F.

Conſtruction of a Teleſcope Plumb-Level.

This Inſtrument is compoſed of two Branches, joined together at Right Angles; whereof *that* carrying the Thread and Plummet, is about a Foot and a half, or two Foot long. Fig. G.

This Thread is hung towards the Top of the Branch, at the Point 2. The Middle of the Branch, where the Thread paſſes, is hollow, that ſo it may not touch in any Place but towards the Bottom, at the Place 3, where there is a little Blade of Silver, on which is drawn a Line perpendicular to the Teleſcope.

The ſaid Cavity is covered by two Pieces of Braſs, making as it were a kind of Caſe, leſt the Wind ſhould agitate the Thread; for which reaſon there is alſo a Glaſs covering the Silver Blade, to the End that we may ſee when the Thread and Plummet play upon the Perpendicular. The Teleſcope 1, is faſtened to the other Branch, which is about two Feet long, and is made like the other Teleſcopes of which we have already ſpoken. All the Exactneſs of this Inſtrument conſiſts in having the Teleſcope at Right Angles with the Perpendicular.

This Inſtrument has a Ball and Socket faſtened behind the aforeſaid Branch, for placing it upon it's Foot.

There are ſome of theſe ſort of Levels made of Braſs or Iron, whoſe Teleſcope and the Cavity, in which is included the Thread carrying the Plummet, is about 4 or 5 Feet long, in order to level great Diſtances at once.

The Teleſcope is about 1 Inch and a half Diameter, and the Caſe in which the Thread, carrying the Plummet, is incloſed, is about 2 Inches wide, and half an Inch thick. This Caſe is faſtened with

with Screws in the Middle, to the Telescope; so that they may be at Right Angles with one another: And at the two Ends of the Telescope are adjusted two broad Circles, in which the Telescope exactly turns; which Circles, being flat underneath, are fastened to a strong Iron Rule.

This Level is supported by two Feet almost like that of Figure E, *Plate* 12, fastened with Screws to the Extremities of the Iron Rule. Also there are two Openings, covered with Glasses, inclosed in little Brass Frames, which open, that so the Thread and Plummet may be hung to the Top of the Case, and play upon two little Silver Blades, in a Line drawn on them perpendicular to the Telescope. These Blades are placed against the Openings of the Case, and the Telescope is like that before spoken of, on the Semi-Circle.

All the Exactness of this Instrument consists in having the Telescope at Right Angles to the Perpendiculars drawn upon the Silver Blades.

To prove this Level, you must place it upon it's Foot, in such manner that the Thread may exactly play upon the Perpendicular, and note some Object cut by the Hair in the Focus of the Telescope. Then taking off the Thread and Plummet, turn the Instrument upside down, and hanging the Thread and Plummet to the Hook at the Bottom of the Case, which will now be uppermost, look thro' the Telescope at the aforesaid Object, and if the Thread exactly plays upon the Perpendicular, it is a sign the Instrument is exact; but if it does not, you must remove the little Hook to the Right-hand or Left, 'till you make the Thread fall upon the Perpendicular, both before you have turned the Instrument upside down, and afterwards. You may likewise raise or lower the Telescope, by means of a Screw. *Note*, Ingenious Workmen may easily supply what I have omitted in this brief Description.

Fig. H. The Instrument H is a little simple Level, founded on the same Principle as the three precedent ones; the Figure thereof is sufficient to shew it's Construction and Use.

Fig. I. The Level I. places itself, and is composed of a pretty thick Brass Rule, about one Foot long, and an Inch broad, having two Sights of the same Height placed at the Ends of the Rule, and in the Middle there is a kind of Beam (almost like those of common Scales) for freely suspending the Level.

At the Bottom of the said Rule is screwed on a Piece of Brass, likewise carrying a pretty heavy Ball of Brass. All the Exactness of this Instrument consists in a perfect *Equilibrium*; to know which, it is easy: for holding the Instrument suspended by it's Ring, and having espied some Object thro' the Sights, you need but turn the Instrument about, and observe whether the aforesaid Object appears of the same Height thro' the Sights; and if it does, the Instrument is perfectly *in equilibrio*: but if the Object appears a little higher or lower, you may remedy it by removing the Piece of Brass carrying the Ball 'till it be exactly in the Middle of the Point of Suspension, and then it must be fixed with a Screw, because, by Experience, the Instrument was found to be level.

Construction of a Level of Mr Huygens'*s.*

Fig. K. The principal Part of this Instrument, is a Telescope *a*, 15 or 18 Inches long, being in form of a Cylinder, and going thro' a Ferrel, in which it is fastened by the Middle. This Ferrel has two flat Branches *b b*, one above, and the other below, each about a fourth Part of the Telescope in length. At the Ends of each of these two Branches are fastened little moving Pieces, which carry two Rings, by one of which the Telescope is suspended to a Hook, at the End of the Screw 3; and by the other a pretty heavy Weight is suspended, in order to keep the Telescope *in equilibrio*. This Weight hangs in the Box 5, which is almost filled with Linseed Oil, Oil of Wallnuts, or any thing else that will not coagulate, for more aptly settling the Ballances of the Weight and Telescope.

This Instrument carries sometimes two Telescopes close, and very parallel to each other, the Eye-Glass of one being on one side, and the Eye-Glass of the other on the opposite side, that so one may see on both sides, without turning the Level. If the Tube of the Telescope being suspended, be not found level, as it will often happen, put a Ferrel or Ring 4 upon it, which may be slid along the Tube, for placing it level, and keeping it so. And this must be, if there be two Telescopes.

There is a human Hair horizontally strained and fastened to a little Fork in the Focus of the Object Glass of each Telescope, which may be raised or lowered, by means of a little Screw, as has been already mentioned.

For proving this Level, having suspended it by one of the Branches, observe some distant Object through the Telescope, with the Weight not hung on, and very exactly mark the Point of the Object cut by the Hair of the Telescope: Now hanging the Weight on, if the horizontal Hair answers to the same Point of the said Object, it is a sign the Center of Gravity of the Telescope and Weight, is precisely in a Right Line joining the two Points of Suspension, which continued would pass thro' the Center of the Earth.

But if it otherwise happens, you must remedy it, by sliding the little Ring backwards or forwards. Having thus adjusted the Telescope, that the same Point of an Object be seen, as well before the Weight is hung on, as afterwards, you must turn it upside down, by suspending it to the Branch that was lowermost, and hanging the Weight upon the other. Then if the Hair in the Telescope cuts the aforesaid Point of the Object, it is manifest, that that

Point

Point of the Object is in the horizontal Plane, with the Center of the Tube of the Telescope: but if the Hair does not cut that Point of the Object, it must be raised or lowered by means of the Screw 'till it does. *Note*, You must every now and then prove this Instrument, for fear least some Alteration has happened thereto.

The Hook on which this Instrument is hung, is fixed to a flat wooden Cross, at the Ends of each Arm of which, there is a Hook serving to keep the Telescope from too much Agitation, when the Instrument is using, and for keeping it steady when it is carrying, in lowering the Telescope by means of the Screw 3, which carries it.

There is applied to the said flat Cross, another hollowed Cross fastened with Hooks, which serves as a Case for the Instrument. But note, The two Ends of the Cross are left open, that so the Telescope being covered from Wind and Rain, may be always in a Condition to use.

The Foot supporting the Instrument, is a round Brass Plate something concave, to which is fastened three Brass Ferrels, moveable by means of Joints, wherein are Staves of a convenient Length put. The Box at the Bottom of the Level is placed upon this Plate, and may be any ways turned; so that the Weight, which ought to be Brass, may have a free Motion in the Box, which must be shut by means of a Screw, that so the Oil may be preserved in Journeys.

Construction of another Level.

This Instrument is a Level almost like that whose Description we have last given, but it is easier to carry from place to place. Fig. L.

Number 1. Is the Case in which the Telescope is enclosed.

2. Is a kind of Stirrup, where the Screw, serving for the Point of Suspension, passes; at the End of which is a Hook, upon which the Ring, at the End of the Plate carrying the Telescope, is hung.

3. Are the Screws above and below for fixing the Telescope, when the Instrument is carrying.

4. Are the Hooks for keeping the Case shut.

5. Is one End of the Telescope.

6. Is the End of the Plate whereon a great Brass Ball 7 is hung, serving to keep the Telescope level.

There are three Ferrels 8, well fixed to the Bottom of the Stirrup, serving as a Foot to support the whole Instrument. *Note*, There are sometimes put two Telescopes on this Level, as well as in that other of which we have last spoken.

CHAP. II.

Of the Uses of the aforesaid Instruments in Levelling.

LEvelling is an Operation shewing the Height of one Place in respect to another. One Place is said to be higher than another, when it is more distant from the Center of the Earth. A Line equally distant from the Center of the Earth, in all it's Points, is called the Line of true Level; whence, because the Earth is round, that Line must be a Curve, and make a part of the Earth's Circumference, as the Line B C F G, all the Points of which are Fig. 1. equally distant from the Center A of the Earth: but the Line of Sight, which the Operations of Levels give, is a right Line perpendicular to the Semi-Diameter of the Earth A B, raised above the true Level, denoted by the Curvature of the Earth, in proportion as it is more extended; for which Reason, the Operations which we shall give, are but of the apparent Level, which must be corrected to have the true Level, when the Line of Sight exceeds 50 Toises.

The following Table, in which are denoted the Corrections of the Points of apparent Level, for reducing them to the true Level, was calculated by help of the Semi-Diameter of the Earth, whose Length may be known by measuring one Degree of it's Circumference. The Gentlemen of the Academy of Sciences, have found by very exact Observations, that one Degree of the Circumference of a great Circle of the Earth, as the Meridian, contains 57292 Toises; and giving 25 Leagues to a Degree, a League will be 2291 $\frac{17}{25}$ Toises.

Now the whole Circumference of the Earth will be 9000 of the same Leagues, and it's Diameter 2863 $\frac{7}{11}$ of them; from whence all Places on the Superficies of the Earth, will be distant from it's Center 1431 $\frac{15}{44}$ Leagues.

The Line A B represents the Semi-Diameter of the Earth, under the Feet of the Observer. The right Line B D E, represents the visual Ray, whose Points D and E are in the apparent Level of the Point B. This Line of apparent Level, serves for determining a Line of true Level, which is done by taking from the Points of the Line of apparent Level, the Height they are above the true Level in respect to a certain Point, as B; for it plainly appears from the Figure, that all the Points D, E, of the apparent Level, are farther distant from the

Center of the Earth, than the Point B; and to find the Difference, you need but consider the right-angled Triangle A B D, whose two Sides A B, B D, being known, the Hypothenuse A D, may be found: from which substracting the Radius A C, the Remainder C D will shew the Height of the Point D of apparent Level, above the Point of true Level.

A TABLE *shewing the Corrections of the Points of apparent Level, for reducing them to the true Level, every* 50 *Toises.*

Distances of the Points of apparent Level.	*Corrections.* *Inches.*	*Lines*
50 Toises.	0.	0
100	0.	1 $\frac{1}{3}$
150	0.	3
200	0.	5 $\frac{1}{3}$
250	0.	8 $\frac{1}{3}$
300	1.	0
350	1.	4 $\frac{1}{3}$
400	1.	9 $\frac{1}{3}$
450	2.	3
500	2.	9
550	3.	6
600	4.	0
650	4.	8
700	5.	4
750	6.	3
800	7.	1
850	7.	11 $\frac{1}{2}$
900	8.	11
950	10.	0
1000	11.	0

The Rule serving to calculate this Table, is to divide the Square of the Distance by the Diameter of the Earth, which is 6,565,179 *Toises; for which Reason the Corrections are to one another, as the Squares of the Distances. Altho' the Foundation of this Calculation be not strictly Geometrical, yet it is nigh enough the Truth for Practice.*

If the Points of apparent Level should be taken instead of the Points of true Level, a Body would err in conducting the Water of a Source, which let be, for Example, at the Point B; for this Source will not run along the Line B D E, but will remain in the Point B; for if it should run along the Line B E, it would run higher than it is, which is impossible, because it cannot be endued with any other Figure but a Circular one, equally distant from the Center of the Earth. On the contrary, a Source in D will have a great Descent down to the Point B; but it cannot run further, because it must be elevated higher than the Source, if it continues it's way in the same right Line, which cannot be done, unless it be forced by some Machine.

How to rectify Levels.

Fig. 2. To rectify Levels, as, for Example, The Air-Level C you must plant two Staffs, as A B, about 50 Toises distant from each other, because of the Roundness of the Earth (take care of exceeding that Distance); then espying from the Station A, the Point B, the Level being placed horizontally, and the Bubble of Air being in the Middle of the Tube, you must raise or lower a Piece of Pasteboard upon the Staff B, in the Middle of which is drawn a black horizontal Line, 'till the visual Ray of the Observer's Eye meets the said Line; after which must be fastened another Piece of Pasteboard to the Staff A, the Middle of which let be the Height of the Eye, when the Piece of Pasteboard B was seen: then removing the Level to the Staff B, place it to the Height of the Center of the Pasteboard, and the Level being horizontally posited for observing the Piece of Pasteboard A, if then the visual Ray cuts the Middle of the Piece of Pasteboard, it is a sign the Level is very just; but if the visual Ray falls above or below, as in the Point C, you must, by always keeping the Eye at the same Height, lower the Telescope or the Sight, 'till the Middle of the visual Ray falls upon the Middle of the Difference, as in D; and the Telescope thus remaining, the Tube of the Level must be adjusted 'till the Bubble of Air fixes in the Middle, which may be done by means of the Screw 4.

Again; Return to the Staff A, and place the Level the Height of the Point D, for looking at the Piece of Pasteboard B; and if the visual Ray falls upon the Middle of the Piece of Pasteboard, it is a sign the Telescope agrees with the Level: if not, the same Operations must be repeated, until the visual Rays fall upon the Centers of the two Pieces of Pasteboard.

Another way to rectify Levels.

Knowing two Points distant from each other, and perfectly level, place the End of the Telescope carrying the Eye-Glass to the exact Height of one of those two Points, the Bubble of Air being fixed in the Middle of it's Tube; then by looking thro' it, if it happens that the

the Hair of the Telescope cuts the second Point, it is a sign the Level is just; but if the Hair falls above or below the Point of Level, you must, in always keeping the Eye at the same Height, raise or lower the End of the Level where the Object Glass is, until the Visual Ray of the Telescope falls upon the exact Point of Level; and leaving it thus, raise or depress the Tube carrying the Level, so that the Bubble of Air may remain in the Middle.

What we have said concerning the Rectification of this Level, may serve likewise for the Rectification of others, the Difference is only to change the Plummets and the Hairs of the Telescopes, according to their Constructions.

The Manner of Levelling.

To find, for Example, the Height of the Point A on the Top of a Mountain, above the Point B at it's foot, place the Level about the middle Distance between the two Points, as in D, and plant Staffs in A and B. Also let there be Persons instructed with Signals, for raising or lowering upon the said Staffs slit Sticks, at the Ends of which are fastened pieces of Paste-Board: The Level being placed upon it's Foot, look towards the Staff A E, and cause one of the Persons to raise or lower the Paste-Board, until the upper Edge or Middle appears in the visual Ray; then measure exactly the perpendicular Height of the Point E above the Point A, which, in this Example, suppose 6 Feet 4 Inches, which set down in a Memorial. Then turn the Level horizontally, so that it may always be at the same Height, for the Eye-Glass of the Telescope to be next to the Eye; but if it be a Sight Level, there is no necessity of turning it about, and cause the Person at the Staff B to raise or lower the Piece of Paste-Board, until the upper Edge of it be seen, as at C, which suppose 16 Feet 6 Inches, which set down in the Memorial above the other Number of the first Station; whence to know the Height of the Point A above the Point B, take 6 Feet 4 Inches from 16 Feet 2 Inches, and the Remainder will be 10 Feet 2 Inches, for the Heighth of A above B. Fig. 3.

Note, If the Point D, where the Observer is placed, be in the Middle between the Point A and the Point B, there is no necessity of regarding the Height of the apparent Level above the true Level, because those two Points being equally distant from the Eye of the Observer, the visual Ray will be equally raised above the true Level, and consequently there needs no Correction to give the Height of the Point A above the Point B.

Another Example of Levelling.

It is required to know, whether there be a sufficient Descent for conducting Water from the Source A to the Receptacle B of a Spring. Now because the Distance from the Point A to B is great, there are several Operations required to be made. Having chosen a proper Height for placing the Level, as at the Point I, plant a Pole in the Point A near the Source, on which slide up and down another, carrying the Piece of Paste-Board L; measure the Distance from A to I, which suppose 1000 Toises. Then the Level being adjusted in the Point K, let somebody move the Paste-Board L up or down, until you can espy it thro' the Telescope or Sights of the Level, and measure the Height A L, which suppose 2 Toises, 1 Foot, 5 Inches. But because the Distance A I is 1000 Toises, according to the aforementioned Table, you must substract 11 Inches, and the Height A L will consequently be but 2 Toises 6 Inches, which note down in the Memorial. Fig. 4.

Now turn the Level about, so that the Object Glass of the Telescope may be next to the Pole planted in the Point H, and the Level being adjusted, cause some Person to move the Piece of Paste-Board G up and down, until the upper Edge of it may be espied thro' the Telescope; measure the Height H G, which suppose 3 Toises, 4 Feet, 2 Inches; measure likewise the Distance of the Points I, H, which suppose 650 Toises; for which Distance, according to the Table, you must substract 4 Inches 8 Lines from the Height H G, which consequently will then be but 3 Toises, 3 Feet, 9 Inches, 4 Lines, which set down in the Memorial.

This being done, remove the Level to some other Eminence, from whence the Pole H G may be discovered, and the Angle of the House D, the Ground about which is level with the Receptacle B of the Spring.

The Level being adjusted in the Point E, look at the Staff H, and the visual Ray will give the Point F; measure the Height H F, which suppose 11 Feet 6 Inches; likewise measure the Distance H E, which suppose 500 Toises, for which Distance the Table gives 2 Inches 9 Lines of abatement, which being taken from the Height H F, and there will remain 11 Feet, 3 Inches, 3 Lines, which set down in the Memorial. Lastly, Having turned the Level for looking at the Angle of the House D, measure the Height of the Point D, where the visual Ray terminates above the Ground, which suppose 8 Feet 3 Inches. Measure also the Distance from the Point D, to the said House, which is 450 Toises, for which Distance the Table gives 2 Inches 3 Lines of Abatement; which being taken from the said Height, there will remain 8 Feet 9 Lines, which set down in the Memorial.

How

How to set down all the different Heights in the Memorial.

Having found proper Places (as we have already supposed) for placing the Level between two Points, you must write on the Memorial, in two different Columns, the observed Heights; namely, under the first Colum those observed by looking thro' the Telescope, when the Eye was next to the Source A; and under the second Column, those observed when the Eye was next to the Receptacle B of the Spring, in the following Manner.

First Column.

	Toises.	*Feet.*	*Inches.*	*Lines.*
First Height Corrected	2	0	6	0
Third Height	1	5	3	3
	3	5	9	3

Second Column.

	Toises.	*Feet.*	*Inches.*	*Lines.*
Second Height	3	3	9	4
Fourth Height	1	2	0	9
	4	5	10	1

Having added together the Heights of the first Column, and afterwards those of the second, substract the first Additions from the second.

Toises.	*Feet.*	*Inches.*	*Lines.*
4	5	10	1
3	5	9	3
1	0	0	10

Whence the Height of the Source A above the Receptacle B is 1 Toise and 10 Lines.

If the Distance be required, you need but add all the Distances measured together; namely,

The First of	1000	Toises
The Second	650	
The Third	500	
The Fourth	450	

The whole Distance 2600 Toises.

Lastly, Dividing the Descent by the Toises of the Distance, there will be for every 100 Toises, about 2 Inches 9 Lines of Descent, nighly.

CHAP. III.

Of the Construction and Use of a Gauge for Measuring Water.

Fig. M. THIS Gauge serves to know the Quantity of Water which a Source or Spring furnishes, and is commonly a Rectangular Parallelepipedon of Brass well soldered, about a Foot long, 8 Inches broad, and as many in height, more or less, according to the Quantity of Water to be measured, having several round Holes very exactly drilled in it, an Inch in Diameter, and others for Half an Inch of Water to pass thro'; and also others for a Quarter of an Inch of Water to pass thro' them. All of which ought to be drilled so as their Centers may be at the same Height. The upper Extremes of the Inch-Holes must be within two Lines of the Top of the Gauge; and the Holes are stopped with little square Brass Plates, adjusted in the Grooves 1, 2 and 3. There is a Brass Partition, crossing the Vessel at the Place 4, fixed about an Inch from the Bottom, and drilled with several Holes, for the Water to pass more freely. This Partition is made to receive the Shock of the Water falling from the Source into the Gauge, and hindring it from making Waves, so that it may more naturally run out thro' the Holes.

Note, The Holes which give a Cylindric Inch of Water, ought to be exactly 12 Lines in Diameter; that giving half an Inch ought to be $8\frac{1}{2}$ Lines, and that giving a Quarter of an Inch must be exactly 6 Lines. This may be easily found by Calculation.

To use this Instrument, it must be placed so as it's Bottom may be parallel to the Horizon, and then let the Water of the Source run thro' a Pipe into the Gauge (as *per* Figure), and when it wants about a Line of the Top, open one of the Holes (for Example) of an Inch. Then if the Water always keeps the same Height in the Gauge, it is manifest that there runs as much into it as goes out of it, and so the Source will furnish an Inch of Water.

But

But if the Water in the Gauge riſes, there muſt be another Hole opened, either of an Inch, half an Inch, or a Quarter of an Inch; ſo that the Water may keep to the ſame Height in the Gauge, that is, to a Line above the Holes of an Inch; and then the Number of Holes opened will give the Quantity of Water furniſhed by the Source.

The little Veſſel receiving the Water running out of the Gauge, is to ſhew how much Water the Source furniſhes in a determinate Space of Time: For having a Pendulum which ſwings Seconds, note how many Seconds there will be in the Time that this Veſſel, ſet under the Hole giving an Inch of Water, is filling; and exactly meaſuring the Quantity of Water it contains, you may have the Quantity of Water the Source furniſhes in an Hour. Fig. N.

There has ſeveral very exact Experiments been made upon this Subject: from whence it has been found, that a Source giving one Inch of Water, will fill 14 Pints of *Paris*, in a Minute.

It follows from hence, that an Inch of Water gives in an Hour 8 *Paris* Muids, and in 24 Hours, 72 Muids.

If, for Example, a Cubic Veſſel be placed under the Gauge, containing a Cubic Foot; and if the Water runs thro' the Hole giving an Inch of Water, that Veſſel will be filled in two Minutes and a half: From whence it follows, that it gives 14 Pints in a Minute, becauſe it furniſhed 35 Pints in two Minutes and a half.

By this means we may know the Inches of Water a Spring or Running-Stream gives: As if, for Example, the Spring gives 7 Pints of Water in a Second; then it is ſaid to furniſh an Inch of Water: If it ſhould give 21 Pints, then it is ſaid to furniſh 3 Inches of Water; and ſo of others.

To meaſure the Running-Water of an Aqueduct or River, which cannot be received in a Gauge, you muſt put a Ball of Wax upon the Water, made ſo heavy with ſome other Matter, as that there may be but a ſmall part of the Ball above the Surface of the Water, that ſo the Wind can have no power on it. And after having meaſured a Length of 15 or 20 Feet of the Aqueduct, you may know by a Pendulum in what time the Ball of Wax will be carried that Diſtance; and afterwards multiplying the Breadth of the Aqueduct or River by the Height of the Water, and that Product by the Space which the Ball of Wax has moved, this laſt Product will give all the Water paſſed, in the noted Time, thro' the Section of the River. Example; Suppoſe in an Aqueduct two Feet wide, and one Foot deep, a Ball of Wax moves, in 20 Seconds, 30 Feet, which will be one Foot and a half in a Second: But becauſe the Water moves ſwifter at the Top than the Bottom, you muſt take but 20 Feet, which will be one Foot in a Second; the Product of one Foot deep, by 2 Feet broad, is 2 Feet, which multiplied by 20, the Length, gives 40 Cubic Feet, or 40 times 35 Pints of Water, which makes 1400 Pints in 20 Seconds; and if 20 Seconds give 1400 Pints, 60 Seconds will give 4200 Pints; and dividing 4200 by 14, which is the Number of Pints an Inch of Water gives, in a Minute or 60 Seconds, the Quotient 300 will be the Number of Inches which the Water of the Aqueduct furniſhes.

Mr *Mariotte*, who has learnedly wrote about the Motion of Water, is of opinion that Springs are nothing but Rain Water, which paſſing thro' the Earth, meets with Haſſock or Clay, which it cannot penetrate; and therefore is obliged to run along the Sides, and ſo form a Spring. For ſupporting this Hypotheſis, he brings the following Experiment.

Having ſet a Cubic Veſſel about a Foot high in a proper place to catch Rain-Water for ſeveral Years, he obſerved that the Water aroſe in the Veſſel each Year, one with another, 18 Inches; but he thought it better to make it but 15 Inches: whence a Toiſe will receive in a Year 45 Cubic Feet of Water; for multiplying 36 Feet by 15 Inches, the Product will be 45 Cubic Feet.

The ſame Author likewiſe computes the Extent of Ground which ſupplies the River *Seine* with Water; and has found that the *Seine* is not the ſixth Part as big as it might be. He has again obſerved, that it has but 10 Inches of Deſcent in 1000 Toiſes over-againſt the *Invalids*. He ſaith likewiſe, that, according to this ſuppoſition, the greateſt Spring of *Montmartre*, when it is moſt abounding, doth not furniſh over and above Water, ſince the Ground overwhelming it ought to ſend Water thereto. Whence he concludes, that there is a great deal of Water loſt in the Earth.

To know the Shock Water produces, Experience has ſhewn that Water accelerates it's Motion, according to the odd Numbers 1, 3, 5, 7, *&c.* that is, if in a fourth Part of a Second it deſcends one Foot in a Pipe, it will deſcend 3 Feet in the next fourth of a Second.

The Quantities of Water ſpouting out thro' equal Holes made at the Bottoms of Reſervatories, of different Heights, are to each other in the ſubduplicate *Ratio* of the Heights. The following Table ſhews the different Expences of Water at different Heights.

A Table of the Expence of Water in a Minute, the Diameter of the Ajutage being three Lines in different Heights of a Reservatory or Cistern.

Heights of Reservatories. Feet.	*Expence of Water 3 Lines in Pipe.* Pints.
6	9
9	11
12	14
18	16
25	19
30	21
40	24
52	28

A Table of the Expence of Water thro' different Ajutages at the same Height of the Reservatory.

Diameters of different Pipes. Lines.	*Expence of Water.* Pints.
1	1
2	6
3	14
4	25
5	39
6	56
7	76
8	110

A Table of the Height of the Spouts of Water at different Heights of Reservatories.

Heights of Spouts. Feet.	Pints.	*Heights of Reservatories.* Inches.
6	5	1
20	10	4
20	21	4
30	33	0
40	45	4
50	58	4
60	72	0
70	86	4

You may see by this Table, that an Ajutage *, double another in Diameter, will expend four times the Water as that other will. Example; That of three Lines will expend in a Minute 14 Pints, and that of 6 Lines will expend 56 Pints. *Note*, The Ajutages must not be made Conical, but Cylindrical.

* Adjutage *is a small Pipe thro' which the Water runs out.*

CHAP. IV.

Of the Construction and Uses of Instruments for Gunnery.

Construction of the Callipers.

Fig. O. THIS Instrument is made of two Branches of Brass, about six or 7 Inches long when shut, each Branch being four Lines broad, and three in Thickness. The Motion of the Head thereof is like that of the Head of a two-Foot Rule, and the Ends of the Branches are bent inwards, and furnished with Steel at the Extremes.

There is a Kind of Tongue fastened to one of the Branches, whose Motion is like that of the Head, for raising or lowering it, that so it's End, which ought to be very thin, may be put into Notches made in the other Branch, on the Inside of which are marked the Dimeters answerable to the Weights of Iron-Bullets, in this Manner: Having gotten a Rule, on which are denoted the Divisions of the Weights, and the Bores of Pieces (the Method of dividing which will be shewn in speaking of the next Instrument), open the Callipers, so that the inward Ends may answer to the Distance of each Point of the Divisions shewing the Weights of Bullets: And then make a Notch at each Opening with a triangular File, that so the End of the Tongue entering into each of these Notches, may fix the Opening of the Branches exactly to each Number of the Weights of Bullets. We commonly make Notches for the Diameters of Bullets weighing from one fourth of a Pound to 48 Pounds, and sometimes to 64 Pounds. And then Lines must be drawn upon the Surface of this Branch against the Notches, upon which must be set the correspondent Numbers denoting the Pounds.

The Use of this Instrument is easy, for you need but apply the two Ends of the Branches to the Diameter of the Bullet to be measured; and then the Tongue being put in a convenient Notch, will shew the Weight of the Bullet.

There ought always to be a certain Proportion observed in the Breadth of the Points of this Instrument; so that making an Angle (as the Figure shews) at each Opening, the Inside may give the Weight of Bullets, and the Outside the Bores of Pieces; that is, that applying the outward Ends of those Points to the Diameter of the Mouths of Cannon, the Tongue, being placed in the proper Notch, may shew the Weights of Bullets proper for them.

Construction of the Gunners Square.

Fig. P. This Square serves to elevate or lower Cannons or Mortars, according to the Places they are to be levelled at, and is made of Brass, one Branch of which is about a Foot long, 8 Lines broad, and one Line in Thickness; the other Branch is 4 Inches long, and of the same Length and Breadth as the former. Between these Branches there is a Quadrant divided into 90 Deg. beginning from the shortest Branch, furnished with a Thread and Plummet.

The

The Uſe of this Inſtrument is eaſy, for there is no more to do but to place the longeſt Branch in the Mouth of the Cannon or Mortar, and elevate or lower it, 'till the Thread cuts the Degrees neceſſary to hit a propoſed Object.

There are likewiſe very often denoted, upon one of the Surfaces of the longeſt Branch, the Diviſion of Diameters and Weights of Iron Bullets, as alſo the Bores of Pieces.

The making of this Diviſion is founded upon one or two Experiments, in examining, with all poſſible Exactneſs, the Diameter of a Bullet, whoſe Weight is very exactly known. For Example, having found that a Bullet, weighing four Pounds, is three Inches in Diameter, it will be eaſy to make a Table of the Weights and Diameters of any other Bullets; becauſe, *per Prop.* 18. *lib.* 12. *Eucl.* Bullets are to one another as the Cubes of their Diameters; from whence it follows, that the Diameters are as the Cube Roots of Numbers, expreſſing their Weights.

Now having found, by Experience, that a Bullet, weighing four Pounds, is three Inches in Diameter; if the Diameter of a Bullet weighing 32 Pounds be required, ſay, by the Rule of Three, As 4 is to 32, So is 27, the Cube of 3, to a fourth Number, which will be 216; whoſe Cube Root, 6 Inches, will be the Diameter of a Bullet weighing 32 Pounds.

Or otherwiſe, ſeek the Cube Root of theſe two Numbers 4 and 32, or 1 and 8, which are in the ſame Proportions, and you will find 1 is to 2, As 3 is to 6, which is the ſame as before.

But ſince all Numbers have not exact Roots, the Table of homologous Sides of ſimilar Solids (in the Treatiſe of the Sector) may be uſed. If now, by help of that Table, the Diameter of an Iron Bullet, weighing 64 Pounds, be required, make a Rule of Three, whoſe firſt Term is 397, the Side of the fourth Solid; the ſecond 3 Inches, or 36 Lines, the Diameter of the Bullet weighing four Pounds; and the third Term 1000, which is the Side of the 64th Solid: the Rule being finiſhed, you will have 90 $\frac{3}{4}$ Lines for the Diameter of a Bullet weighing 64 Pounds. Afterwards to facilitate the Operations of other Rules of Three, always take, for the firſt Term, the Number 1000, for the ſecond 90 $\frac{3}{4}$ Lines, and for the third the Number found in the Table, over againſt the Number expreſſing the Weight of the Bullet. As to find the Diameter of a Bullet weighing 24 Pounds, ſay, As 1000 is to 90 $\frac{3}{4}$ Lines, So is 721, to 65 Lines, which is 5 Inches and 5 Lines for the Diameter ſought. By this Method the following Table is calculated.

A TABLE, *containing the Weights and Diameters of Iron Bullets, and the Bores of the moſt common Pieces uſed in the Artillery.*

Weights of Bullets. Pounds.	*Diameters. Inches.*	*Lines.*	*Bores of Pieces.*	*Inches.*	*Lines.*
$\frac{1}{4}$	1	2 $\frac{1}{4}$	$\frac{1}{4}$	1	3
$\frac{1}{2}$	1	6	$\frac{1}{2}$	1	6 $\frac{1}{2}$
1	1	10 $\frac{5}{8}$	1	1	11 $\frac{6}{8}$
2	2	4 $\frac{1}{2}$	2	2	5 $\frac{3}{4}$
3	2	8 $\frac{2}{3}$	3	2	10
4	3	0	4	3	1 $\frac{1}{4}$
5	3	2 $\frac{3}{4}$	5	3	4 $\frac{1}{4}$
6	3	5	6	3	6 $\frac{7}{8}$
7	3	7 $\frac{1}{4}$	7	3	9 $\frac{1}{8}$
8	3	9 $\frac{3}{8}$	8	3	11 $\frac{1}{8}$
9	3	11	9	4	1 $\frac{1}{4}$
10	4	0 $\frac{3}{4}$	10	4	2 $\frac{3}{4}$
12	4	3 $\frac{3}{4}$	12	4	5 $\frac{3}{4}$
16	4	9	16	4	11 $\frac{1}{2}$
18	4	11 $\frac{1}{3}$	18	5	1 $\frac{2}{3}$
20	5	1 $\frac{1}{2}$	20	5	4
24	5	5	24	5	8
27	5	8 $\frac{7}{8}$	27	5	10 $\frac{2}{3}$
30	5	10 $\frac{1}{2}$	30	6	1 $\frac{1}{3}$
33	6	0 $\frac{3}{4}$	33	6	3 $\frac{1}{2}$
36	6	2 $\frac{3}{4}$	36	6	5 $\frac{3}{4}$
40	6	5 $\frac{1}{2}$	40	6	8 $\frac{1}{2}$
48	6	10	48	7	1 $\frac{1}{4}$
50	6	11 $\frac{1}{2}$	50	7	2 $\frac{3}{4}$
64	7	6 $\frac{3}{4}$	64	7	10 $\frac{1}{4}$

Of the Curved-Pointed Compaſſes.

Theſe Compaſſes do not at all differ in Conſtruction from the others, of which we have already ſpoken, excepting only that the Points may be taken off, and curved ones put on, which Fig. Q.

which serve to take the Diameters of Bullets, and then to find their Weights, by applying the Diameters on the Divisions of the before-mentioned Rule. But when you would know the Bores of Pieces, the curve Points must be taken off, and the strait ones put on, with which the Diameters of the Mouths of Cannon must be taken, and afterwards they must be applied to the Line of the Bores of Pieces, which is also set down upon the aforesaid Rule; by which means the Weights of the Bullets, proper for the proposed Cannon, may be found.

Construction of an Instrument to level Cannon and Mortars.

Fig. R. This Instrument is made of a Triangular Brass Plate, about four Inches high, at the Bottom of which is a Portion of a Circle, divided into 45 Degrees; which Number is sufficient for the highest Elevation of Cannon or Mortars, and for giving Shot the greatest Range, as hereafter will be explained. There is a Piece of Brass screwed on the Center of this Portion of a Circle, by which means it may be fixed or movable, according to Necessity.

The End of this Piece of Brass must be made so, as to serve for a Plummet and Index, in order to shew the Degrees of different Elevations of Pieces of Artillery. This Instrument hath also a Brass Foot to set upon Cannon or Mortars, so that when the Pieces of Cannon or Mortars are horizontal, the whole Instrument will be perpendicular.

The Use of this Instrument is very easy; for place the Foot thereof upon the Piece to be elevated, in such manner that the Point of the Plummet may fall upon a convenable Degree, and this is what we call levelling of a Piece.

Of the Artillery Foot-Level.

Fig. S. The Instrument S is called a Foot-Level, and we have already spoken of it's Construction; but when it is used in Gunnery, the Tongue, serving to keep it at right Angles, is divided into 90 Degrees, or rather into twice 45 Degrees from the Middle. The Thread, carrying the Plummet, is hung in the Center of the aforesaid Divisions, and the two Ends of the Branches are hollowed, so that the Plummet may fall perpendicular upon the Middle of the Tongue, when the Instrument is placed level.

To use it, place the two Ends upon the Piece of Artillery, which may be raised to a proposed Height, by means of the Plummet, whose Thread will give the Degrees.

Upon the Surface of the Branches of this Square, which opens quite strait like a Rule, are set down the Weights and Diameters of Bullets, and also the Bores of Pieces, as we have before explained in speaking of the Gunner's Square.

Fig. T. The Instrument T is likewise for levelling Pieces of Artillery, being almost like R, except only the Piece, on which are the Divisions of Degrees, is movable, by means of a round Rivet: that is, the Portion of the Circle (or Limb) may be turned up and adjusted to the Branch, so that the Instrument takes up less room, and is easier put in a Case. The Figure thereof is enough to shew it's Construction, and it's Uses are the same as those of the precedent Instrument.

Explanation of the Effects of Cannon and Mortars.

Fig. V. The Figure V represents a Mortar upon it's Carriage, elevated and disposed for throwing a Bomb into a Citadel, and the Curve-Line represents the Path of the Bomb thro' the Air, from the Mouth of the Piece to it's Fall. This Curve, according to Geometricians, is a Parabolic Line, because the Properties of the Parabola agree with it; for the Motion of the Bomb is composed of two Motions, one of which is equal and uniform, which the Fire of the Powder gives it, and the other is an uniform accelerate Motion, communicated to it by it's proper Gravity. There arises, from the Composition of these two Forces, the same Proportion, as there is between the Portions of the Axis and the Ordinates of a Parabola; as is very well demonstrated by M. *Blondel*, in his Book, entitled, *The Art of throwing Bombs.*

Maltus, an *English* Engineer, was the first that put Bombs in practice in *France*, in the Year 1634, all his Knowledge was purely experimental; he did not, in the least, know the Nature of the Curve they describe in their Passage thro' the Air, nor their Ranges, according to different Elevations of Mortars, which he could not level but tentively, by the Estimation he made of the Distance of the Place he would throw the Bomb to; according to which he gave his Piece a greater or less Elevation, seeing whether the first Ranges were just or not, in order to lower his Mortar, if the Range was too little; or raise it, if it was too great; using, for that effect, a Square and Plummet, almost like that of which we have already spoken.

The greatest Part of Officers, which have served the Batteries of Mortars since *Maltus*'s Time, have used his Elevations; they know, by Experience, nearly the Elevation of a Mortar to throw a Bomb to a given Distance, and augment or diminish this Elevation in proportion, as the Bomb is found to fall beyond or short of the Distance of the Place it is required to be thrown in.

Yet there are certain Rules, founded upon Geometry, for finding the different Ranges, not only of Bombs, but likewise of Cannon, in all the Sorts of Elevations; for the Line, described in the Air by a Bullet shot from a Cannon, is also a Parabola in all Projections, not only oblique ones, but right ones, as the Figure W shews.

A Bullet

A Bullet going out of a Piece, will never proceed in a straight Line towards the Place it is levelled at, but will rise up from it's Line of Direction the Moment after it is out of the Mouth of the Piece. For the Grains of Powder nighest the Breech, taking fire first, press forward, by their precipitated Motion, not only the Bullet, but likewise those Grains of Powder which follow the Bullet along the Bottom of the Piece; where successively taking fire, they strike as it were the Bullet underneath, which, because of a necessary Vent, has not the same Diameter as the Diameter of the Bore: and so insensibly raise the Bullet towards the upper Edge of the Mouth of the Piece, against which it so rubs in going out, that Pieces very much used, and whose Metal is soft, are observed to have a considerable Canal there, gradually dug by the Friction of Bullets. Thus the Bullet going from the Cannon, as from the Point E, raises itself to the Vertex of the Parabola G, after which it descends by a mixed Motion towards B.

Ranges, made from an Elevation of 45 Deg. are the greatest, and those made from Elevations equally distant from 45 Deg. are equal; that is, a Piece of Cannon, or a Mortar, levelled to the 40th Deg. will throw a Bullet, or Bomb, the same Distance, as when they are elevated to the 50th Degree; and as many at 30 as 60, and so of others, as appears in Fig. X. Fig. X.

The first who reasoned well upon this Matter, was *Galilæus*, chief Engineer to the Great Duke of *Tuscany*, and after him *Torricellius* his Successor.

They have shewn, that to find the different Ranges of a Piece of Artillery in all Elevations, we must, before all things, make a very exact Experiment in firing off a Piece of Cannon or Mortar, at an Angle well known, and measuring the Range made, with all the Exactness possible: for by one Experiment well made, we may come to the Knowledge of all the others, in the following Manner.

To find the Range of a Piece at any other Elevation required, say, As the Sine of double the Angle under which the Experiment was made, is to the Sine of double the Angle of an Elevation proposed, So is the Range known by Experiment, to another.

As suppose, it is found by Experiment that the Range of a Piece elevated to 30 Deg. is 1000 Toises: to find the Range of the same Piece with the same Charge, when it is elevated to 45 Deg. you must take the Sine of 60 Degrees, the double of 30, and make it the first Term of the Rule of Three; the second Term must be the Sine of 90, double 45; and the third the given Range 1000: Then the fourth Term of the Rule will be found 1155, the Range of the Piece at 45 Degrees of Elevation.

If the Angle of Elevation proposed be greater than 45 Deg. there is no need of doubling it for having the Sine as the Rule directs; but instead of that, you must take the Sine of double it's Complement to 90 Degrees: As, suppose the Elevation of a Piece be 50 Degrees, the Sine of 80 Degrees, the double of 40 Deg. must be taken.

But if a determinate Distance to which a Shot is to be cast, is given (provided that Distance be not greater than the greatest Range at 45 Deg. of Elevation), and the Angle of Elevation to produce the proposed Effect be required; as suppose the Elevation of a Cannon or Mortar is required to cast a Shot 800 Toises; the Range found by Experiment must be the first Term in the Rule of Three, as for Example 1000 Toises; the proposed Distance 800 Toises, must be the second Term; and the Sine of 60 Degrees, the third Term. The fourth Term being found, is the Sine of 43 Deg. 52 Min. whose half 21 Deg. 56 Min. is the Angle of Elevation the Piece must have, to produce the proposed Effect; and if 21 Deg. 56 Min. be taken from 90 Deg. you will have 68 Deg. 4 Min. for the other Elevation of the Piece, with which also the same Effect will be produced.

For greater Facility, and avoiding the Trouble of finding the Sines of double the Angles of proposed Elevations, *Galilæus* and *Torricellius* have made the following Table, in which the Sines of the Angles sought are immediately seen.

A TABLE *of Sines for the Ranges of Bombs.*

Degrees.	*Degrees.*	*Ranges.*	*Degrees.*	*Degrees.*	*Ranges.*
90	0	0	0	0	0
89	1	349	66	24	7431
88	2	698	65	25	7660
87	3	1045	64	26	7880
86	4	1392	63	27	8090
85	5	1736	62	28	8290
84	6	2709	61	29	8480
83	7	2419	60	30	8660
82	8	2556	59	31	8829
81	9	3090	58	32	8988
80	10	3420	57	33	9135
79	11	3746	56	34	9272
78	12	4067	55	35	9397
77	13	4384	54	36	9511
76	14	4695	53	37	9613
75	15	5000	52	38	9703
74	16	5299	51	39	9781
73	17	5592	50	40	9848
72	18	5870	49	41	9903
71	19	6157	48	42	9945
70	20	6428	47	43	9976
69	21	6691	46	44	9994
68	22	6947	45	45	10000
67	23	7193			

The Ufe of this Table is thus: Suppofe it be known by Experience, that a Mortar elevated 15 Degrees, charged with three Pounds of Powder, throws a Bomb at the Diftance of 350 Toifes, and it is required with the fame Charge to caft a Bomb 100 Toifes further; feek in the Table the Number anfwering to 15 Degrees, and you will find 5000. Then form a Rule of Three, by faying, As 350 is to 450, So is 5000 to a fourth Number, which will be 6428. Find this Number, or the nigheft approaching to it, in the Table, and you will find it next to 20 Deg. or 70 Deg. which will produce the required Effect, and fo of others.

Of the Conftruction and Ufe of the Englifh Callipers.

Fig. Y.

THESE Callipers, or Gunners Compaffes, confift of two long thin Pieces of Brafs, joined together by a Rivet in fuch a Manner, that one may move quite round the other. The Head or End of one of thefe Pieces is cut Circular, and the Head of the other Semi-Circular, the Center of which being the Center of the Rivet. The Length of each of thofe Pieces from the Center of the Rivet is fix Inches; fo that when the Callipers are quite opened, they are a Foot long.

One half of the Circumference of the Circular Head, is divided into every 2 Degrees, every tenth of which are numbered. And on part of the other half, beginning from the Diameter of the Semi-Circle, when the Points of the Callipers are clofe together, are Divifions from 1 to 10, each of which are likewife fubdivided into four Parts. The Ufe of thefe Divifions and Subdivifions, is, that when you have taken the Diameter of any round thing, as a Cannon-Ball, not exceeding 10 Inches, the Diameter of the Semi-Circle will, amongft thofe Divifions, give the Length of that Diameter taken between the Points of the Callipers in Inches and 4th Parts.

From this Ufe, it is manifeft how the aforefaid Divifions for Inches may be eafily made: For, firft, fet the Points of the Callipers together, and then make a Mark for the Beginning of the Divifions; then open the Points one fourth of an Inch, and where the Diameter of the Semi-Circle cuts the Circumference, make a Mark for one fourth of an Inch. Then open the Points half an Inch, and where the Diameter of the Semi-Circle cuts the Circumference, make another Mark for half an Inch. In this Manner proceed for all the other Subdivifions and Divifions to Ten.

Upon

Upon one of the Branches, on the fame Side the Callipers, are, Firft, half a Foot or fix Inches, each fubdivided into ten Parts: Secondly, a Scale of unequal Divifions, beginning at two, and ending at ten, each of which are fubdivided into four Parts. The Conftruction of this Scale of Lines will be very evident, when it's Ufe is fhewn, which is thus: If you have a Mind to find how many Inches, under 10, the Diameter of any Concave, as the Diameter of the Bore of any Piece of Ordnance is in length, you muft open the Branches of the Callipers, fo that the two Points may be outwards; then taking the Diameter between the faid Points, fee what Divifion, or Subdivifion, the outward Edge of the Branch with the Semicircular Head, cuts on the aforefaid Scale of Lines, and that will be the Number of Inches, or Parts, the Diameter of the Bore of the Piece is in Length. Therefore the Divifions on this Scale may be made in the fame Manner as I have before directed, in fhewing how to make the Divifions for finding the Diameters of round Convex Bodies.

Thirdly, The two other Scales of Lines on the fame Face of the fame Branch, fhew when the Diameter of the Bore of a Piece of Cannon is taken with the Points of the Callipers outward, the Name of the Piece, whether Iron or Brafs, that is, the Weight of the Bullets they carry, or fuch and fuch a Pounder, from 42 Pounds to 1. The Conftruction of thefe Scales are from experimental Tables in Gunnery.

On the other Branch, the fame Side of the Callipers, is, Firft, fix Inches, every of which is fubdivided into 10 Parts. Secondly, A Table fhewing the Weight of a Cubic Foot of Gold, Quick-filver, Lead, Silver, Copper, Iron, Tin, *Purbec*-Stone, Chryftal, Brimftone, Water, Wax, Oil, and dry Wood.

On the other Side of the Callipers, is a Line of Chords to about three Inches Radius, and a Line of Lines on both Branches, the fame as on the Sector. Fig. 2.

There is alfo a Table of the Names of the following Species of Ordnance, *viz.* a Falconet, a Falcon, a Three-Pounder, a Minion, a Sacker, a Six-Pounder, an Eight-Pounder, a Demi-Culverin, a Twelve-Pounder, a Whole-Culverin, a Twenty-four-Pounder, a Demi-Cannon, Baftard-Cannon, and a Whole-Cannon. Under thefe are the Quantities of Powder neceffary for each of their Proofs, and alfo for their Service.

Upon the fame Face is a Hand graved, and a Right Line drawn from the Finger towards the Center of the Rivet. Which Right Line fhews, by cutting certain Divifions made on the Circle, the Weight of Iron-fhot, when the Diameters are taken with the Points of the Callipers, if they are of the following Weights, *viz.* 42, 32, 24, 18, 12, 9, 6, 4, 3, 2, 1, 1 $\frac{1}{2}$, 1, Pounds. Thefe Figures are not all fet to the Divifions on the Circumference, for avoiding Confufion. The aforefaid Divifions on the Circumference may be thus made: Firft, When the Points of the Callipers are clofe, continue the Line drawn from the Finger on the Limb, to reprefent the Beginning of the Divifions. Now, becaufe from Experience it is found, that an Iron Ball or Globe weighing one Pound is 1.8 of an Inch, open the Callipers, fo that the Diftance between the two Points may be 1.8 of an Inch; and then, where the Line drawn from the Finger cuts the Circumference, make a Mark for the Divifion 1. Again, to find where the Divifion 1.5 muft be, fay, As 1 is to the Cube of 1.8, So is 1.5 to the Cube of the Diameter of an Iron Ball weighing 1.5 Pounds, whofe Root extracted will give 2.23 Inches. Therefore open the Points of the Callipers, fo that they may be 2.23 Inches diftant from each other; and then, where the Line drawn from the Finger cuts the Circumference, make a Mark for the Divifion 1 $\frac{1}{2}$. The Reafon of this is, becaufe the Weights of Homogeneous Bodies, are to each other as their Magnitudes; and the Magnitudes of Globes and Spheres, are to each other as the Cubes of their Diameters.

Proceed in the aforefaid Manner, in always making 1 the firft Term of the Rule of Three, and the Cube of 1.8 the fecond, *&c.* and all the Divifions will be had.

Upon the Circle or Head, on the fame Side of the Callipers, are graved feveral Geometrical Figures, with Numbers fet thereto. There is a Cube whofe Side is fuppofed to be 1 Foot, or 12 Inches, and a Pyramid of the fame Bafe and Altitude over it: On the Side of the Cube is graved 470, fignifying that a Cubic Foot of Iron weighs 470 Pounds; and on the Pyramid is graved 156 $\frac{2}{3}$, fignifying that the Weight of it is fo many Pounds.

The next is a Sphere, fuppofed to be infcribed in a Cube of the fame Dimenfions, as the former Cube, in which is writ 246 $\frac{1}{4}$, which is the Weight of that Sphere of Iron. The next is a Cylinder, the Diameter and Altitude of which is equal to the Side of the aforefaid Cube, and a Cone over it, of the fame Bafe and Altitude; there is fet to the Cylinder 369 $\frac{3}{4}$, fignifying, that a Cylinder of Iron of that Bignefs, weighs 369 $\frac{3}{4}$, and to the Cone 121 $\frac{7}{100}$, fignifying, that a Cone of Iron of that Bignefs weighs 121 $\frac{7}{100}$ Pounds.

The next is a Cube infcribed in a Sphere of the fame Dimenfions as the aforefaid Sphere. There is fet to it the Number 90 $\frac{1}{4}$, fignifying, that a Cube of Iron infcribed in the faid Sphere, weighs 90 $\frac{1}{4}$ Pounds.

The next is a Circle infcribed in a Square, and a Square in that Circle, and again a Circle in the latter Square. There is fet thereto the Numbers 28, 11, 22, and 14, fignifying, that if the Area of the outward Square be 28, the Area of it's infcribed Circle is 22, and the Area of the Square infcribed in the Circle 14, and the Area of the Circle infcribed in the latter Square 11.

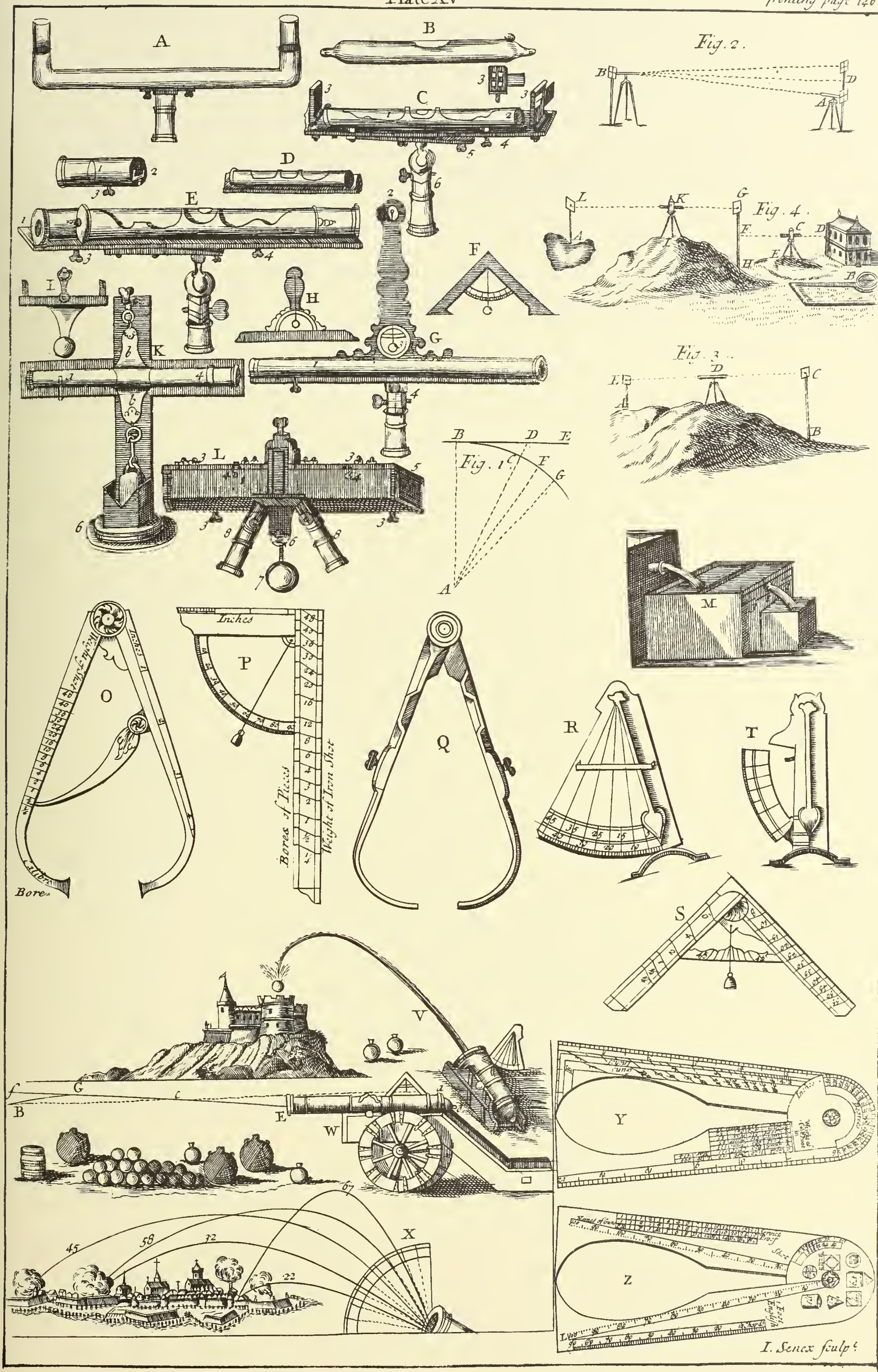
The

The next and laſt, is a Circle croſſed with two Diameters at Right Angles, having in it the Numbers 7, 22, 113 and 355; the two former of which repreſent the Proportion of the Diameter of a Circle to it's Circumference; and the two latter alſo the Proportion of the Diameter to the Circumference. But ſomething nearer the Truth.

I have already, as it were, ſhewn the Uſes of this Inſtrument; but only of the Degrees on the Head, which are to take the Quantity of an Angle, the manner of doing which is eaſy: For if the Angle be an inward Angle, as the Corner of a Room, *&c.* apply the two outward Edges of the Branches to the Walls or Planes forming the Angles, and then the Degrees cut by the Diameter of the Semi-Circle, will ſhew the Quantity of the Angle ſought. But if the Angle be an outward Angle, as the Corner of a Houſe, *&c.* you muſt open the Branches 'till the two Points of the Callipers are outwards; and then apply the ſtraight Edges of the Branches to the Planes, or Walls, and the Degrees cut by the Diameter of the Semi-Circle, will be the Quantity of the Angle ſought, reckoning from 180 towards the Right-Hand.

BOOK

fronting page 146



BOOK VI.

Of the Construction and Uses of Astronomical Instruments.

Taken from the Astronomical Tables of M. de la Hire, *and the Observations of the Academy of Sciences.*

CHAP. I.

Of the Construction and Uses of the Astronomical Quadrant.

THE Quadrants used by Astronomers for Celestial Observations, are usually three Feet, or three Feet and a half (of *Paris*) Radius, that so they may be easily managed and carried from Place to Place. Their Limbs are divided into Degrees and Minutes, that so Observations made with them may be very exact.

This Instrument is composed of several pretty thick Iron or Brass Rulers, whose Breadths ought to be parallel to it's Plane. There are moreover other Iron or Brass Rulers, so adjusted and joined behind the former ones, that their Breadths are perpendicular to the Plane of the Quadrant. These Rulers are joined together by Screws, by means of which the whole Conjunction of the Instrument is made, which ought to be very strait every way, firm, and pretty weighty. The Limb is likewise strengthened with a curved Brass, or Iron Ruler. There is a thick strong circular Blade placed in the Center, serving for the Uses hereafter mentioned; which circular Blade and the Limb must be raised something higher than the Plane of the Instrument, both of which must be covered with well-polished thin Pieces of Brass. But you must take great care that the Surfaces of these Pieces of Brass be both in the same Plane. *Plate* 16. Fig. 1.

The aforesaid circular Iron Blade in the Center must have a round Hole in the Middle thereof, about $\frac{1}{7}$ of an Inch in Diameter, in which is placed a well-turned Brass Cylinder, raised something above the central thin Piece of Brass.

This Cylinder, which is represented in Figure 2, hath the Point of a very fine Needle adjusted in the Center of it's Base, which is supported in going into a little Hole in the Center of the Base, and by lying along a semi-circular Cavity, and is kept therein by means of a little Spring pressing against it; so that when the Needle is taken away, and we have a Mind to put it there again, it may exactly be placed in the little Hole in the Center of the said Cylinder. This little Hole ought to be no bigger than a Hair, but it must be something deep, that so the Point of the Needle may go far enough into it, that at the shaking of the Quadrant it may not come out. At the Point of this Needle is hung a Hair, by means of a Ring made with the same Hair big enough, for fear lest the Knot of the Ring should touch the central Plate, and the Motion of the Hair be disturbed. *Note*, The Base of the central Cylinder A, represented in this Figure, must be such, that the Ring of the Hair, hung Fig. 2.

on the Point of the Needle, may not touch the said Base otherwise than in it's Center, when there is a Plummet hung to the End of the Hair, of about half an Ounce in weight.

The Construction of this central Cylinder ought to be such, that it may be taken away and preserved, and another placed instead thereof, of the same Thickness therewith, but something longer; which coming out beyond the central Blade, sustains the Ruler of the Instrument, in such manner as we are going to explain.

Fig. 3. There is moreover, at the central Brass Blade, which covers the Iron one, a plane Ring A, turning about the Center, but not touching the central Cylinder; in such manner, that the outward Surface thereof is even with the Surface of the said Brass Blade. Upon this Ring is fastened, with two Screws, a flat Tube M, which moves freely along with the Hair and Plummet, which it covers, and so preserves it from the Wind when the Instrument is using.

This Tube carries a Glass, placed against the Divisions of the Limb of the Quadrant, in order to see what Point of Division the Hair falls upon. Behind, and nigh to, the Center of Gravity of the Quadrant, is firmly fixed, with three or four Screws, to the Rulers of the Instrument, the Iron Cylinder I, whose Length is 8 Inches, and Diameter of it's Base two Inches. This Cylinder being perpendicular to the Plane of the Quadrant, may be called it's Axis.

Now because the principal Use of this Instrument is for taking the Altitudes of the Sun or Stars, it must be so ordered, that it's Plane may be easily placed in a vertical Situation;

Fig. 4. therefore an Iron Ruler M N must be prepared, whose Thickness is three Lines, Length eight Inches, and Breadth one Inch, or thereabouts. On one Side of this Ruler are adjusted two Iron Rings Z Z, open a-top with Ears, each of which has a Screw to draw the Ears closer together, which have a Spring. The Bigness of these Rings is nearly equal to the Thickness of the Cylinder I, or Axis of the Quadrant, which being put thro' them, is made fast with the Screws; so that the Axis and Quadrant, which it is fixed to, may remain firm in any Position the Quadrant is put into.

On the other Side of the said Ruler M N is soldered an Iron Cylinder O, of such a Length and Breadth, as to go into the Tube Q, of which we are going to speak.

Now when the Instrument is to be placed so, that it's Plane may be horizontal, for using an Index or moveable Arm to take the Distances of Stars or Places upon Earth, the Cylinder I. must be put into the Tube Q, by which means the Quadrant may be easily turned to what part you please.

Fig. 5. The Foot, or Support of the whole Instrument, is commonly composed of an Iron Tube Q; whose upper Part is capable of containing the Cylinder O, and it's lower Part goes thro' the Middle of an Iron Cross, and is fastened in it by four Iron Arms, at the four Ends of which Cross are four great Screws, to raise or lower the Quadrant, and put it in a convenient Situation. But Monsieur *de la Hire* proposes a Triangular Support in his Tables, which is composed of an Iron or Brass Tube, big enough to contain the Cylinder O, fastened with two Screws to three Iron Rulers R S, bent towards their Tops, and of a pretty good Thickness, which are adjusted and well fixed to a Tee or double Square T X Y. The Screw V, in the Middle of the Tube Q, is for fixing the Cylinder O, according to Necessity.

Now when the Meridian Altitudes of Stars are to be observed, the Ruler T Y ought to be placed in the Meridian Line, and of the three Screws T X Y, which sustain the Weight of the whole Instrument; that which is in X serves to lower the Plane of the Instrument, 'till it answers to the Plane of the Meridian, according as the Observer would have it; and the other two are for raising or lowering the Instrument by little and little, until the Plumb-Line falls upon the requisite Altitude. But it often happens in turning the Screws that are in T and Y, that the Quadrant displaces itself from it's true Position; whence, if the Defect be some Minutes, this may be remedied, by hanging a moveable Weight to the Backside of the Branches of the Instrument, which may alter the Center of Gravity, as likewise change the Inclination of the Quadrant; for the Rulers composing the Foot are not entirely free from Elasticity. Now the nigher to the Foot the Place of Suspension of the Weight is, the less Force will it have to shake the Instrument. *Note*, The Height of the Foot is commonly four Feet and a half, or thereabouts, and the same Use is equally made of the four Branch Support.

The Divisions on the Limb of this Quadrant ought to be made with great Care, that so Observations may thereby be exactly taken. Each Degree is divided into 60 Minutes, by

Fig. 6. means of 11 concentrick Circles, and 6 Diagonal right Lines, as in Figure 6 may be seen. These Diagonal Distances are equal between themselves, but those of the Concentrick Circles are unequal; yet this Inequality is not sensible, if the Radius of the Quadrant be three Feet, and the Distance between the two outmost concentrick Circles be one Inch; for if the Arc A E, of the outmost Circle be 10 Minutes, and there are drawn, from the Center C of the Quadrant, the Radii A D C, E B C, meeting the inner concentrick Circle in the Points D, B, the Arc D B will be likewise 10 Minutes. *Note*, Figure 6 is supposed to be put upon the Limb of the Instrument, Figure 1.

But

But if the Right-lined Diagonals A B, D E, are drawn interſecting each other in the Point F; I ſay F is the middle Point of Diviſion thro' which the middle Circle ought to paſs: For the Arcs A E, B D, which may be taken for ſtrait Lines, are to each other, as A F is to F B: for it is evident, that C A is to C B, as the Diviſions of the Baſe A B of the Right-lined Triangle A C B; but ſince C A is to C B as A E is to D B, therefore A E is to D B, as the Diviſions of the Baſe A B made by a Radius, biſecting the Angle A C B: and conſequently the Point F, before found in the Right-lined Diagonal A B, will be the middle Point of the Diviſions.

Now let us ſuppoſe, that A C be to C B, as 36 Inches is to 35; then A B is to A F, as 71 is to 36. Therefore if the Breadth of one Inch, or 12 Lines, which is the ſuppoſed Meaſure of A B, be divided into 71 equal Parts, the Part A F will be 36 of them, which will be greater by half, or about $\frac{1}{12}$ of a Line, than half of A B, which is but $35\frac{1}{2}$. This Difference is of no conſequence, and may, without any ſenſible Error, be neglected in the Diviſion of the Middle; and much more in the other Diviſions, where it is leſs.

Inſtead of making Right-lined Diagonals, we may make them Portions of Circles paſſing thro' the Center of the Inſtrument, and the firſt and laſt Point of the ſame Diagonal; then we need but divide the firſt Circular Portion into ten equal Parts, and the exact Points will be had thro' which the eleven Concentrick Circles muſt paſs.

The Radius of this firſt Portion may be eaſily found; and then if a thin Ruler be bent into the Curvature thereof, all the other Portions may be drawn by means of it, as we have already mentioned in ſpeaking of the Diviſions of Quadrants, Semi-Circles, *&c.*

Note, It will be proper to leave, at the Bottom of the Limb, the Points that were made for drawing every 10th Minute; for theſe will be a means to take the Correſpondent Altitudes of the Sun, Morning and Evening, much exacter than can be done by the Diagonals, becauſe of the Eſtimation thereby avoided. Moreover, there may be ſome fault in the Diagonals which there cannot be in the Points, if care be taken in making them: for it is difficult enough to draw the Diagonals exactly thro' thoſe Points they ſhould paſs. For which reaſon, if a Micrometer be joined to the fixed Teleſcope of the Inſtrument, the Diagonals need not be uſed, and the aforeſaid Points will be ſufficient; ſince the Micrometer will give, by means of a moveable Hair, the Interval between the neareſt of one of the aforeſaid Points, at every 10th Minute, and the Plummet. And this is done by raiſing or lowering the moveable Hair above or below the horizontal Hair, 10 Minutes of a Degree, or a little more. The Chevalier *de Louville*, of the Academy of Sciences, hath ſatisfactorily uſed a Quadrant for his Obſervations, conſtructed in this Manner.

We now come to ſpeak of the Teleſcopes, and the Manner of finding the firſt Point of the Diviſions of the Limb of the Quadrant.

Theſe Teleſcopes have each two Glaſſes, one of which is the Object-Glaſs, placed towards the viſible Object, and near to the Center of the Quadrant; and the other is the Eye-Glaſs, placed at the other End of the Teleſcope, next to the Eye of the Obſerver.

The Object-Glaſs is firmly faſtened in an Iron Frame, which is fixed with Screws about the Center of the Inſtrument. Near the Eye-Glaſs are placed two fine Hairs, croſſing each other at Right Angles, in an Iron Frame, to which they are faſtened with Wax upon a little piece of Braſs; ſo that the one is perpendicular to the Plane of the Inſtrument, and the other parallel thereto.

The Eye-Glaſs muſt be placed in a Tube, that ſo it may be moved backwards or forwards, according to different Sights; and the Diſtance between the Object-Glaſs and Croſs-Hairs, muſt be the ſaid Glaſs's Focal Length; that is, the Croſs-Hairs muſt be placed in the Focus of the Object-Glaſs. Theſe Teleſcopes muſt be ſo diſpoſed, that the Surfaces of the Lenſes (as Planes) and the Planes in which are the Croſs-Hairs, be parallel to each other, and perpendicular to Right Lines drawn thro' the Centers of the Lenſes, and the Points wherein the Hairs croſs each other. Theſe Teleſcopes are adjuſted behind the Quadrant, that ſo the divided Braſs-Limb may not be incumbered by them.

Between the Frames ſuſtaining the Glaſſes, is a Braſs or Iron Tube, compoſed of two Parts, one of which is inchaſed in the other, that ſo they may eaſily be taken from between the Frames, by means of Ferrels keeping them together.

The Convex Eye-Lens muſt be brought nearer, or removed further from the Croſs-Hairs, according to the diverſe Conſtitutions of Obſervators Eyes; that ſo diſtant Objects may be diſtinctly perceived, as likewiſe the Croſs-Hairs. This Eye-Glaſs is placed in another little moveable Tube, the greateſt part of which lies concealed in another Tube, as may be ſeen in *Fig.* 7.

When the Eye-Glaſs wants cleanſing, or the Croſs-Hairs are broken or diſordered, and others to be put in their place, the before-mentioned Braſs or Iron Tube muſt be taken from between the Frames.

But the Conſtruction of the Eye-Glaſs will be much more convenient, if, inſtead of a Frame only, you uſe a little ſquare Box, about four Lines in Thickneſs, whoſe two oppoſite Sides, which are parallel to the Limb of the Quadrant, have Grooves along them, in which may move a little Plate of a mean Thickneſs, drilled thro' the Middle with a round Hole of a convenient bigneſs. Fig. 7.

Upon

Upon the Surface of this Plate, represented by the Figure *a*, are continued out two Diameters of the aforesaid Hole, crossing each other at Right Angles, one of which is parallel to the Limb, and the other perpendicular thereto, upon which are placed the Cross-Hairs. This Plate is very useful for moving the said Cross-Hairs, strained at Right Angles a-cross the Middle of the Hole, backwards or forwards, according to necessity. And when the Hairs are placed as they should be, the aforesaid Plate is fixed to the Box with Wax, which ought to be furnished with a sliding Cover, for keeping the Cross-Hairs from Accidents.

The Inside of the Tube ought to be blackened with the Smoke of Rosin, in order to preserve the Eye from too strong Rays which come from a luminous Object, that so the Appearance thereof may be more perfect. *Note*, Instead of having Cross-Hairs in the before-mentioned Box, a little Piece of plain Glass may be used, having two fine Lines drawn upon it at Right Angles with the Point of a Diamond.

The Telescope being prepared and placed in a convenient Situation parallel to the Radius, or Side of the Quadrant; the next thing to be done, is to find the first Point of the Divisions of the Limb of the Quadrant, which is 90 Degrees distant from the Line of Collimation or Sight of the Telescope, or a Line parallel to it, passing thro' the Center of the Quadrant. But, First, it will be necessary to say something concerning this Line of Collimation, or Sight, about which M. *de la Hire* says, he had formerly a long Controversy, with very celebrated and great Astronomers, who, for want of duly considering Dioptricks, maintained, that it is impossible to find a settled and constant Line of Collimation in these kind of Telescopes.

It is now manifest, that all the Rays proceeding from any one Point of an Object, after having passed thro' the Glass Lens, will all concur in one and the same Point, which is called the Focus, provided that the Distance of the Radiating Point from the *Lens* be greater than the Semidiameter of either of the Convexities of the *Lens*, which here we suppose equal; that besides, among the Rays coming from a Radiating Point, and falling upon the anterior Surface of the Glass, that which concurs with a Line passing thro' the Centers of the Convexities, will suffer no Refraction at it's going in or coming out of the Glass; therefore the Points of Objects that are in that Right Line, are represented in the same Line, which is called the Axis of the Optick Tube, and the Point of the Axis which is in the Middle of the Glass's thickness, is called the Center of the *Lens*.

If the Right Line passing thro' the Center of the *Lens*, and the Point where the Hairs cross one another, agree with the Axis of the said Optick Tube, it will be the Line of Collimation of the Telescope; and an Object very distant, placed in the Axis produced, will appear in the same Point where the Hairs cross one another: just as in common Indexes, where we take for the Line of Sight, the Right Line, that passing thro' the Slits of the Sights, tends to the Object. But altho' it almost never happens in the Position of Telescopes, which we have established, that the Right Line tending from the Object to the Point wherein the Hairs cross, and whereat the Object is represented, coincides with the Optick Axis; nevertheless we shall not desist finding that Line of Collimation tending from the Object to it's Picture, represented in the Point wherein the Hairs cross each other; which may be done in the following Manner.

Fig. 8. Let X V be a Glass *Lens*, it's Axis A C B, and it's Center C; let F be the Point wherein the Hairs cross one another without the Axis A C B. If from the Point F, which by Construction is at the Focal Distance from the *Lens*, Rays pass thro' the Glass, they will suffer a Refraction at their Entrance into the Glass, and a second Refraction at their going out thereof; after which, they will continue their way parallel to one another. Now there is one of these Rays, namely, F E, which coming from the Point F, after the first Refraction in the Point E, passes thro' the Center C; for after a second Refraction at it's going out of the Glass in the Point D, it will continue it's way from D to O, parallel to F E, according to Dioptrick Rules. But all the Rays separated at their going out of the Glass may be taken as parallel, if they tend to a very distant Point O, therefore they are also parallel to the Ray F E O, which is produced from the Object directly to the Point O; and it is this Right Line F E O, which we call the *Line of Collimation*, in the aforesaid Position of Telescopes, and it will always remain the same, if the Situation of the Glasses be not changed, that is, if the *Lens* and the Cross-Hairs are in the same Position and Distance. The Object O being in one of the extreme Points of the Right Line F E O, will be represented in the Point F.

Note, The Distance between the principal Ray O D, falling from the Point O of the Object upon the *Lens*, and it's refracted Ray E F, is always lesser than the Thickness of the said *Lens* D E, which is insensible, and of no Importance, in the Distance of a very distant Object, and the Distance of the parallel Rays O D, O E F, will be so much the less, as the *Lens* is more directly turned towards the Position of the Cross-Hairs.

We come now to shew how to find the first Point of the Divisions of the Limb of the Quadrant, which is thus: Having fixed the Plane of the Quadrant in a vertical Position, by means of the Plumb-Line C D, direct the Telescope towards a very distant visible Point, nigh to the Sensible Horizon, in respect of the Place where the Telescope of the Instrument is placed; which may be first known by marking the Point B upon the Limb, in the Radius C B, parallel to the Axis of the Tube, which may be nearly done; and by taking the Point D, distant from the Point B 90 Degrees: for when the Plumb-Line falls upon the Point D, the

the Object appearing in the Point wherein the Hairs croſs one another, will be nigh to the Horizon; for the Senſible Horizon muſt be at Right Angles with the Plumb-Line C D. But ſince we are not yet certain whether the Teleſcope be perfectly Horizontal, the Inſtrument muſt be turned upſide down, ſo that the Point D may be above, and the Center below; but it is neceſſary in this Tranſpoſition, that the Line of Collimation be at the ſame Height as it was in the firſt Poſition. Having again directed the Teleſcope towards the Point firſt obſerved, ſo that it may appear in the Point wherein the Hairs croſs, and having adjuſted the Cylinder in the Center of the Inſtrument, faſten the Plumb-Line with Wax upon the Limb in the Point D; and if it exactly falls upon the Center C, it is certain that the *Line of Collimation* is horizontal. For this *Line of Collimation* will remain the ſame in both Situations of the Quadrant, and produced with the Vertical Line C D, the Point D will be the Beginning of the Diviſions of the Limb.

But if, after having turned the Inſtrument upſide down, the Plumb-Line ſuſpended at the Point D, does not preciſely fall upon the Center C, you muſt move it 'till it does paſs thro' it, not any wiſe changing the Poſition of the Quadrant, nor the Glaſſes of the Teleſcope; and then the Point E, upon which the Plumb-Line falls, muſt be marked in the circular Arc D E, deſcribed about the Center C, paſſing thro' the Point D.

Now, I ſay, if the Arc D E be biſected in the Point O, this Point will be the firſt Point of the Diviſions of the Limb, and the Radius C O will be at Right Angles with the *Line of Collimation*. This Operation is very manifeſt, for the *Line of Collimation*, or the Radius C B, parallel to it, will not be changed in either of the Poſitions of the Quadrant, if the Angle B C D, in the natural Situation of the Inſtrument, be greater than a Right Angle; that is, if the Point of an Object the Teleſcope is directed to, be under the Horizon, it is manifeſt that the Vertical Line C D produced, anſwering to the Plumb-Line, makes with the *Line of Collimation* an Angle leſs than a right one, *viz.* the Complement of the Angle B C D, which is equal to the Angle B C E; therefore the Angle B C O, which is a Mean between that which is greater than a right one, and that leſſer, made by the Radius C O, and the *Line of Collimation*, will be a right Angle; which was to be proved.

We may yet otherwiſe have the firſt Point of the Diviſions of the Limb, by knowing a Point perfectly level with the Eye; then placing the Teleſcope in that Point, and that place of the Limb upon which the Plumb-Line plays, will give the firſt Point of Diviſion.

The Proof of this Operation is juſtified, if (the Plumb-Line paſſing thro' the Point O) a very diſtant Object appears in the Point wherein the Hairs croſs one another. For having inverted the Inſtrument, and the Teleſcope being always directed towards the ſame Object, the Plumb-Line will paſs thro' the Points O and C, otherwiſe there will be ſome Error in the Obſervations.

Being well aſſured of the firſt Point of the Diviſions of the Limb, you muſt draw about the Center C two Portions of Circles, an Inch diſtance from each other, between which the Diviſions of the Limb are to be included; to do which, you muſt uſe a Beam-Compaſs, whoſe Points are very fine, one of which, next to the End, moves backwards or forwards, by help of a Screw and Nut, which is adjuſted to the End of the Branch of the Compaſs.

Then one of the Points of the Compaſs being placed in O, the firſt Point of the Diviſions of the Limb, and the other being diſtant therefrom the Length of the Radius of one of the ſaid concentrick Arcs, make a Mark upon the correſpondent concentrick Arc, which exactly divide into two equal Parts, one of which being laid off beyond the Mark, will give the Point B; and ſo the Quadrant O B will be divided into three equal Parts, each being 30 Degrees.

Theſe Parts being each divided into three more, and each of theſe laſt into two; and, finally, each of the Parts ariſing into five more equal ones; the Quadrant will be divided into 90 Degrees, each of which being again divided into ſix equal Parts, every 10th Minute will be had.

The outward and inward concentrick Arcs of the Limb being very exactly divided, as we have directed, very fine Lines muſt diagonally be drawn thereon; that is, from the firſt Point of Diviſion of the inward Arc, to the ſecond Point of Diviſion of the outward Arc; and ſo on from one Diviſion in the inward Arc to the next enſuing Diviſion of the outward Arc, as appears in *Fig.* 6. This being done, the Diſtance between the outward and inward Arcs muſt be divided into 10 equal Parts, thro' each Point of Diviſion of which, muſt nine concentrick Arcs be drawn about the Center of the Quadrant C, which will divide the Diagonals into ten Parts; and ſo the Limb of the Inſtrument will be divided into Degrees and Minutes. Great care ought to be taken, that ſo the Diviſions may be very exactly drawn equal; and that they may be as exact as poſſible, very good and fine Compaſſes exquiſitely to draw the Lines and Circles muſt be uſed; and in making the ſeveral Diviſions, we uſe fine Spring Compaſſes, whoſe Points are as fine as a Needle, and a good dividing Knife. *Note*, The Diviſions of the Limb of the Quadrant for certain Uſes, are continued about 5 Degrees beyond the Point O.

After this Instrument hath been carried in a Coach, or on Horseback, *&c.* care ought to be taken to prove it, for fear lest the Glasses of the Telescope should have been disordered, or the Cross-Hairs removed, which often happens. Likewise when the Tube of the Telescope, if the Instrument be not conveyed, as aforesaid, is exposed to the Heat of the Sun, the Cross-Hairs are too much stretched, and afterwards when the Sun is absent, they relax and become slack, and so are not very fit to be used: yet nevertheless, if you think the Cross-Hairs have not been moved, there is no necessity of proving the Telescope, because the Object-Glass remains immoveable, and always the same; and the Cross-Hairs, which by the Moisture of the Air are slackened, will often become tight again in fine Weather.

NOTE, If a Telescope be placed to an Instrument already divided, it is very difficult to make it agree with the Divisions of the Limb; therefore having proved it, according to the Directions before given, we shall find how much greater or lesser than a Right Angle the Telescope makes, with a Radius passing thro' the first Point of the Divisions of the Limb, and this Difference must be regarded in all Observations made with the Instrument: For if the Angle be greater than a Right one, all Altitudes observed will be greater than the true ones by the Quantity of the said Difference; and contrariwise, if the aforenamed Angle be lesser than a Right Angle, the true Altitudes will be greater than the observed ones. Notwithstanding this, the Cross-Hairs may be so placed, that the *Line of Collimation* of the Telescope may make a Right Angle with the Radius passing thro' the first Point of Division of the Quadrant, in applying the Cross-Hairs on a moveable Plate, as we have mentioned in the Construction. But because in conveying this Instrument to distant Places, the Proof thereof must be often made; and since the Method already laid down is subject to great Inconveniencies, as well on account of the Difficulty of inverting the Instrument, so that the Tube of the Telescope may be at the same Height, as because of the different Refractions of the Atmosphere near the Horizon, at different Hours of the Day; as likewise because of the Agitation and Undulation of the Air, and other the like Obstacles: Therefore we shall here shew two other ways of rectifying these Instruments, that so any one may chuse that which appears most convenient for him.

Now the first of these Methods is this: You must chuse some Place from whence a distant Object may be perceived distinctly, at least 1000 Toises, and whose Elevation above the Horizon does not exceed the Number of Degrees of the Limb of the Quadrant continued out beyond the Beginning of the Divisions. Now after you have observed the Altitude of the said Object, as it appears by the Degrees of the Limb, a Pail brim-full of Water, or some broad-mouthed Vessel, must be placed before, and as nigh to the Quadrant as possible, which must be raised or lowered until the said Object be perceived thro' the Telescope upon the Surface of the Water, as in a Looking-Glass, which will not be difficult to do; provided the Surface of the Water be not disturbed by the Wind; whence the Depression of the said Object will be had in Degrees by Reflexion, and it will appear in an erect Situation, because the Telescope is composed of two convex Glasses, which represent Objects inverted. But by Reflexion inverted Objects appear erect, and erect Objects inverted.

But you ought to observe, that when the Angle made by the Line of Collimation, and the Radius passing thro' the first Point of the Divisions of the Limb, is greater than a right one, the Depression of the aforesaid Object will appear as an Altitude; that is, when you look thro' the Telescope at the Image of the Object in the Surface of the Water, the Plumb-Line of the Quadrant will fall on the left Side of the first Point O of the Divisions of the Limb, and not on the Divisions continued out beyond the Point O. And contrariwise, in other Cases, when the Angle the Line of Collimation makes with the Radius passing thro' the first Point of the Divisions of the Limb, is lesser than a right one, the Altitude of the Object will appear by the Divisions of the Limb, as tho' it was depressed; that is, when you look at the aforesaid Object thro' the Telescope, the Plumb-Line of the Quadrant will fall upon the Divisions of the Limb continued out beyond the Point O. But in all Cases, without regarding the Degrees of Altitude or Depression, denoted by the Plumb-Line, when the Object and it's Image, in the Surface of the Water, is espied thro' the Telescope, the exact middle Point between the two Places whereon the Plumb-Line falls at both Observations on the Limb, is vertical, and answers to the Zenith with respect to the Line of Collimation of the Telescope.

Now having found the Error of the Instrument, that is, the Difference between the first Point of the Divisions of the Limb, and the said middle Point answering to the Zenith, you must try to place the Cross-Hairs in their true Position, if you can conveniently; but if not, regard must be had to the Error in all Observations, whether of Elevation, or Depression.

But note, if the Object be near, and elevated some Minutes above the Horizon, the true Error of the Instrument may be found in the following Manner.

We have three things given in a Triangle, one of which is the known Distance between the Place of Observation and the Object; the other the Distance between the Middle of the Telescope, and the Point of the Surface of the Water, upon which a reflected Ray falls; and the last, the Angle included between those two Sides; that is, the Arc of the Limb contained between the two Places of the Limb upon which the Plumb-Line falls in, observing, as aforesaid, the Object and it's Image on the Surface of the Water thro' the Telescope: I say, we have the ſaid two Sides and included Angle given, to find the Angle oppoſite to the leſſer Side. This being done, if the Arc of the Limb included between the two Places whereon the Plumb-Line falls, in obſerving, as aforeſaid, be diminiſhed, on the Side of the Limb produced, by the Quantity of the Angle found, the Middle of the remaining Arc will be the true vertical Point. *Note*, To find the Diſtance between the Middle of the Tube of the Teleſcope, and the Point of the Surface of the Water upon which the reflected Rays fall, you may uſe a Rod or Thread prolonged from the ſaid Tube to the Surface of the Water.

The other way (which is very ſimple, but yet not eaſy) of proving whether the Line of Collimation of the fixed Teleſcope be right, is thus: We ſuppoſe in this Method, that the Limb of the Quadrant is continued out, and divided into ſome few Degrees beyond 90. Now in ſome ſerene ſtill Night, we take the Meridian Altitude of ſome Star near the Zenith, having firſt turned the divided Face of the Limb of the Quadrant towards the Eaſt. This being done, within a Night or two after, we again obſerve the Altitude of the ſame Star, the divided Face of the Limb being Weſtward. Then the Middle of the Arc of the Limb between the Altitudes at each Obſervation, will be the Point of 90 Deg. that is, a Point thro' which a Radius of the Quadrant paſſes, parallel to the Line of Collimation of the Teleſcope. *Note*, This Method is very uſeful for proving the Poſition of Teleſcopes, which are adjuſted not only to Quadrants, but principally to Sextants, Octants, *&c.* for by means thereof may be found which of the Radii of the ſeveral Inſtruments are parallel to the Lines of Collimation of the Teleſcope.

We ſhall hereafter ſhew the Manner of taking the Altitudes of Celeſtial Bodies; as likewiſe how to obſerve them thro' Teleſcopes.

Of the Index, or moveable Arm of the Quadrant.

I ſhall conclude this Chapter in ſaying ſomething concerning the Conſtruction and Uſe of this Index, which is no more than a moveable Index, with a Teleſcope adjuſted thereto, which produces the ſame Effect as the Indexes of other Inſtruments do; that is, to make any Angle at pleaſure with the Teleſcope fixed to the Quadrant. The principal Part of this Index is an Iron or Braſs Ruler, drilled at one End, and is ſo adjuſted to the Central Cylinder, of which we have already ſpoken, that it has a circular Motion only. Fig. 9.

Upon this Ruler are faſtened two Iron or Braſs Frames, in one of which, *viz.* that which is next to the Center of the Inſtrument, the Object-Glaſs is placed; and in the other, the Eye-Glaſs and Croſs-Hairs, which together make up a Teleſcope, alike in every thing to the other fixed Teleſcope of the Quadrant.

At the End of the Index joining to the Limb, is a little Opening about the bigneſs of a Degree of the Limb, thro' the Middle of which is ſtrained a Hair, which is continued to the Center of the Quadrant. But becauſe in uſing the Index the ſaid Hair is ſubject to divers Inconſtancies of the Air, it is better to uſe a thin Piece of clear Horn, or a flat Glaſs, adjuſted to the aforeſaid little Opening in a Frame, having a Right Line drawn upon that Surface thereof next to the Limb, ſo that it tends to the Center of the Inſtrument. *Note*, The Frame is faſtened in the little Opening by means of Screws.

Now the Index being faſtened to the Center before it is uſed, the Teleſcope muſt be proved, that ſo it may be known whether the fixed Teleſcope agrees therewith. To do which, having placed the Plane of the Inſtrument horizontally, and directed the fixed Teleſcope to ſome Point of a viſible Object, diſtant at leaſt 500 Toiſes; afterwards the moveable Teleſcope muſt be pointed to the ſame Object, that ſo one of the Croſs-Hairs, *viz.* that which is perpendicular to the Plane of the Quadrant, may appear upon the aforeſaid Point of the Object: for it matters not whether the Interſection of the Hairs appear thereon, or the perpendicular Hair only. Then, if the Line drawn upon the Horn or Glaſs on the Index falls upon the 90th Degree of the Limb of the Quadrant, the Teleſcopes agree: if not, either the Horn or Glaſs muſt be removed 'till the Line drawn thereon falls upon the 90th Degree of the Diviſions of the Limb, and then it muſt be faſtened to the Index; or elſe regard muſt be had, in all Obſervations, to the Difference between the firſt Point of the Diviſions of the Limb, and the Line drawn upon the ſaid Piece of Horn or Glaſs.

CHAP. II.

Of the Conſtruction and Uſe of the Micrometer.

THE *Micrometer* is an Inſtrument of great Uſe in Aſtronomy, and principally in meaſuring the apparent Diameters of the Planets, and taking ſmall Diſtances not exceeding a Degree, or Degree and a half. This Inſtrument is compoſed of two rectangular Braſs Frames, one of which, *viz.* A B C D, is commonly 2 $\frac{1}{2}$ Inches long, and 1 $\frac{1}{2}$ broad, having the Sides Fig. 9.

A B and C D, divided into equal Parts, about four Lines diſtant from each other (for this is according to the Turns of the Screw, as ſhall be hereafter explained), but in ſuch manner, that the Lines drawn thro' each Diviſion be perpendicular to the Sides A B and C D, and having human Hairs ſtrained from Diviſion to Diviſion, faſtened with Wax to the Places 2, 2, *&c.*

The other Frame E F G H, whoſe Length E F is one Inch and a half, is ſo adjuſted to the former Frame, that the Sides E F and G H of the one, may move along the Sides A B and C D of the other, without being ſeparated therefrom; which is done by means of Dove-tail Grooves. The Face of this ſecond Frame next to the divided Face of the former, is likewiſe furniſhed with a Hair, ſtrained at the Place 4; ſo that when the Frame is moving, the ſaid Hair may be always parallel to the Hairs on the other Frame. The Screw I, whoſe Cylinder is about four or five Lines in Diameter, goes thro', and turns in the Side B D of one of the Frames, which for this purpoſe is made thicker than the other Sides. The End of this Screw is cut ſo as to go through a round Hole made in the Side F H of the leſſer Frame, which for this purpoſe is likewiſe made thicker than the other Sides; there is alſo a little Pin K put thro' a Hole made in the End of the Screw, that ſo the leſſer Frame can no ways move, but in turning the Screw to the right or left, according as you would have the Frame move forwards or backwards. M N is a circular Plate about an Inch in Diameter, faſtened with two Screws to the Side B D of the Frame. This Plate is commonly divided into 20 or 60 equal Parts, which ſerve to reckon the Revolutions and Parts of the Screw, by means of the Index M, which is adjuſted under the Neck of the ſaid Screw, and turns with it. Now the Diviſions of the Sides of the Frame A B C D, are made according to the Breadth of the Threads of the Screw; for if, for example, the Diviſions are deſired to be 10 Turns of the Screw diſtant from each other, turn the ſaid Screw ten times about, and note how far the Frame hath moved: if it has moved four Lines, the Diviſions muſt be four Lines diſtant from each other; and ſo of others.

Now becauſe Hairs are ſubject to divers Accidents by Heat, and otherwiſe, therefore M. *de la Hire* propoſes a very thin and ſmooth piece of Glaſs to be uſed inſtead of them, adjuſted in Grooves made in the Sides of the Frame, and having very fine parallel Lines drawn thereon, which produce the ſame Effect as the parallel Hairs. All the Difficulty conſiſts in chuſing a very fine and well poliſhed Piece of Glaſs, and drawing the Lines extremely nice; for the Defaults will groſsly appear, when the ſaid Lines are perceived in a Teleſcope.

Note, Theſe Lines muſt be very lightly drawn upon the Glaſs with a ſmall Diamond, whoſe Point is very fine.

This Inſtrument is joined to a Teleſcope, by means of the prominent Pieces L, L, which ſlide in a Kind of parallelogramick Tin-Box, at the two Sides of which are two Circular Openings, wherein are ſoldered two ſhort Tubes; that on one Side being to receive the Tube carrying the Eye-Glaſs; and that on the other Side, the Tube carrying the Object-Glaſs, ſo that the Micrometer may be in the Focus of the ſaid Object-Glaſs.

Uſe of the Micrometer.

In order to uſe this Inſtrument, a lively Repreſentation of Objects appearing thro' the Teleſcope muſt be made in the Point whereat the parallel Hairs are placed; therefore if the Object-Glaſs be placed at it's Focal Diſtance from the Micrometer, more or leſs, according to the Nature and Conſtitution of the Eyes of the Obſervator, the Objects and the parallel Hairs will appear diſtinctly in the ſaid Focus.

If then the Focal Length of the Object-Glaſs be meaſured in Lines or 12th Parts of Inches, or, which is all one, the Diſtance from the Center of the Object-Glaſs to the parallel Hairs of the Micrometer, be meaſured, this Diſtance is to the Length of four Lines, which is the Interval between two fixed parallel Hairs nigheſt each other, as Radius is to the Tangent of the Angle, ſubtended by the two neareſt parallel Hairs. This is evident from Dioptricks: for the Diſtance between the Object and the Obſervator's Eye, is ſuppoſed to be ſo great, that the focal Length of the Object-Glaſs, compared therewith, is of no conſequence; ſo that the Rays proceeding from the Points of the Object directly paſs thro' the Center of the Object-Glaſs in the ſame Manner, as tho' the Obſervator's Eye was placed in the ſaid Object-Glaſs. This may be ſhewn by Experience thus:

Draw two black Lines parallel upon a very ſmooth and white Board, whoſe Interval let be ſuch, that at the Diſtance of 200 or 300 Toiſes, they may be met with or embraced by two parallel Hairs of the Micrometer. This being done, remove the Table in a convenient Place (there being no Wind ſtirring), ſo far from the Teleſcope, until the Lines drawn thereon, which muſt be perpendicular to a Right Line drawn from the Table to the Micrometer, be catched by two fixed parallel Hairs of the Micrometer; and then the Diſtance between the Table and the Object-Glaſs will have the ſame Proportion to the Diſtance between the Lines on the Board, as Radius is to the Tangent of the Angle ſubtended by two Hairs of the Micrometer.

Now move the Frame E F G H, by means of the Screw, 'till it's Hair exactly agrees with one of the parallel Hairs of the other Frame; and when this is done, obſerve the Situation of the Index of the Screw; then turn the Screw until the ſaid Hair of the Frame E F G H agrees

agrees with the next nearest fixed Hair of the other Frame; or, which is the same thing, move the Frame E F G H the Length of four Lines, or one third of an Inch, which may be easily known by means of the Object-Glass, which magnifies Objects, and count the Revolutions and Parts of the Screw, compleated in moving the said Frame that Length. Finally, make a Table, shewing how many Revolutions, and Parts of a Revolution of the said Screw, are answerable to every Minute and Second, by having the Angle subtended by the two black Lines on the Board given, and taking the Revolutions proportional to the Angles; that is, if a certain Number of Revolutions give a certain Angle, half this Number will give half the Angle, *&c.* And this Proportion is exact enough in these small Angles.

Now the Manner of taking the apparent Diameters of the Planets, is thus: Having directed the Telescope, and it's Micrometer, towards a Planet, dispose the Hairs, by the Motion of the Telescope, in such a Manner, that one of the fixed parallel Hairs do just touch one Edge of the Planet, and turn the Screw 'till the moveable Hair just touches the opposite Edge of the said Planet. Then, by means of the Table, you will know how many Minutes or Seconds correspond to the Number of Revolutions or Parts, reckoning from the Point of the Plate over which the Index stood when the fixed Hair touched one Edge of the Planet, to the Point it stands over when the moveable Hair touches the opposite Edge; and consequently, the apparent Diameter of the said Planet will be had. And in this manner may small Angles on Earth be taken, which may be easier done than those of the Celestial Bodies, because of their Immobility.

This Method is convenient enough for measuring the apparent Diameters of the Planets, if the Body of any one of them moves between the parallel Hairs. Yet it ought to be observed, that the Sun and Moon's Diameters appear very unequal upon the account of Refraction; for in small Elevations above the Horizon, by the Space of 30 Minutes, the vertical Diameters appear something lesser than they really are in the Horizon, and the horizontal Diameters cannot be found, unless with much Trouble, and several repeated Observations; as likewise the Distance between two Stars, or the Horns of the Moon, because of their Diurnal Motions, which appear thro' the Telescope very swift.

If two Stars of different Altitudes pass by the Meridian at different Times, the Difference of their Altitudes will be the Difference of their Distances from the Equator towards either of the Poles, which is called their Difference of Declination; and by their Difference of Time in coming to the Meridian, the Difference of their Distance from a determinate Point of the Equator, that is, the first Degree of *Aries* will be had; and this is their Difference of Right Ascension.

If the two Stars are distant from each other, we have Time enough, in the Interval of their Passage by the Meridian and Micrometer, to finish the Operations regarding the first, before proceeding to those of the second; but if they be very near each other, it is extremely difficult to make both the Observations at the same Time, that so the two Stars may be precisely catched in the Meridian. But M. *de la Hire* shews how to remedy this Inconveniency, by only using the common Micrometer: for the Observation of the Passage of Stars between, or upon the Hairs of the Micrometer, will give, by easy Consequences, their Difference of Right Ascension and Declination, without even supposing a Meridian known or drawn.

But if the Difference of Declination and Right Ascension of two Stars that cannot be taken in between the Hairs of the Micrometer be required, this may be found in the following Manner.

We adjust a Cross-Hair to the Micrometer, cutting the parallel ones at Right Angles, which we fasten with Wax to the Middle of the Sides A C and B D. Then the Telescope, and it's Micrometer, being fixed in a convenient Position, so that the Stars may successively pass by the parallel Hairs, as the Stars A and S, in Figure 10; we observe, by a second Pendulum Clock, the Time wherein the first Star A touches the Point in which the aforementioned Cross-Hair A S crosses some one of the parallel Hairs, as A *d*. The Micrometer being disposed for this Observation, which is not difficult to do, reckon the Seconds of Time elapsed between the Observations made in the Point A, and the arrival of the said Star to the Point B, being the Concourse of another parallel Hair B D. We likewise observe the Time wherein the other Star S meets the Cross-Hair at the Point S, and then at the Point D of the parallel Hair B D. *Note*, It is the same thing if the Star S first meets the parallel Hair in D, and afterwards the Cross-Hair in S. Fig. 10.

Now as the Number of Seconds the Star A is moving through the Space A B, is to the Number of Seconds the Star S is moving through the Space S D; so is the Distance A C, known in Minutes and Seconds of a Degree in the Micrometer, to the Distance C S, in Minutes and Seconds of a Degree. But the Horary Seconds of the Motion through the Space A B, must be converted into Minutes and Seconds of a great Circle, by the Rule of Proportion.

Having first converted the Seconds of the Time of the said Motion from A to B, which may be here esteemed as a Right Line, or an Arc of a great Circle, into Minutes and Seconds of a Circle, in allowing 15 Minutes of a Circle to every Minute of an Hour, and

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the same for Seconds: We say, by the Rule of Proportion, As Radius is to the Sine Complement of the Stars known Declination, So is the Number of Seconds in the Arc A B also known, to the Number of Seconds of the same Kind contained in the Arc C A, as an Arc of a great Circle.

Moreover, in the Right-angled Triangle C A B, the Sides C A, and A B being given, as likewise the Right Angle at C, we find the Angle C A B; and supposing C P R perpendicular to the Line A B, A B will be to C A as C A is to A P.

But in the Right-angled Triangle C A P, we have (besides the Right Angle) the Angle A, as likewise the Side C A given; therefore As Radius is to C A, So is the Sine of the Angle C A P, to C P. And as the Number of horary Seconds of the Motion from A to B, is to the horary Seconds in the Motion from S to D, so is C P to C R. Then taking C R from C P, or else adding them together, according as A B or S D is next to the Point C, and we shall have the Quantity of P R in parts of a great Circle, which will be the Difference of the two Stars Declinations. We have no regard here to the Difference of Motion through the Spaces A B and S D, caused by the Difference of Declination, because it is of no Consequence in the Difference of Declinations, as they are observed by the Micrometer.

Finally, As A B is to A P, So is the Number of horary Seconds of the observed Motion of the Star A through the Space A B, to the Number of Seconds of the Motion of the said Star through the Space A P. Wherefore the Time when the Star A comes to P, will be known. But as the Number of Horary Seconds of the Motion through the Space A B is to the Number of Horary Seconds of the Motion through the Space S D; So is the Number of Horary Seconds of the Motion through the Space A P, to the Number of Horary Seconds of the Motion through S R.

Moreover; The Time when the Star S is in S is known, to which if the Time of the Motion through S R be added, when A and S are on the same Side the Point C, or substracted if otherwise, and the Time when the Star is in R will be had. Now the Difference of Time between the Arrivals of the Stars in P and R, that is, the Difference of the Times wherein they come to the Meridian, will be the Difference of their Right Ascensions, which by the Rule of Proportion may be reduced into Degrees and Minutes. *Note*, We have no regard here to the proper Motion of the Stars.

From hence it is easy to know how, instead of the parallel Hair A B, to use another parallel one, passing thro' A, or any other, as also a moveable Parallel, provided that they form Similar Triangles, as will be easily conceived by what hath been already said.

The aforesaid Operation may yet be done by another Method. For the parallel Hairs of the Micrometer being so disposed, that the first of the Stars may move upon one of them; and if the Time wherein the said Star crosses the Cross-Hair of the Micrometer be observed, and if moreover the Time wherein the other Star crosses the said Cross-Hair be observed, and at the same Time the moveable parallel Hair be adjusted to the second Star, no ways altering the Micrometer; we shall have, by means of the Distance between that parallel Hair, the first Star moved upon, and the moveable parallel Hair, the Distance between two parallel Circles, to the Equator, passing thro' the Places of the said Stars, which is their Difference of Declination. And if moreover, the Difference of the Times between the Passages of each of the Stars by the Cross-Hair of the Micrometer be converted into Minutes and Seconds of a Degree, the said Stars ascensional Difference will be had. This needs no Example.

But if this be required between some Star, and the Sun or Moon; as for Example, *Mercury* moving under the Sun's Disk; place the Micrometer so, that the Limb of the Sun may move along one of the parallel Hairs, and observe the Times when the Sun's antecedent and consequent Limbs, and the Center of *Mercury*, touch the Cross-Hair; then the Difference of *Mercury*'s Declination, and the Sun's Limb, by means of the moveable Hair, will be had, the Micrometer remaining fixed. And if to the Time of the Observation of the Sun's antecedent Limb, half the Time elapsed between the Passages of the antecedent and consequent Limb be added, we shall have the Time of the Passage of the Sun's Center by the Cross-Hair of the Micrometer; and by this means the Difference of the Times between the Passage of the Sun's Center and *Mercury* over the Cross-Hair, that is, by the Meridian, will be obtained. And this Difference of Time being converted into Degrees and Minutes, will give their ascensional Difference.

Moreover, since the Sun's Center is in the Ecliptick, if in the same Time as the said Center passes over the Cross-Hair (the Sun's true Place being otherwise known), you seek in Tables, the Angle of the Ecliptick with the Meridian, you will likewise have the Angle that
Fig. 11. the Ecliptick makes with the Sun's Parallel, as in *Fig.* 11. the Angle O C R, of the Ecliptick O C B, and of the Parallel to the Equator R C. Let P C be the Meridian, *Mercury* in M, the Center of the Sun in C, M R parallel to P C, and C R the Difference of Right Ascension between the Center of the Sun C, and *Mercury* in M. Now the Minutes of the Difference of the Right Ascension C R in the Parallel, being reduced to Minutes of a great Circle, say, As Radius is to the Sine Complement of the Sun's or *Mercury*'s Declination; So is the Number of Seconds of the Difference of Right Ascension, to the Number of Seconds

C R,

C R, as the Arc of a great Circle. Then in the Triangle C R T, Right-angled at R, we have the Side C R (now found); as alſo the Angle R C T, *viz.* the Difference between the Right Angle, and the Angle made by the Ecliptick and Meridian; whence the Hypothenuſe C T, and the Side R T may be found. And if R T be taken from M R, which is the Difference of Declination of *Mercury* in M, and the Center of the Sun in C, there will remain T M. Again, As C T is to T R, So is T M to T O; M O will be the Latitude of *Mercury* at the Time of Obſervation: And adding T O to the Side C T, we ſhall have C O, the Difference of Longitude between *Mercury* and the Sun's Center. Therefore the Sun's Longitude being known, that of *Murcury*'s may alſo be found.

If moreover, two or three Hours after the firſt Obſervation of *Mercury* in M, the Difference of Declination and Right Aſcenſion thereof be again obſerved, when he is come to N, we ſhall find, as before, N Q the Latitude of *Mercury*, and C Q the Difference of Longitude of him and the Sun's Center C; whence the Place of the apparent Node of *Mercury* will be had. But note, The Point of Concourſe A, in the Right Line M N, with the Ecliptick C B, is not the Place of the ſaid Node, with regard to the Point C, becauſe between the Obſervations made in the Points M and N, the Sun by it's proper Motion is moved a few Minutes forwards, according to the Succeſſion of Signs, which notwithſtanding we have not regarded in the Obſervations. Therefore ſay, As the Difference of the Latitudes M O and N Q, to O Q, *minus* the proper Motion of the Sun, between the Obſervations made in M and N; So is M O to the Diſtance O A, whence the true Diſtance from the Sun's Center C to *Mercury*'s Node A will be had. *Note*, The proper Motion of the Sun between the Obſervations muſt be taken from O Q, becauſe during that Time *Mercury* is retrograde; but if it's Motion had been direct, the Sun's Motion muſt have been added to O Q.

In the Obſervations of *Mercury*'s Paſſage under the Sun's Diſk, we have had no regard to the proper Motion of the Sun, as being of ſmall conſequence; but if it is required to be brought into Conſideration, C O and C Q muſt be diminiſhed by ſo much of the Sun's proper Motion, as is performed in the Interval of Time between the Paſſage of the Sun's Center and *Mercury*, by the Meridian.

By the ſame Method, the Diſtances of Planets from each other, or from fixed Stars near the Ecliptick, may be obſerved; nevertheleſs, excepting ſome Minutes, not only upon the account of the proper Motions of the Stars, but alſo becauſe of their Diſtance from the Ecliptick or too great Latitude. *Note*, This ſecond Method for finding the Difference of Declination and Right Aſcenſion is not exacter than the former, altho' it is performed with leſs Calculation: for it is ſo difficult to diſpoſe the Hairs of the Micrometer according to the Parallel of the Diurnal Motion, that it cannot be done, but by ſeveral uncertain trials.

M. *de la Hire* hath alſo invented another Micrometer, whoſe Conſtruction is eaſy; for it is only a Pair of proportional Compaſſes, whoſe Legs on one Side, are, for Example, ten Times longer than thoſe on the other Side. The ſhorteſt Legs of theſe Compaſſes muſt be put thro' a Slit made in the Tube of the Teleſcope, and placed ſo in the Focus of the Object-Glaſs, that the two Points, which ought to be very fine, may be applied to all Objects repreſented in the ſaid Focus. Then if the Angle ſubtended by the Diſtance of two Objects in the Focus of the Object-Glaſs be required to be found by means of theſe Compaſſes, you muſt ſhut or open the two ſhorteſt Legs 'till their Points juſt touch the Repreſentations of the Objects; and keeping the Compaſſes to this Opening, if the longeſt Legs be applied to the Diviſions of a Scale, the Minutes and Seconds contained in the Angle ſubtended by the Diſtance of the aforeſaid Objects will be had. The Manner of dividing the ſaid Scale, is the ſame as that for finding the Diſtances of the parallel Hairs of the other Micrometer, in ſaying by the Rule of Proportion, As the Number of Lines contained in the Focal Length of the Object-Glaſs, is to one Line; So is Radius to the Tangent of the Angle ſubtended by one Line in the Focus: therefore if the longeſt Legs be ten Times longer than the others, ten Lines on the Scale will meaſure the ſaid Angle ſubtended by one Line, which being known, it will be eaſy to divide the Scale for Minutes and Seconds. Fig. 12.

This Micrometer may be uſed for taking the apparent Diameters of the Planets; as alſo to take the Diſtances of fixed Stars which are near each other, and meaſure ſmall Diſtances on Earth.

CHAP. III.

Of making Celeſtial Obſervations.

OBſervations of the Sun, Stars, *&c.* made in the Day-Time with long Teleſcopes, are eaſy, becauſe the Croſs-Hairs in the Focus of the Object-Glaſs may then be diſtinctly perceived; but in the Night the ſaid Croſs-Hairs muſt be enlightened with a Link, or Candle,

Candle, that so one may see them with the Stars, thro' the Telescope: and this is done two ways.

First, We enlighten the Object-Glass of the Telescope, in obliquely bringing a Candle near to it, that so it's Smoke or Body do not hinder the Progress of the Rays coming from the Star. But if the Object-Glass be something deep in the Tube, it cannot sufficiently be enlightened, without the Candle's being very near it, and this hinders the Sight of the Star; and if the Telescope is above six Feet long, it will be difficult sufficiently to enlighten the Object-Glass, that so the cross Hairs be distinctly perceived.

Secondly, We make a sufficient Opening in the Tube of the Telescope near the Focus of the Object-Glass, thro' which we enlighten with a Candle the cross Hairs placed in the Focus.

But this Method is subject to several Inconveniences, for the Light being so near the Observator's Eyes, he is often incommoded thereby. And moreover, since the cross Hairs are by that Opening uncovered and exposed to the Air, they lose their Situation, become slack, or may be broken.

Besides this, the said second Method is liable to an Inconveniency for which it ought to be entirely neglected; and that is, that it is subject to an Error, which is, that according to the Position of the Light illuminating the cross Hairs, the said Hairs will appear in different Situations: because, for Example, when the Horizontal Hair is enlightened above, we perceive a luminous Line, which may be taken for the said Hair, and which appears at it's upper Superficies. And contrariwise, when the said Hair is enlightened underneath, the luminous Line will appear at it's lower Superficies, the Hair not being moved; and this Error will be the Diameter of the Hair, which often amounts to more than six Seconds. But M. *de la Hire* hath found a Remedy for this Inconveniency. For he often found, in Observations made in Moonshine Nights, in Weather a little foggy, that the cross Hairs were distinctly perceived; whereas, when the Heavens were serene, they could scarcely be seen: whence he bethought himself to cover that End of the Tube next to the Object-Glass with a Piece of Gawze, or very fine white silken Crape; which succeeded so well, that a Link placed at a good Distance from the Telescope so enlightened the Crape, that the cross Hairs distinctly appeared, and the Sight of the Stars was no way obscured.

Solar Observations cannot be made without placing a smoked Glass between the Telescope and the Eye, which may thus be prepared. Take two equal and well polished round Pieces of flat Glass, upon the Surface of one of which, all round it's Limb, glew a Pasteboard Ring; then put the other Piece of Glass into the Smoke of a Link, taking it several times out, and putting it in again, for fear lest the Heat of the Link should break it, until the Smoke be so thick thereon, that the Link can scarcely be seen thro' it: but the Smoke must not be all over it of the same Thickness, that so that Place thereof may be chosen answering to the Sun's Splendor. This being done, this Glass thus blackened, must be glewed to the before-mentioned Pasteboard Ring, with it's blackened Side next to the other Glass, that so the Smoke may not be rubbed off.

Note, When the Sun's Altitude is observed thro' a Telescope, consisting of but two Glasses, it's upper Limb will appear as tho' it were the lower one.

There are two principal Kinds of Observations of Stars, the one being when they are in the Meridian, and the other when they are in vertical Circles.

If the Position of the Meridian be known, and then the Plane of the Quadrant be placed in the Meridian Circle, by means of the plumb Line suspended at the Center, the Meridian Altitudes of Stars may be easily taken, which are the principal Operations, serving as a Foundation to the whole Art of Astronomy. The Meridian Altitude of a Star may likewise be had by means of a Pendulum Clock, if the exact Time of the Star's Passage by the Meridian be known. Now it must be observed, that Stars have the same Altitude during a Minute before and after their Passage by the Meridian, if they be not in or near the Zenith; but if they be, their Altitudes must be taken every Minute, when they are near the Meridian, which we suppose already known, and then their greatest or least Altitudes will be the Meridian Altitudes sought.

As to the Observations made without the Meridian in Vertical Circles, the Position of a given Vertical Circle must be known, or found by the following Method.

First, The Quadrant and it's Telescope remaining in the same Situation wherein it was when the Altitude of a Star, together with the Time of it's Passage by the Intersection of the cross Hairs in the Focus of the Object-Glass, was taken, we observe the Time when the Sun, or some fixed Star, whose Latitude and Longitude is known, arrives to the Vertical Hair in the Telescope; and from thence the Position of the said Vertical Circle will be had, and also the observed Star's true Place.

But if the Sun, or some other Star, does not pass by the Mouth of the Tube of the Telescope, and if a Meridian Line be otherwise well drawn upon a Floor, or very level Ground, in the Place of Observation, you must suspend a Plumb-Line to some fixed Place, about three or four Toises distant from the Quadrant, under which upon the Floor must a Mark be made in a right Line with the Plumb-Line. This being done, you must put a thin Piece of Brass, or Pasteboard, very near the Object-Glass, in the Middle of which there

is

is a ſmall Slit vertically placed, and paſſing thro' the Center of the Circular Figure of the Object-Glaſs. Now by means of this Slit, the beforementioned Plumb-Line may be perceived thro' the Teleſcope, which before could not be ſeen, becauſe of its Nearneſs thereto. Then the Plumb-Line muſt be removed and ſuſpended, ſo that it be perceived in a right Line with a vertical Hair in the Focus of the Object-Glaſs, and a Point marked on the Floor directly under it. And if a right Line be drawn thro this Point, and that marked under the Plumb-Line before it was removed, the ſaid Line will meet the Meridian drawn upon the Floor; and ſo we ſhall have the Poſition of the vertical Circle the obſerved Star is in, with reſpect to the Meridian, the Angle whereof may be meaſured in aſſuming known Lengths upon the two Lines from the Point of Concourſe; for if thro' the Extremities of theſe known Lengths, a Line or Baſe be drawn, we ſhall have a Triangle, whoſe three Sides being known, the Angle at the Vertex may be found, which will be the Angle made by the Vertical Circle and Meridian.

The Manner of taking the Meridian Altitudes of Stars.

It is very difficult to place the Plane of the Quadrant in the Meridian exactly enough to take Meridian Altitude of a Star; for unleſs there be a convenient Place and a Wall, where the Quadrant may be firmly faſtened in the Plane of the Meridian, which is very difficult to do, we ſhall not have the true Poſition of the Meridian, proper to obſerve all the Stars, as we have mentioned already. Therefore it will be much eaſier, and principally in Journeys, to uſe a portable Quadrant, by means of which the Altitude of a Star muſt be obſerved a little before its Paſſage over the Meridian, every Minute, if poſſible, until its greateſt or leaſt Altitude be had. Now, tho' by this means we have not the true Poſition of the Meridian, yet we have the apparent Meridian Altitude of the Star.

Altho' this Method is very good, and free from any ſenſible Error, yet if a Star paſſes by the Meridian near the Zenith, we cannot have its Meridian Altitude, by repeated Obſervations every Minute, unleſs by chance; becauſe in every Minute of an Hour the Altitude augments about fifteen Minutes of a Degree: and in theſe kind of Obſervations, the inconvenient Situation of the Obſervator, the Variation of the Star's Azimuth ſeveral Degrees in a little time, the Alteration that the Inſtrument muſt have, and the Difficulty in well replacing it vertically again, hinders our making of Obſervations oftner than in every fourth Minute of an Hour; during which Time the Difference in the Star's Altitude will be one Degree. Therefore in theſe Caſes it will be better to have the true Poſition of the Meridian, or the exact Time a Star paſſes by the Meridian, in order to place the Inſtrument in the ſaid Meridian, or move it ſo that one may obſerve the Altitude of the Star the Moment it paſſes by the Meridian.

Of Refractions.

The Meridian Altitudes of two fixed Stars, which are equal, or a ſmall matter different, the one being North, and the other South, being obſerved, and alſo their Declination otherwiſe given; to find the Refraction anſwering to the Degrees of Altitude of the ſaid Stars, and the true Height of the Pole, or Equator, above the Place of Obſervation.

Having found the apparent Meridian Altitude of ſome Star near the Pole (by the aforegoing Directions) if the Complement of the ſaid Star's Declination be added thereto, or taken therefrom, we ſhall have the apparent Height of the Pole. After the ſame manner may alſo the apparent Height of the Equator be found, by means of the Meridian Altitude of ſome Star near the Equator, in adding or ſubſtracting its Declination.

Then theſe Heights of the Pole and Equator being added together, their Sum will always be greater than a Quadrant; but 90 Degrees being taken from this Sum, the Remainder will be double the Refraction of either of the Stars obſerved at the ſame height: and therefore taking the ſaid Refraction from the ſaid apparent Height of the Pole, or Equator, we ſhall have their true Altitude.

Example.

Let the Meridian Altitude of a Star obſerved below the North Pole, be 30 Deg. 15 Min. and the Complement of its Declination 5 deg. whence the apparent Height of the Pole will be 35 Deg. 15 Min. Alſo let the apparent Meridian Altitude of ſome other Star, obſerved near the Equator, be 30 Deg. 40 Min. and its Declination 40 Deg. 9 Min. whence the apparent Height of the Equator will be 54 Deg. 49 Min. Therefore the Sum of the Heights of the Pole and Equator thus found, will be 90 Deg. 4 Min. from which ſubſtracting 90 Deg. and there remains 4 Min. which is double the Refraction at 30 Deg. 28 Min. of Altitude, which is about the middle between the Heights found: therefore at the Altitude of 30 Deg. 15 Min. the Refraction will be ſomething above 2 Min. *viz.* 2 Min. 1 Sec. and at the Altitude of 30 Deg. 40 Min. the Refraction will be 1 Min. 59 Sec.

Laſtly, If 2 Min. 1 Sec. be taken from the apparent Height of the Pole 35 Min. 15 Sec. the Remainder 35 Deg. 12 Min. 59 Sec. will be the true Height of the Pole; and ſo the true Height of the Equator will be 54 Deg. 47 Min. 1 Sec. as being the Complement of the Height of the Pole to 90 deg.

Note, The Refraction and Height of the Pole found according to this way, will be so much the more exact, as the Altitude of the Stars is greater; for if the Difference of the Altitudes of each Star should be even 2 Deg. when their Altitudes are above 30 Deg. we may by this Method have the Refraction, and the true Height of the Pole, because in this Case the Difference of Refraction in Altitudes differing two Degrees, is not sensible.

Another Way of observing Refractions.

The Quantity of Refraction may also be found by the Observations of one Star only, whose Meridian Altitude is 90 Deg. or a little less; for the Height of the Pole or Equator above the Place of Observation being otherwise known, we shall have the Star's true Declination, by it's Meridian Altitude; because Refractions near the Zenith are insensible.

Now if we observe by a Pendulum the exact Times when the said Star comes to every Degree of Altitude, as also the Time of it's Passage by the Meridian, which may be known by the equal Altitudes of the Star being East and West, we have three things given in a spherical Triangle, *viz.* the Distance between the Pole and Zenith, the Complement of the Star's Declination, and the Angle comprehended by the aforesaid Arcs; namely, the Difference of mean Time between the Passage of the Star by the Meridian and it's Place, converted into Degrees and Minutes; to which must be added, the convenable proportional Part of the mean Motion of the Sun in the Proportion of 59 Min. 8 Sec. *per* Day: therefore the true Arc of the Vertical Circle between the Zenith and the true Place of the Star may be found.

But the apparent Arc of the Altitude of the Star is had by Observation, and the Difference of these Arcs will be the Quantity of Refraction at the Height of the Star. By a like Calculation the Refraction of every Degree of Altitude may be found.

The same may be done by means of the Sun, or any other Star, provided it's Declination be known, to the End that at the Time of Observation the true Distance of the Sun or Star from the Zenith may be found.

The Refractions of Stars being known, it will then be easy to find the Height of the Pole; for having observed the Meridian Altitude of the Polar Star, as well above as below the Pole, the same Day, and having diminished each Altitude by it's proper Refraction, half of the Difference of the corrected Altitudes, added to the lesser Altitude corrected, or substracted from the greater Altitude thus corrected, will give the true Height of the Pole.

M. *de la Hire* has observed with great Care for several Years the Meridian Altitudes of fixed Stars, and principally of *Sirius*, and *Lucida Lyra*, with Astronomical Quadrants very well divided, and very good Telescopes at different Hours of the Day and Night, and at different Seasons of the Year; and he assures us, that he never found any Difference in their Altitudes, but what proceeded from their proper Motion.

And because *Sirius* comes to about the 26th Degree of the Meridian, we might doubt whether in the lesser Altitudes the Refractions in the Winter would be greater than those in the Summer; hence he also observed, with the late M. *Picard*, the lesser Meridian Altitudes of the Star *Capella*, which is about 4 ½ Degrees at several different Times of the Year.

Having compared these different Observations together, and made the necessary Reductions, because of the proper Motion of that Star, there was scarcely found one Minute of Difference, that could proceed from any other Cause but Refraction. Therefore he made but one Table of the Refraction of the Sun, Moon, and the Stars, for all Times of the Year, conformable to the Observations that he made from them.

Notwithstanding this, one would think that Refractions nigh the Horizon are subject to divers Inconstancies, according to the Constitution of the Air, and the Nature of high or low Grounds, as M. *de la Hire* has often found; for observing the Meridian Altitudes of Stars at the Foot of a Mountain, which seemed to be even with the Top of it, they appeared to him a little higher, than if he had observed them at the Top: But if the Observations of others may be depended upon, Refractions are greater, even in Summer, in the frozen Zones, than in the temperate Zones.

How to find the Time of the Equinox and Solstice by Observation.

Having found the Height of the Equator, the Refraction and the Sun's Parallax at the same Altitude, it will not afterwards be difficult to find the Time in which the Center of the Sun is in the Equator; for if from the apparent Meridian Altitude of the Center of the Sun, the same Day as it comes to the Equinox, be taken the convenient Refraction, and then the Parallax be added thereto, the true Meridian Altitude of the Sun's Center will be had. Now the Difference of this Altitude, and the Height of the Equinoctial, will shew the Time of the true Equinox before or after Noon: and if the Sum of the Seconds of that Difference be divided by 59, the Quotient will shew the Hours and Fractions which must be added or substracted from the true Hour of Noon, to have the Time of the true Equinox.

The Hours of the Quotient must be added to the Time of Noon, if the Meridian Altitude of the Sun be lesser than the Height of the Equator about the Time of the vernal Equinox; but

but they muſt be ſubſtracted, if it be found greater. You muſt proceed contrariwiſe, when the Sun is near the autumnal Equinox.

Example. The true Height 41 Deg. 10 Min. of the Equator being given, and having obſerved the true Meridian Altitude 41 Deg. 5 Min. 15 Sec. of the Sun, found by the apparent Altitude of it's upper or lower Limb, corrected by it's Semidiameter, Refraction, and Parallax, and the Difference will be 4 Min. 45 Sec. or 285 Seconds, which being divided by 59, the Quotient will be $4\frac{49}{59}$; that is, 4 Hours 48 Minutes, which muſt be added to Noon, if the Sun be in the vernal Equinox, and conſequently the Time of the Equinox will happen 4 Hours 48 Minutes after Noon. But if the Sun was in the autumnal Equinox, the Time of the ſaid Equinox would happen 4 Hours 48 Minutes before Noon, that is, at 12 Minutes paſt Seven in the Morning.

As to the Solſtices, there is much more Difficulty in determining them than the Equinoxes, for one Obſervation only is not ſufficient; becauſe about this Time the Difference between the Meridian Altitudes in one Day, and the next ſucceeding Day, is almoſt inſenſible.

Now the exact Meridian Altitude of the Sun muſt be taken, 12 or 15 Days before the Solſtice, and as many after, that ſo one may find the ſame Meridian Altitude by little and little; to the End that by the proportional Parts of the alteration of the Sun's Meridian Altitude, we may more exactly find the Time wherein the Sun is found at the ſame Altitude, before and after the Solſtice, being in the ſame Parallel to the Equator.

Now having found the Time elapſed between both the Situations of the Sun, you muſt take half of it, and ſeek in the Tables the true Place of the Sun at theſe three Times. This being done, the Difference of the extreme Places of the Sun muſt be added to the mean Place, in order to have the mean Place with Compariſon to the Extremes; but if the mean Place found by Calculation, does not agree with the mean Place found by Compariſon, you muſt take the Difference, and add to the mean Time, the Time anſwering to that Difference, if the mean Time found by Calculation be leſſer; but contrariwiſe, it muſt be ſubſtracted if it be greater, in order to have the Time of the Solſtice.

Example. The laſt Day of *May*, the apparent Meridian Altitude of the Sun was found at the Royal Obſervatory, 64 Deg. 47 Min. 25 Sec. and the 22d Day of *June* following, the apparent Meridian Altitude was found 64 Deg. 28 Min. 15 Sec. from whence we know, by having the Difference of Declination at thoſe Times, that the Sun came to the Parallel of the firſt Obſervation, the 22d of *June*, at 4 Hours 12 Minutes in the Morning; and conſequently the mean Time between the Obſervations, was on the 22d of *June*, at 2 Hours 6 Minutes in the Morning.

Now by Tables, the true Place of the Sun at the Time of the firſt Obſervation, was 2 Signs 18 Deg. 58 Min. 23 Sec. and at the Time of the laſt it was 3 Signs, 11 Deg. 4 Min. 52 Sec. and in the middle Time 3 Signs, 1 Min. 56 Sec. But the Difference of the two extreme Places is 22 Deg. 6 Min. 29 Sec. half of which is 11 Deg. 3 Min 15 Sec. which added to the mean Place, makes 3 Signs, 1 Min. 38 Sec. which is the mean Place with compariſon to the Extremes. Again, The Difference between the mean Place, by Calculation 3 Signs, 1 Min. 56 Sec. and the mean Place by Compariſon, is 18 Seconds, which anſwers to 7 Min. 18 Sec. of Time, which muſt be taken from the mean Time, becauſe the mean Place by Calculation is greater than the mean Place by Compariſon. Therefore the Time of the Solſtice was the 11th of *June*, at 1 Hour, 58 Min. 18 Sec. in the Morning.

Note, The Error of a few Seconds, in the obſerved Altitude of the Sun, will cauſe an alteration of an Hour in the true Time of the Solſtice; as in the propoſed Example, 10 Seconds, or thereabouts, in Altitude, will cauſe an Error of an Hour; whence the true Time of the Solſtice cannot be had but with Inſtruments well divided, and ſeveral very exact Obſervations.

Obſervations made in the Royal Obſervatory at Paris, *about the Time of the Solſtice for finding the Height of the Pole, and the Sun's greateſt Declination or Obliquity of the Ecliptick.*

	Deg.	*Min.*	*Sec.*
The apparent Meridian Altitude of the upper Limb of the Sun at the Time of the Summer Solſtice, gathered from ſeveral Obſervations, is found -	64	55	24
Refraction to be ſubſtracted - - - - - - -	00	00	33
Parallax to be added - - - - - - - -	00	00	01
True Altitude of the upper Limb of the Sun - - - -	64	54	52
Semidiameter of the Sun - - - - - - - -	00	15	49
True Meridian Altitude of the Sun's Center - - - -	64	39	03
At the Time of the Winter Solſtice, the apparent Meridian Altitude of the upper Limb of the Sun - - - - - - - - - -	18	00	24
Refraction to be ſubſtracted - - - - - -	00	03	12
Parallax to be added - - - - - - - -	00	00	05
True Altitude of the Sun's upper Limb - - - -	17	57	17
Semidiameter of the Sun - - - - - - - - -	00	16	21

True

	Deg.	Min.	Sec.
True Meridian Altitude of the Sun's Centre - - - - - -	17	40	56
Then the true Distance of the Tropicks is - - - - -	46	58	7
The half, which is the greatest Declination of the Sun, is - -	23	29	3½
The Height of the Equator above the Observatory - - -	41	09	59½
Its Complement, which is the Height of the Pole - - -	48	50	00½

Observations of the Polar Star.

By divers Observations of the greatest and least apparent Meridian Altitudes of the Polar Star, which is in the end of the Tail of the Little Bear, it is concluded that the apparent Altitude of the Pole, as M. *Picard* has denoted it in his Book of the Dimensions of the Earth, between *St James*'s and *St Martin*'s Gates (about *S. Jaques de la Boucherie*, at *Paris*) is 48 Deg. 52 Min. 20 Sec.

	Deg.	Min.	Sec.
The Reduction being made according to the Distance of the Places, the apparent Height of the Pole at the Royal Observatory will be - -	48	51	02
The Convenable Refraction to that Height - - - - - -	00	01	04
Then the true Height of the Pole at the Observatory. - - -	48	49	58
For which let us take - - - - - - - - - - -	48	50	00
And consequently the Height of the Equator will be - - -	41	10	00

The true or apparent Time in which a Planet or fixed Star passes by the Meridian, being given, to find the Difference of Right Ascension between the fixed Star, or Planet, and the Sun.

The given Time from Noon to or from the time of the Passage of the Star or Planet by the Meridian, must be converted into Degrees, and what is required will be answered.

Example. *Jupiter* passed by the Meridian at 10 Hours, 23 Min. 15 Sec. in the Morning, whose Distance in time from Noon, which is 1 Hour, 36 Min. 45 Sec. being converted into Degrees of the Equator, will give 24 Deg. 11 Min. 15 Sec. for the Difference of Right Ascension between the Sun and *Jupiter*, in that Moment the Center of *Jupiter* passed by the Meridian.

In this, and the following Problem, we have proposed the true or apparent Time, and not the mean Time; because the true Time is easier to know by Observations of the Sun, than the mean Time. We shall explain what is meant by mean Time, as likewise true or apparent Time, in the next Chapter.

The true Time between the Passages of two fixed Stars by the Meridian being given, or else of a fixed Star and a Planet, to find their Ascensional Difference.

The given Time between their Passages by the Meridian must be converted into Degrees of the Equator, and the Right Ascension of the true Motion of the Sun answering to that time, must be added thereto; then the Sum will be the Ascensional Difference sought.

Example. Suppose between the Passages of the Great Dog, called *Sirius*, by the Meridian, and the Heart of the Lion named *Regulus*, there is elapsed 3 Hours, 20 Min. of time, and the Right Ascension of the true Motion of the Sun, let be 7 Min. 35 Sec.

Whence converting 3 Hours, 20 Min. into Degrees of the Equator, and there will be had 50 Deg. to which adding 7 Min. 35 Sec. and the Sum 50 Deg. 7 Min. 35 Sec. will be the Ascensional Difference between *Sirius* and *Regulus*.

You must proceed thus for the Ascensional Difference of a fixed Star and a Planet, or of two Planets; yet note, if the proper Motion of the Planet or Planets be considerable between both their Passages by the Meridian, regard must be had thereto.

How to observe Eclipses.

Amongst the Observations of Eclipses, we have the Beginning, the End, and the Total Emersion, which may exactly enough be estimated by the naked Eye, without Telescopes, except the Beginning and the End of Eclipses of the Moon, where an Error of one or two Minutes may be made, because it is difficult certainly to determine the Extremity of the Shadow. But the Quantity of the Eclipse, that is, the eclipsed Portion of the Sun and Moon's Disk, which is measured by Digits, or the 12th parts of the Sun and Moon's Diameter, and Minutes, or the 60th parts of Digits, cannot be well known without a Telescope joined to some Instrument. For an Estimation made with the naked Eye is very subject to Error, as it is easy to see in the History of ancient Eclipses, altho they were observed by very able Astronomers.

The Astronomers who first used Telescopes furnished with but two Glasses, namely, a Convex Object-Glass, and a Concave Eye-Glass, in the Observations of Eclipses, observed those of the Sun in the following manner. They caused a hole to be made in the Window-shutter of a Room, which Room in the Day-time, when the Shutters were shut, was darkened thereby; thro' which Hole they put the Tube of a Telescope, in such manner, that the Rays of the Sun, passing thorough the Tube, might be received upon a white piece of Paper, or a Table-Cloth, upon which was firſt deſcribed a Circle of a convenable bigneſs, with five other Concentric Circles, equally diſtant from one another, which, with the Center, divided a Diameter of the outward Circle into 12 equal Parts. Then having adjuſted the Table-Cloth perpendicular to the Situation of the Tube of the Teleſcope, the luminous Image of the Sun was caſt upon the Table-Cloth, which would ſtill be greater according as the Table-Cloth was more diſtant from the Eye-Glaſs of the Teleſcope; whence by moving the Tube forwards and backwards, they found a Place where the Image of the Sun appeared exactly equal to the outward Circle, and at that Diſtance they fixed the Table-Cloth, with the Tube of the Teleſcope, which compoſed the Inſtrument for the ſaid Obſervation. Afterwards they moved the Tube according to the Sun's Motion, to the End that the luminous Limb of it's Diſk might every where touch the outward Circle deſcribed upon the Table-Cloth, by which means the Quantity of the eclipſed Portion was ſeen, and it's greateſt Obſcurity meaſured by the Concentric Circles; they denoted the Hour of every Phaſe, by a Second Pendulum Clock, rectified and prepared for that purpoſe. The ſame Method is ſtill obſerved by many Aſtronomers, who uſe alſo a Circular Reticulum, made with ſix Concentric Circles upon very fine Paper, which muſt be oiled, to render the Sun's Image more ſenſible. The greateſt of the Circles ought exactly to contain the Image of the Sun in the Focus of the Object-Glaſs of a Teleſcope of 40 or 60 Feet; the ſix Circles are equally diſtant, and divide the Diameter of the Sun in twelve equal Digits. When the Paper is placed in the Focus of a great Teleſcope, the enlightened part of the Sun will very diſtinctly be ſeen; then the Eye-Glaſs is not uſed.

There are others who uſe a Teleſcope furniſhed with two Convex-Glaſſes, from whence the ſame Effect follows. But altho' the Uſe of a Teleſcope in this manner be very proper to obſerve Eclipſes of the Sun, yet it is not fit to obſerve Eclipſes of the Moon, becauſe it's Light is not ſtrong enough. Laſtly, Others place a Micrometer in the common Focus of the Convex Lenſes. Beſides the Quantity of the Phaſes of the Eclipſes of the Sun and Moon (eaſily known by the ſaid Micrometer), we may have the Diameters of the Luminaries, and the Proportion of the Earth's Diameter to the Moon's, as well by the obſcure Protion of it's Diſk, as by the luminous Portion and the Diſtance between it's Horns.

The Method of obſerving Eclipſes by means of the Micrometer will be much better, if the Diviſions to which the parallel Hairs are applied be made ſo, that ſix Intervals of the Hairs, may contain the Diameter of the Sun or Moon. For the moveable Hair poſited in the Middle of the Diſtance between the immoveable ones (which is not difficult to do), will ſhew the Digits of the Eclipſe.

The ſame Teleſcope and Micrometer may ſerve for all the other Obſervations, and to meaſure Eclipſes; as, to obſerve the Paſſage of the Earth's Shadow over the Spots of the Moon, in Lunar Eclipſes.

There yet remains one conſiderable Difficulty, and that is, to make a new Diviſion of the Micrometer ſerving as a common Reticulum for all Obſervations; for it ſcarcely happens in an Age in two Eclipſes, that the apparent Diameters of the Sun and Moon are the ſame.

Therefore M. *de la Hire* has invented a new Reticulum, which having all the Uſes of the Micrometer, may ſerve to obſerve all Eclipſes, it being adapted to all apparent Diameters of the Sun and Moon, and it's Diviſions are firm and ſolid enough to reſiſt all the Viciſſitudes of the Air, altho' they are as fine as Hairs.

The Conſtruction and Uſe of this Reticulum is thus: Firſt, Take two Object Lenſes of Teleſcopes of the ſame Focus, or nighly the ſame, which join together. As for Example, The Focus of two Lenſes together of eight Feet, which is the fit Length of a Teleſcope for obſerving Eclipſes, unleſs the Beginning and the End of Solar ones, which require a longer Teleſcope exactly to determine them.

Secondly, We find from Tables, that the greateſt Diameter of the Moon at the Altitude of 90 Deg. is 34 Min. 6 Sec. To which adding 10 Sec. and there will ariſe 34 Min. 16 Sec. Therefore ſay, As Radius is to the Tangent of 17 Min. 8 Sec. (the half of 34 Min. 16 Sec.) So is 8 Feet, or the focal Length of the two Lenſes to the Parts of a Foot, which doubled will ſubtend an Angle of 34 Min. 16 Sec. in the Focus of the Teleſcope, and this will be the Diameter of the ſaid Circular Reticulum.

Thirdly, Upon a very flat, clear, and well poliſhed Piece of Glaſs, deſcribe lightly with the Point of a Diamond, faſtened to one of the Legs of a Pair of Compaſſes, ſix Concentric Circles, equally diſtant from each other. The Semidiameter of the greateſt and laſt let be equal to the fourth Term before found. Likewiſe draw two Diameters to the greateſt Circle at Right Angles. The flat Piece of Glaſs being thus prepared and put into the Tube, of which we have before ſpoken, and in the Focus of the Teleſcope, will be a very proper Reticulum for obſerving Solar and Lunar Eclipſes, and it will divide all the apparent Diameters into twelve equal Parts or Digits, as we are now going to explain.

It is manifeſt from Dioptricks, that all Rays coming from Points of a diſtant Object, after their Refraction by two convex Lenſes, either joined or ſomething diſtant from each other, will be painted in the common Focus of the ſaid Lenſes, which will appear ſo much the greater, according as the Lenſes be diſtant from one another; ſo that they will appear the smallest when the Lenses are joined together. Therefore if the Object-Glasses used in this Construction, be each put into a Tube, and one of these Tubes slides within the other; then the said Lenses being thus joined, the Image of a distant Object, whose Rays fall upon the Lenses under an Angle of 34 Min. 16 Sec. will exceed the Moon's greatest apparent Diameter by 10 Sec. Therefore in moving the Lenses by little and little, such a Position may be found, wherein the Diameter of the greatest Circle on the Reticulum posited in the Focus, will answer to an Angle of 34 Min. 16 Sec. For the Image of an Object perceived under a less Angle, may be equal to the Image of the same Object perceived under a greater Angle, according to the different Lengths of the Foci. But the Reticulum is in a separate Tube, and so it may be removed at a Distance at pleasure from the Object-Glasses. We now proceed to lay down two different Ways of finding the Positions of [the Lenses and Reticulum, proper to receive the different Diameters of the Sun and Moon.

First, In a very level and proper Place for making Observations with Glasses, place a Board, with a Sheet of Paper thereon, directly exposed to the Tube's Length, having two black Lines drawn upon it parallel to each other, and at such a Distance from each other, that it subtends an Angle of 34 Min. 6 Sec. so that the Distance of the said two Lines, represented iu the Focus of the Object-Glasses, may likewise subtend an Angle of 34 Min. 6 Sec. And this may be found in reasoning thus, (as we have already done for the Micrometer). As Radius is to the Tangent of 17 Min. 3 Sec. So is the Distance from the Tube of the Object-Glasses to the Broad, To half the Distance that the parallel Lines on the Paper must be at. And thus we shall find by Experience the Place of each Object-Glass, and the Reticulum in the common Focus, in such manner that the Representation of the two black Lines on the Paper, embraces entirely the Diameter of the greatest Circle of the said Reticulum. Now we set down 34 Min. 6 Sec. upon the Tubes, in each Position of the Lenses and their Foci, or the Reticulum, that so the Lenses and Reticulum may be adjusted to their exact Distance, every time an Angle of 34 Deg. 6 Min. is made use of.

Again, Let the said Board and white Paper be placed further from the Tube, in such manner, that the Distance between the parallel Lines on the Paper subtend, or is the Base of an Angle of 33 Min. for example, whose Vertex is at the Lenses of the Telescope: which may be done, in saying, As the Targent of 16 Min. 30 Sec. is to Radius; So is half the Interval of the parallel Lines on the Paper, To the Distance of the Board from the Lenses. Now in this Position of the Telescope and Board, the Position of the Lenses and Reticulum between themselves must be found; so that the Representation of the parallel Lines, which appear very distinctly in the Focus of the Lenses, occupies the whole Diameter of the greatest Circle on the Reticulum. This being done, the Number 33 Min. must be made upon the Tubes, in the Places wherein each of the Lenses and Reticulum ought to be. Proceed in this manner for the Angles of 32 Min. 31 Min. 30 Min. and 29 Min.

If the Distances, denoted upon the Tubes between the different Positions of the Lenses and the Reticulum, answering to a Minute, be divided into 60 equal Parts, we shall have their Positions for every Second; and by this means the same Circle of the Reticulum may be accommodated to all the different apparent Diameters of the Sun and Moon, and the Diameter of the greatest Circle being divided into 12 equal Parts, it will serve to measure the Quantities of all solar and lunar Eclipses.

The second Method taken from Opticks, being not founded upon so great a Number of Experiments as the former, may perhaps appear easier to some Persons; for the Foci of both the Lenses being known, say, As the Sum of the focal Lengths of the Lenses (whether they be equal or not) less the Distance between the Lenses, is to the focal Length of the outward Lense, less the Distance between the Lenses; So is this same Term, to a fourth, which being taken from the focal Length of the outward Lens, there remains the Distance from the outward Lens, to the common Focus of the Lenses, which is the Place of the Reticulum.

The Position of the common Focus of the Lenses may also be known by this Method; when they be joined, in using the aforesaid Analogy, without having any regard to the Distance between the Lenses, which is computed from the Places of the Lenses Centers; therefore in supposing several different Distances between the Object-Lenses, the Length of their Foci will he had, that is, the Place of the Reticulum, correspondent to each Distance.

Again, say, As the known focal Length is to the Semidiameter of the Reticulum, be it what it will; So is Radius, to the Tangent of the Angle answering to the Semidiameter of the Reticulum. By this Method we may likewise have the Magnitude of the said Reticulum, in saying, As Radius is to the Tangent of an Angle of 17 Min. 3 Sec. So is the focal Length of the Lenses, to the Semidiameter of the outward concentrick Circle. Having thus found the Minutes and Seconds subtended by the Diameter of the greatest Circle of the Reticulum, according to the different Intervals of the Lenses, they must be wrote upon each Tube of the Lenses and Reticulum, and the Distances between the Terms found, divided into Seconds, as is mentioned in the former Method. And thus may the Positions of the Lenses and Reticulum be soon found, which shall contain the apparent Diameters of the Sun or Moon, according as they appear. If it be found very difficult to draw exactly the

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concentrick Circles upon the Piece of Glaſs, you need but draw thirteen right Lines thereon with the Point of a Diamond, equally diſtant and parallel to each other, with another right Line perpendicular to them; but the Length of this Perpendicular between the two extreme Parallels, muſt be equal to the Diameter of the Reticulum, found in the manner aforeſaid. This Reticulum may be uſed inſtead of one compoſed of Hairs.

A plain thin Piece of Glaſs, having Lines drawn thereon with a very fine Point of a Diamond, may likewiſe be uſed in an Aſtronomical Teleſcope, *&c.* for if it be adjuſted in its proper Frame, in the manner as is directed in the Micrometer, the Lines drawn thereon may be uſed inſtead of the parallel Hairs. I am of opinion, that the aforeſaid Reticula are very uſeful in practical Aſtronomy, they not being ſubject to the Inconſtancies of the Air, of being gnawed by Inſects, or to the Motions of the Inſtrument, which the Hairs are.

There are thoſe who prefer Hairs, to Lines drawn upon a Piece of Glaſs, whoſe Surface may cauſe ſome Obſcurity to the Objects, or if it be not very flat, there may ſome Error ariſe: but if they have a mind to avoid theſe Difficulties, which are of no conſequence, as we know by Experience, they may uſe ſtraight Glaſs-Threads, inſtead of Hairs: for ſome of theſe may be procured as fine as Hairs, and of Strength enough to reſiſt the Inconſtancies of the Air.

Although the Phaſes or Appearances of the Eclipſes of the Moon, apply'd by Aſtronomers to Aſtronomical and Geographical Uſes, may be obſerved much eaſier and exacter by our Reticulum, than by the antient Methods; yet it muſt be acknowledged, that the Immerſions into, and Emerſions of the Moon's Spots out of the Earth's Shadow, may more conveniently be obſerved, becauſe of their great Number, than the Phaſes, and that there is leſs Preparation is uſing a Teleſcope, which need be only ſix Feet in length: and in order for this, a Map of the Moon's Diſk, when it is at the full, muſt be procured, wherein are denoted the proper Names of the Spots, and principal Places appearing on its Diſk. This may be found in the reformed Aſtronomy of *R. P. Riccioli*, &c.

There are great Advantages ariſing from Obſervations of Eclipſes, for if the exact Time of the Beginning of an Eclipſe of the Moon, of it's total Immerſion in the Shadow, of it's Emerſion and it's End, as likewiſe of the Paſſage of the Earth's Shadow by the Spots on it's Surface, be obſerved, we ſhall have the Difference of Longitude of the two Places wherein the Obſervations are made; this is known to all Aſtronomers. But ſince Lunar Eclipſes ſeldom happen, ſo as that the Difference of Longitude may thereby be concluded, the Eclipſes of *Jupiter*'s Satellites may be obſerved inſtead of them; but principally of the firſt, whoſe Motion about *Jupiter* being very ſwift, one may make ſeveral Obſervations thereof during the Space of one Year; and from thence the Difference of Longitude of the two Places, wherein the ſaid Obſervations are made, may be had.

Nevertheleſs you muſt take notice, that Lunar Eclipſes may much eaſier be obſerved, than the Eclipſes of *Jupiter*'s Satellites, which cannot be eaſily and exactly done without a Teleſcope of twelve Feet in length; whereas the Phaſes of the Beginning or End, or of the Immerſion and Emerſion of Lunar Eclipſes, may be obſerved without a Teleſcope, and the Immerſions and Emerſions of its Spots with one of an indifferent length.

M. *Caſſini*, a very excellent Aſtronomer of the Academy of Sciences, publiſhed in the Year 1693, exact Tables of the Motions of *Jupiter*'s Satellites; therefore in comparing the Times of the Immerſion or Emerſion of *Jupiter*'s firſt Satellite, found by the Tables fitted for the Obſervatory (at *Paris*) with the Obſervations thereof made in any other Place, we ſhall have, by the Difference of Time, the Difference of Longitude of the Obſervatory, and the Place wherein the Obſervations were made: which may be confirmed in obſerving the ſame Phænomena in both Places.

It is proper here to inform Obſervators of one Caſe, which often hinders an exact Obſervation of *Jupiter*'s Satellites; which is, that in a ſerene Night, we often find the Light of *Jupiter* and its Satellites, obſerved thro' the Teleſcope, to diminiſh by little and little, ſo that it is impoſſible to determine exactly the true Times of the Immerſion and Emerſion of the Satellites. Now the Cauſe of this Accident proceeds from the Object-Glaſs of the Teleſcope, which is covered over with Dew, and thereby a great Number of Rays of Light, coming from *Jupiter* and its Satellites, is hinder'd from coming thro' the Object-Glaſs to the Eye. A very ſure Remedy for this, is, to make a Tube of blotting Paper; that is, a Tube about two Feet long, and big enough to go about the End of the Tube of the Teleſcope next to the Object-Glaſs, muſt be made, in rolling two or three Sheets of ſinking Paper upon each other. This Tube being adjuſted about the Tube of the Teleſcope, will ſuck in or drink up the Dew, and hinder its coming to the Object-Glaſs; and by this Means we may make our Obſervations conveniently.

CHAP.

CHAP. IV.

Of the Construction and Use of an Instrument shewing the Eclipses of the Sun and Moon, the Months and Lunar Years, as also the Epacts.

Fig. 13. THIS Instrument was invented by M. *de la Hire*, and is composed of three round Plates of Brass, or Pieces of Paste-board, and an Index which turns about a common Center upon the Face of the upper Plate, which is the least. There are two circular Bands, the one blue, and the other white, in which are made little round Holes; the outward of which shews the New Moons, and the Image of the Sun; and the inward ones, the Full Moons, and the Image of the Moon. The Limb of this Plate is divided into 12 lunar Months, each containing 29 Days, 12 Hours, 44 Minutes; but in such manner, that the End of the 12th Month, which makes the Beginning of the second lunar Year, may surpass the first New Moon by the Quantity of 4 of 179 Divisions, denoted upon the middle Plate.

Upon the Limb of this Plate is fastened an Index, one of whose Sides, which is in the *fiducial Line*, makes part of a right Line, tending to the Center of the Instrument; which Line also passes thro' the Middle of one of the outward Holes, shewing the first New Moon of the lunar Year. *Note*, The Diameter of the Holes is equal to the Extent of about 4 Degrees.

The Limb of the second Plate is divided into 179 equal Parts, serving for so many lunar Years, each of which is 354 Days, and about 9 Hours. The first Year begins at the Number 179, at which the last ends.

The Years accomplished are each denoted by their Numbers 1, 2, 3, 4, *&c.* at every fourth Division, and which make four times a Revolution to compleat the Number 179, as may be seen in the Figure of this Plate. Each of the lunar Years comprehend four of the aforesaid Divisions: So that in this Figure they anticipate one upon the other four of the said 179 Divisions of the Limb.

Upon the Limb of the same Plate, under the Holes of the first, there is a Space coloured black, answering to the outward Holes, and which shews the Eclipses of the Sun, and another red Space, answering to the innermost Holes, shewing the Eclipses of the Moon. The Quantity of each Colour appearing through the Holes, shews the Bigness of the Eclipse. The Middle of the two Colours, which is the Middle of the Moon's Node, answers on one Side to the Division marked $4\frac{2}{3}$ of a Degree; and on the other Side it answers to the opposite Number.

The Figure of the coloured Space is shewn upon this second Plate, and it's Amplitude or Extent shews the Limits of Eclipses.

The third and greatest Plate, which is underneath the others, contains the Days and Months of common Years. The Divisions begin at the first Day of *March*, to the End that a Day may be added to the Month of *February*, when the Year is *Bissextile*. The Days of the Year are described in form of a Spiral, and the Month of *February* goes out beyond the Month of *March*, because the lunar Year is shorter than the solar one; so that the 15th Hour of the 10th Day of *February* answers to the Beginning of *March*: But after having reckoned the last Day of *February*, you must go back again to have the first of *March*. There are thirty Days marked before the Month of *March*, which serve to find the Epacts.

Note, That the Days, as they are here taken, are not accomplished pursuant to the Use of Astronomers, but as they are vulgarly reckoned, beginning on a Minute, and ending at the Minute of the following Day. Therefore every time that the first Day, or any other of a Month is spoken of, we understand the Space of that Day marked in the Divisions; for we here reckon the current Days according to vulgar Use.

In the Middle of the upper Plate are wrote the *Epochs*, shewing the Beginning of the lunar Years, with respect to the solar Years, according to the *Gregorian* Calendar, and for the Meridian of *Paris*. The Beginning of the first Year, which must be denoted by 0, and answers to the Division 179, happened in the Year 1680 at *Paris*, the 29th of *February* at $14\frac{1}{2}$ Hours. The End of the first lunar Year, being the Beginning of the second, answers to the Division marked 1, which happened at *Paris* in the Year 1681, the 27th of *February*, at $23\frac{1}{2}$ Hours, in counting successively 24 Hours from one Minute to the other. And lest there should be an Error in comparing the Divisions of the Limb of the second Plate with the Divisions of the *Epochs* of the lunar Years answering them, we have put the same Numbers to them both.

We

We have set down successively the *Epochs* of all the lunar Years, from the Year 1700 to the Year 1750, to the End that the Use of this Instrument may more easily serve to make each of the aforesaid solar and lunar Years agree together. As to the other Years of our Cycle of 179 Years, it will be easy to render it compleat, in adding 354 Days, 8 Hours, 48 $\frac{2}{3}$ Minutes for each lunar Year.

The Index extending itself from the Center of the Instrument to the Limb of the greatest Plate, serves to compare the Divisions of one Plate with those of the two others. And if this Instrument be applied to a Clock, a perfect and accomplished Instrument in all it's Parts will be had.

The Table of *Epochs*, which is fitted for the Meridian of *Paris*, may easily be reduced to other Meridians; if for the Places eastward of *Paris*, the Time of the Difference of Meridians be added; and for Places westward, the Time of the Difference of Meridians be substracted.

It is proper to place the Table of *Epochs* in the Middle of the upper Plate, to the End that it may be seen with the Instrument.

How to make the Divisions upon the Plates.

The Circle of the greatest Plate is so divided, that 368 Deg. 2 Min. 42 Sec. may comprehend 354 Days, and something less than 9 Hours; from whence it is manifest, that the Circle must contain 346 Days, 15 Hours, which may without sensible Error be taken for $\frac{2}{3}$ of a Day. Now to divide a Circle into 346 $\frac{2}{3}$ equal Parts, reduce the whole into third Parts, which in this Example make 1040; then seek the greatest Multiple of 3 less than 1040, which may be halved. Such a Number will be found in a double Geometrical Progression, whose first Term is 3; as for Example, 3, 6, 12, 24, 48, 96, 192, 384, 768.

Now the 9th Number of this Progression is the Number sought. Then substract 768 from 1040, there will remain 272, and find how many Degrees, Minutes, and Seconds, this remaining Number makes; by saying, As 1040 is to 360 Deg. So is 272 to 94 Deg. 9 Min. 23 Sec.

Therefore take an Angle of 94 Deg. 9 Min. 23 Sec. from the said Circle, and divide the remaining Part of the Circle always into half, after having made 8 Subdivisions, you will come to the Number 3, which will be the Arc of one Day; by which likewise dividing the Arc of 94 Deg. 9 Min. 23 Sec. the whole Circle will be found divided into 346 $\frac{2}{3}$ Days; for there will be 256 Days in the greatest Arc, and 90 $\frac{2}{3}$ Days in the other. Each of these Spaces answer to 1 Deg. 2 Min. 18 Sec. as may be seen in dividing 360 by 346 $\frac{2}{3}$, and ten Days make 10 Deg. 23 Min. And thus a Table may be made, serving to divide the Plate.

Those Days are afterwards distributed to each of the Months of the Year, according to the Number corresponding to them, in beginning at the Month of *March*, and continuing on to the 15th Hour of the 10th of *February*, which answers to the Beginning of *March*, and the other Days of the Month of *February* go on farther above *March*.

The Circle of the Second Plate must be divided into 179 equal Parts; to do which, seek the greatest Number which may be continually bisected to Unity, and be contained exactly in 179: you will find 128 to be this Number, which take from 179, and there remains 51. Now find what part of the Circumference of the Circle the said Remainder makes; in saying, As 179 Parts is to 360 Deg. So is 51 Parts to 102 Deg. 34 Min. 11 Sec.

Therefore having taken from the Circle an Arc of 102 Deg. 34 Min. 11 Sec. divide the remaining Part of the Circle always into half; and after having made seven Subdivisions, you will come to Unity: whence this part of the Circle will be divided into 128 equal Parts; and then the remaining 51 Parts may be divided, by help of the last Opening of the Compasses. Wherefore the whole Circumference will be found divided into 179 equal Parts, every of which answers to 2 Deg. 40 Sec. as may be seen in dividing 360 by 179.

Lastly, To divide the Circle of the upper Plate, take one fourth of it's Circumference, and add to it one of the 179 Parts or Divisions of the Limb of the middle Plate; the Compasses opened to the Extent of the Quadrant thus augmented, being turned four Times over, will divide the Circle in the Manner as it ought to be: for in subdividing every of the Quarters into three equal Parts, one will have twelve Spaces for the twelve Lunar Months, in such manner, that the End of the 12th Month, which makes the Beginning of the Lunar Year, exceeds the first New Moon by 4 of the 179 Divisions, marked upon the middle Plate.

Use of this Instrument.

A Lunar Year being proposed, to find the Days of the Solar Year corresponding to it, in which the New and Full Moons, together with the Eclipses, ought to happen.

For Example; Let the 24th Lunar Year of the Table of *Epochs* be proposed, which answers to the Division 24 of the middle Plate. Fix the Fiducial Line of the Index on the upper Plate, over the Division marked 24, in the middle Plate, wherein the Beginning of the 25th Lunar Year is; and seeing by the Table of *Epochs*, that *that* Beginning falls upon the 14th Day of *June*, of the Year 1703, at 9 Hours, 52 Minutes, turn the two upper

Plates together, in the Position they are in, till the Fiducial Line of the Index, fastened to the upper Plate, answers to the 10th Hour, or thereabouts, of the 4th of *June*, denoted upon the undermost Plate; at which time, the first New Moon of the proposed Lunar Year happens: for then the Fiducial Line passes thro' the middle of the Hole of the first New Moon of the said Lunar Year.

Afterwards, without changing the Situation of the three Plates, extend a Thread from the Center of the Instrument, or the moveable Index, making it pass thro' the middle of the Hole of the first Full Moon; and the Fiducial Line will answer to the beginning of the 29th Day of *June*, at 4 Hours and a Quarter; which is the time that *that* Full Moon was totally eclipsed, as appears by the red Colour quite filling the Hole, shewing the Full Moon.

By the same means we may know, that at the time of the Full Moon, which happened about the third Hour in the Morning, of the 14th of *July*, there was a partial Eclipse of the Sun.

If we proceed farther, the Eclipses may be known which happened in the Month of *December*, in the Year 1703, and towards the beginning of the following Year. But because the 10th New Moon goes out beyond the 28th Day of *February*, having brought the Index to the 28th Day of *February*, move the two upper Plates backwards, conjointly with the Index (in the Posture they are found in) until the Fiducial Line happens over the beginning of *March*; whence moving the Index over all the Holes of the New and Full Moons, and the last Plate will shew the times in which the Eclipses ought to happen.

But because the 13th New Moon is the first of the succeeding Lunar Year, which answers to the Number 25 of the Divisions of the middle Plate, leave the two undermost Plates in the posture they are found, and move forwards the upper Plate 'till the Fiducial Line meets with the Number 25 of the middle Plate, at which Point it will shew upon the greatest Plate, the first New Moon of the 26th Lunar Year, according to the order of our Epoch, which happened the 2d Day of *June*, 18 Hours 40 Minutes of the Year 1704; and afterwards moving the Index over the middle of the Holes of the New and Full Moons, it will shew upon the last Plate the Days they happened on, as well as the Eclipses to the End of *February:* after which, the same Operation must be made for the preceding Year, that is, that after having come to the last Day of *February*, you must proceed backwards to the first Day of *March.*

We might likewise find the Beginnings of all the Lunar Years without using the Table of Epochs; but since it is not possible to adjust the Plates and the Index so exactly one upon another, as that some Error may not happen, which will augment itself from Year to Year, the said Table of Epochs will serve to rectify the Use of this Instrument.

In placing the Fiducial Line of the Index upon the Moon's Age, between the Days of the Lunar Months, denoted upon the Limb of the upper Plate, the correspondent Days of the common Months will be shewn, and the Hours nearly, upon the Limb of the lower Plate.

Note, That the Calculations of the Table of Epochs are made for the mean Time of the Full Moons, which supposes the Motions of the Sun and Moon always equable; from whence there will be found some Difference between the apparent Times of the New Moons, Full Moons, and Eclipses, as they appear from the Earth, and the times found by that Table.

The proper Motions of the Sun and Moon, as well as those of the other Planets, appear to us sometimes swift, and sometimes slow; which apparent Inequality in part proceeds from their Orbits being not concentric with the Earth, and in part from hence, that the equal Arcs of the Ecliptick, which are oblique to the Equator, do not always pass thro' the Meridian with the equal Parts of the Equator. Astronomers, for the ease of Calculation, have ficted a Motion which they call mean or equable, in supposing the Planets to describe equal Arcs of their Orbits, in equal Times. That Time which they call true or apparent, is the measure of true or apparent Motion, and mean Time is the measure of mean Motion. They have likewise invented Rules for reducing mean Time to true or apparent Time, and contrariwise, for reducing true or apparent Time to mean Time.

To find by Calculation whether there will happen an Eclipse at the time of the New or Full Moon.

For an Eclipse of the Sun, multiply by 7361, the Number of Lunar Months accomplished from that which begun the 8th of *January*, 1701, according to the *Gregorian* Calendar, to that which you examine, and add to the Product the Number 33890; then divide the Sum by 43200; and after the Division, without having regard to the Quotient, examine the Remainder, or the difference between the Divisor and the Remainder: for if either of them be less then 4060, there will happen an Eclipse of the Sun.

But to find an Eclipse of the Moon, likewise multiply by 7361, the Number of Lunar Months, accomplished from that which begun the 8th of *January*, 1701, to the New Moon preceding the Full Moon examined; add to the Product 37326, and divide the Sum by 43200. The Division being made, if the Remainder, or the difference between the Remainder and the Divisor, be less than 2800, there will be an Eclipse of the Moon.

Note,

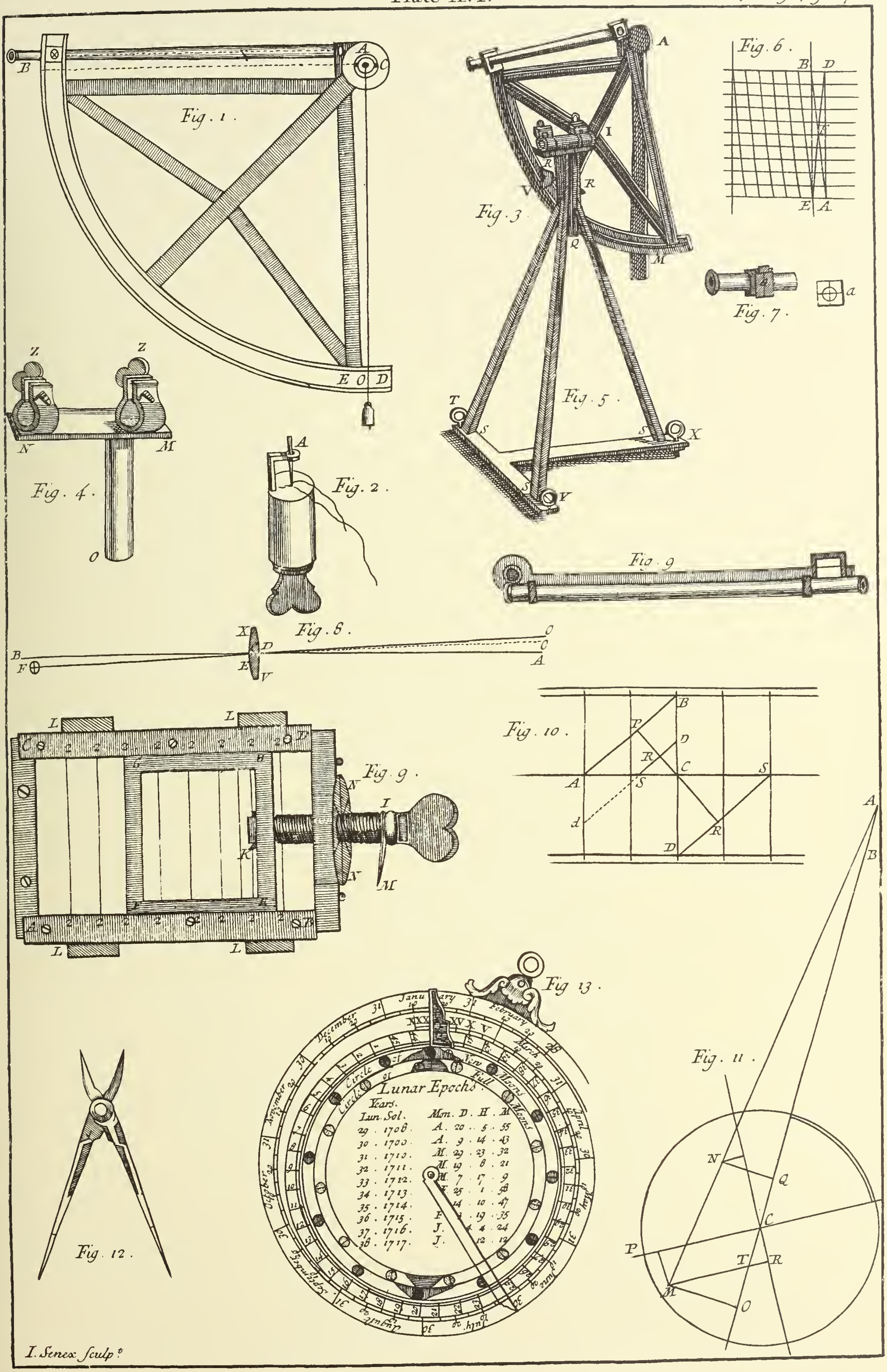


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Lunar Epochs.
Years.
Lun. Sol.
Mon. D. H. M.
29 . 1708. A. 20 . 5 . 55
30 . 1700. A. 9 . 14 . 43
31 . 1710. M. 29 . 23 . 32
32 . 1711. M. 19 . 8 . 21
New Moons
Full Moons
January
February
March
April
May
June
July
August
September
October
November
December
I. Senex sculp.

Note. An Eclipſe of the Sun or Moon will be ſo much the greater, as the Remainder or Difference is leſſer; and contrariwiſe.

Example of an Eclipſe of the Sun.

It is required to find, whether at the New Moon of the 22d of *May*, in the Year 1705, there happened an Eclipſe of the Sun.

From the 8th of *January*, 1701. to the 22d of *May*, 1705, there were accompliſhed 54 Lunations. Multiply, according to the Rule, the Number 54 by 7361, and add to the Product 33890: the Sum being divided by 43200, there will remain 42584, which is greater than 4060; and the Difference between the Remainder 42584, and the Diviſor 43200, is 616, which is leſs than 4060: therefore there was then an Eclipſe of the Sun.

Example of an Eclipſe of the Moon.

It is required to find whether the Full Moon of the 27th of *April*, in the Year 1706, was eclipſed.

From the 8th of *January*, in the Year 1701, to the New Moon preceding the Full Moon in queſtion, there were 65 Lunar Months accompliſhed; therefore having multiplied, according to the Rule, the Number 65 by 7361, and added to the Product 37326, the Sum will be 515791; which being divided by 43200, without having any regard to the Quotient, the Remainder will be 40591, greater than 2800. The Difference between the Diviſor and the Remainder is 609, which is leſs than 2800; therefore there was an Eclipſe of the Moon the 27th day of *April*, 1706.

CHAP. V.

The Deſcription of a Second-Pendulum-Clock for Aſtronomical Obſervations.

THE Figure here adjoined, ſhews the Compoſition of a Second-Pendulum-Clock, whoſe two Plates A A and BB, are about half a Foot long, and two Inches and a half broad, having four little Pillars at the four Corners, that ſo they may be an Inch and a half diſtant from each other. Theſe Plates ſerve to ſuſtain the Axes of the principal Wheels, the firſt of which being the loweſt, and figured C C, hath 80 Teeth. The Axis of this Wheel hath a little Pulley, having ſeveral Iron Points D D round about the ſame, in order to hold the Cord to which the Weights are hung, in the manner as we ſhall explain by and by. The Wheel C C, being turned by the Weight, likewiſe turns the Pinion, E of eight Teeth, and ſo moves the Wheel F, which is faſtened to the Axis of the Pinion E; this Wheel hath forty-eight Teeth, which falling into the Teeth of the Pinion G, whoſe Number is eight, moves the Wheel H, (made in figure of a Crown) conſiſting of forty-eight Teeth. Again, The Teeth of this laſt Wheel fall into the Teeth of the Pinion I, whoſe Number is twenty-four, and the Axis thereof being upright, carries the Wheel K of 15 Teeth, which are made in Figure of a Saw: Over this Wheel is a croſs Axis, having two Palats L L, ſuſtained by the Tenons N, Q and P, which are faſtened to the Plate B B. It muſt be obſerved, that as to the Tenons N and Q, the lower part Q appearing, hath a great Hole drilled therein, that the Axis L M may paſs thro' it; this part Q, which is faſtened to the lower part of the Tenon N, likewiſe holds the Wheel K, and the Pinion I. There is a great Opening in the Plate B B, in order for the Axis and the Palats to go out beyond it. One end of this Axis (as I have already mentioned) goes into the Tenon, P, and ſo moves eaſier than if it was ſuſtained by the Plate B B, and then go out beyond the ſaid Plate, which it muſt neceſſarily do, that ſo the little Stern S, fixed thereto, may freely vibrate with the ſaid Axis, and the Teeth of the Wheel K alternately meet the Palats L L, as in common Clocks. *Plate* 17. Fig. 1.

The lower part of the little Stern S is bent, and a ſlit made therein, thro' which goes an Iron-Rod, ſerving as a Pendulum, having the Lead X at the End thereof. This Rod is faſtened in V to a very thin Piece of Braſs or Steel, which vibrates between two cycloidal Cheeks T T, (one of which is ſeen in Fig. 1, and both in Fig. 2.) of which more hereafter.

It is eaſy to perceive in what manner this Clock goes by the Force of the Wheels carried round by the Weight: for the Motion is continued by the Pendulum V X, when the ſaid Pendulum is ſet a-going; becauſe the little Stern S, altho' very light, being in motion, not only goes with the Pendulum, but likewiſe by its Vibrations ſtill aſſiſts the Motion ſome ſmall matter, and ſo renders it perpetual, which otherwiſe by Friction and the Air's reſiſtance, would come to nothing. But becauſe the Property of the Pendulum is to move equably always, provided its length be the ſame, the ſaid Pendulum will cauſe the Wheel K to go neither too faſt nor too ſlow, (as happens to Clocks not having Pendulums) every

every Tooth is obliged to move equably; therefore the other Wheels, and the Hands of the Dial-plate, are necessarily constrained to perform their Revolutions equably. Whence if there should be some Default in the Construction of the Clock, or if the Axes of the Wheels do not move freely on account of the Intemperance of the Air, provided the Clock does not stand still; we have nothing to fear from these Inequalities, for the Clock will always go true.

As to the Hands for shewing the Hours, Minutes, and Seconds, we dispose them in the following manner. The third Plate Y Y is parallel to the two precedent ones, and is three Lines distant from A A. We describe a Circle about the Center *a*, which is the middle of the Axis, carrying the Wheel C, continued out beyond the Plate A A. This Circle is divided into 12 equal Parts, for the Hours. We likewise describe another Circle about the said Center, and divide it into 60 equal Parts, for the Minutes in an Hour. We place the Wheel *b* upon the Axis R, continued out beyond the Plate A A, fastened to a little Tube, going out beyond the Plate Y Y to *e*. This Tube is put about the Axis R, and turns about with it, in such manner nevertheless, that it may be turned only when there is necessity. We place the Hand shewing the Minutes in *e*, which makes one Revolution in an Hour. The beforementioned Wheel *b* moves the Wheel *b*, having the same Number of Teeth as that, *viz.* 30; and the Teeth of the Wheel *f*, falls into the Teeth of the Pinion *b*, whose Number is 6, and they have a little Axis common to them, which is partly sustained by the Tenon *d*. This Pinion moves round the Wheel *f*, having 72 Teeth, fastened to a little Tube *g*, which is put about the Tube carrying the Wheel *b*. Now the Hand shewing the Hours must be placed upon the Extremity of the Tube *g*, and will be shorter than that denoting the Minutes. But that one may not be deceived in reckoning of Seconds, we place a round Plate *m m* upon the Extremity of the Axis of the Wheel H, divided into 60 equal Parts, and make an opening Z in the Plate Y, in the upper Part of which Opening is a small Point *o*, which, as the said Plate turns about, shews the Seconds. The Disposition of the Hands and Circles will be easier seen in Figure 3, which represents the Outside of the Clock.

Now having spoken of the Disposition of the Wheels, the next thing is to determine the Length of the Pendulum, which must be such, that every of its Vibrations be made in a Second of Time. This Length must be 3 Feet 8 $\frac{1}{2}$ Lines (of *Paris* *) from the Point of Suspension, which is the Center of the cycloidal Cheeks, to the Center of the Weight X.

We now proceed to say something concerning the Times of the Revolutions of the Wheels and the Hands, in order to confirm what we have already said of the Number of Teeth. Now one Revolution of the Wheel C C, makes ten Revolutions of the Wheel F, sixty of the Wheel H, and one hundred and twenty of the upper Wheel K, which having 15 Teeth, and alternately pushing the Palats L L, makes thirty Vibrations, which are so many goings and comings of the Pendulum V X. Whence 120 Revolutions of the Wheel K, is equal to 3600 Vibrations of the Pendulum, which are the Seconds contained in one Hour; and so the Wheel C makes one Revolution in an Hour, and the Hand *e* fastened thereto, shews the Minutes; and because the Wheel *b* makes it's Revolution in the same time, (*viz.* an Hour) the Wheel *b* hath the same Number of Teeth as *b*, and the Pinion on the same Axis hath six Teeth; and since the Number of Teeth of the Wheel *f* is twelve times greater, the said Wheel will go round once in 12 Hours, as likewise the Hand *g* fastened thereto. Finally, Because the Wheel H is making sixty Revolutions in the same time the Wheel C C is making one, therefore the circular Plate Z, having the Seconds denoted thereon, will move once round in a Minute; and so every 60th part of the said Plate will shew one Second.

The Weight X, at the end of the Pendulum, must weigh about 3 Pounds, and be of Lead covered with Brass. Regard must not only be had to its Weight, but likewise to its Figure, which is of Consequence, because the least Resistance of the Air is prejudicial thereto; whence we make it in form of a Convex Cylinder *a*, whose ends are pointed, as appears in Figure 3. wherein the Pendulum is represented, tho' the Weights at the end of the Pendulums made for these Clocks used at Sea are in the Figure X, in form of a Lens, this Figure being found more proper than the other.

Fig. 3. In the same Figure may likewise be seen the manner of the Disposition of the Weight *b*, in order to so move the Clock, that it may not stand still while the Weight *b* is drawing up; and this is done by means of a Cord, one end of which must first be fastened to a piece of Iron fixed to the Plate A A, (of Figure 1.) and then it must be put about the Pulley *c*, of the Weight *b*; afterwards over the Pulley *d*, (which hath Iron Points round it in figure of the Teeth of a Saw, for hindering, lest the Weight *b* should pull the Cord down all at once) then about the Pulley *f* of the Weight *g*, and last of all the other end of the said Cord must be fixed to some proper Place. Things being thus disposed, it is manifest that half of the Weight *b* moves the Wheels round, and that the Motion of the Clock doth not cease, when the Cord *e* is pulled with one's Hand in order to draw the Weight *b* up. *Note*, The Weight *g* is for sustaining the Weight *b*, and need not be near so big.

The

* *Or* 3 *Feet* 9 $\frac{1}{4}$ *Inches of our* English *Measure.*

The Weight of *b* cannot be certainly determined by Reaſoning, but the leſs it is the better, provided it be ſufficient to make the Clock go. They weigh generally about ſix Pounds in the beſt Kind of theſe Clocks that have yet been made, whereof the Diameter of the Pulley D is one Inch, the Weight of the Pendulum X three Pounds, and it's Length three Feet 8 ½ Lines. *Note*, If this Clock be at the Height of a Man above the Ground, it will go 30 Hours.

We now proceed to ſhew the Manner of making the Cycloidal Cheeks between which the Pendulum ſwings, and in which the whole Exactneſs of the Clock conſiſts. In order to do which, deſcribe the Circle A F B K, whoſe Diameter A B let be equal to half of the Length of the Pendulum; aſſume the equal Parts of the Circumference A C, C D, D E, E F and A G, G H, H I, I K, and draw the Lines G C, H D, I E and K F, from one Diviſion to the other, which Lines will be parallel. Now make the Line L M equal to the Arc A F, which divide into the ſame Number of equal Parts as A F, and aſſume one of theſe Parts, which lay off upon the Line C G, from C to N, and G to O. Again, Lay off two of the ſaid equal Parts of the Line L M, upon the Line D H, from D to P, and from H to Q. Moreover, Aſſume three of the ſaid equal Parts upon the Line L M, which lay off upon I E from E to R, and I to S. And finally, Aſſume four of the ſaid Parts (which is the whole Length of the Line L M), and lay off upon K F, from F to T, and K to V; and ſo of other Parts, if there had been more of them aſſumed upon the Periphery of the Circle A F B K. Now if the Points N, P, R, T, as alſo O, Q, S, V, be joined, we ſhall have the Figure of the Cycloidal-Cheeks (between which the Pendulum ſwings), which muſt be afterwards cut out in Braſs. To draw the Line L M equal to the Arc A F, aſſume the two Semi-Chords of the Arcs A F, which lay off upon the Line X V, from X to Y; this being done, take the whole Chord of the Arc A F, and lay off from X to Z, and divide Z Y into three equal Parts; one of which being laid off from Z to V, and the Line X V will be nearly the Length of the Arc A F. Fig. 4.

The Uſe of this Inſtrument ſufficiently appears from what hath been already ſaid.

The principal Inſtruments that an Aſtronomer ought to have, beſides a good Quadrant, and Pendulum Clock, is a Teleſcope ſeven or eight Feet long, having a Micrometer adjuſted thereto, for obſerving the Digits of Solar and Lunar Eclipſes, as likewiſe another of 15 or 16 Feet, for the Obſervation of *Jupiter*'s Satellites; and, if poſſible, a parallactick Inſtrument to take the Parallaxes of the Stars.

ADDITIONS of *Engliſh* Instruments.

Of Globes, Spheres, the Aſtronomical Quadrant, a Micrometer, and Gunter'*s Quadrant.*

CHAP. I.

Of the Globes.

SECTION I.

Of Globes there are two Kinds, *viz.* Celeſtial and Terreſtrial. The firſt is a Repreſentation of the Heavens, upon the Convex Surface of a material Sphere, containing all the known Stars, after the manner that Aſtronomers, for the eaſier knowing them, have divided them into Conſtellations, or Figures of Men, Beaſts, Fowls, Fiſhes, *&c.* according to the Reſemblance they fancied each ſelect Number of Stars formed. The other is the Terreſtrial Globe, which is the Image of the Earth, on the Convex Surface of a material Sphere, exhibiting all the Kingdoms, Countries, Iſlands, and other Places ſituated upon it, in the ſame Order, Figure, Dimenſions, Situation, and Proportion, reſpecting one another as on the Earth itſelf. Fig. 5, & 6.

There are ten eminent Circles upon the Globe, ſix of which are called *greater*, and the four other *leſſer Circles*.

A lesser Circle is that which is parallel to a *greater*, as the Tropicks and Polar Circles are to the Equator, and as the Circles of Altitude are to the Horizon.

The great Circles are,

I. The *Horizon*, which is a broad wooden Circle encompassing the Globe about, having two Notches, one in the North, the other in the South part thereof, for the *Brazen Meridian* to stand, or move round in, when the Globe is to be set to a particular Latitude.

There are usually reckoned two *Horizons:* First, The *Visible* or *Sensible Horizon*, which may be conceived to be made by some great Plane, or the Surface of the Sea; and which divides the Heavens into two *Hemispheres*, the one above, the other (apparently) below the Level of the Earth.

This Circle determinates the Rising and Setting of the Sun, Moon, or Stars, in any particular Latitude: for when any one of them comes just to the Eastern edge of the *Horizon*, then we say it Rises; and when it doth so at the Western edge, we say it Sets. And from hence also is the Altitude of the Sun or Stars reckoned, which is their height in Degrees above the Horizon.

Secondly, The other *Horizon* is called the *Real* or Rational Horizon, and is a Circle encompassing the Earth exactly in the middle, and whose Poles are the *Zenith* and *Nadir*, *viz.* two Points in its Axis, each 90 Deg. distant from its Plane, (as the Poles of all Circles are) the one exactly over our Heads, and the other directly under our Feet. This is the Circle that th wooden Horizon on the Globe represents.

On which *Broad Horizon* several Circles are drawn, the innermost of which is the Number of Degrees of the *Twelve Signs* of the *Zodiack*, *viz.* 30 to each Sign: for the ancient Astronomers observed the Sun in his (apparent) *Annual Course*, always to describe one and the same Line in the Heavens, and never to deviate from this *Track* or *Path* to the North or South, as all the other Planets did, more or less: and because they found the Sun to shift as it were backwards, thro' all the Parts of this Circle, so that in one whole Year's Course he would *Rise*, Culminate, and *Set*, with every Point of it; they distinguished the fixed Stars that appeared, in or near this Circle, into 12 Constellations or Divisions, which they called *Signs*, and denoted them with certain Characters; and because they are most of them usually drawn in the form of Animals, they called this Circle by the Name of *Zodiack*, which signifies an *Animal*, and the very middle Line of it the *Ecliptick*; and since every Circle is divided into 360 Degrees, a twelfth part of this Number will be 30, the Degrees in each Sign.

Next to this you have the Names of those Signs; next to this the Days of the Months, according to the *Julian* Account, or Old Style, with the Calendar; and then another *Calendar*, according to the *Foreign Account* or New Style.

And without these, is a Circle divided into thirty two equal Parts, which make the 32 Winds or Points of the Mariners Compass, with the Names annexed.

The Uses of this Circle in the Globe are,

1. To determine the Rising and Setting of the Sun, Moon, or Stars, and to shew the time of it, by help of the Hour-Circle and Index; as shall be shewn hereafter.

2. To limit the Increase and Decrease of the Day and Night: for when the Sun rises due East, and sets West, the Days are equal.

But when he Rises and Sets to the North of the East and West, the Days are longer than the Nights; and contrariwise, the Nights are longer than the Days, when the Sun Rises and Sets to the Southwards of the East and West Points of the Horizon.

3. To show the Sun's Amplitude, or the Amplitude of a Star; and also on what Point of the Compass, it Rises and Sets.

II. The next Circle, is the *Meridian*, which is represented by the brazen Frame or Circle, in which the Globe hangs and turns. This is divided into four Nineties or 360 Degrees, beginning at the Equinoctial.

This Circle is called the Meridian, because when the Sun comes to the South part of it, it is Meridies, Mid-day, or High noon; and then the Sun hath its greatest Altitude for that Day, which therefore is called the *Meridian Altitude.* The Plane of this Circle is perpendicular to the *Horizon*, and passeth thro' the South and North Parts thereof, thro' the *Zenith* and *Nadir*, and thro' the Poles of the World. In it, each way from the *Equinoctial* on the *Celestial Globe*, is accounted the North or South Declination of the Sun or Stars; and on the *Terrestrial*, the Latitude of a Place *North* or *South*, which is equal to the elevation or height of the Pole above the Horizon: Because the Distance from the *Zenith* to the *Horizon*, being the same as that between the *Equinoctial* and the *Poles*, if from each you imagine the Distance from the Pole to the Zenith to be taken away, the Latitude will remain equal to the Pole's Altitude.

There are two Points of this Circle, each 90 Degrees distant from the Equinoctial, which are called the *Poles* of the World, the upper one the North Pole, and the under one the South Pole. A Diameter continued thro' both the Poles in either Globe and the Center,

is

is called the Axis of the Earth or Heavens, on which they are ſuppoſed to turn about.

The Meridians are various, and change according to the Longitude of Places; for as ſoon as ever a Man moves but one Degree, or but a Point to the Eaſt or Weſt, he is under a New Meridian: But there is or ſhould be one fixed, which is called the firſt *Meridian.*

And this on ſome Globes, paſſes thro' one of the *Azores* Iſlands: but the *French* place the firſt Meridian at *Fero*, one of the *Canary* Iſlands.

The Poles of the Meridian are the Eaſt and Weſt Points of the Horizon. On the *Terreſtial Globe*, are uſually drawn 24 Meridians, one thro' every 15 Degrees of the *Equator*, or every 15 Degrees of Longitude.

The Uſes of the Meridian Circle are,

Firſt, To ſet the Globe to any particular Latitude, by a proper Elevation of the Pole above the Horizon of that Place. And, Secondly, To ſhew the Sun or Stars Declination, right Aſcenſion, and greateſt Altitude; of which more hereafter.

III. The next great Circle, is the *Equinoctial Circle*, as it is called on the *Celeſtial*, and the *Equator*, on the Terreſtrial Globe. This is a great Circle whoſe Poles are the Poles of the World: it divides the Globe into two equal Parts or Hemiſpheres, as to North and South; it paſſes thro' the Eaſt and Weſt Points of the Horizon, and at the Meridian is always as much raiſed above the Horizon, as is the Complement of the Latitude of any particular Place. Whenever the Sun comes to this Circle, it makes equal Days and Nights all round the Globe, becauſe it then Riſes due Eaſt, and Sets due Weſt, which it doth at no other time of the Year. All Stars alſo which are under this Circle, or which have no Declination, do always Riſe due Eaſt, and Set full Weſt.

All People living under this Circle (which by Navigators is called the *Line*) have their Days and Nights conſtantly equal. And when the Sun is in the Equinoctial, he will be at Noon in their *Zenith*, or directly over their Heads, and ſo their erect Bodies can caſt no Shadow.

From this Circle both ways, the Sun, or Stars Declination on the Celeſtial, or Latitude of all Places on the *Terreſtrial Globe*, is accounted on the Meridian: and ſuch leſſer Circles as run thro' each Degree of Latitude or Declination parallel to the Equinoctial, are called Parallels of Latitude or Declination.

Through every 15 Degrees of this Equinoctial, the Hour-Circles are drawn at right Angles to it on the *Celeſtial Globe*, and all paſs thro' the Poles of the World, dividing the Equinoctial into 24 equal Parts.

And the Equator on the *Terreſtial Globe*, is divided by the Meridians into 36 equal Parts; which Meridians are equivalent to the Hour-Circles on the other Globe.

IV. The *Zodiack* is another *great Circle* of the *Globe*, dividing the Globe into two equal Parts (as do all great Circles): When the Points of *Aries* and *Libra* are brought to the Horizon, it will cut *that* and the Equinoctial obliquely, making with the former an Angle equal to 23 Degrees 30 Minutes, which is the Sun's greateſt Declination. This Circle is accounted by Aſtronomers as a Kind of broad one, and is like a Belt or Girdle: Through the Middle of it is drawn a Line called the *Ecliptick*, or *Via Solis*, the *Way of the Sun*; becauſe the Sun never deviates from it, in it's annual Courſe.

This Circle is marked with the Characters of the *Twelve Signs*, and on it is found out the Sun's Place, which is under what Star or Degree of any of the *Twelve Zodiacal Conſtellations*, he appears to be in at Noon. By this are determined the four Quarters of the Year, according as the Ecliptick is divided into four equal Parts; and accordingly as the Sun goes on here, he has more or leſs Declination.

Alſo from this Circle the Latitude of the Planets and fixed Stars are accounted from the Ecliptick towards the Poles.

The Poles of this Circle are 23 Degrees, 30 Minutes diſtant from the Poles of the World, or of the Equinoctial; and by their Motion round the Poles of the World, are the Polar Circles deſcribed.

V. If you imagine two great Circles both paſſing thro' the Poles of the World, and alſo one of them thro' the Equinoctial Points *Aries* and *Libra*, and the other thro' the *Solſtitial Points*, *Cancer* and *Capricorn*: Theſe are called the two *Colures*, the one the *Equinoctial*, and the other the *Solſtitial Colure.* Theſe will divide the Ecliptick into four equal Parts, which are denominated according to the Points they paſs thro', called the four Cardinal Points, and are the firſt Points of *Aries*, *Libra*, *Cancer*, and *Capricorn.*

Theſe are all the great Circles.

VI. If you ſuppoſe two Circles drawn parallel to the Equinoctial at 23 Degrees, 30 Minutes, reckoned on the Meridian, theſe are called the *Tropicks*, becauſe the Sun appears, when in them, to turn backward from his former Courſe; the one

one the Tropick of *Cancer*, the other the Tropick of *Capricorn*, becauſe they are under theſe Signs.

VII. If two other Circles are ſuppoſed to be drawn thro' 23 Degrees, 30 Minutes, reckoned in the Meridian from the Polar Points, theſe are called the *Polar Circles:* The Northern is the Arctick, and the Southern the Antarctick Circle, becauſe oppoſite to the former.

Theſe are the four leſſer Circles.

And theſe on the *Terreſtrial Globes*, the Ancients ſuppoſed to divide the Earth into five *Zones*, *viz.* two *Frigid*, two *Temperate*, and the *Torrid Zone.*

Beſides theſe ten Circles already deſcribed, there are ſome other neceſſary Circles to be known, which are barely imaginary, and only ſuppoſed to be drawn upon the Globe.

1. *Meridians*, or *Hour-Circles*, which are great Circles all meeting in the Poles of the World, and croſſing the Equinoctial at right Angles; theſe are ſupplied by the brazen Meridian Hour-Circle and Index.

2. *Azimuths*, or *Vertical Circles*, which likewiſe are great Circles of *the Sphere*, and meet in the *Zenith* and *Nadir*, as the Meridians and Hour-Circles do in the Poles; theſe cut the Horizon at right Angles, and on theſe is reckoned the Sun's Altitude, when he is not in the Meridian. They are repreſented by the Quadrant of Altitude, by and by ſpoken of, which being fixed at the Zenith, is moveable about the Globe thro' all the Points of the Compaſs.

3. There are alſo *Circles of Longitude* of the Stars and Planets, which are great Circles paſſing thro' the Poles of the Ecliptick, and in that Line determining the Stars or Planets Place or Longitude, reckoned from the firſt Point of *Aries.*

4. *Almacanters*, or *Parallels of Altitude*, are Circles having their Poles in the Zenith, and are always drawn parallel to the Horizon. Theſe are leſſer Circles of the Sphere, diminiſhing as they go further and further from the Horizon. In reſpect of the Stars, there are alſo Circles ſuppoſed to be *Parallels of Latitude*, which are Parallels to the Ecliptick, and have their Poles the ſame as that of the Ecliptick.

5. *Parallels of Declination* of the Sun or Stars, are leſſer Circles, whoſe Poles are the Poles of the World, and are all drawn parallel to the Equinoctial, either North or South; and theſe (when drawn on the *Terreſtrial Globe*) are called *Parallels of Latitude.*

VIII. There are belonging to Globes a Quadrant of Altitude, and Semi-Circle of Poſition. The firſt is a thin pliable Piece of Braſs, whereon is graduated 90 Degrees anſwerable to thoſe of the Equator, a fourth Part of which it repreſents; with a Nut and Screw, to faſten it to any part of the brazen Meridian as occaſion requires. There is, or ſhould be likewiſe, a Compaſs belonging to a Globe, that ſo it may be ſet North and South.

The Semi-Circle of Poſition is a narrow Plate of Braſs, inſcribed with 180 Degrees, and anſwerable to juſt half the Equator.

Laſtly, The Braſs Circle, faſtened at right Angles on the brazen Meridian, and the Index put on the Axis, is called the Index and Hour-Circle.

SECTION II.

Having now deſcribed the Circles of the Globes, I proceed to their Conſtruction.

The Body of the Globe is compoſed of an *Axle-Tree*, two *Paper-Caps* ſewed together, a Compoſition of Plaiſter laid over them, and laſt of all globical Papers or Gores (of which more by and by), ſtuck or glewed on the Plaiſter.

The *Axle-Tree* is a Piece of Wood which runs thro' the middle of the Globe, turned ſometimes of an equal Thickneſs, but oftner ſmaller in the Middle than at the Ends; where two Pieces of thick hardened Wire are ſtruck in, which is the Axis, that appears without the Globe, on which it turns within the brazen Meridian.

The Paper-Caps incloſe this *Axle-Tree*, and are made in the following Manner. You muſt have a Ball of Wood turned round, about a Quarter of an Inch leſs in Diameter, than the Size you intend to make your Globe of, with two Pieces of Wire ſtuck into it, diametrically oppoſite to each other, for Conveniency of turning in a Frame, which may be made of two Pieces of Stick fixed upright in a Board, with Notches on the Tops to lay the Wire in. Round this wooden Ball you muſt paſte waſte Paper, both brown and white, 'till you judge it to be of the Thickneſs of Paſteboard; and before it be quite dry, cut it in the Middle, ſo that it may come off in two Hemiſpheres: to prevent the Paper faom ſticking, let the Ball at firſt making be thick painted, and every time before you paſte Paper on it, greaſe or oil it a little.

The Holes at the Tops of the Caps, occaſioned by the *Axis* on which the Ball turned, are very convenient for the *Axis* of the Globe to go thro' in covering of it. Then having faſtened the Top of the Caps with ſmall Nails to each end of the wooden Axle-Tree, ſew them cloſe together in the Middle with ſtrong Twine.

That

That the Caps may meet exactly, obſerve two things: 1ſt, That the *Axle-tree* be juſt in the Diameter of the Ball. 2dly, That before you take the Caps off the Ball, you make Scores a-croſs the parting all round, about an Inch aſunder, whereby to bore the Holes for ſewing them even together, and leave a Mark to direct how to join them again in the ſame Points: for Inſtance, make a Croſs over any one of the Scores in the upper Cap, and another Croſs upon the ſame Score in the under Cap; and when you cloſe them, bring the two Croſſes together, by which means the Caps in ſewing will come as cloſe together as before they were parted. This Care muſt be taken, that there may be no Openings between; in which caſe, Paper muſt be crammed in to ſtop up the Gaps: but whether there be any Gaps or no, there muſt be Paper paſted all over it's ſewing, to prevent any of the Plaſter from falling in.

The Plaſter is made with Glue, diſſolved over the Fire in Water and Whiting mixed up thick, with ſome Hemp ſhred ſmall; the Uſe of which is to bind the Plaſter, and keep it from cracking (as Hair is put into Mortar for the ſame End): a Handful will ſerve two or three Gallons of Stuff. There is no neceſſity for mixing the whole over the Fire, except the Whiting runs into Lumps not eaſily to be broken with the Hand.

For laying on this Plaſter over the Caps in a globular Form, you muſt have a Steel Semi-Circle exactly half the Circumference you intend the Globe to have, fixed flat-ways in a level Table made for that purpoſe, with a Notch at each end for the *Axis* (which muſt nicely fit it) to turn in, and two Buttons to cover it, to prevent the *Axis* from being forced out of the Notches, when the Globe is clogged with Plaſter, and ſo requires ſome Violence to turn it.

Then fixing your Paper-Sphere within this Semi-Circle, lay Plaſter on it with your Hands, turning the Globe eaſily round, 'till it be covered ſo as to fill the Semi-Circle: But before it comes to touch the Semi-Circle in all it's Parts, and be equally ſmooth all round, it will require a great many Layings on of the Plaſter, letting it dry between every ſuch Application.

The ſecond or third time of laying on Stuff, it will begin to touch the Semi-Circle in ſome parts, and to appear round; the fourth time it will touch in more parts, and look rounder; 'till at laſt it will touch in all parts, and become perfectly round and ſmooth, like a Ball of poliſhed Marble.

The next thing to be done is to poiſe the Globe; for it generally happens, by reaſon of the Plaſter lying thicker in one place than in another, that ſome ſide weighs ſtill downwards. To remedy this, a Hole muſt be cut in that part, and a convenient Quantity of Shot put in, in a Bag, to bring it to a due Balance with the reſt; after which the Place muſt be ſtopped up with a Cork, and covered again with Plaſter. The Bag that holds the Shot may be glewed or ſewed to the Cap within, or faſtened to the Cork: ſometimes after one part is balanced, the Weight will incline to another; in which caſe the ſame Remedy muſt be applied again, as often as there will be neceſſity.

This done, by help of another Semi-Circle, divided into 18 equal Parts, draw the Equator and Parallels of Latitude, placing a Black-lead Pencil at the Graduation, and turning the Globe againſt the Point of it to make a Line. Then divide the Equator with a Pair of Compaſſes into ſo many Parts as there are globical Papers or Gores to lay on, and draw Lines thro' each from Pole to Pole by the Side of the Semi-Circle. Within each of theſe Spaces ſo marked out, you have only to lay one of the Gores, which (being cut out ſo exact, as neither to lap over, nor leave a Vacancy between them) by the Aſſiſtance of the Lines drawn upon the Plaſter, may be fitted, ſo as to fall in with each other with the greateſt Exactneſs. In applying the Gores, you may uſe a good binding Paſte, but Mouth Glue is better.

SECTION III.

Conſtruction of the Circles of the Globe on the Globical Papers or Gores.

As 7 is to 22, So is the Diameter of a Globe to the Circumference of any one of it's greateſt Circles. The Diameter of the Globe is uſually given, from whence it often happens that the Circumference conſiſts of odd Numbers and Parts. Whereas if the Circumference was given in even Numbers, as Inches, it might more eaſily be divided into Parts. For Example, if the Circumference was 36 Inches, each 10 Degrees of Longitude on the Equator will be one Inch; if the Circumference be 54, each 10 Degrees will be one Inch and a half; if 72, every 10 Degrees of Longitude will be two Inches.

The Diameter of a Globe being given, ſuppoſe 24 Inches, to find the Circumference, ſay, As 7 is to 22, So is 24 to 75.43 Inches, the Length of the Circumference ſought.

The Length of each Gore, from the North Pole to the South Pole, will be exactly half the Circumference of the Globe, which is 37.71 Inches, and the Length from the Equator to either Pole will be $\frac{1}{4}$, *viz.* 18.86 Inches.

If each of the Globical Papers contain in their greatest Breadth 30 Degrees of the Equator, 12 of them will cover the Globe, and by dividing the Circumference 75.43 by 12, the Quotient will give 6.28 Inches for the Breadth of the Gore.

If 18 of the Gores go to cover the Globe, the Breadth of each will be 20 Degrees of the Equator, or 4.19 Inches.

If 24, each will contain 15 Degrees of the Equator, or 3.14 Inches of the Circumference.

If 36, each Paper will contain 10 Degrees of the Circumference, or 2.09 Inches.

If the Globe be so large as to take up 360 Papers, that is, one to every Degree of Longitude, then will the Breadth of each Gore be 23 parts of an Inch.

Again, If the Circumference of a Globe be given, suppose 72 Inches, divide it by 2 (for the Length of the Gores from Pole to Pole) and the Quotient will be 36 Inches; and consequently half that Length, or the Distance from the Equator to either Pole, will be 18 Inches: as the Distance from N. to S. taken from a supposed Scale of Inches, is 36 Inches, or one half of the Circumference of the Globe; and the Distance from C to N or S, 18 Inches, or $\frac{1}{4}$ of the Circumference.

If each Gore contains 30 Degrees of the Equator in Breadth, or $\frac{1}{12}$ of the Circumference, it will take up 6 Inches thereof as IK.

If 18 of the Gores go to cover a Globe of the aforesaid Circumference, each will contain 20 Degrees in Longitude of the Equator, or 4 Inches, as L M.

If your Papers be $\frac{1}{24}$ of the Circumference, each will contain 15 Degrees of the Equator, or or 3 Inches, as *a b*.

If they be $\frac{1}{36}$ of the Circumference, each will contain 10 Degrees of the Equator, or 2 Inches, as *c d*.

If there be 72 Papers for covering the Globe, each will contain 5 Degrees of the Equator, or 1 Inch, that is $\frac{1}{72}$ of the Circumference.

If, lastly, the Globe requires 360 Papers, each will contain 1 Degree, or $\frac{1}{5}$ of an Inch.

This being premised, I now proceed to give the Manner of drawing the Circles of the Globes upon the aforesaid Gores.

Fig. 7. Draw the Diameter W E, and cross it with another at right Angles to it, as N S. From the Scale of Inches set off from C to N, and to S, (the North and South Poles) 18 Inches or $\frac{1}{4}$ of the Circumference, which divide into 9 equal parts, each of which likewise subdivide into 10 more (for the 90 Degrees of North and South Latitude). Upon C, as a Center, describe the Circle N E, S W, and divide each Quadrant into 90 Degrees, numbering each 10th Degree with Figures from the Equator towards the Poles, as 10, 20, 30, *&c.* Thus the three Points are found, thro' which the parallel Circles to the Equator must be drawn, *viz.* two of them are in the Quadrants N E, N W. and S E, S W, and the third is in the Diameter N S.

To find the Centers of any of the said Parallels, suppose of the Parallel of 60 Degrees, set one Foot of your Compasses in the Point 60, or F, of the Quadrant N E, and extend the other to the Point 60, or D, in the Diameter N S; then describe the little Arcs A, B, and removing the Foot of your Compasses to the Point D, describe two other Arcs, cutting those before described, and thro' the Points of Intersection draw a right Line, which will cut the Diameter C N, produced in the Point G, the Center of the 60th Parallel. Having thus found the Centers of all the Parallels, and drawn them in the Northern Hemisphere, transfer the central Points in the Line C N continued, into the Line C S continued also, and draw the Parallels of the Southern Hemisphere. Note, That whether the polar Papers extend to the 80th or 70th Parallel, those Circles in the meridional Papers, or those that encompass the Body of the Globe, must be described as is here ordered; but in the polar Papers the Pole must be the Center, as you see in the Figure, where one Point of the Compasses being set in the South Pole S, and the other extended to the 80th or 70th Degree of Latitude in the Diameter, strikes those Parallels in the polar Papers. See more concerning the polar Papers hereafter.

Then because the polar Circles and Tropicks are but Parallels 23 Deg. 30 Min. distant from the Poles and Equator; at those Distances describe double Lines, representing such Circles, to distinguish them from other Parallels.

To draw the Meridians.

Having chosen one of the Proportions beforementioned for the Breadth of each Paper on the Equator, suppose $\frac{1}{12}$ of the Equator, which is the common Proportion in globical Papers, and the greatest Breadth that can be allowed them, let the Globe be of what Magnitude soever: then because $\frac{1}{12}$ of the Equator contains 30 Degrees, which in the Gores for a Globe of 72 Inches Circumferences, are six Inches in Breadth; from a Scale of Inches take three Inches between your Compasses, and lay them off on the Diameter W C E, from C to K, and from C to I, the Length from I to K being six Inches, or 3 Degrees of the Equator, into which it must be divided, and numbered at each 5th or 10th Degree, with the Degrees of Longitude.

Now

Now becaufe a fingle Degree cannot be well divided into Parts in fo fmall a Projection, and feeing that any Number of Degrees of Longitude in any Parallel has the fame Proportion to one Degree in that Parallel, as the fame Number of Degrees of Longitude under the Equator has to one Degree of Longitude; therefore take 15 Degrees of the Equator, *viz.* I C or I K, in your Compaffes, and having divided it feparately, as you would a fingle Degree, into 60 equal Parts, look in the following Table what Proportion a Degree (or 15 Degrees) in each 5th or 10th Parallel of Latitude, hath to a Degree (or 15 Degrees) on the Equator. For example, in the firft Column of the Table towards the Left-Hand, are the Degrees of Latitude; over againft the 10th Degree, I find 59 Miles in the fecond Column, and 00 Minutes, or Fractions of a Mile, in the third Column, which fignifies that a Degree (or 15 Degrees) in the 10th Parallel of Latitude, contains but 59 Miles 00 Minutes of a Degree (or 15 Degrees of the Equator) which Length I take from the Scale I C or C K between my Compaffes, and fet off on each fide the Meridian, or Diameter N S, on the 10th Parallel.

Again, in the Parallel of 20 Degrees, I find a Degree to contain 56 Miles 24 Minutes, or parts of a Mile, of a Degree in the Equator, and transfer that Length from the aforefaid Scale upon the 20th Parallel; the like is to be underftood of all the reft, and thofe Points being found and joined, will form the Meridians on the Gores. The fame Directions muft be followed in all other Proportions for the Breadth of the Gores; in chufing of which, obferve, that as it is manifeft from the Figure of the Globe, that a Paper fo large as $\frac{1}{12}$ of the Circumference of the Globe, cannot lie upon its Convexity, without crumbling, lapping over, or tearing, in the Application; therefore it will be better to ufe fome leffer Proportion, as L M, *a b*, or *c d*: for note, the narrower they are, the more exactly they will fit the Globe. Note alfo, in drawing the Parallels from 10 to 30 Degrees of Latitude, right Lines will do well enough.

A TABLE *fhewing in what Proportion the Degrees of Longitude decreafe in the Parallels of Latitude.*

Lat.	Mil.	Min.	Lat.	Mil.	Min	Lat.	Mil	Min.	Lat.	Mil.	Miu.	Lat.	Mil.	Min.	Lat.	Mil	Min.	Lat.	Mil.	Min.
0	60	0	13	58	28	25	54	24	37	47	56	49	39	20	61	29	4	73	13	32
1	59	56	14	58	12	26	54	0	38	47	16	50	38	32	62	28	8	74	16	32
2	59	54	15	58	0	27	53	28	39	46	36	51	37	44	63	27	12	75	15	32
3	59	52	16	57	40	28	53	0	40	46	0	52	37	0	64	26	16	76	14	32
4	59	50	17	57	20	29	52	28	41	45	16	53	36	8	65	25	20	77	13	32
5	59	46	18	57	4	30	51	56	42	44	36	54	35	26	66	24	24	78	12	32
6	69	40	19	56	44	31	51	24	43	43	52	55	34	24	67	23	28	79	11	28
7	59	37	20	56	24	32	50	52	44	43	8	56	33	32	68	22	32	80	10	24
8	59	24	21	56	0	33	50	20	45	42	24	57	32	40	69	21	32	81	9	20
9	59	10	22	55	36	34	49	44	46	41	40	58	31	48	70	20	32	82	8	20
10	59	0	23	55	12	35	49	8	47	41	0	59	31	0	71	19	32	83	7	20
11	58	52	24	54	48	36	48	32	48	40	8	60	30	0	72	18	32	84	6	12
12	58	40																85	5	12
																		86	4	12
																		87	3	12
																		88	2	4
																		89	1	4
																		90	0	0

The exact Geometrical Way of drawing the Parallels and Meridians on the Gores.

Becaufe in the Method before laid down, the true Centers of the Parallels are not exactly in thofe Points found as there directed; nor the Points in them the Points by which the Meridians muft pafs: therefore I think it proper here to exhibit the Geometrical Manner of drawing them truly. *Plate* 18. Fig 1.

Suppofe S B to be the Semidiameter of the Globe, with which defcribe the Quadrant B I, and continue out the Semidiameter S I, both ways. Make S A equal to $\frac{1}{4}$ of the Circumference; the Point A of which, will be the Pole of the Gore. Then divide the Quadrant B I into 90 equal Parts or Degrees, to every of which draw the Tangents *i* 80, *k* 70, *l* 60, *m* 50, &c. until they meet the Radius S I continued. Again, having divided the Line A S (equal to $\frac{1}{4}$ of the Circumference of the Globe) into 90 equal Parts, (I have only divided it into 9) and numbred them as *per* Figure; take the Length of the Tangent *i* 80 between your Compaffes, and fetting one Foot in the Point 80 of the Line A S, the other will fall upon the Point *a* in the faid Line continued out beyond A, which will be the Center of the 80th Parallel paffing thro' the Point 80 in the Line A S.

Moreover, to find the Center of the 70th Parallel, take the Tangent *k* 70 between your Compasses, and setting one Foot in the Point 70 of the Line A S, the other will fall on the Point *b* in the Line A S continued, which will be the Center of the 70th Parallel, passing thro' the Point 80 in the Line A S.

In like manner, to find the Center of the 60th Parallel, take the Tangent *l* 60 between your Compasses, and set it off from the Point 60 in the Line A S, and you will have the Center *c* for the 60th Parallel, passing thro' the Point 60. Proceed thus for finding the Centers *d*, *e*, *f*, *g*, *&c.* of the Parallels 50, 40, 30, 20, *&c.* about each of which Centers respective Arcs being drawn, the Parallels will be had.

The Reason of this Operation for finding the Centers of the Parallels, is this; If a Sphere or Globe hath revolved upon a Plane, in such manner that every Point of the Periphery of some lesser Circle of it, has touched the said Plane, and the Point which in the beginning of the Motion was contiguous to the Plane, became to be contiguous to it again; then the Points on the Plane, that were contiguous to the Points of the Periphery of the aforesaid lesser Circle, will be in the Circumference of a Circle, whose Center will be the Vertex of a right Cone, lying on the aforesaid Plane, the Base of which will be the said Circle; and consequently the Vertex will be determined in the Plane, by continuing a right Line raised on the Circle's Center perpendicularly 'till it cuts the aforesaid Plane.

How to draw the Meridians.

Having drawn the Sines 10 *p*, 20 *q*, 30 *r*, 40 *s*, *&c.* divide the Radius BS into 360 equal Parts, or make a Diagonal Scale of that Length, whereby 360 may be taken off. Then having assumed SC for half the Breadth of the Gore, suppose $\frac{1}{48}$ of the Circumference of the Equator, take S *x* (the Sine Complement of 80 Deg.) between your Compasses, and applying this Extent on the Radius B S, or the Diagonal Scale, see how many of those Parts that the Diameter is divided into, that Extent takes up. Then take $\frac{1}{48}$ of those Parts, and with the Quotient as so many Degrees make the Arc 10 L off, which will give the Point L in the Parallel of 10 Degrees, thro' which the Meridian must pass.

Again, take S *w* between your Compasses, and see how many of the Parts that the Radius B S is divided into, it contains; then take $\frac{1}{48}$ of those Parts, and with the Quotient, as so many Degrees, make the Arc 20 M off, which will give another Point M, thro' which the same Meridian must pass in the 20th Parallel.

In like manner, to find the Point N in the Parallel of 30 Degrees, thro' which the Meridian must pass, take S *u* (the Sine Complement of 60 Degrees) in your Compasses, and see how many of the Parts that the Radius B S is divided into, it contains; then taking $\frac{1}{48}$ of those Parts, with the Quotient as so many Degrees, make the Arc 30 N off.

Proceeding in this manner, you may find other Points in the other Parallels, thro' which the Meridian must pass. Which Points being afterwards joined, the quarter of the Meridian A N C will be drawn; and therefore one quarter of the Gore; and consequently the other three Quarters of the Gore will be easily limited.

Method of ordering the Circumpolar Papers.

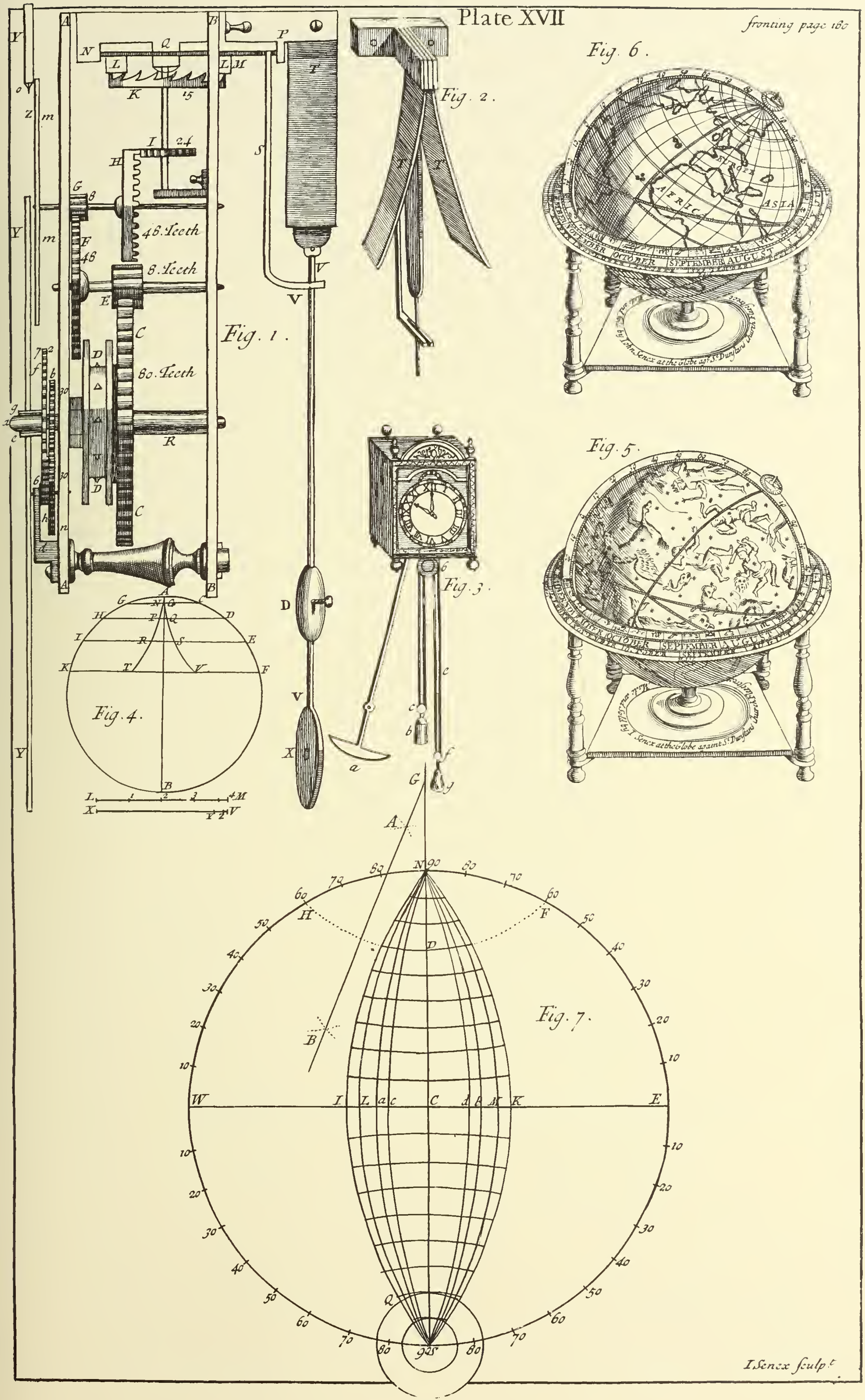
The Circumpolar Papers were formerly not cut out by themselves, 'till Artists found it hard to make the Poles, or Points of the Gores, fall nicely in the North and South Poles; whence, to help that Inconveniency, they made Circular Papers serve to cover the Superficies of the Globe between the Polar Circles, the Parallels on which Papers are all Concentric Circles, and the Meridians Right Lines: yet finding still so big Papers not to fit the Globe's Convexity, but wrinkle about the edges, they have extended them from the Poles only to the Parallels of 70 Degrees. But neither will it do yet, because the Longitude decreases disproportionally, the further off the Poles. If the Diameter of a Polar Paper extends to 10 Degrees from the Pole only, that Paper will lie flat upon the Globe's Convexity, without any sensible stretching or contracting: But if it extend to or beyond the 70th Parallel, you must take another Course.

Fig. 7. Suppose A P B to be half of a Gore, 12 of which will cover a Globe. About the Point P with an extent to the 70th Parallel, describe a Circle, which from the Points G or F, divide into 12 equal Parts; or, which is the same, continue every other Meridian in the Parallel 80 to the Parallel 70, and by the aforementioned Table set off on each Side these 12 Meridians, the true Longitude of each 10 Degrees in the Parallel of 80; or, which will save that trouble, transfer the Distance from C to G, or from G to D upon the Parallel of 70 Deg. in the Polar Paper, for that is the extent of 10 Degrees in that Parallel; and, as is manifest from the Figure, there will lie between each twelfth part of the Circumference F G, a narrow slip of Paper which must be cut out, and then the Paper being laid upon the Globe, the Parts will naturally close: whereas, for want of this Care taken, we commonly see the Polar Papers wrap over and wrinkle; besides, the Points of the Meridians on the Polar Papers seldom meet those of the Meridians of the Gores, except now and then by chance.

From this one rough Draught you may transfer the rest of the Gores that are to make up the Surface of the Globe; by which the trouble of projecting a New Scheme for every

Plate XVII

fronting page 160



I. Senex sculp.t

Gore will be avoided. Obſerve to do it with great care, for a ſmall Error will, when the Gores are all joined, appear very ſenſible. Then becauſe the Gores in all make 12, you may divide your Projections upon three Sheets of large Paper, allowing four Gores to each Sheet.

Draw an Eaſt and Weſt Diameter thro' every Sheet, in each of which ſet off the Diſtance from I to K, of *Fig.* 7. *Plate* 17. with your Compaſſes four times, without ſhifting the Points. In the middle of each erect Perpendiculars, and transfer 70 Degrees thereon (allowing the Polar Papers to include 20 Degrees from the Poles) Northwards and Southwards from the Center, which is the Interſection of the Equator with the ſtraight Meridians or Perpendiculars, for Northern and Southern Latitude. Fig. 2.

From the aforeſaid Semi-gore, take the Diſtance between the Point of each 10th Parallel in the Perpendiculars, and in the Meridians A C, B D, and in the fair Draught deſcribe Arcs to the Right and Left, upon the Points in the Perpendiculars.

Then placing one foot of your Compaſſes in the Point A or B, extend the other to the Point of the Meridians and Parallels Interſection; and as you go along, transfer the Diſtances upon the Copies from the correſpondent Points of the Equator into the Arcs, and the Places where they cut will be the Points thro' which the Meridians and Parallels muſt be drawn. And that Meridian, among all the Papers which is pitched upon for the firſt, let be divided equally from the Equator to G, and then in the Polar Papers to the Poles, into Degrees or Minutes, numbering each 10th or 5th Degree, with the Degrees of Latitude, minding to draw three Lines to diſtinguiſh it from other Meridians. The ſame muſt be obſerved in deſcribing the Ecliptick or Equator; on which laſt every 5th or 10th Degree, 'till you come to 180 Degrees, muſt be figured Eaſtward and Weſtward from the firſt Meridian.

When all the Papers are finiſhed ſo far as relates to the Meridians and Parallels, you muſt next draw the Ecliptick; and becauſe that Circle interſects the Meridians in ſuch and ſuch Parallels of Declination, and the Meridians cut the Equator in the Degrees of Right Aſcenſion; therefore by help of a Table of the Declination of thoſe Points of the Ecliptick that cut the Meridian, and the Right Aſcenſion of the ſame Points, find the Declination over-againſt the Right Aſcenſion, which ſhews thro' what parts of the Meridians the Ecliptick Arcs muſt paſs; and draw Right Lines thro' the Points of Interſection, which Lines will form the Ecliptick on the Globe.

A TABLE *of Right Aſcenſion and Declination of every* 15 *Degrees of the Signs.*

	Deg.	Deg.	Min.		Deg.	Min.
Aries	15	13	48	Add 180 Degrees for the ſix other Signs.	5	56
Taurus	0	27	54		11	30
Taurus	15	42	31		16	23
Gemini	0	57	48		20	12
Gemini	15	73	43		22	39
Cancer	0	90	00		23	30
Cancer	15	106	17		22	39
Leo	0	122	12		20	12
Leo	15	137	29		16	23
Virgo	0	152	6		11	30
Virgo	15	166	12		5	56

Seek the Right Aſcenſion as Longitude, and the Declination as Latitude, and where they interſect is the reſpective Point of the Ecliptick.

Proceed next to inſert the Stars on the Gores for the *Celeſtial Globe*, and *Places* on thoſe for the Terreſtrial Globe, by help of moſt approved Aſtronomical and Geographical Tables and Maps, according to their reſpective Longitude and Latitude, which may eaſily be affected by finding the Meridian and Parallel of the Star or Place; and the Point where they interſect each other, will be the exact Situation thereof.

The Rhumb Lines (which always make the ſame Angles with the Parallels they are dawn thro') may be inſcribed by *Wright*'s Card, or Loxodromick Tables, found in ſome Books of Navigation, as thoſe in *Newhouſe*. Trade Winds are beſt deſcribed from Dr *Halley* in the *Philophical Tranſactions*: the Conſtellations may be drawn by a Celeſtial Globe.

Your Projectures of the Heaven and Earth being finiſhed, you may either apply them to a particular Pair of Globes, or have them engraved in Copper-Plates.

CHAP. II.

Of Aſtronomical and Geographical Definitions, and the Uſes of the Globes.

BEfore I lay down the Uſes of the Globe, it will be proper to exhibit the following Definitions, neceſſary to be known in order to underſtand their Uſes.

Definition I. The *Latitude of any Place*, is an Arc of the Meridian of that Place, intercepted between the Zenith and the Equator; and this is the ſame as an Arc of the Meridian intercepted between the Pole and the Horizon; and therefore the Latitude of any Place is often expreſſed by the Pole's Height, or Elevation of the Pole: the Reaſon of which is, that from the Equator to the Pole, there always being the Diſtance of 90 Degrees, and from the Zenith to the Horizon the ſame Number, and each of theſe 90 containing within it the Diſtance between the Zenith and the Pole; that Diſtance therefore being taken away from both, muſt leave the Diſtance from the Zenith to the Equator equal to the Diſtance between the Pole and the Horizon, or the Elevation of the Pole above the Horizon.

Definition II. Latitude of a Star or Planet, is an Arc of a great Circle reckoned on the Quadrant of Altitude, laid through the Star and Pole of the Ecliptick, from the Ecliptick towards its Pole.

Definition III. Longitude of a Place is an Arc of the Equator intercepted between the Meridian; or it is more properly the Difference, either Eaſt or Weſt, between the Meridians of any two Places, accounted on the Equator.

Definition IV. Longitude of a Star, is an Arc of the Ecliptick, accounted from the beginning of *Aries* to the Place where the Star's Circle of Longitude croſſeth the Ecliptick; ſo that it is much the ſame as the Star's Place in the Ecliptick, accounted from the beginning of *Aries*.

Definition V. Amplitude of the Sun or of a Star, is an Arc of the Horizon intercepted between the true Eaſt or Weſt Points of it, and that Point upon which the Sun or Star riſes or ſets.

Definition VI. Right Aſcenſion of the Sun, or of a Star, is that part of the Equinoctial reckoned from the begining of *Aries*, which riſeth or ſetteth with the Sun or Star in a Right Sphere: but in an Oblique Sphere it is that part of a Degree of the Equinoctial, which comes to the Meridian with it, (as before) reckoned from the beginning of *Aries*.

Definition VII. A right or direct Sphere, is when the Poles are in the Horizon, and the Equator in the Zenith: the Conſequence of being under ſuch a Poſition of the Heavens as this (which is the caſe of thoſe who live directly under the Line) is, that the Inhabitants have no Latitude nor Elevation of the Pole; they can nearly ſee both the Poles of the World. All the Stars in the Heaven do once in twenty-four Hours riſe, culminate, and ſet with them; the Sun always riſes and deſcends at Right Angles with the Horizon, which is the Reaſon they have always equal Days and Nights, becauſe the Horizon doth exactly biſect the Circle of the Sun's Diurnal Revolution.

Definition VIII. A Parallel Sphere, is where the Poles are in the Zenith and Nadir, and the Equinoctial in the Horizon; which is the Caſe of ſuch Perſons, if any ſuch there be, who live directly under the North or South Poles.

And the Conſequence of ſuch a Poſition are, that the Parallels of the Sun's Declination will alſo be Parallels of his Altitude, or Almacanters to them. The Inhabitants can ſee only ſuch Stars as are on their ſide the Equinoctial; and they muſt have ſix Months Day, and ſix Months continual Night every Year; and the Sun can never be higher with them than 23 Degrees, 30 Minutes, which is not ſo high as it is with us on *February* the 10th.

Definition IX. An oblique Sphere, is where the Pole is elevated to any Number of Degrees leſs than 90: and conſequently the Axis of the Globe can never be at Right Angles to, nor in the Horizon; and the Equator and Parallels of Declination, will all cut the Horizon obliquely, from whence it takes its Name.

Oblique Aſcenſion of the Sun or Stars, is that Part or Degree of the Equinoctial reckoned from the beginning of *Aries*, which riſes and ſets with them in an oblique Sphere.

Aſcenſional Difference, is the Difference between the right and oblique Aſcenſion, when the leſſer is ſubſtracted from the greater.

On the Terreſtrial Globe.

Definition X. A Space upon the Surface of the Earth, reckoned between two Parallels to the Equator, wherein the Increaſe of the longeſt Day is a quarter of an Hour, is by ſome Writers called a Parallel.

Definition XI. And the Space contained between two ſuch Parallels, is called a Climate: Theſe Climates begin at the Equator; and when we go North or South, till the Day becomes half an Hour longer than it was before, they ſay we are come into the firſt Climate; when the Days are an Hour longer than they are under the Equator, we are come to the Second Climate, *&c.* theſe Climates are counted in Number 24, reckoned each way from the Poles.

The Inhabitants of the Earth are divided into three ſorts, as to the falling of their Shadows.

Definition XII. *Amphiſcii,* who are thoſe which inhabit the Torrid Zone, or live between the Equator and Tropicks, and conſequently have the Sun twice a Year in their Zenith; at which time they are *Aſcii,* i. e. have no Shadows, the Sun being vertical to them: theſe have their Shadows caſt to the Southward, when the Sun is in the Northern Signs, and to the Northward when the Sun is in the Southern Signs reckoned in reſpect of them.

Definition XIII. *Heteroſcii,* who are thoſe whoſe Shadows fall but one way, as is the Caſe of all ſuch as live between the Tropicks and Polar Circles; for their Shadows at Noon are always to the Northward in North Latitude, and to the Southward in South Latitude.

Definition XIV. *Periſcii,* are ſuch Perſons that inhabit thoſe Places of the Earth that lie between the Polar Circles andthe Poles, and therefore have their Shadows falling all manner of ways, becauſe the Sun at ſome time of the Year goes clear round about them. The Inhabitants of the Earth, in reſpect to one another, are alſo divided into three Sorts.

Periœcei, who are ſuch as inhabiting the ſame Parallel (not a great Circle), are yet directly oppoſite to one another, the one being Eaſt or Weſt from the other exactly 180 Degrees, which is their Difference of Longitude. Now theſe have the ſame Latitude and Length of Days and Nights, but exactly at contrary Times; for when the Sun riſeth to one, it ſets to the other.

Antœci, who are Inhabitants of ſuch Places, as being under a Semi-circle of the ſame Meridian, do lie at equal Diſtance from the Equator, one towards the North, and the other towards the South. Now theſe have the ſame Degree of Latitude, but towards contrary Parts, the one North and the other South; and therefore muſt have the Seaſons of the Year directly at contrary Times one to the other.

Antipodes, who are ſuch as dwell under the ſame Meridian, but in two oppoſite and equidiſtant Parallels, and in the two oppoſite Points of thoſe two Parallels; ſo that they go Feet againſt Feet, and are diſtant from each other an intire Diameter of the Earth, or 180 Degrees of a great Circle. Theſe have the ſame Degree of Latitude, but the one South, the other North, and accounted from the Equator a quite contrary way; and therefore theſe will have all things, as Day and Night, Summer and Winter, directly contrary to one another.

USE I. *To find the Latitude of any Place.*

Bring the Place to the Braſs Meridian, and the Degrees of that Circle, intercepted between the Place and the Equinoctial, are the Latitude of that Place either North or South.

Then to fit the Globe ſo that the wooden Horizon ſhall repreſent the Horizon of that Place, elevate the Pole as many Degrees above the wooden Horizon, as are contain'd in the Latitude of that Place, and it is done; for then will that Place be in the Zenith.

If after this you rectify the Globe to any particular time, you may by the Index know the time of Sun-riſing and Setting with the Inhabitants of that Place, and conſequently the preſent Length of their Day and Night, *&c.*

USE II. *To find the Longitude of a Place.*

Bring the Places ſeverally to the Braſs Meridian, and then the Number of Degrees of the Equinoctial, which are between the Meridians of each Place, are their Difference of Longitude either Eaſt or Weſt.

But if you reckon it from any Place where a firſt Meridian is ſuppoſed to be placed, you muſt bring the firſt Meridian to the Brazen one on the Globe; and then turn the Globe about 'till the other Place comes thither alſo: reckon the Number of Degrees of the Equinoctial intercepted between the firſt Meridian, and the proper one of the Place, and that is the Longitude of that Place, either Eaſt or Weſt.

USE III. *To find what Places of the Earth the Sun is Vertical to, at any time aſſigned.*

Bring the Sun's Place found in the Ecliptick on the Terreſtrial Globe to the brazen Meridian, and note what Degree of the Meridian it cuts; then by turning the Globe round about, you will ſee what Places of the Earth are in that Parallel of Declination (for they will all come ſucceſſively to that Degree of the brazen Meridian); and thoſe are the Places and Parts of the Earth to which the Sun will be Vertical that Day, whoſe Inhabitants will then be *Aſcii*; that is, their erect Bodies at Noon will caſt no Shadow.

Of

Of the Celestial Globe.

USE IV. *To find the Sun's place in the Ecliptick in any given Day of the Month, by means of the Circle of Signs on the wooden Horizon.*

Seek the Day of the Month upon the Horizon, observing the Difference between the *Julian* and *Gregorian* Calendars; and then against the said Day you will find, in the Circle of Signs, the Sign and Degree the Sun is in the said Day. This being done, find the same Sign and Degree upon the Ecliptick on the Superficies of the Globe, and the Sun's place will be had. *Note*, If the Sun's place be required more exactly, you must consult an Ephemeris for the given Year, or else calculate it from Astronomical Tables.

USE V. *The Sun's Place for any Day being given, to find his Declination.*

Bring the Sun's Place for that Day to the Meridian, and then the Degrees of the Meridian, reckoned from the Equinoctial either North or South to the said Place, shew the Sun's Declination for that Day at Noon, either North or South, according to the time of the Year, *viz.* from *March* the 10th to *September* the 12th, North; and from thence to *March* again, South.

USE VI. *To find the Sun's Amplitude either Rising or Setting.*

Having rectified the Globe to the Latitude of the Place, that is, moved the brazen Meridian 'till the Degree of the Latitude thereon be cut by the Plane of the wooden Horizon, bring the Sun's Place to the said Horizon either on the East or West side, and the Degrees of the Horizon, reckoned from the East Point, either North or South, give the Amplitude sought, and at the same time you have in the Circle of Rhumbs the Point that the Sun rises or sets upon.

USE VII. *To find the Sun's Right Ascension.*

Bring the Sun's Place to the brazen Meridian, and the Degrees intercepted between the beginning of *Aries*, and that Degree of the Equinoctial which comes to the Meridian with the Sun, is the Right Ascension; which if you would have in time, you must reckon every 15 Degrees for one Hour, and every Degree four Minutes.

Note, The Reason of bringing the Sun's place to the Meridian in this Use, is to save the trouble of putting the Globe into the Position of a Right Sphere: for properly Right Ascension is that Degree of the Equinoctial, which rises with the Sun in a Right Sphere. But since the Equator is always at Right Angles to the Meridian, if you bring the Sun's place thither, it must in the Equinoctial cut his Right Ascension.

USE VIII. *To find the Sun's Oblique Ascension.*

Having rectified the Globe to the Latitude, bring the Sun's Place to the East-side of the Horizon, and the Number of Degrees intercepted between that Degree of the Equinoctial, which is now come to the Horizon and the beginning of *Aries*, is the Oblique Ascension. Now the lesser of these two Ascensions being taken from the greater, the Remainder is the ascensional Difference; which therefore is the Difference in Degrees between the Right or Oblique Ascension, or the Space between the Sun's Rising or Setting, and the Hour of six. Wherefore the ascensional Difference being converted into Time, will give the time of the Sun's Rising and Setting before or after six.

USE IX. *To find the time of the Sun's Rising or Setting in any given Latitude.*

Having first brought his Place to the Meridian, and the Hour-Index to twelve at Noon, bring his Place afterwards to the Horizon, either on the East or West-side thereof; then the Hour Index will either shew the time of his Rising and Setting accordingly. Now the time of the Sun's Setting being doubled, gives the Length of the Day; and the time of his Rising doubled, gives the Length of the Night.

USE X. *The find the Sun's Meridian Altitude, or Depression at Midnight, in any given Latitude.*

Bring his Place to the Meridian above the Horizon, for his Noon Altitude, which will shew the Degrees thereof, reckoning from the Horizon; and to find his midnight Depression below the North Point of the Horizon, the Point in the Ecliptick opposite to the Sun's present Place, must be brought to the South part of the Meridian above the Horizon, and the Degrees there intercepted between that Point and the Horizon, are his midnight Depression.

USE XI. *To find the Sun's Altitude at any time of the Day given.*

Rectify the Globe, that is, bring the Sun's Place to the Meridian, and set the Hour-Index to twelve, and raise the Pole to the Latitude of the Place above the Horizon. This being done, fit the Quadrant of Altitude, that is, screw the Quadrant of Altitude to the Zenith, or in our Latitude ſcrew it ſo that the divided Edge cuts 51 Deg. 32 Min. on the Meridian reckoned from the Equinoctial. Then turn the Globe about 'till the Index ſhews the given Time, and ſtay the Globe there; after which, bring the Quadrant of Altitude to cut the Sun's Place in the Ecliptick, and then that Place or Degree of the Ecliptick will ſhew the Sun's Altitude on the Quadrant of Altitude.

USE XII. *To find the Sun's Altitude, and at what Hour he is due Eaſt or Weſt.*

Rectify the Globe, and fit the Quadrant of Altitude. Then bring the Quadrant to cut the true Eaſt Point, and turn the Globe about 'till the Sun's Place in the Ecliptick cuts the divided Edge of the Quadrant of Altitude; for then that Place will ſhew the Altitude, and the Index the Hour.

USE XIII. *The Sun's Azimuth, or when he is on any Point of the Compaſs being given; to find his Altitude and the Hour of the Day.*

Set the Quadrant of Altitude to the Azimuth given, and turn the Globe about 'till his Place in the Ecliptick touches the divided Edge of the Quadrant; ſo ſhall that Place give the Altitude on the Quadrant, and the Hour-Index the Time of the Day.

USE XIV. *To find the Declination, and the Right Aſcenſion of any Star.*

Bring the Star to the brazen Meridian, and then the Degrees intercepted between the Equinoctial and the Point of the Meridian cut by the Star, gives its Declinations. And the Meridian cuts, and ſhews its Right Aſcenſion on the Equinoctial, reckoning from the beginning of *Aries.*

USE XV. *To find the Longitude and Latitude of any Star.*

Bring the Solſtitial Colure to the brazen Meridian, and there fix the Globe; then will the Pole of the Ecliptick be juſt under 23 Deg. 30 Min. reckoning from the Pole above the North Point of the Horizon, and upon the ſame Meridian; there ſcrew the Quadrant of Altitude, and then bring its graduated Edge to the Star aſſigned, and there ſtay it: ſo will the Star cut its proper Latitude on the Quadrant, reckoned from the Ecliptick; and the Quadrant will cut the Ecliptick in the Star's Longitude, or its Diſtance from the firſt Point of *Aries.*

USE XVI. *To find the time of any Star's riſing, ſetting, or culminating, that is, being on the Meridian.*

Rectify the Globe, and Hour-Index, and bring the Star to the Eaſt or Weſt part of the Horizon, or to the brazen Meridian, and the Index will ſhew accordingly the Time of the Star's riſing, ſetting, or culminating, or of its being on the Meridian.

USE XVII. *To know, at any time aſſigned, what Stars are riſing or ſetting, which are on the Meridian, and how high they are above the Horizon; on what Azimuth or Point of the Compaſs they are; by which means the real Stars in the Heaven may eaſily be known by their proper Names, and rightly diſtinguiſhed from one another.*

Rectify the Globe, and fit the Quadrant of Altitude, and ſet the Globe, by means of the Compaſs, due North and South; then turn the Globe and Hour-Index to the Hour of the Night aſſigned; ſo will the Globe, thus fixed, repreſent the Face or Appearance of the Heavens for that time: whereby you may readily ſee what Stars are in or near the Horizon; what are on or near the Meridian; which are to the North, or which to the South, *&c.* and the Quadrant of Altitude being laid over any particular Star, will ſhew its Altitude and Azimuth, or on what Point of the Compaſs it is, whereby any Star may eaſily be known; eſpecially if you have a Quadrant to take the Altitude of any real Star ſuppoſed to be known by the Globe, to ſee whether it agrees with that Star which is its Repreſentative on the Globe or not.

USE XVIII. *The Sun's Place given, as alſo a Star's Altitude, to find the Hour of the Night.*

Rectify the Globe, and fit the Quadrant of Altitude; then move the Globe backwards or forwards, till the Quadrant cuts the Star in its given Altitude: for then the Hour-Index will ſhew the Hour of the Night. And thus may the Hour of the Night be known by a Star's Azimuth, or its Azimuth by its Altitude.

USE XIX. *To find the Diſtance between any two Stars.*

If the Stars lie both under the ſame Meridian, bring them to the brazen Meridian, and the Degrees of the ſaid Meridian comprehended between them, are their Diſtance.

If they are both in the Equinoctial, or have both the ſame Declination, that is, are both in the ſame Parallel, then bring them one after another to the brazen Meridian, and the Degrees of the Equinoctial intercepted between them, when thus brought to the Meridian ſeverally, are their Diſtance.

If the Stars are neither under the ſame Meridian or Parallel, then either lay the Quadrant of Altitude from one to the other (if it will reach), and that will ſhew the Diſtance between them in Degrees; or elſe take the Diſtance with Compaſſes, and apply that to the Equinoctial, or to the Meridian.

This Method of Proceeding will alſo ſhew the Diſtance of any two Places on the Terreſtrial Globe in Degrees. Wherefore to find how far any Place on the Globe is from another, you need only take the Diſtance between them on the Globe with a Pair of Compaſſes, and applying the Compaſſes to the Equator at the Beginning of *Aries*, or at the firſt Meridian, you will there find the Degrees of their Diſtance, which multiplied by 70 (or 69 $\frac{4}{11}$, *Engliſh* Miles), and that will be their Diſtance in Miles.

CHAP. III.

Of SPHERES.

SECTION I.

Of the Ptolemaick Sphere.

Fig. 3. THE third Figure of *Plate* 18, repreſents a Ptolemaick Armillary Sphere, made of Braſs, or Wood, conſiſting of the ſame Circles that have been deſcribed in Chapter I. aforegoing, and having a round Ball fixed in the Middle thereof, upon the Axis of the World, repreſenting the Earth. Upon the Surface of this Ball are drawn Meridians, Parallels, *&c.* as likewiſe as many Kingdoms, Countries, Seas, *&c.* with their Names, as can conveniently be depicted thereon. This Sphere revolves about the ſaid Axis, between the Meridian, and by this means not only ſhews the Sun's diurnal and annual Courſe, *&c.* about the Earth, according to the Ptolemaick Hypotheſis, which ſuppoſes the Earth to be at reſt, and the Sun to move about the ſame; but likewiſe by it any Problem relating to the Sun, may be ſolved, that can be done by the Globes. And this any one that knows the Uſe of the Globes may likewiſe do.

SECTION II.

Of the common Copernican Sphere.

Fig. 4. This Sphere ſtands upon four braſs or wooden Feet, upon each of which are fixed the four Ends of a braſs or wooden Croſs, upon which Croſs is faſtened a large hollow braſs or wooden Circle, whoſe Center is exactly over the Center of the Croſs. Upon the upper Plane of this Circle are the Calendars, and Circle of Signs deſcribed, the ſame as on the Horizon of the Globes. Cloſe within the Inſide of this Circle is fitted a flat moveable Rundle, whoſe Center is common with the Center of the Croſs. The outmoſt Limb of this Rundle is divided into 24 equal Parts, repreſenting the 24 Hours of Day and Night, numbered from the Index (of which more hereafter) towards the Right-hand with Numerical Letters from I to XII, and then beginning again with I, II, *&c.* to XII again.

There is a round Wheel fixed upon the Croſs, under the ſaid Rundle, whoſe Convex Side is cut into a certain Number of Teeth. Thro' the Rundle, the Wheel on the Croſs, and the Croſs itſelf, is fitted a perpendicular Axis, about which the Rundle moves. This repreſents part of the Axis of the Ecliptick, and at the Top thereof is placed a little Golden Ball, repreſenting the Sun.

On the under Side of the moveable Rundle moves another Wheel, whoſe Convex Side is cut into Teeth, and as the Rundle is turned about upon it's Center, this Wheel is alſo turned about upon it's Center, by the falling in of the Teeth on that Wheel fixed on the Croſs. Likewiſe near the outmoſt Limb of the Rundle is fitted another Wheel, into which is fitted a Pedeſtal, holding up a Sphere of ſeveral Parts, having a Terreſtrial Globe incloſed therein, as ſhall be ſhewn hereafter. The outmoſt Limb of this Wheel is likewiſe cut into Teeth, fitted into the Teeth of the fixed Wheel; and ſo as the Rundle moves round, this Wheel is carried about, and with it likewiſe the Earth, and all the Circles faſtened upon the aforeſaid Pedeſtal.

On one Side of this Rundle is faſtened a little round Pin to turn about the Rundle by, and near this Pin, is an Index upon the Rundle, reaching to the outward Limb of the great hollow Circle, and ſo at once may be applied to the Day of the Month in both Calendars, and alſo to the Degree of the Ecliptick the Sun is in that Day at Noon. *Note*, This Index is called the Index of the moveable Rundle. On each Side of the Croſs is placed a Pillar, ſupporting a broad Circle, repreſenting the Zodiack, with the Ecliptick in the Middle

Middle thereof, as in the Ptolemaick Sphere. *Note*, This is called the Zodiack, in the Uſe of the Sphere.

Upon the aforeſaid Pedeſtal are faſtened two Circles cutting each other at Right Angles, repreſenting the two Colures ſo placed, that the Points wherein they interſect each other ſtand directly upwards and downwards, and repreſent the Poles of the Ecliptick, the uppermoſt being the North, and the other the South. One of theſe Colures, *viz.* the Solſtitial, hath a ſmall Hour-Circle placed thereon, at the Extremity of the Axis of the Earth. In the Middle, between the two Poles of the Ecliptick, is a Circle broader than the Colures, cutting them at Right Angles; and this repreſents the Ecliptick, ſo called in the Uſe of the Sphere, and is divided into Degrees, figured with the Names and Characters of the Signs, and having on the inward Edge thereof ſeveral of the moſt notable fixed Stars, with the Names affixed to them, and each Star placed to the Degree and Minute of Longitude thereon, that it hath in Heaven.

Oblique to this Ecliptick $23\frac{1}{2}$ Degrees, on the Inſide, is fitted a thin Circle, repreſenting the Equinoctial, and is divided into 360 Degrees, and having two parallel leſſer Circles at $23\frac{1}{2}$ Degrees equally diſtant therefrom, repreſenting the Tropicks. On the Inſide of all theſe Circles, two thin Semi-Circles (called Semi-Circles of Latitude) are fitted in the Poles of the Ecliptick, ſo as one of them may move thro' one half the Ecliptick, *viz.* from *Cancer* thro' *Aries* to *Capricorn*; and the other from *Cancer* thro' *Libra* to *Capricorn*: the former of theſe may be called the vernal Semi-Circle of Latitude, and the other the autumnal Semi-Circle of Latitude. On the Edge of theſe Semi-Circles are depicted the ſame fixed Stars in their proper Longitude and Latitude, as are placed on the Ecliptick Circle aforeſaid, with their ſeveral Names affixed to them.

Thro' the ſolſtitial Colure at $23\frac{1}{2}$ Degrees from each Pole of the Ecliptick, goes a Wire, repreſenting the Earth's Axis, having an Index placed on the End thereof, for pointing at the Hour, on the Hour-Circle placed on the ſolſtitial Colure, as aforeſaid. In the Middle of this Axis is fixed a round Ball, repreſenting the Earth, having Meridians, Parallels, *&c.* and the Bounds of the Lands and Waters depicted thereon, as alſo the Names of as many Countries and Towns as can be placed with conveniency thereon. And in two oppoſite Points of the Equinoctial of this Ball, *viz.* 90 Degrees diſtant from the firſt Meridian, are fixed two ſmall Pins, whereon a moveable Horizon is placed, in the Eaſt and Weſt Points thereof; ſo that theſe Pins ſerve for an Axis to the Horizon: for on theſe Pins the Horizon may be elevated or depreſſed to any Degree the Pole is elevated above the Horizon. This Horizon ſlides on the North and South Points, within a brazen Meridian, hung upon the Axis of the Earth.

Round this Meridian, on the outmoſt Side, is made a Groove, having a ſmall braſs Ring fitted therein, ſo as the upper Side thereof is even with the upper Side of the brazen Meridian. This ſmall braſs Ring is faſtened to two oppoſite Points in the Horizon, *viz.* in the North and South, and ſerves as a Spring to keep it to the Degree of the Meridian you elevate the Horizon to. Upon two Pins on this ſmall Ring, are likewiſe faſtened two Semi-Circles of Altitude, yet not ſo faſtened, but that they may move as upon Centers, the one moving from North to South, thro' the Eaſt-Side of the Horizon, and the other the ſame way thro' the Weſt-Side. This Motion is performed upon the two Pins aforeſaid, as upon two Poles, which they repreſent, *viz.* the Poles of the Horizon, and therefore are ſo placed, that they may divide the upper and lower half of the Horizon into two equal Parts, and as the Horizon is moved, ſlide always into the Zenith and Nadir, and ſo become the Poles of the Horizon. Theſe two Semi-Circles of Altitude are divided into twice 90 Degrees, numbered at the Horizon upwards and downwards, and ending at 90 in the Zenith and Nadir.

SECTION III.

The Uſe of the Copernican Sphere.

USE I. *The Day of the Month given; to rectify the Sphere for Uſe in any given Latitude, and to ſet it correſpondent to the Situation of the Heavens.*

Bring the Index of the moveable Rundle to the Day of the Month, and elevate the Horizon to the Latitude of the Place; then bring the Meridian to the Sun's Place in the Ecliptick, and the Index of the Hour-Circle to 12. Laſtly, Bring the Center of the Earth, the Sun, or Golden Ball, in the Sphere, and the Sun in Heaven into a Right Line. Then will the Earth be rectified to it's Place in Heaven, the Horizon to it's Latitude on Earth, the Circles on the Sphere agreeable to thoſe in Heaven, and the whole correſpondent with the Heavens for that Day at Noon.

USE II. *The Day of the Month being given, to find the Sun's Declination.*

Rectify the Earth's Place (according to *Uſe* I.) and then you will have the Sun's Place in the Zodiack; then bring the Meridian to the Sun's Place in the Ecliptick on the Sphere; and the Number of Degrees comprehended between the Equinoctial and the Sun's Place, are the Sun's Declination for that Day at Noon.

USE III. *To find the Sun's Right or Oblique Ascension for any Day at Noon.*

Rectify the Earth's place to the Day of the Month, and bring the Meridian to the Sun's place in the Ecliptick; and the Number of Degrees on the Equinoctial contained between the vernal Colure, and the Sun's place, are the Right Ascension sought.

Now to find the Oblique Ascension, turn the Earth 'till the East side of the Horizon stands against the Sun, and the Degree of the Equinoctial then at the Horizon, shews the Oblique Ascension.

USE IV. *To find the Sun's Meridian Altitude.*

Bring the Index of the Rundle to the Day of the Month, and rectify the Horizon to the Latitude of the Place. This being done, bring the Meridian to the Sun's place in the Ecliptick, and the Number of Degrees on the Meridian comprehended between the Horizon and the Sun's place, gives the Meridian Altitude sought.

USE V. *To find the Sun's Altitude at any time of the Day.*

Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Horizon, and Hour-Index: then turn the Earth 'till the Hour-Index comes to the given Hour of the Day, and bring the vertical Circle to the Sun's place, and the Number of Degrees of the vertical Circle that transite the Sun's place, are his Altitude above the Horizon.

USE VI. *The Sun's Altitude being given, to find the Hour of the Day.*

Bring the Index of the Rundle to the Day of the Month, and rectify the Horizon and Hour-Index (as by *Use* I.); then turn the Earth 'till the you can fit the Horizon to the given Altitude upon the vertical Circle, directly against the Sun's place; then the Hour-Index will give the Hour of the Day, respect being had to the Morning or Afternoon.

USE VII. *To find at what Hour the Sun comes to the East or West Points of the Horizon.*

Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Horizon and Hour-Index (as by *Use* I.); then bring the vertical Circle to the East Point of the Horizon, if it be the Sun's Easting you enquire; or to the West Point of the Horizon, if it be the Sun's Westing. This being done, turn the Earth 'till the vertical Circle comes to the Sun's place; then will the Index point to the Hour of the Day.

USE VIII. *To find the time of the Sun's rising or setting.*

Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Horizon, and Hour-Index, Then turn the Earth Eastwards, 'till some part of the East-side of the Horizon stands directly against the Sun's place; then will the Hour-Index point to the time of the Sun's rising. Again, Turn the Earth 'till some part of the West-side of the Horizon stands directly against the Sun's place, then the Index of the Hour-Circle will shew the time of the Sun's setting.

USE IX. *The Hour of the Day given, to find the Sun's Azimuth.*

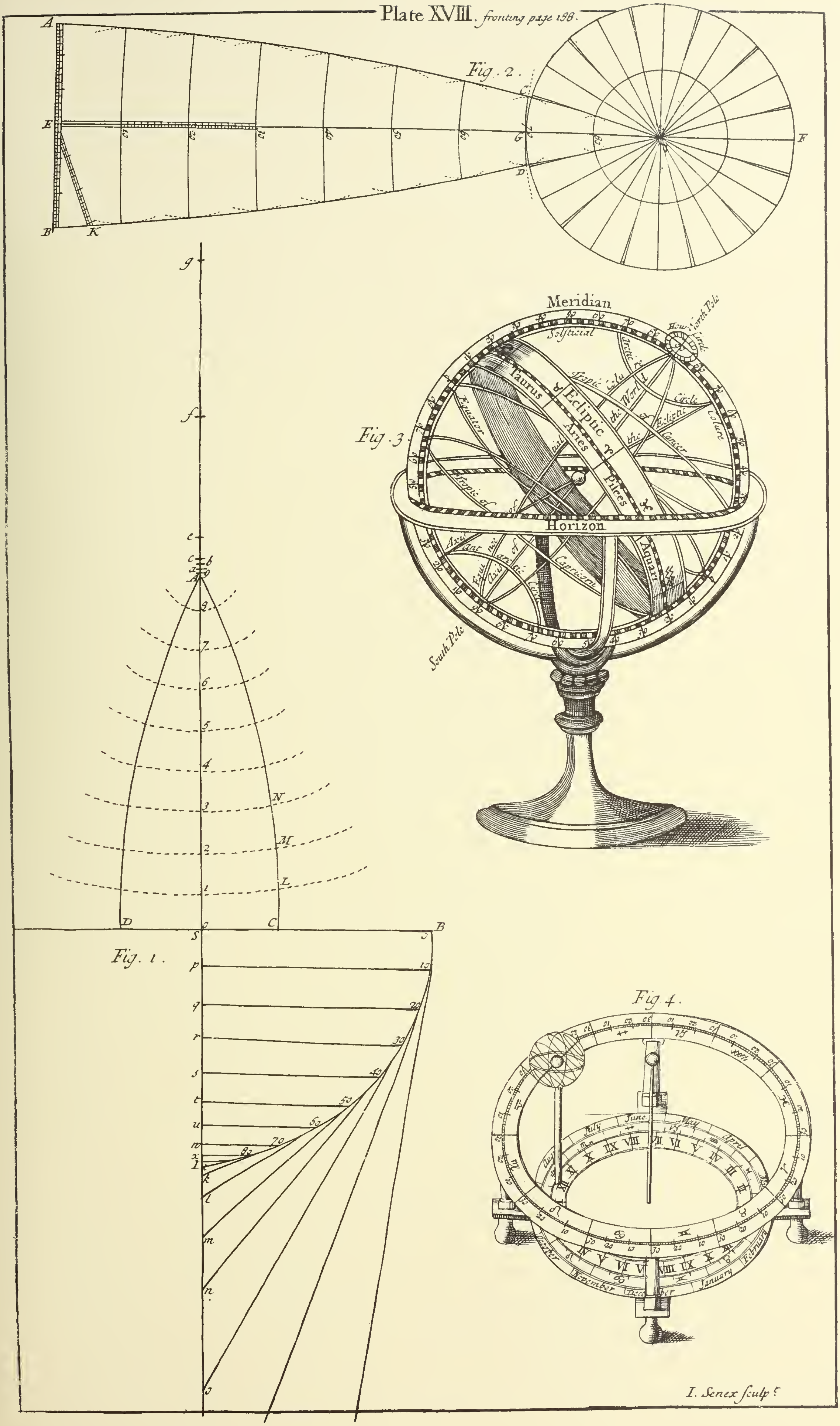
Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Horizon and Hour-Index. Then turn the Earth 'till the Hour-Index points to the Hour of the Day given. This being done, bring the vertical Circle to the Sun's place, and the Number of Degrees of the Horizon, that the vertical Circle cuts, counted from the East Point, either Northwards or Southwards, are the Degrees of the Sun's Azimuth before Noon. Or the Number of Degrees of the Horizon that the vertical Circle cuts, counted from the West-side of the Horizon, either Northwards or Southwards, give the Sun's Azimuth after Noon.

USE X. *To find in what Place of the Earth the Sun is in the Zenith, at any given time; as also in what several Places of the Earth the Sun shall stand in the Horizon at the same time.*

Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Hour-Index; then seek the Sun's Declination, and turn the Earth eastwards 'till the Index points to the given Hour; so shall the Number of Degrees of the Equinoctial that the Meridian passes thro' while the Earth is thus turning, be the Number of Degrees of Longitude, eastwards from your Habitation, the Place shall have in the Parallel of the Sun's Declination.

Now if you open a Pair of Calliper Compasses to 90 Degrees on the Equinoctial, and place one Foot in this Point of the Earth thus found, and turn the other Foot round about the Earth, all the Places that the Foot passes thro' will at that time have the Sun in their Horizon.

Plate XVIII. *fronting page 188.*



USE XI. *How to find the true Places of the Stars on the Sphere*; *as likewise their Longitude and Latitude.*

Round the Plane of the Ecliptick, are placed ſeveral of the moſt noted fixed Stars, according to their true Longitude; and along the two Semi-circles of Latitude, are the ſame Stars placed according to their Latitude from the Ecliptick. Whence if you would find the true place of any given Star in the Sphere; Firſt ſeek the Star in the Ecliptick, and likewiſe the ſame Star on one of the Semi-circles of Latitude, and bring the edge of that Semi-circle to the Star in the Ecliptick; then will the Star on the Semi-circle of Latitude ſtand in the ſame Place and Situation on the Sphere, that it does in Heaven.

USE XII. *To find the Declination, right and oblique Aſcenſion of a Star.*

Bring the proper Semi-circle of Latitude to the Star on the Ecliptick, and the Meridian to the Star on the Semi-circle of Latitude; and then the Number of Degrees on the Meridian, comprehended between the Equinoctial and the Star, are its Declination. Likewiſe the Degree of the Equator, cut by the Meridian, is the Star's right Aſcenſion. But to find a Star's oblique Aſcenſion, rectify the Horizon (as by *Uſe* I.) and bring the proper Semi-circle of Latitude to the Star in the Ecliptick, and turn the Eaſt-ſide of the Horizon to the Star; then will the Degree of the Equator cut by the Horizon be the Star's oblique Aſcenſion.

USE XIII. *To find the Time of the Riſing and Setting of any Star in any given Latitude.*

Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Horizon and Hour Index; then bring the proper Semi-circle of Latitude to the Star on the Ecliptick, and the Eaſt-ſide of the Horizon to the Star; this being done, the Hour-Index will ſhew the Hour the Star riſes at: and if you bring the Weſt-ſide of the Horizon to the Star, the Index of the Hour-Circle will ſhew the Time that the Star ſets.

USE XIV. *The Day of the Month, Hour of the Night, and Latitude of the Place being given, to know any remarkable Star obſerved in the Heavens.*

Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Horizon and Hour-Index; then turn the Earth 'till the Index of the Hour-Circle comes to the Hour of the Night, and obſerve the Altitude of the Star, and what Point of the Compaſs it bears upon. Afterwards bring the vertical Circle to the ſame Point of the Compaſs, and number the Star's Altitude on the vertical Circle, and try with the Semi-circle of Latitude what Star you can fit to that Altitude, for that is the Star in the Heavens.

USE XV. *The Azimuth of any known Star being given, to find the Hour of the Night, and Almicanter of that Star.*

Bring the Index of the moveable Rundle to the Day of the Month, and rectify the Horizon and Hour-Index; afterwards bring the Star to its place, and the vertical Circle to its known Degree of Azimuth. This being done, turn the Earth 'till the vertical Circle comes to the Star; then the Index of the Hour-Circle will ſhew the Hour of the Night, and the Degree of the vertical Circle cut by the Star will be its Almicanter.

SECTION IV.

The Deſcription and Uſe of the Copernican Sphere, called the Orrery.

The Outſide of this Inſtrument, as appears by the figure thereof, is very beautiful, the Frame being of fine Ebony adorned with 12 Silver Pilaſters, in the form of *Caryatides*; and with all the Signs of the Zodiack caſt of the ſame Metal, and placed between them: the Handles are alſo of Silver finely wrought, with very nice Joints. On the top of the Frame, which is exactly circular, is a broad Silver Ring, on which the Figures of the twelve Signs are exactly graved, with two Circles accurately divided; one ſhewing the Degrees of each Sign, and the other the Sun's Declination againſt his place in the Ecliptick each Day at Noon. *Plate* 19. Fig. 1.

The aforeſaid Silver Plate, repreſents the Plane of the great Ecliptick of the Heavens, or that of the Earth's annual Orbit round the Sun; which, as it paſſes thro' the Center of the Sun, ſo its Circumference is made by the Motion of the Earth's Center; and which, for the better advantage of View and Sight, is in the Figure placed parallel to the Horizon.

S is a large gilded Ball, ſtanding up in the middle, whoſe Support A B makes with the Plane of the Ecliptick an Angle of about 82 Degrees. This Support repreſents the Sun's Axis continued, about which he revolves in about 25 Days, and the Golden Ball repreſents the Sun itſelf placed pretty near the Center of the Earth's Orbit; ſo that

 when

when the Inſtrument is ſet a-going, the Excentricity of the Earth, and the other Planets, may be in the ſame Proportion as they are in the Heavens.

The two little Balls M and V, which ſtand upon two Wires at different Diſtances from the Sun, repreſent *Mercury* and *Venus*: The reaſon why they are placed upon the ſaid two Wires, is only that their Centers may be ſometimes in, and always pretty near the Plane of the great Ecliptick; and this Poſition is contrived in order to ſhew what Appearances they do really exhibit in their ſeveral Revolutions round the Sun.

The Globe E is of Ivory, and repreſents the Earth. The Pin or Wire that ſupports it, repreſents the Earth's Axis continued, and makes an Angle of 66 $\frac{1}{2}$ Degrees, with the Plane of the Ecliptick. And as the Earth in each of her annual Revolutions round the Sun, always keeps her own Axis parallel to itſelf; ſo when this Inſtrument is ſet a-going, the little Ivory Earth will likewiſe do ſo too, in it's Revolution round the Golden Sun S.

The little Ball *m* ſtanding upon a Wire, repreſents the Moon, and *a b* is a Silver Circle repreſenting her Orbit round about the Earth, the Plane whereof always paſſes thro' the Center of the Earth; and there are ſeveral Figures graved upon it, ſhewing the Moon's Age, from one New Moon to the other.

One half of the Moon's Globe is white, and the other black, that ſo her Phaſes may be repreſented: for this Inſtrument is ſo contrived, that this little Moon will turn round it's own Axis, at the ſame time as it moves in the Silver Orbit round the Earth E.

The whole Movement, which conſiſts of near 100 Wheels, is covered by a great Braſs Plate, having a Hole in it, and there is a moveable Index on the Silver Ecliptick, on the former of which, are the common Solar Years denoted; and by taking the Inſtrument to pieces, it may be ſet to this preſent time; and the Planets, by means of an Ephemeris, may be ſet to any particular time alſo. So that if a Weight or Spring, as in a Clock, were applied to the Axis of the Movement, ſo as to make it move round once in juſt twenty-four Hours, the repreſentative Planets in the Inſtrument, *viz. Mercury*, *Venus*, the *Earth*, and the *Moon*, would all perform their Motions round the Sun, and one another, exactly in the ſame Order as their Originals do in the Heavens; and ſo the Aſpects, Eclipſes, *&c.* of the Sun and Planets, would thereby be ſhewn for ever. But becauſe this would be inſtructive only in that ſlow and tedious way, to ſuch as could have daily recourſe to it, therefore there is a Handle fitted to it, by which the Axis may be ſwiftly turned round; and ſo all the Appearances ſhewn in a very little time: for by turning the Handle backwards or forwards, what Eclipſes, Tranſits, *&c.* have happened in any time paſt, or what will happen for any time to come, will be ſhewn, without doing any injury to the Inſtrument.

One entire Turn of the Handle of this Inſtrument, anſwers to the diurnal Motion of the Earth about it's Axis, and is meaſured by means of an Hour-Index, placed at the Foot of the Wire whereon the Earth is fixed, moving once round in the ſame time. Alſo obſerve that the Contrivance of this Inſtrument is ſuch, that the Motion may be made to tend either way, forwards or backwards; and ſo the Handle may be turned about 'till the Earth be brought to any Degree or Point of the Ecliptick required.

Again, As the Earth moves round, by turning the Handle, the Moon's Orbit riſes and falls about 5 Degrees above and below the great Ecliptick, that ſo her North or South Latitude may be exactly repreſented; and there are two little Studs placed in two oppoſite Points of the Moon's Orbit, repreſenting the Moon's Nodes.

Now if the Handle, one Turn of which anſwers to one Natural Day, or twenty-four Hours, be turned twenty-five times about, then the Sun will have moved once round about it's Axis. Again, 365 $\frac{1}{4}$ of the Turns of the Handle will carry the Earth quite round the Sun; 88 will carry *Mercury* quite round; 244 will make *Venus* move once round the Sun; and about 27 $\frac{1}{4}$ Turns will carry the Moon round the Earth in her Orbit, which will likewiſe at the ſame time always turn the ſame Hemiſphere towards the Earth.

And by thus revolving the Earth and Planets round the Sun, the Inſtrument may be brought to exhibit *Mercury*, and ſometimes *Venus*, as directly interpoſed between the Earth and the Sun; and then they will appear as Spots in the Sun's Diſk: and this Inſtrument ſhews alſo very clearly the Difference between the Geocentrick and Heliocentrick Aſpects, according as the Eye is placed in the Center of the Earth or Sun.

This Inſtrument likewiſe very plainly ſhews the Difference between the Moon's Periodick and Synodick Months, and the reaſon thereof; for if the Earth be ſet to the firſt Point of *Aries*, at which time ſuppoſe the firſt New Moon happens, and afterwards the Handle be turned 27 $\frac{1}{4}$ times about, we ſhall have the ſecond New Moon; and if at the Earth's Place in the Ecliptick where this laſt New Moon happens, ſome Mark be made, and then the Handle be turned 27 $\frac{1}{4}$ times more, the Moon will be exactly brought again to interpoſe between the Earth and the Sun, that is, it will be New Moon with us: but the Line of the *Syzygy* will not be right againſt the aforeſaid Mark in the Ecliptick, but behind it; and it will require two Days time, or two Turns more of the Handle, before it gets thither. The reaſon of this is plain, becauſe in this 27 $\frac{1}{4}$ Days, the Earth advances ſo far forwards in her annual Courſe, as is the Quantity of the Difference in time between the Moon's two Months.

If

If the Handle be turned about 'till the Conjunction or Oppoſition of the Sun and Moon happens in or near the Nodes, then there will be an Eclipſe of the Sun or Moon. But in order yet further to ſhew the Solar Eclipſes, and alſo the ſeveral Seaſons of the Year, the Increaſe and Decreaſe of Day and Night, and the different Lengths of each in different Parts of our Earth, there is a little Lamp contrived to put on upon the Body of the Sun, which caſting, by means of a Convex Glaſs (the Room wherein the Inſtrument is, being a little darkened), a ſtrong Light upon the Earth, will ſhew at once all theſe things: Firſt, how one half of our Globe is always illuminated by the Sun, while the other Hemiſphere is in the dark, and conſequently how Day and Night are formed by the Revolution of the Earth round her Axis. Alſo by turning round the Handle, you will ſee how the Shadow of the Moon's Body will cover ſome part of the Earth, and thereby ſhew, that to the Inhabitants of that part of the Earth there will be a Solar Eclipſe.

When the Earth is brought to the firſt Degree of *Aries* or *Libra*, the reaſon of the Equality of Days and Nights all over the Earth, will be plainly ſhewn by this Inſtrument; for in theſe Poſitions, as the Earth turns about her Axis, juſt one half of the Equator, and all Parallels thereto, will be in the Light, and the other half in the Dark; and therefore the Days and Nights muſt be every where equal: for the Horizon of the Earth's Diſk will be parallel to the Plane of the Solſtitial Colure.

And when the Earth is brought to *Cancer*, the Horizon of the Diſk, or that Plane which divides the Earth's enlightened Hemiſphere from the darkened one, will not then be parallel to, but lie at Right Angles to the Plane of the Solſtitial Colure. The Earth being now in *Cancer*, the Sun will appear to be in *Capricorn*, and conſequently it will be our Winter Solſtice. And as the Earth is turned either way about it's Axis, the entire Northern frigid Zone, or all Parts of the Earth lying within the Artick Circle, are in the dark Hemiſphere; and by making a Mark in any given Parallel, by the Earth's diurnal Revolution, you will know how much longer the Nights are than the Days in that Parallel. And the contrary of this will happen, when the Earth is brought to *Capricorn*.

Therefore this Inſtrument delightfully and demonſtratively ſhews, how thereby all the Phænomena of the different Seaſons of the Year, and the Varieties and Viciſſitudes of Night and Day, are ſolved and accounted for.

CHAP. IV.

Of an Aſtronomical Quadrant, Micrometer, and Gunter's *Quadrant.*

SECTION I.

THIS Figure repreſents an Aſtronomical Quadrant upon it's Pedeſtal, with it's Limb curiouſly divided diagonally, and furniſhed with a fixed and moveable Teleſcope. Fig. 2.

This Quadrant may be moved round horizontally, by turning a perpetual Screw fitted into the Pedeſtal: For as this Screw is turned about by means of a Key, at the ſame time it cauſes the Axis A to turn, by the falling in of its Threads between the Teeth of a ſtrong thick Circle on the ſaid Axis.

Behind the Quadrant is fixed, at Right Angles to its Plane, a ſtrong thick Portion of a Circle greater than a Semicircle, having one Semi-circle of the outſide thereof cut into Teeth. There is likewiſe another ſtrong thick Portion of a Circle ſomething greater than a Semi-circle behind the Quadrant, which is moveable upon two fixed Studs, at Right Angles to the former Portion; ſo that the Plane of this Portion may be parallel, inclined, or at Right Angles to the Plane of the Quadrant. On the ſide of this Portion, which is made flat next to the other fixed Portion, is a contrivance with a Screw and perpetual Screw, ſuch that in turning the Screw the Threads of the perpetual Screw may be locked in betwen the Teeth of the fixed circular Portion; and by this means the Quadrant fixed to any Point, according to the direction of the Plane of the fixed Portion. And when the Quadrant is to be moved but a ſmall matter in the aforeſaid Direction, this may be done by turning the perpetual Screw with a Key.

The Outſide of the abovementioned moveable circular Portion is cut into Teeth, and about the Center thereof the Axis A is moveable, according to the Direction of the Plane of the ſaid Portion. In this Axis ſlides a little Piece carrying a perpetual Screw, whoſe Threads, by means of a Trigger, may be locked in between the Teeth of the moveable circular Portion. And ſo when the Axis is ſet in the Pedeſtal, the Quadrant may be fixed to any Point, according to the Direction of the Plane of the ſaid moveable Portion.

*

Therefore

Therefore by these Contrivances the Quadrant may be readily fixed to any required Situation, for observing Celestial Phænomeua, without moving the Pedestal.

There is a Piece sliding on the Index, upon which the moveable Telescope is fastened, carrying a Screw and perpetual Screw; so that when the Telescope and Index are to be fixed upon any Point in the Limb of the Instrument, this may be done, by means of the Screw which locks the Threads of the perpetual Screw in between some of the Teeth cut round the curve Surface of the Limb of the Instrument: and when the Index and Telescope is to be moved a very minute Space backwards or forwards along the Limb, this is done by means of a Key turning a small Wheel fastened upon the aforenamed Piece, which is cut into a certain Number of Teeth, and whose Axis is at Right Angles to the Plane of the Quadrant; for this Wheel moves another (having the same Number of Teeth as that) which is at the end of the Cylinder whereon the perpetual Screw is: and by this means the perpetual Screw is turned about; and so the Index and Telescope may be moved a very minute Space backwards or forwards along the Limb. *Note*, The Number of Teeth the Curve Surface of the Limb is divided into, must be as great as possible, and the Threads of the perpetual Screw falling between them very fine; for the Exactness of the Instrument very much depends upon thus.

These Quadrants are commonly two Feet Radius, and all Brass, except the Pedestal and the perpetual Screws; the Telescopes have each two Glasses and Cross-hairs in their *Foci*; and for the Manner of dividing their Limbs, &c. See our Author's Quadrants.

SECTION II.

Concerning a Micrometer.

Fig. 3. This Micrometer is made of Brass: A B C *g* is a rectangular Brass Frame, the Side A B being about 3 Inches long, and the Side B C, as likewise the opposite Side A *g*, are about 6 Inches; and each of these three Sides are $\frac{8}{10}$ of an Inch deep. The two opposite Sides of this Frame are screwed to the circular Plate, which we shall speak of by and by.

The Screw P having exactly 40 Threads in an Inch, being turned round, moves the Plate G D E F, along two Grooves made near the Tops of the two opposite Sides of the Frame; and the Screw Q having the same Number of Threads in an Inch as P, moves the Plate R N M Y along two Grooves made near the bottom of the said Frame, in the same direction as the former Plate moves, but with half the Velocity as that moves with. These Screws are both at once turned, and so the said Plates moved along the same way, by means of a Handle turning the perpetual Screw S, whose Threads fall in between the Teeth of Pinions on the Screws P and Q. *Note*, Two and a half Revolutions of the perpetual Screw S, moves the Screw P exactly once round.

The Screw P turns the Hand *a*, fastened thereto over 100 equal Divisions made round the Limb of a circular Plate, to which the abovenamed two opposite Sides of the Frame are screwed at Right Angles. The Teeth of the Pinion of the Screw P, whose Number are 5, takes into the Teeth of a Wheel, on the backside of the circular Plate, whose Number are 25. Again, On the Axis of this Wheel is a Pinion of two, which takes into the Teeth of another Wheel moving about the Center of the circular Plate, without side the same, having 50 Teeth. This last Wheel moves the lesser Hand *b* once round the abovenamed circular Plate, in the $\frac{1}{100}$ part of the time the Hand *a* is moving round: for because the Number of Teeth of the Pinion on the Screw P, are 5, and the Number of Teeth of the Wheel this Pinion moves round, are 20; therefore the Screw P moves four times round in the same time the said Wheel is moving once round. Again, Since there is a Pinion of two takes into the Teeth of a Wheel, whose Number are 50, therefore this Wheel with 50 Teeth will move once round in the same time that the Wheel of 20 Teeth hath moved twenty-five times round; and consequently the Screw P, or Hand *a*, must move a hundred times round in the same time as the Wheel of 50 Teeth, or the Hand *b*, hath moved once round.

It follows from what hath been said, that if the circular Plate W, which is fastened at Right Angles to the other circular Plate, be divided into 200 equal Parts, the Index *x* to which the Handle is fastened, will move five of these Parts in the same time that the Hand *a* has moved one of the hundred Divisions round the Limb of the other circular Plate: and so by means of the Index *x*, and Plate W, every fifth Part of each of the Divisions round the other Plate may be known.

Moreover, Since each of the Screws P and Q have exactly 40 Threads in an Inch; therefore the upper Plate G D E F will move 1 Inch, when the Hand *a* hath moved forty times round, the four thousandth part of a Inch, when the said Hand hath moved over one of the Divisions round the Limb, and the twenty thousandth part of an Inch, when the Index *x* hath moved one part of the 200 round the Limb of the circular Plate W; and the under Plate R N M Y, half an Inch, the two thousandth part of an Inch, and the ten thousandth part of an Inch the same way, in the said respective times.

Hence,

Hence, if the under Plate, having a large round Hole therein, be fixed to a Teleſcope, ſo that the Frame may be moveable together with the whole Inſtrument, except the ſaid lower Plate, and the ſtrait ſmooth Edge H I, of the fixed narrow Plate A B I H, as likewiſe the ſtrait ſmooth Edge D E of the moveable Plate G D E F, be perceivable thro' the round Hole in the under Plate, in the Focus of the Object-Glaſs; then when the Handle of the Micrometer is turned, the Edge H I of the narrow Plate ABIH, fixed to the Frame, and D E of the moveable Plate, will appear thro' the Teleſcope equally to accede to, or recede from, each other. And ſo theſe Edges will ſerve to take the apparent Diameters of the Sun, Moon, *&c.* the manner of doing which is thus: Suppoſe in looking at the Moon thro' the Teleſcope, you have turned the Handle 'till the two Edges D E and H I are opened, ſo as to juſt touch or claſp the Moon's Edges; and that there was twenty-one Revolutions of the Hand *a* to compleat that Opening. Firſt ſay, As the focal Length of the Object-Glaſs, which ſuppoſe 10 Feet, is to Radius, So is 1 Inch to the Tangent of an Angle ſubtended by 1 Inch in the Focus of the Object-Glaſs, which will be found 28 Min. 30 Sec. Again, Becauſe there are exactly 40 Threads of the Screws in one Inch, ſay, If forty Revolutions of the Hand *a* give an Angle of 28 Min. 38 Sec. what Angle will twenty-one Revolutions give? The Anſwer will be 15 Min. 8 Sec. and ſuch was the Moon's apparent Diameter, and ſo may the apparent Diameters of any diſtant Objects be taken.

It is to be obſerved, that the Diviſions upon the top of the Plate G D E F, are Diagonal Diviſions of the Revolutions of the Screws, with Diagonal Diviſions of Inches againſt them; and ſo as the ſaid Plate ſlides along, theſe Diagonals are cut by Diviſions made on the Edge of the narrow Plate KL, fixed to the oppoſite Sides of the Frame by means of two Screws. Theſe Diagonal Diviſions may ſerve to count the Revolutions of the Screws, and to ſhew how many there are in an Inch, or the Parts of an Inch.

SECTION III.

Of the Conſtruction of Gunter's *Quadrant.*

This Quadrant, which is partly a Projection, that is, the Equator, Tropicks, Ecliptick, and Horizon, are ſtereographically projected upon the Plane of the Equinoctial, the Eye being ſuppoſed to be placed in one of the Poles, may be thus made. Fig. 4.

About the Center A deſcribe the Arc C D, which may repreſent either of the Tropicks. Again, Divide the Semidiameter A T ſo in E, that A E being Radius, A T may be the Tangent of 56 Deg. 46 Min. half the Sun's greateſt Declination above the Radius or Tangent of 45 Deg. To do which, ſay, As the Tangent of 56 Deg. 46 Min. is to 1000; So is Radius to 655: therefore if A T be made 1000 equal Parts, A E, the Radius of the Equator, will be 655 of thoſe Parts. And if about the Center A, with the Diſtance A E, the Quadrant E F be deſcribed, this will ſerve for the Equinoctial.

Now to find the Center of the Ecliptick, which will be ſomewhere in the left Side of the Quadrant A D (repreſenting the Meridian) you muſt divide A D ſo in G, that if A F be the Radius, A G may be the Tangent of 23 Deg. 30 Min. the Sun's greateſt Declination; therefore if A F be 1000, A G will be 434. And if about the Center G, with the Semidiameter G D, an Arc E D be deſcribed, this will be $\frac{1}{4}$ of the Ecliptick. And to divide it into Signs and Degrees, you muſt uſe this Canon, *viz.* As Radius is to the Tangent of any Degree's diſtance from the neareſt Equinoctial Point, So is the Co-ſine of the Sun's greateſt Declination to the Tangent of that Degree's Right Aſcenſion, which muſt be counted on the Limb from the Point B, by which means the Quadrant of the Ecliptick may be graduated.

As, for Example, The Right Aſcenſion of the firſt Point of ♉ being 27 Deg. 54 Min. lay a Ruler to the Center A, and 27 Deg. 54 Min. on the Limb, from B towards C, and where it cuts the Ecliptick, will be the firſt Point of ♉; and ſo for any other.

The Line E T, between the Equator and the Tropick, which is called the Line of Declination, may be divided into 23 Deg. 30 Min. in laying off from the Center A, the Tangent of each Degree added to 45 Deg, the Line A E being ſuppoſed the Radius of the Equinoctial. As ſuppoſe the Point for 10 Degrees of Declination be to be found, add 5 Deg. (half 10.) to 45 Deg. and the Sum will be 50 Deg. the Tangent of which will be (ſuppoſing the Radius 1000) 1192: therefore laying 1192 Parts from A, or 192 from E, and you will have a Point for 10 Degrees of Declination; and ſo for others.

Moſt of the principal Stars between the Equator and Tropick of *Cancer*, may be put on the Quadrant by means of their Declination, and Right Aſcenſion. As ſuppoſe the Wing of *Pegaſus* be 13 Deg. 7 Min. and the Right Aſcenſion 358 Deg. 34 Min. from the firſt Point of *Aries*. Now if about the Center A, you draw an occult Parallel thro' 13 Deg. 7 Min. of Declination, and then lay a Ruler from the Center A thro' 1 Deg. 26 Min. (the Complement of 358 Deg. 34 Min. to 360 Deg.) in the Limb B C, the Point where the Ruler cuts the Parallel, will be the Place for the Wing of *Pegaſus*, to which you may ſet the Name, and the Time when he comes to the South.

There being Space sufficient between the Equator and the Center, you may there describe the Quadrant, and divide each of the two Sides furthest from the Center into 100 Parts; so shall the Quadrant be generally prepared for any Latitude. But before the particular Lines can be drawn, you must have four Tables fitted for the Latitude the Quadrant is to serve in.

First, A Table of Meridian Altitudes for the Division of the Circles of Days and Months, which may be thus made: Consider the Latitude of the Place, and the Sun's Declination for each Day of the Year; if the Latitude and Declination be both North, or both South, add the Declination to the Complement of the Latitude; if they be one North, and the other South, substract the Declination from the Complement of the Latitude, and you will have the Meridian Altitude for that Day. As, in the Latitude of 51 Deg. 32 Min. North, whose Complement is 38 Deg. 28 Min. the Declination on the 10th of *June* will be 23 Deg. 30 Min. North; therefore add 23 Deg. 30 Min. to 38 Deg. 28 Min. and the Sum will be 61 Deg. 58 Min. the Meridian Altitude on the 10th of *June*. Again, The Declination on the 10th of *December* will be 23 Deg. 30 Min. South; wherefore take 23 Deg. 30 Min. from 38 Deg. 28 Min. and the Remainder will be 14 Deg. 58 Min. the Meridian Altitude on the 10th of *December*. And in this manner may the Meridian Altitude for each Day in the Year be found, and put in a Table.

Your Table being made, you may inscribe the Months and Days of each Month on the Quadrant, in the Space left below the Tropick. As, Laying a Ruler upon the Center A, and 16 Deg. 42 Min. the Sun's Meridian Altitude on the 1st of *January*, in the Limb BC, you may draw a Line for the end of *December* and beginning of *January*. Again, Laying a Ruler to the Center A, and 24 Deg. 34 Min. the Sun's Altitude at Noon the end of *January*, or first of *February*, on the Limb, and you may draw a Line for that Day. And so of others.

Now to draw the Horizon, you must find its Center, which will be in the Meridian Line A C; and if the Point H be taken such, that if A H be the Tangent Complement of the Latitude, *viz.* of 38 Deg. 28 Min. A F being supposed Radius; or if A F be supposed 1000, and A H 776 of those Parts, then will H be the Center of the Horizon. Therefore if about the Center H, with the Distance H E, an Arc be described cutting the Tropick T D, the said Arc will represent the Horizon.

The next thing done, must be to make a Table for the Division of the Horizon, which may be done by this Canon, *viz.* As Radius is to the Sine of the Latitude, So is the Tangent of any Number of Degrees in the Horizon (which will be not more than 40 in our Latitude) to the Tangent of the Arc in the Limb which will divide the Horizon.

As in our Latitude, 7 Deg. 52 Min. belong to 10 Deg. of the Degrees of the Horizon; therefore laying a Ruler to the Center A, and 7 Deg. 52 Min. in the Limb B C, the Point where the Ruler cuts the Horizon, will be 10 Deg. in the Horizon; and so of the rest. But the Lines of Distinction between every 5th Degree are best drawn from the Center H.

The third Table for drawing the Hour-Lines, must be a Table of the Sun's Altitude above the Horizon at every Hour, especially when he comes to the Equator, Tropicks, and other intermediate Declinations. If the Sun be in the Equator, and so have no Declination, as Radius to the Co-sine of the Latitude, so is the Co-sine of any Hour from the Meridian to the Sine of the Sun's Altitude at that Hour.

But if the Sun be not in the Equator, you must say, As the Co-sine of the Hour from the Meridian is to Radius, So is the Tangent of the Latitude to the Tangent of a 4th Arc. Then consider the Sun's Declination, and the Hour proposed; if the Latitude and Declination be both alike, and the Hour fall between Noon and Six, substract the Declination from the aforesaid 4th Arc, and the Remainder will be a 5th Arc.

But if the Hour be either between Six and Midnight, or the Latitude and Declination unlike, add the Declination to the 4th Arc, and the Sum will be a 5th Arc. Then as the Sine of the fourth Arc is to the Sine of the Latitude, so is the Co-sine of the 5th Arc to the Sine of the Altitude sought.

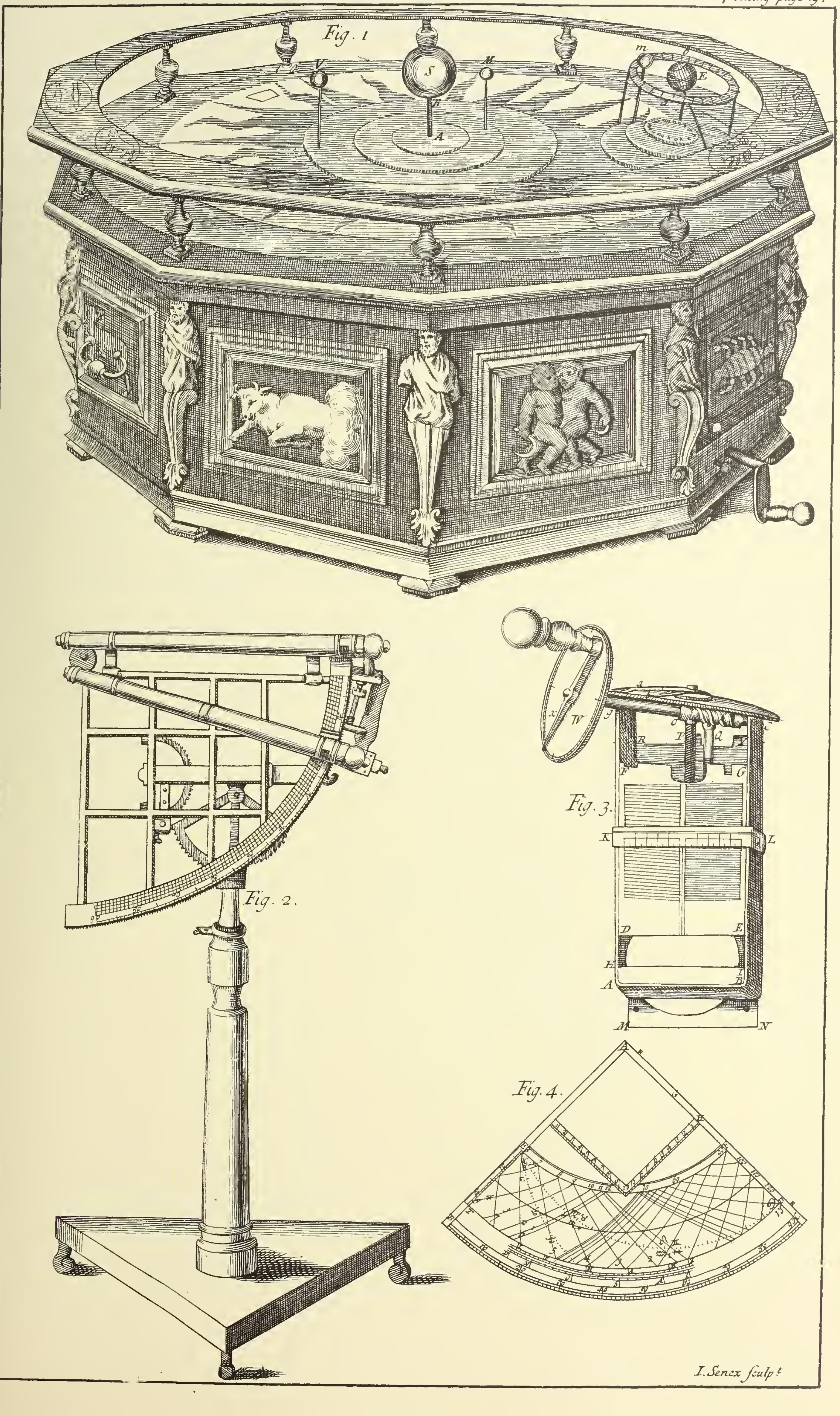
Lastly, You may find the Sun's Declination when he rises or sets, at any Hour, by this Canon, *viz.* As Radius is to the Sine of the Hour from Six, So is the Co-tangent of the Latitude to the Tangent of the Declination.

As in our Latitude you willl find, that when the Sun rises at five in the Summer, or seven in the Winter, his Declination is 11 Deg. 36 Min. whence you willfind the Sun's Meridian Altitude in the beginning of ♋ will be 61 Deg. 58 Min. in ♊ 58 Deg. 40 Min. in ♉ 49 Deg. 58 Min. in ♈ 38 Deg. 30 Min. *&c.* but the beginning of ♋ and ♑ is represented by the Tropick T D, drawn thro' 23 Deg. 30 Min. of Declination, and the beginning of ♈ and ♎ by the Equator E F. Now if you draw an occult Parallel between the Equator and the Tropick, at 11 Deg. 30 Min. of Declination, it shall represent the beginning of ♉, ♍, ♏, and ♓. If you draw another occult Parallel thro' 20 Deg. 12 Min. of Declination, it will represent the beginning of ♊, ♌, ♐ and ♒.

Then lay a Ruler from the Center A thro' 61 Deg. 58 Min. of Altitude in the Limb B C, and note the Point where it crosses the Tropick of ♋. Then move the Ruler to 58 Deg. 40 Min.

Plate XIX

fronting page 194.



40 Min. and note where it croſſes the Parallel of ♊; then to 49 Deg. 58 Min. and note where it croſſes the Parallel of ♉; and again to 38 Deg. 28 Min. noting where it croſſes the Equator; and a Line drawn thro' theſe Points will repreſent the Line of 12 in the Summer, while the Sun is in ♈, ♉, ♊, ♋, ♌, or ♍. In like manner, if you lay a Ruler to A and 26 Deg. 58 Min. in the Limb, and note the Point where it croſſes the Parallel of ♓; then move it to 18 Deg, 16 Min. and note where it croſſes the Parallel of ♒. And again, to 14 Deg, 58 Min. noting where it croſſes the Tropick of ♑; the Line drawn thro' theſe Points ſhall ſhew the Hour of twelve in the Winter. And in this manner may the reſt of the Hour-Lines be drawn, only that of ſeven from the Meridian in Summer, and five in the Winter, will croſs the Line of Declination, at 11 Deg. 35 Min. and that of eight in the Summer, and four in the Winter, at 21 Deg. 38 Min.

The fourth Table for drawing of the Azimuth Lines muſt alſo be made for the Altitude of the Sun above the Horizon, at every Azimuth, eſpecially when the Sun comes to the Equator, Tropicks, and ſome other intermediate Declinations.

If the Sun be in the Equator, and ſo has no Declination, as Radius to the Co-ſine of the Azimuth from the Meridian; ſo is the Tangent of the Latitude to the Tangent of the Sun's Altitude at the Azimuth in the Equator.

If the Sun be not in the Equator, as the Sine of the Latitude is to the Sine of the Declination, ſo is the Co-ſine of the Sun's Altitude at the Equator, at a given Azimuth, to the Sine of a 4th Arc.

Now when the Latitude and Declination are both alike in all Azimuths, from the Prime Vertical to the Meridian, add this 4th Arc to the Arc of Altitude at the Equator. But when the Azimuth is above 90 Degrees diſtant from the Meridian, take the Altitude at the Equator from this 4th Arc. When the Latitude and Declination are unlike, take the ſaid 4th Arc from the Arc of Altitude at the Equator, and then you will have the Sun's Altitude for a propoſed Azimuth.

Laſtly, When the Sun riſes or ſets upon any Azimuth, to find his Declination, ſay, As Radius to the Co-ſine of the Latitude, So is the Co-ſine of the Azimuth from the Meridian, to the Sine of the Declination.

Now a Table being made according to the aforeſaid Directions, if you would draw the Line of Eaſt or Weſt, which is 90 Degrees from the Meridian, lay a Ruler to the Center A, and 30 Deg. 38 Min. numbered in the Limb from C towards B, and note the Point where it croſſes the Tropick of ♋; then move the Ruler to 26 Deg. 10 Min. and note where it croſſes the Parallel of ♊; then to 14 Deg. 45 Min. and note where it croſſes the Parallel of ♉; then to 0° and 0°, and you will find it croſs the Equator in the Point F; then a Line drawn thro' theſe Points will be the Eaſt and Weſt Azimuth. And ſo may all the other Azimuths be drawn.

Theſe Lines being thus drawn, if you ſet two Sights upon the Line A C, and at the Center A hang a Thread and Plummet, with a Bead upon the Thread, the foreſide of the Quadrant will be finiſhed.

SECTION IV.

The Uſe of Gunter's *Quadrant.*

USE I. *The Sun's Place being given, to find his Right Aſcenſion, and contrariwiſe.*

Let the Thread be laid upon the Sun's Place in the Ecliptick, and the Degrees which it cuts in the Limb, will be the Right Aſcenſion ſought.

For example; Suppoſe the Sun's Place be the 4th Degree of ♊, the Thread laid on this Degree will cut 62 Deg. in the Limb, which is the Right Aſcenſion required. But if the Sun's Place be more than 90 Deg. from the beginning of *Aries*, the Right Aſcenſion will be more than 90 Deg. And in this Caſe the Degrees cut by the Thread muſt be taken from 180, to have the Right Aſcenſion.

Now if the Sun's Right Aſcenſion be given, to find its Place, lay the Thread on the Right Aſcenſion, and it will croſs the Sun's Place in the Ecliptick.

USE II. *The Sun's Place being given, to find his Declination, and contrariwiſe.*

Lay the Thread, and ſet the Bead to the Sun's Place in the Ecliptick; then move the Thread to the Line of Declination, and there the Bead will fall upon the Degrees of the Line of Declination ſought.

For example; Let the Sun's Place be the 4th Degree of ♊, the Bead being firſt ſet to this Place, move the Thread to the Line of Declination, and there you will find the Sun's Declination 21 Deg. from the Equator.

Now the Sun's Place being ſought, in having the Declination given, you muſt firſt lay the Thread and Bead to the Declination, and then the Bead moved to the Ecliptick will give the Sun's Place ſought.

USE

USE III. *The Day of the Month being given, to find the Sun's Meridian Altitude, and contrariwise.*

Lay the Thread to the Day of the Month, and the Degrees which it cuts in the Limb will be the Sun's Meridian Altitude.

Suppose the Day given be *May* the 15th, the Thread laid upon this Day will cut 59 Deg. 30 Min. the Meridian Altitude sought.

Again, If the Thread be set to the Meridian Altitude, it will fall upon the Day of the Month.

As, suppose the Sun's Meridian Altitude be 59 Deg. 30 Min. the Thread set to this Altitude falls upon the 15th Day of *May*, and the 9th of *July*; and which of those two is the true Day, may be known by the Quarter of the Year, or by another Day's Observation: for if the Sun's Altitude be greater, the Thread will fall upon the 16th of *May*, and the 8th of *July*; and if it prove lesser, then the Thread will fall on the 14th of *May*, and the 10th of *July*; whereby the Question is fully answered.

USE IV. *The Sun's Altitude being given, to find the Hour of the Day, and contrariwise.*

Having put the Bead to the Sun's Place in the Ecliptick, observe the Sun's Altitude by the Quadrant; and then if the Thread be laid over the same in the Limb, the Bead will fall upon the Hour required. For example; Suppose on the 10th of *April*, the Sun being then in the beginning of *Taurus*, I observe his Altitude by the Quadrant to be 36 Deg. place the Bead to the beginning of *Taurus* in the Ecliptick, and afterwards lay the Thread over 36 Degrees of the Limb; then the Bead will fall upon the Hour-Line of 9 and 3: and so the Hour is 9 in the Morning, or 3 in the Afternoon. Again, If the Altitude be near 40 Degrees, the Bead will fall half way between the Hour-Line of 9 and 3, and the Hour-Line of 10 and 2. Wherefore it must be either half an Hour past 9 in the Morning, or half an Hour past two in the Afternoon; and which of these is the true Time of the Day, may be known by a second Observation: For if the Sun rises higher, it is Morning, and if it becomes lower, it is Afternoon.

Now to find the Sun's Altitude by having the Hour given, you must lay the Bead upon the Hour given (having first rectified or put it to the Sun's Place) and then the Degrees of the Limb cut by the Thread, will be the Sun's Altitude sought.

Note, The Bead may be rectified otherwise, in bringing the Thread to the Day of the Month, and the Bead to the Hour-Line of 12.

USE V. *To find the Sun's Amplitude either rising or setting, when the Day of the Month or Sun's Place is given.*

Let the Bead rectified for the time, be brought to the Horizon; and there it will shew the Amplitude sought. If, for example, the Day given be the 4th of *May*, the Sun will then be in the 4th Degree of *Gemini*. Now if the Bead be rectified and brought to the Horizon, it will there fall on 35 Deg. 8 Min. and this is the Sun's Amplitude of rising from the East, and of his setting from the West.

USE VI. *The Day of the Month or Sun's Place being given, to find the Ascensional Difference.*

Rectify the Bead for the given time, and afterwards bring it to the Horizon; then the Degrees cut by the Thread in the Limb will be the ascensional Difference. And if the ascensional Difference be converted into time, in allowing an Hour for 15 Degrees, and four Minutes of an Hour for one Degree, then we shall have the time of the Sun's rising before six in the Summer, and after six in the Winter: and consequently the Length of Day and Night may be known by this means.

USE VII. *The Sun's Altitude being given, to find his Azimuth, and contrariwise.*

Rectify the Bead for the time, and observe the Sun's Altitude. Then bring the Thread to the Complement of that Altitude, and so the Bead will give the Azimuth sought upon or among the Azimuth Lines.

And to find the Altitude by having the Azimuth given, having rectified the Bead to the Time, move the Thread 'till the Bead falls on the given Azimuth; then the Degrees of the Limb cut by the Thread, will be the Sun's Altitude at that time.

USE VIII. *The Altitude of any one of the five Stars on the Quadrant being given, to find the Hour of the Night.*

First, Put the Bead to the Star, which you intend to observe, and find how many Hours he is from the Meridian by *Use* IV. then from the Right Ascension of the Star, substract the Sun's Right Ascension converted into Hours, and mark the Difference: for this Difference added to the observed Hour of the Star from the Meridian, will shew how many Hours the Sun is gone from the Meridian, which is in effect the Hour of the Night.

For

For Example; The 15th of *May*, the Sun being in the 4th Degree of *Gemini*, I set the Bead to *Arcturus*, and obferving his Altitude, find him to be in the Weft, about 52 Deg. high, and the Bead to fall upon the Hour-Line of two after Noon; then the Hour will be 11 Hours 50 Min. paft Noon, or 10 Minutes fhort of Midnight. For 62 Deg. the Sun's right Afcenfion, converted into Time, makes 4 Hours 8 Min. which if we take out of 13 Hours 58 Min. the right Afcenfion of *Arcturus*, the Difference will be 9 Hours 50 Min. and this being added to two Hours, the obferved Diftance of *Arcturus* from the Meridian, fhews the Hour of the Night to be 11 Hours 50 Min.

Thus have I briefly fhewn the Manner of folving feveral of the chief and moft ufeful Aftronomical Problems, by means of this Quadrant. As for the Manner of taking Altitudes in Degrees, as likewife the Ufe of the Quadrat, fee our Author's Quadrant.

There are other Quadrants made by Mr *Sutton* long fince; one of which (being in my Opinion the beft) is a Stereographical Projection of $\frac{1}{4}$ of thofe Circles, or quarter of the Sphere between the Tropicks, upon the Plane of the Equinoctial, the Eye being in the North Pole.

The faid quarter on the Quadrant, is that between the South part of the Meridian, and Hour of Six, which will leave out all the outward Part of the Almicanters between it and the Tropick of *Capricorn*; and inftead thereof, there is taken in fuch a like Part of the depreffed Parallels to the Horizon, between the fame Hour of Six and Tropick of *Capricorn*, for the Parallels of Depreffion have the fame Refpect to the Tropick of *Capricorn*, as the Parallels of Altitude have to the Tropick of *Cancer*, and will produce the fame Effect.

This Projection is fitted for the Latitude of *London:* and thofe Lines therein that run from the Right-hand to the Left, are Parallels of Altitude; and thofe which crofs them, are Azimuths. The leffer of the Circles that bounds the Projection, is one fourth of the Tropick of *Capricorn*, and the other one fourth of the Tropick of *Cancer*. There are alfo the two Eclipticks drawn from the fame Point in the left Edge of the Quadrant, with the Characters of the Signs upon them; as likewife the two Horizons from the fame Point. The Limb is divided both into Degrees and Time, and by having the Sun's Altitude given, we may find the Hour of the Day to a Minute by this Quadrant.

The Quadrantal Arcs next the Center contain the Calendar of Months; and under them in another Arc is the Sun's Declination: fo that a Thread laid from the Center over any Day of the Month, will fall upon the Sun's Declination that Day in this laft Arc, and on the Limb upon the Sun's right Afcenfion for that fame Day. There are feveral of the moft noted fixed Stars between the Tropics, placed up and down in the Projection; and next below the Projection is the Quadrat and Line of Shadows, being only a Line of natural Tangents to the Arcs of the Limb; and by help thereof, the Heights of Towers, Steeples, *&c.* may be pretty exactly taken.

Now the Manner of ufing this Projection in finding the Time of the Sun's rifing or fetting, his Amplitude, Azimuth, the Hour of the Day, *&c.* is thus: Having laid the Thread to the Day of the Month, bring the Bead * to the proper Ecliptick (which is called rectifying it), and afterwards move the Thread, and bring the Bead to the Horizon: then the Thread will cut the Limb in the Time of the Sun's rifing or fetting, before or after Six. And at the fame Time the Bead will cut the Horizon in the Degrees of the Sun's Amplitude. Again, Suppofe the Sun's Altitude on the 24th of *April* be obferved 45 Degrees, What will the Hour and Azimuth then be? Having laid the Thread over the 24th of *April*, bring the Bead to the Summer Ecliptick, and then carry it to the Parallel of the Altitude 45 Degrees: and then the Thread will cut the Limb at 55 Deg. 15 Min. and fo the Hour will be either 41 Min. paft Nine in the Morning, or 19 Min. paft Two in the Afternoon. And the Bead among the Azimuths fhews the Sun's Diftance from the South to be 50 Deg. 41 Min.

Note, If the Sun's Altitude be lefs than that which it hath at fix o'Clock, on any given Day; then the Operation muft be performed among thofe Parallels above the upper Horizon, the Bead being rectified to the Winter Ecliptick.

There are a great many other Ufes of this Quadrant, which I fhall omit, and refer you to *Collins*'s Sector upon a Quadrant, wherein it's Defcription and Ufe, together with thofe of two other Quadrants, are fully treated of.

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* *The Bead is a little round Leaden or Brafs Shot, with a fmall Hole thro' it, that moves ftiffly up and down, fo as to remain at any Point or Part of the Thread of a Plummet faftened to the Centre of the Quadrant.*

BOOK VII.

Of the Construction and Uses of Instruments for Navigation.

CHAP. I.

Of the Construction and Use of the Sea-Compass, and Azimuth Compass.

SECTION I.

Plate 20. Fig. 1.

THE first Figure shews the Compass Card, whose Limb represents the Horizon of the World. It is divided into four times 90 Degrees, and very often but into 32 equal Parts; for the 32 Points, whereof the four principal Points, which are called Cardinal ones, cross each other at Right Angles, *viz.* the North, distinguished by the *Flower-de-Luce*, the South opposite thereto, and the East and West. Now if each of these Quarters be bisected, we shall have the eight Rhumbs. Again, Bisecting each of these last Spaces, we shall have the eight Semi-Rhumbs. And lastly, Bisecting each of these last Parts, we shall have the sixteen Quarter-Rhumbs. The four Collateral Rhumbs take their Name from the four Principal Rhumbs, each assuming the two Names of those that are nighest them: as, the Rhumb in the Middle, between the North and the East, is called North-East; that between the South and the East, is called South-East; that between the South and the West, is called South-West; and that in the Middle between the North and the West, is called North-West.

Also every of the eight Semi-Rhumbs assumes it's Name from the two Rhumbs that be nighest it; as that between the North and North-East, is called North North-East; that between the East and North-East, is called East North-East; that between the East and South-East, is called East South-East; and so of others.

Finally, Each of the Quarter-Rhumbs has it's Name composed of the Rhumbs or Semi-Rhumbs which are nighest to it, in adding the Word one-fourth after the Name of the Rhumb nearest to it. For Example; The Quarter-Rhumb nearest to the North, and next to the North-East, is called North one-fourth North-East; that which is nearest the North-East towards the North, is called North-East, one-fourth North; and so of others, as they appear abbreviated round the Card. Each Quarter-Rhumb contains 11 Deg. 15 Min. the Semi-Rhumbs 22 Deg. 30 Min. and the whole Rhumbs 45 Deg.

The Inside of this Card, which is supposed double, is likewise divided into 32 equal Parts, by a like Number of Radii, denoting the 32 Points, and the Middle thereof, which is glewed upon a Pasteboard, hath a free Motion upon it's Pivot, that so it may be used when the Declination or Variation of the Needle is found. *Note*, The Outside of this Card is placed upon the Limb of the Box.

The

The second Figure represents a piece of Steel in form of a Rhumbus, which serves for the Needle, and is fastened under the moveable Card with two little Pins, so that one of the ends of the longest Diameter of the said Rhumbs be precisely under the *Flower-de-luce.* This piece of Steel must be touched by a good Load-stone; so that one end may direct itself towards the North part of the World. The manner of doing which, we have already shewn in speaking of the Load-stone, and the Compass. *Note,* It is not so well to glew the said Needle under the Card, as some do, as otherwise to fasten it; because that causes a Rust very contrary to the magnetick Virtue.

The little Figure B, in the middle of the Rhumbus, which is called the Cap of the Needle, is made of Brass, and hollowed into a Conical Form. This Cap is applied to the Center of the Card, and is fastened thereto with Glew.

The third Figure represents the whole Compass, whereof A is a round wooden Box, about six or seven Inches Diameter, and four deep; (we sometimes make these Boxes square) *b b* and *c c* are two Brass Hoops, the greater of which being *b b*, is fastened to the Sides of the Box at the opposite Places B B. The other Hoop *c* C is fastened by two other Pivots, at the Places C C, diametrically opposite to the Hoop *b b*; and these two Pivots go into Holes made towards the top of another kind of wooden Box, in which the Card is put. And by this means, this last Box, and the two Hoops will, have a very free Motion; so that when the great Box A is placed flat in a Ship, the lesser Box will be always horizontal, and *in equilibrio,* notwithstanding the Motion of the Ship. In the middle of the Bottom of this last Box, is placed a very strait and well pointed brass Pivot, on which is placed the Cap B of the Card, which Card having a very free Motion, the *Flower-de-luce* will turn towards the North, and all the other Points towards the other correspondent Parts of the World. Finally, the Card is covered with a Glass, that so the Wind may have no power on it.

Use of the Sea-Compass.

The Course that a Ship must take to sail to a proposed Place, being known by a Sea-Chart, and the Compass placed in the Pilot's Room, so as the two parallel Sides of the square Box to be fixed according to the length of the Ship, that is, parallel to a Line drawn from the Poop to the Prow; make a Cross, or other Mark, upon the middle of that Side of the square Box perpendicular to the Ship's length, and the most distant from the Poop, that so the Stern of the Ship by this means may be directed accordingly.

Example. Departing from the Island of *Ushant,* upon the Confines of *Britany,* we desire to sail towards Cape *Finisterre* in *Galicia.* Now in order to do this, we must first seek (according to the manner hereafter directed) in a *Mercator*'s Chart, the Direction or Course of the Ship leading to that Place; and this we find is between the South-West and the South South-West; that is, the Ship's Course must be South-West, one-fourth to the South. Therefore having a fair Wind, turn the Stern of the Ship, so that a Line tending from the South-West, one-fourth South, exactly answers to the Cross marked upon the middle of the Side of the square Box; and then we shall have our Desire. And by this means, which is really admirable, we may direct a Ship's Course as well in the Night as in the Day, as well being shut up in a Room in the Ship, as in the open Air, and as well in cloudy Weather as fair; so that we may always know whether the Ship goes out of her proper Direction.

Of the Variation or Declination of the Needle.

It is found by experience, that the touched Needle varies from the true North, that is, the *Flower-de-luce* does not exactly tend to the North part of the World, but varies therefrom, sometimes towards the East, sometimes towards the West, more or less, according to different Times, and at different Places.

About the Year 1665, the Needle at *Paris* did not decline or vary at all; whereas now its Variation is there above 12 Degrees North-westwardly, Therefore every time a favourable Opportunity offers, you must endeavour to observe carefully the Variation of the Needle, that so respect thereto may be had in the steering of Ships. If, for example, the Variation of the Needle in the Island of *Ushant,* which was the supposed Place of Departure in the abovementioned Example, was 10 Degrees; and if the Ship should exactly keep the Course of South-West, one-fourth to the South, instead of arriving at the Cape *Finisterre,* it would to another Country 10 Degrees more to the East.

Now to remedy this, you need only remove the Cross, upon the Side of the Box, shewing the Rhumb of Direction, more easterly by the Quantity of the Degrees of the Needle's Variation westwardly; and so as often as a new Declination or Variation of the Needle be found, the place of the said Cross must be altered. *Note,* When the Box is quite round, a Mark must be made against the North and South on the Body of the said Box.

If likewise a Vessel departs from the *Sorlings* in *England,* in order to sail to the Island of *Madeira,* you will find by a Sea-Chart that her Course must be South South-West; but if at the same time the Variation of the Needle be six Degrees North Easterly, the the Cross denoted upon the Edge of the Compass must be removed six Degrees towards the West, in order to direct the Ship according to her true Course found in the Chart.

But if a Sea-Compaſs be uſed, wherein the Poſition of the Needle may be altered, as that which hath a double Card, the *Flower-de-luce* of the Card muſt be fixed, ſo that its Point may ſhew the true North; and then you will have it to alter every time there is a new Variation obſerved. Now in this Caſe the Croſs upon the Edge of the Compaſs muſt not be altered.

It is very neceſſary, and principally in long Courſes, for Seamen to make Celeſtial Obſervations often, in order to have the Variation of the Needle exactly, that the Direction of the Veſſel may thereby be truly had, as likewiſe that they may know where they are, after having eſcaped a great Storm, during which they were obliged to leave the true Courſe, and let the Veſſel run according as the Wind or Currents drove her.

SECTION II.

Of the Azimuth Compaſs.

Fig. 4. This Compaſs is ſomething different from the common Sea-Compaſs before ſpoken of. For upon the round Box, wherein is the Card, is faſtened a broad braſs Circle A B, one Semi-circle whereof is divided into 90 equal Parts or Degrees, numbered from the middle of the ſaid Diviſions both ways, with 10, 20, *&c.* to 45 Degrees; which Degrees are alſo divided into Minutes by Diagonal Lines and Circles: But theſe graduating Lines are drawn from the oppoſite part of the Circle, *viz.* from the Point *b*, wherein the Index turns in time of Obſervation.

b c is that Index moveable about the Point *b*, having a Sight *b a* erected thereon, which moves with a Hinge, that ſo it may be raiſed or laid down, according to neceſſity. From the upper part of this Sight, down to the middle of the Index, is faſtened a fine Hypothenuſal Lute-ſtring, or Thread *d e*, to give a Shadow upon a Line that is in the middle of the ſaid Index.

Note, The reaſon of making the Index move on a Pin faſtened in *b*, is, that the Degrees and Diviſions may be larger; for now they are as large again as they would have been, if they had been divided from the Center, and the Index made to move thereon.

The abovenamed broad braſs Circle A B, is croſſed at Right Angles with two Threads, and from the ends of theſe Threads are drawn four ſmall black Lines on the Inſide of the round Box; alſo there are four Right Lines drawn at Right Angles to each other on the Card.

This round Box, thus fitted with its Card, graduated Circle, and Index, *&c.* is to be hung in the braſs Hoops B B; and theſe Hoops are faſtened to the great ſquare wooden Box C C.

The Uſe of the Azimuth Compaſs in finding the Sun's Magnetical Azimuth or Amplitude, and from thence the Variation of the Compaſs.

There are ſeveral ways of finding the Variation of the Needle, as by the riſing and ſetting of the ſame Star, or by the Obſervation of two equal Altitudes of the Star above the Horizon, ſince the ſaid Star, at each of thoſe Times, will be equally diſtant from the true Meridian of the World; or elſe by a Star's paſſage over the Meridian.

But theſe Methods are not much uſed at Sea: Firſt, becauſe the Time wherein the Sun, or a Star, paſſes over the Meridian, cannot be known preciſely enough; for there is a great deal of Time taken in making Obſervations of the Sun's Altitudes, 'till he is found to have the greateſt, that is, his Meridian Altitude.

Secondly, Becauſe the Sun's Declination may be conſiderably altered, and alſo the Ship's Latitude between the Times of the two Obſervations of his equal Altitudes above the Horizon, Morning and Evening, or of his Riſing and Setting.

Therefore the Variation of the Compaſs may much better be found by one Obſervation of the Sun's magnetical Amplitude, or Azimuth. But the Sun's Declination, and the Latitude of the Place the Ship is in, muſt be known, that ſo his true Amplitude may be had; his Altitude muſt alſo be given, when the magnetick Azimuth is taken, that ſo his true Azimuth may be had at that Time alſo.

Now if the Obſervation be for an Amplitude at Sun-riſing, or an Azimuth before Noon, you muſt put the Center of the Index *b c* upon the Weſt Point of the Card within the Box, ſo that the four Lines on the Edge of the Card, and the four Lines on the Inſide of the Box, may agree or come together. But if the Obſervation be for the Sun's Amplitude, Setting, or an Azimuth in the Afternoon, then you muſt turn the Center of the Index right-againſt the Eaſt Point of the Card, and make the Lines within the Box concur with theſe on the Card. Having thus fitted the Inſtrument for Obſervation, turn the Index *b c* towards the Sun, 'till the Shadow of the Thread *de* falls directly upon the ſlit of the Sight, and upon the Line that is along the middle of the Index; then will the inner Edge of the Index cut the Degree and Minute of the Sun's magnetical Azimuth, from the North or South.

But note, that if the Compaſs being thus placed, the Azimuth be leſs than 45 Deg. from the South, and the Index be turned towards the Sun, it will then paſs off the Diviſions of the

the Limb, and ſo they become uſeleſs as it now ſtands: therefore you muſt turn the Inſtrument juſt a Quarter of the Compaſs, that is, place the Center of the Index on the North or South Point of the Card, according as the Sun is from you, and then the Edge thereof will cut the Degree of the Magnetick Azimuth, or Sun's Azimuth from the North, as before.

The Sun's Magnetical Amplitude (that is, the Diſtance from the Eaſt or Weſt Points of the Compaſs, to that Point in the Horizon whereat the Sun riſes or ſets), being obſerved by this Inſtrument, the Variation of the Compaſs may be thus found.

Example. Being out at Sea the 15th Day of *May*, in the Year 1715, in 45 Degrees of North Latitude, I find from Tables that the Sun's Declination is 19 Deg. North, and his Eaſt Amplitude 27 Deg. 25 Min. North. Now I obſerve by the Azimuth Compaſs, the Sun's Magnetical Amplitude at his riſing and ſetting, and find that he riſes between the 62d and 63d Deg. reckoning from the North towards the Eaſt part of the Compaſs, that is, between the 27th and 28th Degrees from the Eaſt; and ſince in this Caſe the magnetical Amplitude is equal to the true Amplitude, I conclude that at this Place and Time, the Needle has no Variation.

But if the Sun at his riſing ſhould have appeared between the 52d and 53d Degrees from the North towards the Eaſt, his magnetical Amplitude would then be between 37 and 38 Degrees, that is, about 10 Degrees greater than the true Amplitude: and therefore the Needle would vary about 10 Degrees North-Weſterly. If, on the contrary, the magnetical Eaſt Amplitude found by the Inſtrument ſhould be leſs than the true Amplitude, their Difference would ſhew that the Variation of the Needle is North-Eaſterly. For if the magnetical Amplitude be greater than the true Amplitude, this proceeds from hence, that the Eaſt part of the Compaſs is drawn back from the Sun towards the South, and the *Flower-de-luce* of the Card approaches to the Eaſt, and ſo gives the Variation North-Eaſterly. The reaſon for the contrary of this is equally evident.

If the true Eaſt Amplitude be Southwardly, as likewiſe the magnetical Amplitude, and this laſt be the greater; then the Variation of the Needle will be North Weſt; and if, on the contrary, the magnetical Amplitude be leſs than the true Amplitude, the Variation of the Needle will be North-Eaſterly, as many Degrees as are contained in their Difference.

What we have ſaid concerning North-Eaſt Amplitudes, muſt be underſtood of South-Weſt Amplitudes, and what we have ſaid of South-Eaſt Amplitudes, muſt be underſtood of North-Weſt Amplitudes.

Finally, If Amplitudes are found of different Denominations; for Example, when Amplitudes are Eaſt, if the true Amplitude be 6 Deg. North, and the magnetical Amplitude 5 Deg. South; then the Variation, which in this Caſe is North-Weſt, will be greater than the true Amplitude, it being equal to the Sum of the magnetical and true Amplitudes: and ſo adding them together, we ſhall have 11 Degrees of North-Weſt Variation. Underſtand the ſame for Weſt Amplitudes.

The Variation of the Compaſs may likewiſe be found by the Azimuth; but then the Sun's Declination, the Latitude of the Place, and his Altitude muſt be had, that ſo his true Azimuth may be found.

CHAP. II.

Of the Conſtruction and Uſe of Inſtruments for taking the Altitudes of the Sun or Stars at Sea.

Of the Sea-Aſtrolabe.

THE moſt common Inſtrument for taking of Altitudes at Sea is the Aſtrolabe, which Fig. 5.
conſiſts of a braſs Circle, about one Foot in Diameter, and ſix or ſeven Lines in Thickneſs, that ſo it may be pretty weighty: there is ſometimes likewiſe a Weight of ſix or ſeven Pounds hung to this Inſtrument at the Place B, that ſo when the Aſtrolabe is ſuſpended by it's Ring A, which ought to be very moveable, the ſaid Inſtrument may turn any way, and keep a perpendicular Situation during the Motion of the Ship.

The Limb of this Inſtrument is divided into four times 90 Degrees, and very often into halves, and fourths of Degrees.

It is abſolutely neceſſary, that the Right Line CD, which repreſents the Horizon, be perfectly level, that ſo the beginning of the Diviſions of the Limb of the Inſtrument may be made therefrom. Now to examine whether this be ſo or no, you muſt obſerve ſome diſtant Object thro' the Slits or little Holes of the Sights F and G, faſtened near the Ends of the Index, freely turning about the Center E, by means of a turned-headed Rivet: I ſay, you muſt obſerve the ſaid diſtant Objects, in placing the Eye to one of the ſaid Sights, for Example, to G: then if the Aſtrolabe be turned about, and the ſame Object appears thro' the other Sight F, without moving the Index, it is a ſign the Fiducial Line of the Index is horizontal. But if at the ſecond time of Obſervation, the Index muſt be raiſed or lowered before the Object be eſpied thro' the Sights, then the middle Point between the two Poſitions will ſhew the true horizontal Line paſſing thro' the Center of the Inſtrument, which muſt be verified by ſeveral repeated Obſervations, before the Diviſions of the Limb are begun to be made, in the Manner as we have elſewhere explained.

Uſe of the Aſtrolabe.

The Uſe of this Inſtrument is for obſerving the Sun or Stars Altitude above the Horizon, or their Zenith Diſtance. The Manner of effecting which, is thus: Holding the Aſtrolabe ſuſpended by it's Ring, and turning it's Side towards the Sun, move the Index 'till the Sun's Rays paſs thro' the Sights F and G; then the Extremes of the Index will give the Altitude of the Sun in H, upon the divided Limb, from C to F, comprehended between the horizontal Radius E C and the Rays E F of the Sun, becauſe the Inſtrument in this Situation repreſents a Vertical Circle. Now the Diviſions of the Arcs B G or A F, ſhew the Sun's Zenith Diſtance.

The Construction of the Ring.

Fig. 6. This Figure repreſents a braſs Ring or Circle, about 9 Inches in Diameter, which it is neceſſary ſhould be pretty thick; that ſo being weighty, it may keep it's perpendicular Situation better than when it is not ſo heavy, having the Diviſions denoted on the Concave Surface thereof. The little Hole C, made thro' the Ring, is 45 Degrees diſtant from the Point of Suſpenſion B, and is the Center of the Quadrant D E, divided into 90 Degrees, one of whoſe Radius's C E, is parallel to the Vertical Diameter B H of the Ring, and the other horizontal Radius C D, is perpendicular to the ſaid Vertical Diameter.

Now having found the ſaid horizontal Radius C D very exactly, by ſuſpending the Ring, *&c.* Radius's muſt be drawn from the Center C, to each Degree of the Quadrant D E, and upon the Points wherein the ſaid Radius's cut the Concave Surface of the Ring, the correſpondent Numbers of the Degrees of the Quadrant muſt be graved; and ſo the Concave Surface of the Ring, will be divided from F to G. This Diviſioning may be firſt made ſeparately upon a Plane, and afterwards transferred upon the Concave Surface of the Ring.

This Inſtrument is reckoned better than the Aſtrolabe, becauſe the Diviſions of the Degrees upon the Concave Surface are larger in proportion to it's bigneſs, than thoſe on the Aſtrolabe.

The Uſe of the Ring.

When this Inſtrument is to be uſed, you muſt ſuſpend it by the Swivel B, and turn it towards the Sun A; ſo that it's Rays may paſs thro' the Hole C. This being done, the little Spot will fall between the horizontal Line C F, and vertical Line C G, upon the Degrees of the Sun's Altitude on the Inſide of the Ring, reckoned from F to I.

Of the Quadrant.

Fig. 7. The Inſtrument of Figure 7, is a Quadrant about one Foot Radius, having it's Limb divided into 90 Degrees, and very often each Degree into every 5th Minute by Diagonals. There are two Sights fixed upon one of the Sides A E, and the Thread to which the Plummet is faſtened, is fixed in the Center A. I ſhall not here mention the Construction of this Inſtrument, becauſe we have ſufficiently ſpoken thereof in Chap. V. Book IV.

Now to uſe this Inſtrument, you muſt turn it towards the Sun D, in ſuch manner that it's Rays may paſs thro' the Sights A and E, and then the Thread will fall upon the Degrees of the Sun's Altitude on the Limb, in the Point C, reckoned from B to C, and the Complement of his Altitude reckoned from E to C.

Of the Fore-Staff, or Croſs-Staff.

Fig. 8. This Inſtrument conſiſts of a ſtrait ſquare graduated Staff A B, between two and three Foot in length, and four Croſſes or Vanes F F, E E, D D, C C, which ſlide ſtiffly thereon. The firſt and ſhorteſt of theſe Vanes F F, is called the Ten-croſs or Vane, and belongs to that Side of the Staff whereon the Diviſions begin at about 3 Degrees from the End A, (whereat the Eye is placed in time of Obſervation) to 10 Degrees. *Note*, Sometimes the Thirty-croſs E F is ſo made, as that the Breadth thereof ſerves inſtead of this Ten-croſs.

The next longer Vane E E, is called the Thirty-croſs, and belongs to that Side of the Staff, whereon the Diviſions begin at 10 Degrees, and end at 30 Degrees, and this is called the Thirty-ſide: Half the Length of the Thirty-vane will reach on this Thirty-ſide, from 30 Deg. to 23 Deg. 52 Min. and the whole Length from 30 Deg. to 19 Deg. 47 Min.

The

The next longer Vane D D, is called the Sixty-crofs, and belongs to that Side of the Staff whereon the Divifions begin at 20 Deg. and end at 60 Deg. and is called the Sixty-fide. The length of this Crofs will reach on this Sixty-fide, from 60 Deg. to 30 Deg.

The longeft Crofs C C, is called the Ninety-crofs, and belongs to that Side whereon the Divifions begin at 30 Degrees, and end at 90 Degrees, and is called the Ninety-fide of the Staff: the Degrees on the feveral Sides of the Staff, are numbered with their Complements to 90 Degrees in fmall Figures.

This Staff may be graduated Geometrically thus: Upon a Table, or on a large Paper pafted fmoothly upon fome Plane, draw the Line F G, the length of the Staff to be graduated, and on F and G raife the Perpendiculars F C and G D; upon which lay off the Length you intend for the half Length of one of the four Croffes, from F to C, and G to D, and draw the Line C D reprefenting the Staff to be graduated. This being done, about the Center F, with the Semidiameter F G or C D, defcribe an eighth part of a Circle, which divide into 90 equal Parts. Then if Right Lines be drawn from the Point F, to each of the aforefaid Divifions, thefe Lines will divide the Line C D, as the Staff ought to be graduated. Fig. 9.

But if this Staff is to be graduated by the Table of natural Tangents, you muft firft obferve, that the Graduations are only the natural Co-tangents of half Arcs, the half Crofs being Radius; therefore divide the length of the half Crofs into 1000 equal Parts, or 100000 if poffible, according to the Radius of the Tables of natural Tangents: then take from this the Co-tangents, as you find them in the Table, and prick them from F fucceffively, and your Staff will be graduated for that Vane. So do for the reft feverally. If it be required to prick down the 80th Degree, the half of 80 is 40, and the natural Co-tangent of 40 Deg. is 119175, which take from the Scale or half Crofs fo divided, and prick it from F to P, and that will be the Point for 80 Degrees, *&c.* So again, To put on the 64th Degree, half of 64 is 32, and its Co-tangent is 160033, which take from the divided Crofs (prolonged), prick it from E, and you will have the 64th Degree.

Now that the Crofs C D, when transferred to B, fhall make the Angles C A D eighty Degrees, is demonftrable thus: Since C B the half Crofs is Radius, and A B is by Conftruction the Tangent of 50 Deg. the Angle A C B is 50 Degrees; and fince the Triangle A B C is Right Angled, the Angle B A C will be 40 Degrees: but the Angle D A C is double the Angle B A C; therefore the Angle D A C is 80 Degrees, and the Point B the true Point on the Staff for 80 Degrees. The fame Demonftration holds, let the Crofs be what it will. Fig. 10.

If the Staff be to be graduated by any Diagonal Scale, meafure half the Length of the Vane by the Scale, and fay, As the Radius of the Tables 100000, is to the Meafure of half the Crofs, So is the natural Co-tangent of the half any Number of Degrees defired to be pricked on the Staff, to the Space between the Center of the Staff F, and the Point for the Degrees fought.

For Example; Suppofe half the Length of the Vane, meafured on a Diagonal Scale, be 945; to find what Number muft be taken off the Diagonal Scale for the 80th Degree. The Co-tangent of 40 Degrees (half of 80) is 1191753, which being multiplied by 945, and divided by Radius, gives 11261. And this being taken from the Diagonal Scale, will give the Degrees defired.

The Ufe of the Fore-Staff.

The chief Ufe of this Inftrument, is to take the Altitude of the Sun or Stars, or the Diftance of two Stars; and the Ten, Thirty, Sixty and Ninety Croffes are to be ufed, according as the Altitude is greater or leffer; that is, if it be lefs than 10 Degrees, the Ten Crofs muft be ufed; if above 10, but lefs than 30 Degrees, the Thirty Crofs muft be ufed; and if the Altitude be judged to be above 30, but lefs than 60 Degrees, the Sixty Crofs muft be ufed. But when Altitudes are greater than 60 Degrees, this Inftrument is not fo convenient as others.

To obferve an Altitude.

Place the flat End of the Staff to the outfide of your Eye, as near the Eye as you can, and look at the upper End *b* of the Crofs for the Center of the Sun or Star, and at the lower End *a* for the Horizon. But if you fee the Sky inftead of the Horizon, flide the Crofs a little nearer to your Eye; and if you fee the Sea inftead of the Horizon, move the Crofs a little further from your Eye, and fo continue moving the Crofs till you fee exactly the Sun or Star's Center by the top of the Crofs *b*, and the Horizon by the bottom thereof *a*. Then the Degrees and Minutes cut by the inner Edge *c* of the Crofs, upon the Side of the Staff peculiar to the Crofs you ufe, is the Altitude of the Sun or Star: But if it be the Meridian Altitude you are to find, you muft continue your Obfervation as long as you find the Altitude increafe, ftill moving the Crofs nearer to your Eye; but when you perceive the Altitude is diminifhed, forbear any farther Obfervation, and do not alter your Crofs; but as it ftands, count the Degrees and Minutes on the Side proper to the Crofs, and you will have the Meridian Altitude required, as alfo the Zenith Diftance, by fubftracting Fig. 11.

ſtracting the ſaid Altitude from 90 Degrees, if it be not graduated on the Staff. To which Zenith Diſtance add the Minutes allowed for the Height of your Eye above the Surface of the Sea, according to the little Table in the Margin, or ſubſtract it from the Altitude, and then you will have the true Zenith Diſtance and Altitude.

Height of the Eye.	*Allowance.*
Engliſh Feet.	*Min.*
1	1
2	1½
3	2
4	2
5	2½
6	3
7	3
8	3
9	3½
10	3½
12	4
16	4
20	5
24	5½
28	6
32	6½
36	7
40	7
44	7½
48	8

If it be hazy or ſomewhat thick Weather, the Fore-Staff may be uſed as above; but if the Sun ſhines out, the upper Limb of the Sun muſt be either obſerved, and afterwards his Semidiameter muſt be ſubſtracted from the Altitude found, or elſe a coloured Glaſs on the top of the Croſs muſt be uſed, to defend the Sight from the Splendor of the Sun.

To obſerve the Diſtance of two Stars, or the Moon's Diſtance from a Star, place the Staff's flat end to the Eye, as before directed, and looking to both ends *a* and *b* of the Croſs, move it nearer or farther from the Eye, 'till you can ſee the two Stars, the one on one end, and the other on the other end of the Croſs. Then ſee what Degrees and Minutes are cut by the Croſs on the ſide of the Staff proper to that Vane in uſe; and thoſe Degrees ſhew the obſerved Star's Diſtance.

But that there may be no Miſtake in placing the Staff to the Eye, which is the greateſt Difficulty in the Uſe of this Inſtrument: Firſt, before Obſervation, put on the Sixty-croſs, and place it to 30 Degrees on its proper Side, and alſo the Ninety-croſs, ſliding to it 30 Degrees likewiſe on his Right Side: this being done, place the end of the Staff to the corner of your Eye, moving it ſomething higher or lower about the Eye, 'till you ſee the upper ends of the two Croſſes at once exactly in a Right Line, and alſo their lower ends; and that is the true Place of your Staff in Time of Obſervation.

If the Sun's Altitude is to be obſerved backwards by this Inſtrument, you muſt have an horizontal Vane to fix upon the Center or Eye-end of the Croſs, as alſo a Shoe of Braſs for a Sight Vane, to fit on to the end of any of the Croſſes; then when you would obſerve, having put on the horizontal Vane, and fixed the Shoe to the end of a convenient Croſs, turn your back to the Sun, and looking through the Sight in the braſs Shoe, lift the end of the Staff up or down, 'till the Shadow made by the upper end of the Croſs falls upon the ſlit in the Horizon-Vane, and at the ſame time you can ſee the Horizon through the Horizon-Vane. Then the Degrees and Minutes cut by the Croſs on the proper Side, are the Altitude. But if there be fixed a Lens, or ſmall double Convex-Glaſs, to the upper end of the Vane, to contract the Sun-beams, and caſt a ſmall bright Spot on the Horizon-Vane, this will be found more convenient than the Shadow, which is commonly imperfect and double.

Of the Engliſh Quadrant, or Back-Staff.

Fig. 12. This Inſtrument is commonly made of Pear-Tree, and conſiſts of three Vanes A, B, C, and two Arcs. The Vane at A is called the Horizon-Vane, that at B the Shade-Vane, becauſe it gives the Shadow upon the Horizon-Vane in Time of Obſervation, and that at C the Sight-Vane, becauſe in Time of Obſervation it is placed at the Eye. The leſſer Arc D E is the Sixty Arc, and that marked F G is the Thirty Arc, both of which together make 90 Degrees, but they are of different Radius's. The Sixty Arc D E is divided into 60 Degrees, commonly by every five, but ſometimes by ſingle Degrees. In Time of Obſervation, the Shadow-Vane is placed upon this Arc always to an even Degree.

The Thirty Arc G F, is divided into 30 Degrees, and each Degree into Minutes by Diagonal Lines, and Concentrick Arcs. The Manner of doing which, I have already laid down elſewhere.

The Uſe of the Engliſh Quadrant.

If the Sun's Altitude be to be taken by this Inſtrument, you muſt put the Horizon-Vane upon the upper End or Center A of the Quadrant, the Shade-Vane upon the Sixty Arc D E, to ſome Number of Degrees leſs than you judge the Co-altitudes will be by 10 or 15 Degrees, and the Sight-Vane upon the Thirty Arc F G. This being done, lift up the Quadrant, with your Back towards the Sun, and look through the ſmall Hole in the Sight-Vane C; and ſo raiſe or lower the Quadrant 'till the Shadow of the upper Edge of the Shade-Vane B falls upon the upper Edge of the ſlit in the Horizon-Vane A: if then at the ſame time the Horizon appears thro' the ſaid ſlit, the Obſervation is finiſhed; but if the Sea appears inſtead of the Horizon, then remove the Sight-Vane lower towards F; but if the Sky appears inſtead of the Horizon, then ſlide the Sight-Vane a little higher: and ſo continue removing the Sight-Vane, 'till the Horizon appears thro' the ſlit of the Horizon-Vane,

Vane, and the Shadow of the Shade-Vane falls at the same time on the said Slit of the Horizon-Vane. This being done, see how many Degrees and Minutes are cut by the Edge of the Sight-Vane C, which answers to the Sight-Hole, and to them add the Degrees that are cut by the upper Edge of the Shade-Vane, and the Sum is the Zenith Distance or Complement of the Altitude. But to find the Sun's Meridian or greatest Altitude on any Day, you must continue the Observation as long as the Altitude be found to increase, which you will perceive, by having the Sea appear instead of the Horizon, removing the Sight-Vane lower; but when you perceive the Sky appear instead of the Horizon, the Altitude is diminished: therefore desist from farther Observation at that Time, and add the Degrees upon the Sixty Arc to the Degrees and Minutes upon the Thirty Arc, the Sum is the Zenith Distance, or Co-altitude of the Sun's upper Limb.

And because it is the Zenith Distance or Co-altitude of the upper Limb of the Sun, that is given by the Quadrant, when observing by the upper Edge of the Shade-Vane, as it is customary to do, and not the Center; you must add 16 Min. the Sun's Semi-diameter, to that which is produced by your Observation, and the Sum is the true Zenith Distance of the Sun's Center. But if you observe by the lower part of the Shadow of the Shade-Vane, then the lower Limb of the Sun gives the Shadow; and therefore you must substract 16 Min. from what the Instrument gives: but considering the Height of the Observator above the Surface of the Sea, which is commonly between 16 and 20 Feet, you may take five or six Minutes from the 16 Minutes, and make the allowance but 10 Min. or 12 Min. to be added instead of 16 Min.

Note also, The Refraction of the Sun or Stars causes them to appear higher than they are; therefore after having made your Observation, you must find the convenient Refraction, and substract it from your Altitude, or add it to the Zenith Distance, in order to have the true Altitude or Zenith Distance.

If a Lens or double Convex-Glass be fixed in the Shade-Vane, which contracts the Rays of Light, and casts them in a small bright Spot on the Slit of the Horizon-Vane instead of a Shade, this will be an Improvement to the Instrument if the Glass be well fixed; for then it may be used in hazy Weather, and that so thick an Haze, that an Observation can hardly be made with the Forestaff: also in clear Weather the Spot will be more defined than the Shadow, which at best is not terminated.

Of the Semi-circle for taking Altitudes at Sea.

This Semi-circle is about one Foot in Diameter, and the Limb thereof is divided into 90 Degrees only, each of which are quartered for 15 Min. At A and B are two Sights fixed to the Extremes of the Diameter, and another at C, so adjusted as to slide on the Limb of the Semi-circle, that so the Sun's Rays may pass thro' it when the Instrument is using. Fig. 13.

The Use of the Semi-circle.

If an Altitude is to be taken forwards by this Instrument, the Eye must be placed at the Sight A, and then you must look thro' the Sights A and B at the Horizon, and slide the Sight C on the Limb, 'till the Sun's Rays passing thro' it, likewise come thro' the Sight A to the Eye. This being done, the Degrees of the Arc between B and C, shew the Sun's Altitude.

But if the Sun's Altitude is to be taken backwards, which is the best way, because of its Splendor offending the Eye, you must place the Eye to B, and looking thro' the Sights B and A, at the Horizon, you must slide the Sight C along the Limb, 'till the Sun's Rays coming thro' it, fall upon the Sight A, and then the Arc B C will be the Sun's Altitude above the Horizon, as before.

The Meridian Altitude or Zenith Distance of the Sun or Stars being found by Observation, to find the Latitude of the Place.

Having observed with some one of the Instruments before spoken of, the Meridian Altitude or Zenith Distance of the Sun, or some Star, seek the Sun's Declination the Day of Observation: if it be North, substract the Sun's Declination found from the Sun's Altitude, and you will have the Height of the Equinoctial above the Horizon, and this Height taken from 90 Degrees, and you will have the Latitude of the Place. But if the Zenith Distance be added to the Declination of the Sun or Star, the Sum will be the Latitude of the Place.

Again, If the Sun or Star have South Declination, you must add the observed Altitude to the Declination, and the Sum will be the Height of the Equinoctial above the Horizon, which taken from 90 Degrees, and the Latitude will be had. But if the Zenith Distance be taken from the Declination, the Remainder will be the Latitude of the Place.

Lastly, If the Sun has no Declination, his Altitude taken from 90 Degrees, will be the Latitude; and so in this Case the Zenith Distance itself is the Latitude.

Example. The Sun being in the first Degree of *Cancer*, his Meridian Altitude at *Paris* is 64 Deg. 30 Min. Zenith Distance 25 Deg. 30 Min. his Declination 23 Deg. 30 Min. North. Now if 23 Deg. 30 Min. be taken from 64 Deg. 30 Min. the Remainder is 41 Deg. for the Altitude of the Equinoctial, and so the Complement of 41 Deg. to 90 Deg. is 49 Deg. the

Height of the Pole or Latitude of *Paris*; but if the Zenith Diftance 25 Deg. 30 Min. be added to the Sun's Declination 23 Deg. 30 Min. the Sum will be 49 Deg. the Latitude of *Paris*, as before.

Again, Suppofe the 22d of *December* (New Stile) the Sun's Meridian Altitude at *Paris* is obferved 17 Deg. 30 Min. and his Zenith Diftance 72 Deg. 30 Min. his Declination is then 23 Deg. 30 Min. South, which added to 17 Deg. 30 Min. and the Sum is 41 Deg. whofe Complement 49 Deg. is the Latitude of *Paris*. Again, If from the Zenith Diftance 72 Deg. 30 Min. be taken the Declination, the Remainder will be 49 Deg. the Latitude of *Paris*, as before.

CHAP. III.

Of the Conftruction and Ufes of the Sinecal Quadrant.

Plate 21. Fig 1.

THIS Inftrument is compofed of feveral Quadrants, having the fame Center A, and feveral parallel ftrait Lines croffing each other at Right Angles, both Quadrants and Right Lines being equally diftant from each other. Now one of thefe Quadrants, as BC, may be taken for a quarter or fourth part of any great Circle of the Sphere, and principally for a fourth Part of the Horizon and the Meridian.

If the Quadrant BC be taken for one-fourth part of the Horizon, either of the Sides, as AB, may reprefent the Meridian, that is, the Line of North and South. And then the other Side AC, being at Right Angles with the Meridian, will reprefent the Line of Eaft and Weft. All the other Lines parallel to AB are alfo Meridians, and all thofe parallel to the Side AC, are Eaft and Weft Lines.

The aforefaid Quadrant is firft divided into eight equal Parts by feven Radius's drawn from the Center A, which reprefent the eight Points of the Compafs contained in one-fourth of the Horizon, each of which is 11 Deg. 15 Min. the Arc BC is likewife divided into 90 Degrees, and each Degree divided into 12 Minutes, by means of Diagonals, drawn from Degree to Degree, and fix Concentrick Circles. There is likewife a Thread, as AL, fixed to the Center A, which being put over any Degree of the Quadrant, ferves to divide the Horizon as is neceffary. The Conftruction of the reft of this Inftrument, is enough manifeft from the Figure thereof.

The Ufe of the Sinecal Quadrant.

There are formed Triangles upon this Inftrument fimilar to thofe made by a Ship's Way with the Meridians and Parallels, and the Sides of thefe Triangles are meafured by the equal Intervals between the Concentrick Quadrants, and the Lines N and S, E and O.

Thefe Circles and Lines are diftinguifhed, by marking every fifth with broader Lines than the others; fo that if every Interval be taken for one League, there will be five Leagues from one broad Line to the other; and if every Interval be taken for four Leagues, then there will be twenty Leagues, which make a Sea Degree, from one broad Line to the other.

Let us fuppofe, for example, that a Ship has failed 150 Leagues North-Eaft, one-fourth North, which is the third Point, and makes an Angle of 33 Deg. 45 Min. with the North part of the Meridian. Now we have two things given, *viz.* the Courfe, and Diftance failed, by which a Triangle may be found on this Inftrument fimilar to that made by the Ship's Courfe, and her Latitude and Longitude; and fo the other unknown Parts of the Triangle found. And this is done thus:

Let the Center A reprefent the Place of Departure, and count, by means of the Concentrick Arcs, along the Point that the Ship failed on, as AD, 150 Leagues from A to D; then the Point D will be the Place the Ship is arrived at, which note with a Pin. This being done, let DE be parallel to the Side AC, and then there will be formed a Right-angled Triangle AED, fimilar to that made by the Ship's Courfe, difference of Latitude and Longitude: the Side AE of this Triangle gives 125 Leagues for the difference of Latitude Northwards, which make 6 Deg. 15 Min. reckoning 20 Leagues to a Degree, and one League for three Minutes. And laftly, the Side ED will give 83 leffer Leagues towards the Eaft, which being reduced in the manner hereafter fhewn, will give the difference of Longitude, and fo the whole Triangle will be known.

Note, We call leffer Leagues thofe that anfwer to the Parallels of Latitude between the Equator and the Poles, which continually decreafe the nearer they are to the Pole, and confequently alfo the Degrees of Longitude; and therefore the nearer a Ship fails to either of the Poles, the lefs way muft fhe make to alter her difference of Longitude any determinate Number of Degrees.

Since

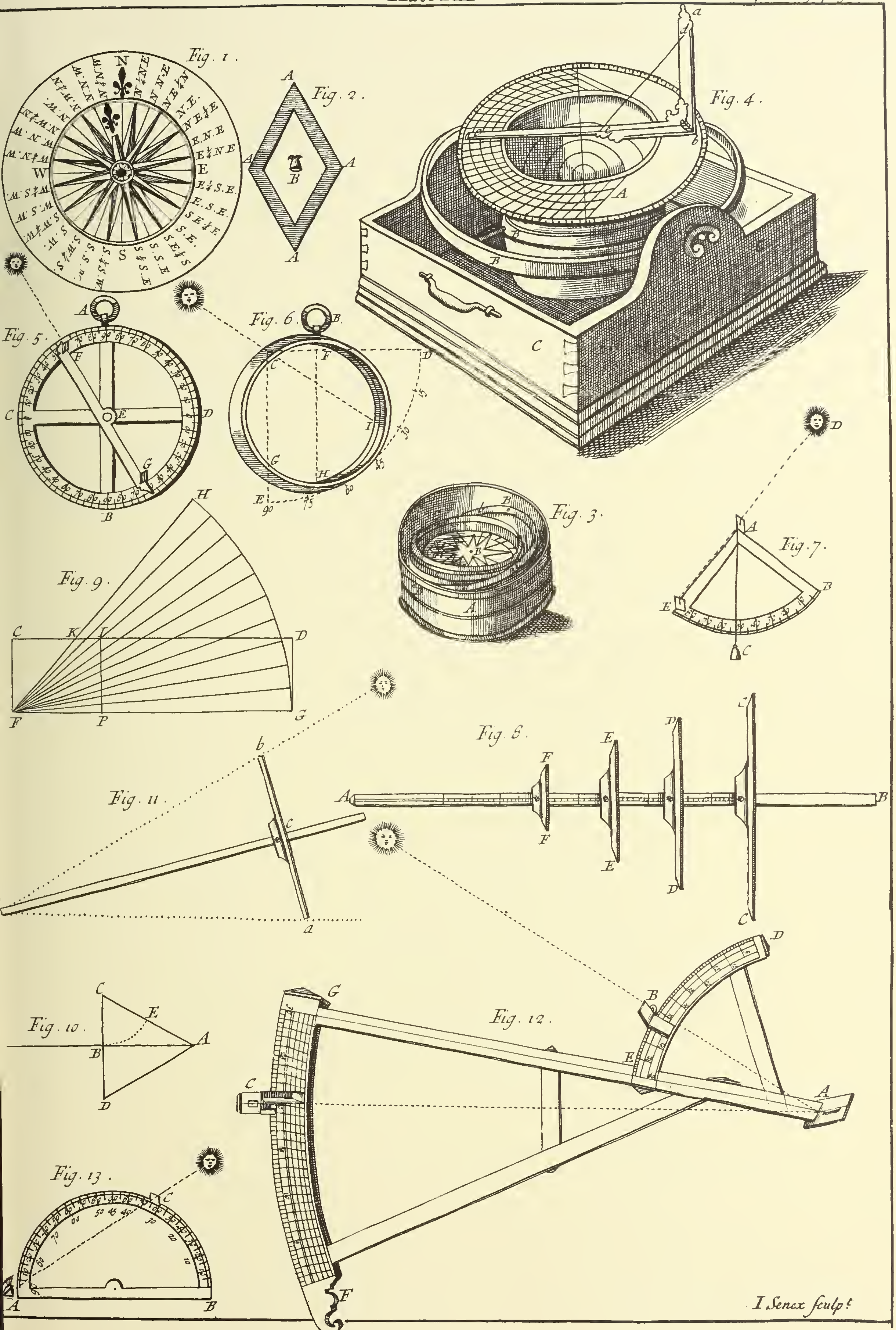
Fig. 1.
N
E
S
W
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
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Since the Center A always reprefents the Place of departure, it is manifeft that when the Point D of arrival is found, be it in what manner foever, all the Parts of the Triangle A E D will afterwards be eafily determined.

If the Sinecal Quadrant be taken for a fourth part of the Meridian, one Side thereof, as A B, may be taken for the common Radius of the Meridian, and the Equator; and the other fide A C, will then be half the Axis of the World. The Degrees of the Circumference BC, will reprefent the Degrees of Latitude, and the Parallels to the Side A B perpendicular to AC, affumed from every Point of Latitude to the Axis A C, will be the Radius's of the Parallels of Latitude, as likewife the Sine-Complements of thofe Latitudes.

If, for Example, it be required to find how many Degrees of Longitude 83 leffer Leagues make in the Parallel of 48 Deg. you muft firft extend a Thread from the Center A, over the 48th Deg. of Latitude on the Circumference; and keeping it there, count the 83 Leagues propofed on the Side A B, beginning at the Center A. Thefe will terminate at H, in allowing every fmall Interval four Leagues, and the Interval between the broad Lines twenty Leagues. This being done, if the Parallel H G be traced out from the Point H to the Thread, the part A G of the Thread, fhews that 125 greater Leagues, or the equinoctial Leagues, makes 6 Deg. 15 Min. in allowing 20 Leagues to a Degree, and three Minutes for one League; and therefore the 83 leffer Leagues A H, which make the difference of Longitude of the fuppofed Courfe, and which are equal to the Radius of the Parallel G I, make 6 Deg. 15 Min. of the faid Parallel.

Let it be required, for a fecond example, to reduce 100 leffer Leagues into Degrees of Longitude on the Parallel of 60 Degrees. Having firft extended the Thread from the Center A over the 60th Degree on the Circumference, count the 100 Leagues of Longitude on the Side AB, and the Parallel terminating thereon being directed to the Thread, the part of the Thread affumed from the Center, fhews that 200 Leagues under the Equator make 10 Degrees; that is, 100 Leagues in the Parallel of 60 Degrees make 10 Degrees of Longitude, fince every Degree of a great Circle is double to any Degree of the Parallel of 60 Degrees.

On one Side of this Inftrument is put a Scale, called a Scale of *Crofs Latitudes*, whofe Conftruction and Divifion is the fame as that of the Meridian Line of *Mercator*'s Chart, of which we fhall fpeak by and by. The Ufe of this Scale is to find a mean Parallel between that of Departure and that of Arrival.

When a Ship has failed on an oblique Courfe, that is, neither exactly North, South, Eaft, or Weft; thefe Courfes, befides the North and South greater Leagues, give *leffer Leagues* eaftwardly and weftwardly, which muft be reduced to Degrees of Longitude. But thefe Leagues were made neither upon the Parallel of Departure, nor upon that of Arrival; for they were made upon all the Parallels between thofe of Departure and Arrival, and are all unequal between themfelves, and confequently we are neceffitated to find a mean proportional Parallel between that of Departure and that of Arrival, which for this reafon is called a mean Parallel, and ferves to reduce Leagues made in failing a-crofs divers Parallels, into Degrees and Minutes of the Equator.

Now there are feveral ways of finding fuch a mean Parallel; but I fhall only fpeak of that here, which is done by means of the Scale of *increafed Latitude*, without Calculation, and is thus: Let it be required, for example, to find a mean Parallel between that of 40 Deg. and that of 60 Deg.

Take, by means of a Pair of Compaffes, the middle between the 40th and 60th Deg. upon this Scale, and the faid middle Point will terminate againft the 51ft Deg. which confequently will be the mean Parallel fought.

Note, Becaufe this Scale is in two Lines, you muft take the Diftance from 40 Deg. of Latitude to 45 Deg. which is on one Side, and lay it off upon fome feparate Right-Line. This being done, you muft take the Diftance from 45 Deg. to 60 Deg. which is on the other Side, and join thefe two Spaces together; then half of thefe two Lines being taken between your Compaffes, you muft fet one Foot upon the Number 60, and the other Point will fall upon 51 Deg. which will be the mean Parallel fought. After which, it will be eafy to reduce the Leagues failed Eaftwardly into Degrees of Longitude, by the Sinecal Quadrant, confidered as a quarter of the Meridian, in the manner as we have laid down in the two Examples abovementioned.

Of Mercator's Charts.

This Figure reprefents a *Mercator*'s Chart. But before we give the Conftruction and Ufes thereof, it is neceffary to obferve that when a Ship fails upon any determinate Point of the Compafs, fhe always makes the fame Angle with all the Meridians fhe paffes over upon the Surface of the Terraqueous Globe. *Fig. 2.*

If a Ship fails North and South, fhe makes an infinitely acute Angle with the Meridian fhe defcribes, that is, fhe runs parallel to it, or rather fails upon it.

If a Ship fails due Eaft and Weft, fhe cuts all the Meridians at Right Angles; for fhe either defcribes the Equator, or fome leffer Circle which is parallel thereto. But if her Courfe be on any Point between the North and Eaft, North and Weft, South and Eaft, or South and Weft, then fhe will not defcribe a Circle; becaufe a Circle drawn oblique to the Meridians,

Meridians, will cut all of them at unequal Angles, which the Ship must not do while she sails upon any determinate Point, unless North and South, or East and West; therefore she describes a Curve, not circular, whose essential Property is to cut all the Meridians at the same Angle. And this is called a Loxodromick Curve, or only Loxodromy, and is a kind of Spiral, making an Infinity of Revolutions towards a certain Point, which is the Pole, and every Turn thereof approaches nigher and nigher thereto. A Ship's Course then, except the two first abovenamed, is always a Loxodromick Curve, and is the Hypothenuse of a Right-angled spherical Triangle, whose two other Sides are the Ship's Way in Longitude and Latitude. Now we have the Latitude commonly given by Observation, and the Loxodromick Angle by the Compass; therefore by Trigonometry we may find the Hypothenuse, or the Way that the Ship has sailed, *&c.*

But because the Calculation of a Ship's Way by means of the Loxodromick Curve is troublesome, therefore the Ancients sought after some Method whereby a Ship's Way might be a strait Line, which might nearly preserve the Property of the Loxodromick Curve, which is, to cut all the Meridians under the same Angle. But they found this absolutely impossible upon the account of the Meridians not being parallel between themselves, as in reality they are not. And therefore they supposed the Meridians to be parallel strait Lines; and so from this supposition it follows, that the Degrees of Longitude unequally distant from the Equator, are of the same bigness; whereas they really always diminish from the Equator, in a certain known Proportion, which is as Radius is to the Sine-Complement of the Latitude. But to retrieve this Error, they have supposed the Degrees of Latitude, which by the Nature of the Sphere are every where equal, to be augmented in the same Proportion as the Degrees of Longitude diminish. And so the Inequality which ought to be in the Degrees of Longitude of different Parallels, is thrown upon the Degrees of Latitude in the manner we are going to lay down.

Now Charts made in this manner are called *Mercator*'s Charts, because *Mercator* was the first that made them; and they are commonly esteemed the best: for by the Experience of several Ages, it is found that Seamen ought to have very simple Charts, wherein the Meridians, Parallels, and Rhumb-Lines, may be represented by strait Lines, that so they may prick down their Courses easily.

CHAP. IV

Of the Construction and Uses of Mercator's *Charts.*

Fig. 2. If the Degrees of Latitude are to be augmented as much as those of Longitude are found enlarged by making them equal to the Degrees of the Equinoctial, the Secants must be used, which increase in the same Proportion as the Sine-Complements of the Latitudes (which ought to represent the Degrees of Longitude) have been increased, by making them equal to the Radius of the Equator, because of the Parallelism of the Meridians: for the Sine-Complement of an Arc is to Radius, as Radius is to the Secant of that Arc.

As, assuming for one Degree of the Equator, and for the first Degree of Latitude, the whole Radius, or some aliquot part thereof; take for the 2d Degree of Latitude, the Secant of one Degree, or a similar aliquot part of this Secant; and for the 3d Degree of Latitude, take the Secant of two Degrees, or the similar aliquot part thereof, and so on.

When a Chart is to be made large, you must take, for 30 Minutes of Latitude, and 30 Minutes of the Equator, the Radius of a Circle or some aliquot part thereof, for one Degree of Latitude. This being done, you must add continually the Secant of 30 Min. for $1\frac{1}{2}$ Degree of Latitude, the Secant of 1 Degree for 2 Degrees of Latitude, the Secant of $1\frac{1}{2}$ Degree for $2\frac{1}{2}$ Degrees of Latitude, or their similar aliquot parts; and so proceed on. In doing of which, we use a Scale of equal parts, from which the Secants as they are found in Tables are taken off, by taking away some of the last Figures.

In these Charts the Scale is changed, according as the Latitude is; as, for example, if a Ship sails between the 40th and 50th Parallel of Latitude, the Degrees of the Meridians between those two Parallels will serve for a Scale to measure the Ship's Way; whence it follows, that there are fewer Leagues on the Parallels, the nearer they are to the Poles, because they are measured by a Magnitude likewise continually increasing from the Equator towards the Poles.

If, for example, a Chart of this kind be to be drawn from the the 40th Degree of North Latitude to the 50th, and from the 6th Degree of Longitude to the 18th: First draw the Line A B, representing the 40th Parallel to the Equator, which divide into twelve equal Parts, for the 12 Degrees of Longitude, which the Chart is to contain. This being done, take a Sector or Scale, one hundred Parts whereof are equal to each of these Degrees of Longitude, and at the Points A and B raise two Perpendiculars to A B, which will represent two parallel

parallel Meridians, and muſt be divided by the continual Addition of Secants. As, for the Diſtance from 40 Deg. to 41 Deg. of Latitude, take 131 $\frac{1}{2}$ equal Parts from your Scale, which is the Secant of 40 Deg. 30 Min. For the Diſtance from 41 Deg. to 42 Deg. take 133 $\frac{1}{2}$ equal Parts from your Scale, which is the Secant of 41 Deg. 30 Min. For the Diſtance from 42 Deg. to 43 Deg. take 136, which is the Secant of 42 Deg. 30 Min. and ſo on to the laſt Degree of your Chart, which will be 154 equal Parts, *viz.* the Secant of 49 Deg. 30 Min. and will give the Diſtance from 49 Deg. of Latitude to 50 Deg. and by this means the Degrees of Latitude will be augmented in the ſame Proportion as the Degrees of Longitude on the Globe do really decreaſe.

Having divided the Meridians, you may place the Card upon the Chart, for doing of which, chuſe a convenient Place towards the Middle thereof, as the Point R, about which, as a Center, deſcribe a Circle ſo big that it's Circumference may be divided into 32 equal Parts, for the 32 Points of the Compaſs. Then having drawn a Line towards the Top of the Chart, parallel to the two divided Meridians, this will be the North Rhumb, and upon it a *Flower-de-Luce* muſt be put, that thereby all the other Rhumbs or Points may be known, the principal of which ought to be diſtinguiſhed from the others by broader Lines.

After this, all the Towns, Ports, Iſlands, Coaſts, Sands, Rocks, *&c.* which form the Chart, muſt be laid down upon the ſame, according to their true Latitudes and Longitudes. And if the Chart be large, there may ſeveral Cards be placed thereon, always with their North and South Lines parallel between themſelves.

The Uſe of Mercator's *Charts.*

The chief Uſe of a Sea-Chart, is to find the Point of Departure therein, the Point arrived at, the Courſe, the Diſtance ſailed, the Longitude and the Latitude, as we ſhall now explain by ſome Examples.

Example I. Suppoſe a Ship is to ſail from the Iſland *de Oueſſant*, in 48 Deg. 30 Min. of North Latitude, and 13 Deg. 30 Min. of Longitude, to Cape *Finiſter* in *Galicia*, which is in 43 Deg. of Latitude, and 8 Deg. of Longitude. Now the Point of the Compaſs the Ship muſt keep to, as alſo the Diſtance between the ſaid two Places is required. In order to do this, you muſt imagine a Line drawn from the Iſland *de Oueſſant* to Cape *Finiſter*, and with a Pair of Compaſſes examine what Point on the Chart that Line is parallel to, and this Point, which is South-Weſt, one-fourth South, is that which the Ship muſt ſail on.

But to find the Diſtance of the two Places, take between your Compaſſes the Extent of five Degrees on the Meridian againſt the beforenamed Courſe, that is, from the 43d Deg. to the 48th; and this will be a Scale of 100 Leagues. This being done, ſet one Foot of your Compaſſes thus opened upon the Iſland *de Oueſſant*, and the other Foot upon the occult Line tending to Cape *Finiſter*, making a little Mark thereon; and this Extent of the Compaſſes will give 100 Leagues of Diſtance. Then take the Diſtance from the aforeſaid Mark to Cape *Finiſter* between your Compaſſes, and placing one Foot upon the 43d Deg. of the Meridian, and the other Foot will fall upon 44 Deg. 45 Min. which amounts to 35 Leagues; and conſequently the whole Diſtance between Cape *Finiſter* and the Iſland *de Oueſſant* is 135 Leagues.

Example II. A Ship ſailing from the Iſland *de Oueſſant* South-Weſt, one-fourth South, towards Cape *Finiſter*, and the Maſter-Pilot having examined the Force of the Wind, and the Number of Sails ſpread, and knowing by experience the Swiftneſs of his Ship, has eſtimated her Way to have been 50 Leagues in 20 Hours. Now to find the Point upon the Chart wherein the Ship is, he muſt take the Extent of 2 $\frac{1}{2}$ Degrees, equivalent to 50 Leagues, between his Compaſſes, upon the Meridian, from the 46th Deg. to the 48 $\frac{1}{2}$ Deg. This being done, if one Foot of the Compaſſes thus opened be ſet upon the Place of Departure, the other Foot will fall upon the Point T, the Place wherein the Ship is, on the Line of the Ship's Way. But if the Longitude and Latitude of the Point T, or Place wherein the Ship is, be ſought, he muſt place one Foot of the Compaſſes upon the Point T, and the other upon the neareſt Parallel, and then conduct the Compaſſes thus opened perpendicularly along the Parallel to the Meridian and the Degree thereof whereat the Point of the Compaſſes comes to, will be the Latitude of the Point T. And to find the Longitude of this Point, he muſt ſet one Foot of the Compaſſes therein, and the other upon the neareſt Meridian. Then if this Foot be ſlid along the Meridian (ſo that a Line joining the two Points be always parallel to itſelf) to the divided Parallel, he will have, upon that Parallel, the Longitude of the Point T.

Becauſe Meridians and Parallels are not drawn a-croſs the Chart, to the end that the Rhumb-Lines may not be confuſed, therefore you may uſe a Ruler, which will produce the ſame Effect.

Example III. The Courſe being given, and the Latitude by Obſervation; to find the Diſtance ſailed, and to prick down the Place of the Ship upon the Chart. Suppoſe a Ship departed from the Iſland *de Oueſſant* is arrived to a Place whoſe Latitude, by Obſervation, is found to be 46 Degrees; take, between your Compaſſes, the Diſtance from the 46th Degree of the Meridian to the 48 $\frac{1}{2}$, which is the Latitude of the Place of Departure, over which 48 $\frac{1}{2}$ Degree and the Iſland *de Oueſſant* having laid a Ruler, ſlide one Foot of the Compasses thus opened along the Side of this Ruler, 'till the other Foot intersects the Line of the Ship's Way; then the Point of Intersection S will be that whereat the Ship was at the Time of Observation. Now to find the Distance sailed, you must extend the Compasses from this Point S to the Place of Departure, and lay off this Extent upon the Meridian, which will reach from the 46th Degree to the 49th; and consequently the Distance sailed will be 60 Leagues, allowing 20 Leagues to a Degree.

Example IV. The Latitude and Longitude of a Place being given, to find that Place in the Chart. Having placed one Foot of a Sea-Chart Compass upon the known Degree of Latitude, and the other upon the nighest Parallel, you must place with your other Hand one Foot of another Pair of Compasses upon the known Degree of Longitude on the Meridian, and the other Foot upon the nearest Meridian; and then slide both these Pair of Compasses until their two Points meet each other: for then the Point of Concourse will be that sought. This Operation is very much used by Seamen; for the Point where they are, being first found by Calculation, or the Sinecal Quadrant, they can by this means prick down the Place of the Ship upon the Chart, and so it will be easy for them to find what Course the Ship must steer to continue on her Voyage.

BOOK

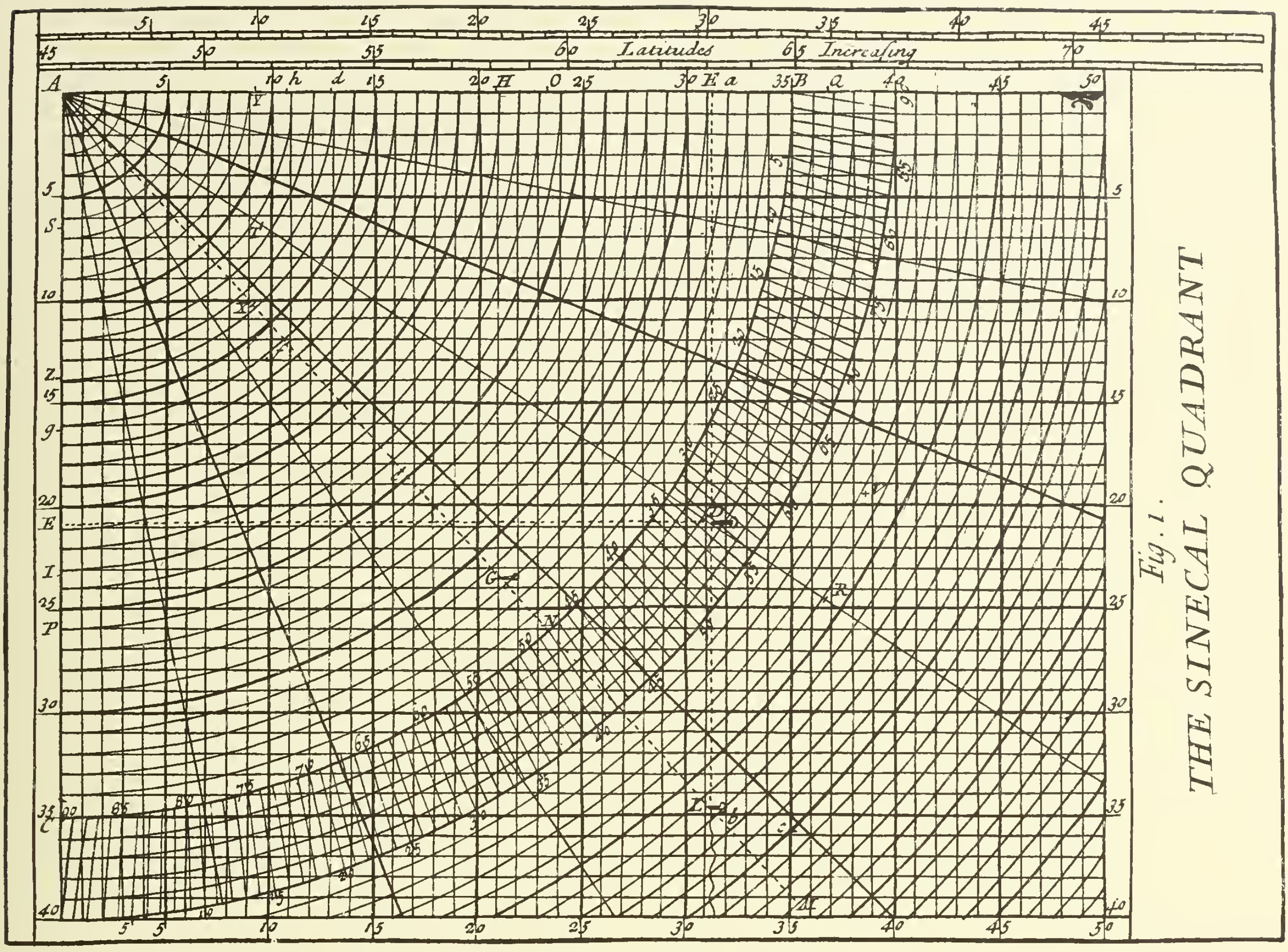
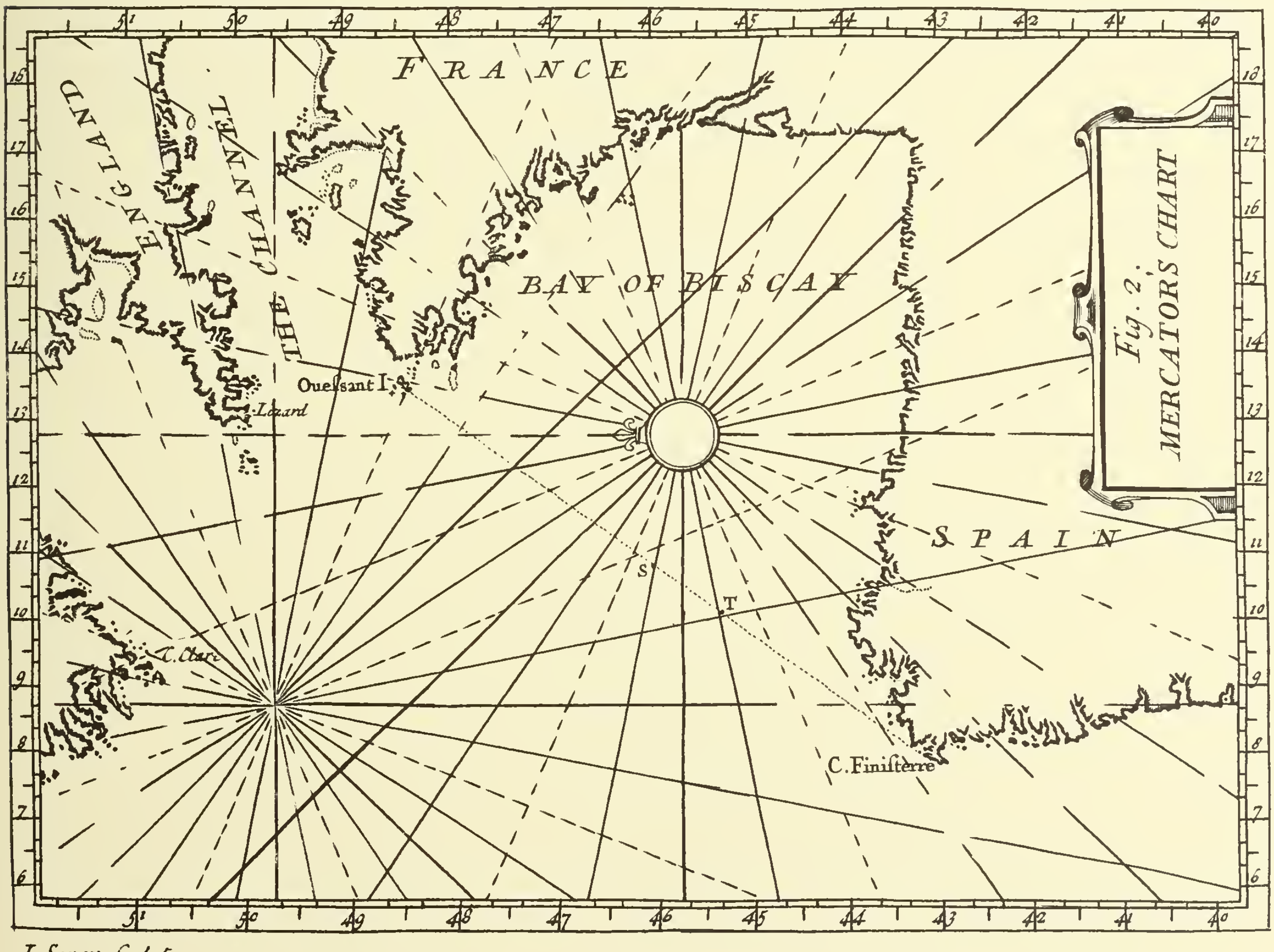


Plate XXI

fronting page 210



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BOOK VIII.

Of the Conſtruction and Uſes of Sun-Dials.

Remarks and Definitions appertaining to Dialling.

SUN-Dials take their Name from the principal Circles of the Sphere to which they are parallel: as, a Horizontal-Dial is one parallel to the Horizon; an Equinoctial-Dial one parallel to the Equinoctial; a Vertical-Dial one that is parallel to a Vertical Circle; and ſo of others.

There are two ſorts of Styles placed on the Surfaces of Dials; one is called a Right Style, which is a pointed Iron-Rod, that ſhews the Hour or Part on a Dial by the Shadow of its Extremity; and the other is called an oblique or inclined Style, or elſe the Axis, which ſhews the Time of Day upon a Dial by the Shadow of the whole Length thereof.

The Extremity of the right Style of any Dial, repreſents the Center of the World and Equator, and the Plane of a Dial is ſuppoſed to be as far diſtant from the Center of the Earth, as is the Length of the right Style. For becauſe the Sun's Diſtance from the Center of the Earth is ſo great, and the Diſtance of any Point in the Earth's Superficies from the Center is ſo ſmall, compared with the Sun's Diſtance; therefore any Point on the Earth's Surface may without any ſenſible Error be taken for its Center: and ſo the Extremity of the Style of any Dial may be taken for the Center of the Earth; and a Line parallel to the Axis of the World, which paſſes thro' the Extremity of the Style, may be conſidered as the Axis of the World.

The Hour-Lines, which are drawn upon Dial-Planes, are the Interſections of the ſaid Planes made by the Hour-Circles of the Sphere.

The Center of a Dial, is the Interſection of its Surface with the Axis of the Dial paſſing thro' the Extremity of the Style parallel to the Axis of the World; and in this Center all the Hour-Lines meet each other.

All Dial-Planes may have Centers, except Eaſt, Weſt, and Polar ones; for on theſe the Hour-Lines are all parallel between themſelves.

The Vertical Line of a Dial-Plane, is a Pependicular drawn from the Extremity of the Style to the Foot thereof; but the Vertical Line of the Place wherein the Dial is, is a right Line perpendicular to the Horizon drawn thro' the Extremity of the Style.

Dials have likewiſe two Meridians; one of which is the ſubſtylar Line or proper Meridian of the Dial-Plane, becauſe its Circle paſſes thro' the Vertical Line of the Dial-Plane; and the other, which is the Meridian of the Place, hath its Meridian Circle paſſing thro' the Vertical Line of the Place.

When a Dial declines neither to the Eaſt or Weſt, the ſubſtylar Line, or Meridian of the Plane, coincides with the Meridian of the Place or Hour-Line of 12, let the Surface of the Dial be Vertical, Horizontal, or even inclined upwards or downwards.

The Horizontal Line of a Dial-Plane, is the common Section of the ſaid Plane, and a horizontal or level Line paſſing thro' the Extremity of the Style; and the Equinoctial Line is the common Section of the Dial-Plane and Equinoctial Circle: and this Line is always perpendicular to the ſubſtylar Line; and conſequently if the Poſition of the ſubſtylar Line be known, and a Point of the Equinoctial Line be given, we may likewiſe have the Poſition of the Equinoctial Line: and contrariwiſe, if the Equinoctial Line be given, we may have the

ſubſtylar

ſubſtylar Line, which is perpendicular thereto. *Note*, This ſubſtylar Line muſt paſs thro' the Foot of the Style and the Center of the Dial.

The Hour-Line of ſix always paſſes thro' the Interſection of the Horizontal and Equinoctial Lines in declining Dials; and ſo the ſaid Point of Interſection is one Point of the Hour-Line of ſix. *Note*, The Point wherein the Subſtyle and Meridian Lines meet, is the Center of the Dial.

When a Dial is to be drawn upon a Plane, you muſt firſt find the Poſition of the ſaid Plane, or of the Wall it is to be ſet up againſt, with regard to the Sun and the principal Circles of the Sphere: And this may be done, in obſerving ſeveral Times the ſame Day, at every 3 or 4 Hours interval, where the Shadow of the Extremity of a Style falls upon the Dial-Plane: for by this means the Poſition of the Dial-Plane may be determined, and afterwards all the Hour-Lines, *&c.* may be drawn thereon in the manner we ſhall hereafter ſhew. *Note*, The Exactneſs of a Dial very much depends upon theſe Points.

CHAP. I.

Of Regular and Irregular Dials, *drawn upon Planes and Bodies of different Figures.*

Plate 22. Fig. 1. THIS Inſtrument repreſents a hollow Body, having 14 Planes, upon each of which a Dial may be drawn.

The upper Plane A, is parallel to the Horizon; and ſo upon this a Horizontal-Dial is drawn, as well as upon the under Plane E, whereon the Sun ſhines but a very little. The Plane B is parallel to the Axis of the World, and makes an Angle of 49 Degrees with the Horizon of *Paris*; for the Latitude of which, all the Dials are ſuppoſed to be drawn. Now upon this Plane is drawn an upper Polar Dial, and upon the Plane F, which is oppoſite thereto, is drawn an under Polar Dial. The Plane C is parallel to the Prime Vertical, and ſince it faces the South, there is drawn thereon a South Vertical Dial, And upon the oppoſite Plane to this, which is towards G, and faces directly to the North, is drawn a Vertical North Dial, which cannot be repreſented in this Figure.

The Plane H, which is parallel to the Equinoctial, and ſo makes an Angle with the Horizon of 41 Deg. *viz.* the Complement of the Latitude of *Paris*, hath an upper Equinoctial Dial drawn upon it; and upon the oppoſite Plane D, is drawn an under Equinoctial Dial. The Plane K is parallel to the Plane of the Meridian, and becauſe it directly faces the Weſt, a Meridional Weſt Dial is drawn thereon, and upon the oppoſite Plane to this is drawn a Meridional Eaſt Dial. The Plane I makes an Angle 45 Deg. with the Meridian; and therefore there is drawn upon it a vertical Decliner, declining Southweſtwardly 45 Deg. and upon the oppoſite Plane to this is drawn a North-Eaſt Decliner of 45 Deg. Finally, The Plane L declines North-Weſt 45 Deg. and its Oppoſite 45 Deg. South-Eaſt; and ſo upon theſe two Planes are drawn North-Weſt and South-Eaſt Decliners.

The firſt Nine of the abovementioned Dials, are called Regular ones; and the Four others, which decline, are called Irregular Dials.

The Axes of all theſe Dials are parallel to each other, and to the Axis of the World. We ſhall hereafter give the Conſtruction of all theſe Dials, as well as of thoſe on the following Inſtrument, of which we are going to ſpeak.

The Conſtruction of Dials drawn upon a Dodecahedron.

Fig. 2. This Figure is one of the five Regular Bodies, of which we have ſpoken in the firſt Book. This Body is called a Dodecahedron, and is terminated by 12 equal Pentagons, upon every of which may be drawn a Dial, except on the undermoſt.

The Plane A being Horizontal, hath a Horizontal-Dial drawn thereon, whoſe Hour-Line of 12 biſects one of the Angles of the Pentagon. Upon the Plane B, which faces the South, is drawn a direct South-Dial, inclining towards the Zenith, or upwards 63 Deg. 26 Min. The Center of this Dial is upwards, and the ſubſtylar Line is the Hour-Line of 12. The oppoſite Plane to this, is a North vertical one, inclining downwards or towards the Nadir 63 Deg. 26 Min. and ſo there is drawn thereon a North inclining Dial, whoſe Center is downwards.

The Dial C, is a South-Eaſt inclining Recliner, whoſe Declination is 36 Deg. and Inclination to the Zenith 63 Deg. 26 Min. and its Center is downwards. The Dial D is a North-Eaſt Decliner of 72 Deg. inclining towards the Nadir 63 Deg. 26 Min. the Center being upwards, and its oppoſite is a South-Weſt Decliner of 72 Deg. inclining towards the Zenith 63 Deg 26 Min. the Center being downwards.

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The Dial E is a North-East Decliner of 36 Deg. and inclines towards the Zenith 63 Deg. 26 Min. the Center being downwards. The oppofite Dial to this, is a South-Weft Decliner of 36 Deg. and inclines towards the Nadir 63 Deg. 26 Min. it's Center being upwards. Finally, the Dial F is a South-Eaft Decliner of 72 Deg. inclining towards the Zenith 63 Deg. 26 Min. the Center being downwards; and it's oppofite is a North-Weft Decliner of 72 Deg. inclining towards the Nadir 63 Deg. 26 Min. the Center thereof being upwards.

All thefe Dials are furnifhed with their Axes, which are parallel between themfelves, and to the Axis of the World.

Now if one of thefe Bodies of Dials be fet upon a Pedeftal, in a Place well expofed to the Sun, and then be fet right by means of a Compafs or Meridian Line, drawn in the Manner we fhall hereafter fhew; all the Dials that the Sun fhines upon will fhew the fame Hour or Part at the fame Time by the Shadows of the Styles.

But if a Dodecahedron of Dials be to be placed upon a Pedeftal fixed in a Garden, it ought to be made of folid Matter, as Stone or good Wood, well painted to preferve it from Rain, *&c.* therefore it will be here neceffary to fhew how to cut out a Dodecahedron.

Take a Stone cut out into a perfect Cube, and divide each of the four Sides of it's Faces into two equal Parts, by two Diameters A C, B D. And at the Points A and C, make the Angles E A F, and H C G, each 116 Deg. 34 Min. that is, make Angles at the Points A and C, on each fide the Diameter A C, of 58 Deg. 17 Min each: becaufe all the Surfaces of the Dodecahedron make Angles of 116 Deg. 34 Min. with each other; therefore two Faces thereof being horizontally placed, all the others will incline 63 Deg. 26 Min. the Complement of 116 Deg. 34 Min. to 180 Deg. Now the Space between F and G, or E H, will be the Length of each fide of the Pentagons, half of which, *viz.* B F, muft be taken and laid off both ways from the Point I to the Points Q and X. And this muft be done upon the Diameters croffing each other on all the other Faces of the Cube. Afterwards the Stone muft be cut away along the Diameters to the Extremities of the Sides of the Pentagons: for Example, you muft cut away the Stone down, or all along the Diameter K M, in a Right Line to the Point Q in the firft Surface of the Cube, as likewife all along the Diameter L N ftrait forwards to the Point S, and again all along the Diameter B D directly forward to the Point T. And proceeding in this manner with the other Faces of the Cube, you may compleat your Dodecahedron. But it will be very proper for a Perfon that has a mind to cut out one of thefe Bodies, to have a Pafteboard one before him, thereby to help his Imagination, that fo he may know better what Angles and Sides to cut away. Fig. 3.

Cylinders may be cut likewife into Dodecahedrons, but let the Method above given fuffice.

We make alfo very curious Dials on the Faces of fmall brafs Dodecahedrons.

The Conftruction of an Horizontal Dial.

The fourth Figure is an Horizontal Dial: To make which, firft draw the two Lines A B, C D, cutting each other at Right Angles in the Point E, which will be the Center of the Dial, the Line A B the Meridian or Hour-Line of 12, and the Line C D the Hour-Line of 6. This being done, make the Angle B E F, 49 Deg. equal to the Elevation of the Pole at *Paris* (the Elevation of the Pole at *Paris* is but 48 Deg 51 Min. but we neglect the nine Minutes, as being but of fmall Confequence in the Conftruction of Dials) and the Line E F will reprefent the Axis of the World. In this the Point G muft be chofen at pleafure, reprefenting the Center of the Earth, and G H muft be drawn at Right Angles to E F, cutting the Meridian or Hour-Line of 12 in the Point H. This Line G H reprefents the Radius of the Equinoctial. Now take H G between your Compaffes, which lay off from H to B on the Meridian Line, and draw the Right Line L H K perpendicular to the Meridian, which will reprefent the common Section of the Equinoctial, and the Plane of the Dial: then about the Point B, as a Center, defcribe the Quadrant M H, which divide into fix equal Arcs, each of which will be 15 Deg. and draw the dotted Lines B 5, B 4, B 3, B 2, B 1. Thefe will divide the Line L H into the Points 1, 2, 3, 4, 5, thro' which Points, if Lines be drawn from the Center E of the Dial, you will have the Hour-Lines of 1, 2, 3, 4, and 5, on one fide the Meridian; and becaufe the Hour-Lines equally diftant on both fides from the Meridian make equal Angles with the Meridian, therefore if the Divifions 1, 2, 3, 4, 5, on one fide the Meridian, be laid off from H towards K on the other fide, and thro' the Points where they terminate are drawn Lines from the Center E; thefe will be the Hour-Lines of 11, 10, 9, 8, 7. And if the Hour-Lines of 7 and 8 in the Morning are continued out beyond the Center, they will give the Hour-Lines of 7 and 8 in the Evening, and likewife the Hour-Lines of 4 and 5 in the Afternoon continued out in the fame manner, will give thofe of 4 and 5 in the Morning. *Note*, Inftead of drawing the Quadrant M H, we might, for greater facility, have only drawn an Arc greater than 60 Deg. for then if an Arc of 60 Deg. had been taken upon it from the Point H, by means of it's Chord, which is equal to Radius, and the faid Arc had been divided in four equal Arcs, each of 15 Deg. and another Arc of 15 Deg. had been added to that of 60 Deg. for the Hour of 5, we might have drawn the Lines B 1, B 2, B 3, *&c.* as we have already done. Fig. 4.

Now to draw the Half-hours, you must bisect each of the Arcs of 15 Deg. on the Quadrant M H, in order to have Arcs of 7 Deg. 30 Min. and for the Quarters, each of these last Arcs must be again bisected; and thro' each Point of Division occult Lines must be drawn from the Center B, cutting the Equinoctial Line K L. Then if the Edge of a Ruler be laid thro' these Points of Concourse and the Center E of the Dial, the Halfs and Quarters of Hours may be drawn.

The Hour-Lines being drawn upon your Dial, you may give it what Figure you please, as a Parallelogram, regular Pentagon, *&c.*

This Dial being fixed upon a very level Plane, that is, set parallel to the Horizon, exposed to the Sun, and it's Hour-Line of 12 placed exactly North and South; as also the Style or Axis E H F being raised perpendicularly upon the Hour-Line of 12, so as E F be parallel to the Axis of the World: I say, if these things be so ordered, the Shadow of the Axis or Style will shew the Hour of the Day from Sun-rising to Sun-setting.

The Construction of a Non-declining Vertical Dial.

Fig. 5. This Dial is parallel to the Prime Vertical, which cuts the Meridian at Right Angles, and passes thro' the East and West Points of the Horizon. The Manner of drawing it is thus: First draw the Lines E B and C D at Right Angles, the first of which shall be the Hour-Line of 12, and the other the Hour-Line of 6; then make the Angle B E F at the Point E, the Center of the Dial, equal to the Complement of the Elevation of the Pole, which at *Paris* is 41 Deg. and raise the Line I G perpendicularly on the Meridian; this will be the right Style, and the Point I is the Foot thereof, and G the Extremity, which, as above said, may be taken for the Center of the Earth: and this Line both ways produced, will be the Horizontal-Line.

From the Point G, in the Right Line E G F, which represents the Axis of the World, raise the Line G H at Right Angles thereto, cutting the Meridian in B. This Line G H shall represent the Radius of the Equinoctial, and the Line L H K, drawn thro' the Point H, cutting the Meridian at Right Angles, represents the common Section of the Equinoctial and the Plane of the Dial. Now make H B equal to H G, and about the Point B, as a Center, describe the Quadrant of a Circle M H, which divide into 6 equal Arcs, each of which will be 15 Deg. by dotted Lines, dividing the Line L K into unequal Parts, which shall be the Tangents of the said Arcs. Finally, If thro' those Points of Division and the Center E, you draw Lines, they will be the Hour-Lines on one side of the Meridian; and for drawing the Hour-Lines on the other side the Meridian, as also the Halves and Quarters of Hours, you must do as is shewn in the Horizontal-Dial.

This Dial is set up against a Wall, or on a very upright Plane, directly facing the South; for which reason it is called a Meridional Vertical Dial: it's Meridional or Hour-Line of 12 must be perfectly upright, and it's Horizontal-Line level. The Center thereof is upwards, and it's Axis points towards the under Pole. The opposite Dial to this, is a Vertical North one, having the Center downwards, and the Extremity of it's Axis pointing to the upper Pole of the World. The Construction of this latter Dial is the same as that of the other, the Hour-Lines and the Axis making the same Angles with the Meridian, as they do on that. But the Sun shines but a small time upon this Dial, and this only in the Summer-time, *viz.* in the Morning from his rising 'till he has passed the Prime Vertical, and in the Evening from the time he has again passed the Prime Vertical 'till his setting. When the Sun describes the Summer Tropick, he rises at *Paris*, at 4 in the Morning, and comes to the Prime Vertical between 7 and 8 in the Morning; and in the Afternoon he repasses the Prime Vertical between 4 and 5, and sets at 8. Therefore we need only draw the Hour-Lines upon this Dial from 4 in the Morning to 8, and from 4 in the Afternoon to 8; at which time the Sun shines upon the Meridional Vertical Dial, but from about 8 in the Morning to about 4 in the Afternoon. But when the Sun by his annual Motion is again come back to the Equinoctial, he will not shine at all upon the Vertical North Dial 'till after he has crossed the Equinoctial again; and all this time he will shine upon the Meridional Vertical Dial from his rising to his setting.

The Construction of a Polar Dial.

Fig. 6. The 6th Figure represents an upper Polar Dial, which is one that inclines upwards, but does not decline: for it is parallel to the Axis of the World, and the Hour-Circle of 6, which cuts the Meridian at Right Angles. And for this reason the Hour of 6 in the Morning or Evening can never be shewn by this Dial; for the Shadow of the Style being then parallel to the Plane of the Dial, cannot be cast upon it. This Dial likewise hath no Center, and the Hour-Lines are all parallel between themselves, and to the Axis of the World. The Plane therefore being parallel to the Horizon of a right Sphere, passes thro' the two Poles of the World, from whence comes the Name of a Polar Dial.

The Manner of drawing this Dial is thus: First draw the Line A B representing the Equinoctial, and I D at Right Angles thereto, for the Meridian or Hour-Line of 12. Then assume the Length of the Style at pleasure, according to the bigness of the Plane the Dial is to be drawn on; let this be C D, about the Extremity of which D describe a Quadrant,

which

which divide into ſix equal Arcs (or only deſcribe an Arc of 60 Degrees, which divide into four Parts, of 15 Degrees each, for the four firſt Hours after Noon, and then add an Arc of 15 Degrees for the Hour of 5.). This being done, draw dotted Lines from the Point D, thro' the Diviſions of the Circumference of the ſaid Arc, to the Line A B; and then if Lines are drawn thro' the Points wherein the dotted Lines cut the Line A B, parallel to the Meridian, theſe Lines will be the Hour-Lines on one ſide the Meridian: and if there be as many Parallels drawn on the other ſide the Meridian, at the ſame Diſtances therefrom as the reſpective parallel Hour-Lines are on the other ſide, theſe will be the Hour-Lines on the other ſide of the Meridian. The Style of this Dial muſt be equal in Length to C F, the Diſtance from the Hour-Line of 3 to the Hour-Line of 12, and may be made in figure of a Right-angled Parallelogram, as is that marked above the Letter K in the Figure of the Dial. This Style is ſet upon the Hour-Line of 12, which for this Reaſon is called the Subſtylar Line.

If a ſingle Rod only be uſed for a Style, as that which is in the Point C of the Meridian, then the Hour will be ſhewn upon this Dial by the Shadow of the Extremity of the Style; whereas when a Parallelogram is uſed, we have the Hour ſhewn by the Shadow of one of its Sides, that is, by a right Line.

An upper Polar Dial may ſhew the Hour from ſeven in the Morning to five in the Afternoon; and an under Polar one is uſeleſs, unleſs in the Summer, wherein the Hour is ſhewn thereby, from the Sun's riſing to five in the Morning, and from ſeven in the Evening 'till his ſetting: and ſo for the Elevation of the Pole of *Paris*, the Hours of four and five in the Morning, and ſeven and eight in the Afternoon, are only ſet down upon this Dial; and theſe may be drawn as thoſe on the upper Polar Dial, for the Diſtances of the Hour-Lines of four and five in the Afternoon from the Subſtyle, on the upper Polar Dial, are equal to the Diſtances of the Hour-Lines of four and five in the Morning from the Subſtyle on the under Polar Dial. Underſtand the ſame for the Hours of ſeven and eight in the Afternoon; and therefore there is no need of drawing the Figure of this Dial. *Note*, The Diſtance of the Hour-Lines on theſe Dials depend upon the Breadth of the Style, or the Diſtance of the Point D from the Equinoctial Line.

To ſet up this Dial at *Paris*, the Plane thereof muſt make an Angle of 49 Deg. with the Horizon, the upper one facing the Sky directly South, that ſo the Axis thereof may be parallel to that of the World, and the oppoſite Dial to this, *viz.* the under Polar one faces downwards, the Morning Hours being towards the Weſt, and the Afternoon ones towards the Eaſt, on both the upper and under ones.

Now if the Horizontal Line is to be drawn upon this Dial, deſcribe the Arc G H, about the Point F, the Extremity of the Style, equal to the Elevation of the Pole, *viz.* 49 Deg. for the Latitude of *Paris*, and draw the Right Line F H, cutting the Meridian in the Point I, thro' which draw the Horizontal Line L K, at Right Angles. Now by means of this Line, we may know whether the Dial be well placed, and have its convenient Inclination; for if the Dial be inclined rightly, a Plane laid along the Horizontal Line, and ſupported by the Edge of the Style, will be level or parallel to the Horizon.

A Polar Dial in a right Sphere is parallel to the Horizon, and in a parallel Sphere it is vertical or upright.

The Conſtruction of an Equinoctial Dial.

An upper Equinoctial Dial ſhews the Hour but only ſix Months in the Year, *viz.* from the Vernal Equinox to the Autumnal one; and the oppoſite Dial to this, which is an under Equinoctial one, ſhews the Hour during the other ſix Months of the Year, *viz.* from the Autumnal Equinox to the Vernal one. Fig. 7.

The Plane of this Dial is parallel to the Equinoctial Circle, and is cut at Right Angles through the Center thereof by the Axis of the World.

The Conſtruction of this Dial is thus: Draw two Right Lines A H, and E D, croſſing each other at Right Angles, the firſt of which ſhall be the Hour-Line of 12, and the other the Hour-Line of 6; then about the Point A of Interſection deſcribe a Circle, each quarter of which divide into ſix equal Parts, thro' which, if ſtrait Lines be drawn from the Center A, theſe Lines will be the Hour-Lines, becauſe they each make equal Angles of 15 Deg. and if each of theſe Angles be halved and quartered, the halves and quarters of Hours will be had.

The Conſtruction of an under Equinoctial Dial is the ſame as of an upper one; and in a parallel Sphere, *viz.* where the Pole is in the Zenith, there is but one Equinoctial Dial, which will likewiſe be *there* an Horizontal one. And in a right Sphere, *viz.* where the two Poles are in the Horizon, theſe Dials are non-declining Vertical ones, and are ſet up againſt Walls, one of which faces the North Pole, and the other the South Pole, the Sun ſhining upon each ſix Months in the Year. But in an oblique Sphere, as that which we inhabit, theſe Dials are inclined to the Horizon, and make an Angle therewith equal to the Complement of the Latitude, *viz.* at *Paris*, an Angle of 41 Deg.

The Axis of an Equinoctial Dial is a ſtrait Iron Rod going thro' the Center of the Dial perpendicular to the Plane thereof, and parallel to the Axis of the World. The Length of this Rod may be at pleasure, when it hath no other Use but shewing the Hour by the Shadow thereof; but when the Length of the Days, and the Sun's Place are to be shewn thereby, the said Rod must have a determinate Length, as we shall shew hereafter.

The Construction of East and West Dials.

Fig. 8. These Dials are parallel to the Plane of the Meridian; one of which directly faces the East, and the other the West. The 8th Figure is a West Dial, having the Hour-Lines parallel to each other, and to the Axis of the World, as in a Polar Dial, and their Construction is nearly the same as of the Hour-Lines on a Polar Dial.

This Dial is made thus: First draw the right Line A B, representing the Horizontal Line, and about the Point A, assume the Arc B C of a Radius at pleasure in this Line, equal to the Complement of the Latitude, or Height of the Equator above the Horizon, which at *Paris* is 41 Deg. Then draw the Line C D, produced, as is necessary, from the Point C, and this Line shall represent the common Section of the Equinoctial and Plane of the Dial; after this, draw E D from the Point D, parallel to the Equinoctial Line, and this Line E D will be the Place of the Substyle, that is, the Line on which the Style must be placed; as likewise the Hour-Line of Six. Now to draw the other Hour-Lines, assume the Point E at pleasure on the substylar Line, about which, as a Center, describe an Arc of 60 Deg. which divide into four equal Parts for 15 Deg. each, beginning from the substylar Line. After this, lay off as many Arcs of 15 Deg. as is necessary upon the said Arc both ways continued, and draw dotted Lines from the Center E thro' all the Divisions of the Arc to the Equinoctial Line: then if right Lines be drawn thro' the Points in the Equinoctial Line, made by the dotted Lines, parallel to the Hour-Line of 6, and perpendicular to the Equinoctial Line; these Lines will be the Hour-Lines. *Note*, This Dial shews the Time of Day after Noon to the Setting of the Sun; and since the Sun sets (at *Paris*) at Eight o'Clock in the Summer, we have pricked down the Hour-Lines from One to Eight in this Dial, as appears *per* Figure.

The Construction of an East Dial is the same as of this; and there are pricked down the Hour-Lines upon it from the Sun's rising in Summer, *viz.* from Four in the Morning to Eleven. The reason that the Hour-Line of Twelve cannot be drawn upon these Dials, is, because when the Sun is in the Meridian, his Rays are parallel to their Planes.

If a West Dial be drawn upon a Sheet of Paper, and then the said Paper is rendered Transparent by oiling, you will perceive thro' the backside of the Paper an East Dial drawn entirely; only the Figures of the Hours must be altered, that is, you must put 11 in the Place of 1; 10 in the Place of 2; and so of others.

The Style of these Dials is a Brass or Iron Rod, in Length equal to E D, which is likewise equal to the Distance of the Hour of 3 from the Hour of 6. This Style is set upright in the Point D, and shews the Hour by the Shadow of it's Extremity. These Dials, which may have likewise a Style in figure of a Parallelogram, as we have mentioned in speaking of Polar Dials, are set upright against Walls or Planes, perpendicular to the Horizon, and parallel to the Meridian, one of which directly faces the East, and the other the West, in such manner, that the Horizontal Line be perfectly level.

The Construction of Vertical Declining Dials.

A Vertical Dial is one that is made upon a Vertical Plane, that is, a Plane perpendicular to the Horizon, as a very upright Wall.

Among the nine Regular Dials of which we have spoken, there are four of them Vertical ones, which do not decline at all, since they directly face the four Cardinal Parts of the World. It now remains that we here speak of Irregular Dials, some of which are vertical Decliners, others undeclining Decliners, and finally, others declining Incliners. Vertical Decliners are of four Kinds: for some decline South-eastwardly, the opposite ones to these, North-westwardly, others decline South-westwardly, and the opposite ones to these, North-eastwardly.

Now among the Irregular Dials, the vertical Decliners are most in use, because they are made upon or set up against Walls (which commonly are built upright), or else upon Bodies whose Planes are upright; but before these Dials can be made, the Declinations of the Walls or Planes, on which they are to be made or set up against, must first be known or found exactly: and this may be done by some one of the Methods hereafter mentioned.

Fig. 9. Now suppose we know that a Plane (as that marked I of Figure 1.) or upright Wall, declines 45 Deg. South-westwardly at *Paris*, or thereabouts, where the Pole is elevated 49 Deg. above the Horizon. It is required to draw a Dial for this Declination.

First, draw the Lines A B, C D, crossing each other at Right Angles in the Point E, the former of which shall be the Hour-Line of 12, and the other the Horizontal Line. About the Point E, as a Center, draw the Arc F N of 45 Deg. because the Plane's Declination is such, and since it is South-westwardly, the said Arc must be drawn on the Right-side of the Meridian; but if the Declination had been South-eastwardly, that Arc must have been drawn

drawn on the Left-fide the Meridian. This being done, raife the Perpendicular F H from the Point F to the horizontal Line, that fo we may have one Point of the Style therein, *viz.* the Foot of the Style. Then take the Diftance E F between your Compaffes, and lay it off upon the horizontal Line from E to O, and about the Point O, as a Center, defcribe the Arc E G equal to the Height of the Pole, *viz.* in this Cafe 49 Deg. and draw the dotted Line O A to the Hour-Line of 12; then A will be the Center of the Dial thro' which the Subftyle A H muft be drawn of an indeterminate Length. *Note*, This Subftyle is one of the principal Lines, by means of which a Dial of this kind is drawn, and upon which the whole Exactnefs thereof almoft depends.

Upon the Point H raife the right Line H I equal to H F, perpendicular to the Subftyle A H, and draw the right Line A I, prolonged, for the Axis of the Dial. Then let fall the Perpendicular K I to the Axis, cutting the fubftylar Line in K, and make K L equal to K I, and draw a right Line both ways thro' the Point K, perpendicular to the Subftyle A K; this will reprefent the Equinoctial Line, and cuts the Horizontal Line in a Point thro' which the Hour-Line of 6 muft pafs. Thus having already the Hour-Lines of 12 and 6, if the Operations hitherto performed have been done right, two dotted Lines L 6, and L N being drawn, will be at Right Angles to each other. Again, About the faid Point L, as a Center, defcribe the Quadrant of a Circle between the faid dotted Lines, whofe Circumference divide into 6 equal Arcs, of 15 Degrees each, and draw occult Lines thro' the Points of Divifion to cut the Equinoctial Line; but to have the Morning Hour-Lines, and thofe after 6, prolong the Arc of the Quadrant both ways, and lay off as many Arcs of 15 Degrees upon it, as is neceffary, that fo occult Lines may be drawn from the Center L to cut the Equinoctial Line. Then if Lines are drawn from the Center A thro' all the Points wherein the occult Lines cut the Equinoctial Line, thefe Lines thus drawn will be the Hour-Lines. *Note*, There muft be but 12 Hour-Lines drawn upon any vertical declining Plane, for the Sun will fhine on any one of them but 12 Hours.

Points in the horizontal Line D C, thro' which the Hour-Lines muft pafs, may be found otherwife, by applying the Center of a horizontal Dial to the Point F, in fuch manner, that the Meridian Line thereof coincides with the Line F E, and it's Hour-Line of 6, with the Line F 6: for then the Points where the Hour-Lines of the horizontal Dial cut the faid Line D C, will be the Points therein thro' which the Hour-Lines muft be drawn from the Center A.

The Hour-Lines of fix Hours fucceffively being given upon the Plane of any Dial whatfoever, the other Hour-Lines may be drawn after the following Manner: Suppofe, in this Example, that the Hour-Lines from 6 to 12 are drawn; now if you have a mind to draw the Hour-Lines of 9, 10 and 11 in the Morning, which may be pricked down upon this Dial, draw a Parallel, as S V, from the Point V, taken at pleafure on the Hour-Line of 12, to the Hour-Line of 6, which fhall cut the Hour-Lines of 1, 2, and 3, in the Afternoon. This being done, the Diftance from V to the Hour-Line of 1 taken on this Parallel, and laid off on the other Side, will give a Point in the faid Parallel thro' which the Hour-Line of 11 muft be drawn; likewife the Diftance V 2 will give a Point thereon, thro' which the Hour Line of 10 muft be drawn; and the Diftance V 3 will give a Point thro' which the Hour-Line of 9 muft pafs. And fo if Lines are drawn from the Center of the Dial A thro' the faid Points, they will be the Hour-Lines.

In this manner likewife may be found the Points thro' which the Hour-Lines of 7 and 8 in the Evening are drawn, in firft drawing a Parallel to the Hour-Line of 12, cutting the Hour-Line of 6 in one Point, and meeting the Hour-Lines of 4 and 5 produced; for the Diftance from the Points where the Hour-Lines of 6 and 5 are cut by this Parallel, laid off on the other Side from the Point where the Hour-Line of 6 cuts the Parallel, will give a Point upon it thro' which the Hour-Line of 7 muft be drawn. And the Diftance from the Points where the Parallel cuts the Hour-Lines of 6 and 8, laid off on the other Side on that Parallel, will give a Point therein thro' which the Hour-Line of 8 muft pafs; and if Lines are drawn from the Center A thro' thofe two Points found, they will be the Hour-Lines of 7 and 8 in the Evening. This is a very good way of drawing thofe Hour-Lines that are pretty diftant from the fubftylar Line, becaufe thereby we avoid cutting the Equinoctial very obliquely.

The Conftruction of a South-Eaft vertical Decliner is the fame as of that which we have defcribed, excepting only that what was there made on the Right muft here be on the Left, and the Figures for the Morning Hours fet to thofe for the Afternoon: fo that if a South-Weft declining Dial be drawn upon a Sheet of Paper, and afterwards the Paper be oiled, that you may fee thro' it, you will fee a South-Eaft Decliner thro' the Paper; only the Figures fet to the Hour-Lines muft be altered; as, where the Figure of 1 ftands, you muft fet 11; where the Figure of 2, 10; where the Figure of 3, 9; and fo on. By this means the fubftylar Line, which falls between the Hour-Lines of 3 and 4 Afternoon, in Figure 9, will fall in this Dial between 8 and 9 in the Morning. And if the Plane's Declination had been lefs than 45 Deg. the Subftyle would have fallen yet nearer to the Meridian: but if, on the contrary, the Declination thereof had been greater, the Subftyle would have fallen more diftant from the Meridian, and pretty near the Hour-Line of 6. But when this happens, the Hour-Lines fall so close together near the Substyle, that we are obliged to make the Model of a Dial upon a very large Plane, that so the Hour-Lines may be very long, and the part of the Dial towards the Center taken away.

After the abovenamed manner, likewise may be drawn North-East and North-West Dials; but these have their Centers downwards underneath the Horizontal Line, and properly are no other but South-East or South-West Decliners inverted, as may be seen in Figure 10, which represents a North-West Decliner of 45 Deg. drawn for the Plane L of Figure 1. and the substylar Line of this Dial must be between the Hours of 8 and 9 in the Evening, whence one Decliner only may serve for drawing four, if they have an equal Declination, tho' to different Coasts; two of which will have their Centers upwards, and the other two their Centers downwards.

To draw the Substylar Line upon a Plane by means of the Shadow of the Extremity of an Iron-Rod, observed twice the same Day.

Suppose the Substylar Line is to be found on the Decliner of Figure 9, first place obliquely upon the Dial-Plane, a Wire or Iron Rod, sharp at the end, so that the Extremity thereof be perpendicularly over the Point H in the Plane. This may be done by means of a Square.

Fig 9. Now since this Figure is a South-West vertical Decliner, therefore the Substylar Line thereon must be found among the Afternoon Hours, to the Right-hand of the Meridian; and consequently, let us suppose the Shadow of the Extremity of the Iron-Rod at the first Observation to fall on the Point P; then about the Point H, the Foot of the Style, with the Distance HP, describe the circular Arc PR. This being done, some Hours after the first Observation the same Day, observe when the Shadow of the Extremity of the Rod falls a second time upon the aforesaid Arc, which suppose in the Point Q: then if the Arc PQ be bisected in the Point R, and a Right-line be drawn thro' the Points R and H; this Line will be the Substyle, which being exactly drawn, and the Height of the Pole above the Horizon of the Place where the Dial is made for, being otherwise known, it will not then be difficult to compleat the Dial; for first, the Meridian or Hour-Line of 12 is always perpendicular to the Horizon, in vertical Planes, and the Point wherein the Meridian and Substylar Line produced meet each other, (as the Point A) will be the Center of the Dial. The Horizontal Line is a level Line passing thro' the Foot of the Style, as DHC.

And to draw the Equinoctial Line, you must first form the Triangular Style AHI on the Substyle, whose Hypothenuse AI is the Axis, and Side HI the right Style; then if IK be drawn from the Point I perpendicular to the Axis, meeting the Substylar Line in the Point K; and if thro' K a Right Line MKN be drawn at Right Angles to the Stylar Line, this Line will be the Equinoctial, and the Point wherein it cuts the Horizontal Line will be always the Point thro' which the Hour-Line of 6 must pass. Moreover, the Distance KL, laid off on the Stylar Line, will give the Point L the Center of the Equinoctial Circle. Now what remains to be done, may be compleated as before explained; and even the whole Dial may be drawn in one's Room, after the Positions and Concourses of the principal Lines are laid off upon a Sheet of Paper, and the Angle which the Substylar Line makes with the Meridian or Horizontal Line be taken; for one is the Complement of the other.

Now to prove whether the Equinoctial Line be drawn right, make the Angle BAO equal to the Complement of the Elevation of the Pole, *viz.* 41 Deg. for the Latitude of *Paris*, draw the Line AO to the Horizon, and make the Angle AON a Right one, that so the Point N may be had in the Meridian or Hour-Line of 12, thro' which the Equinoctial Line must pass. Thus having several Ways for finding the principal Points, one of them will serve to prove the other.

When a Dial Plane declines South-eastwardly, the Substylar Line will be on the right Side of the Meridian. In which Case it is proper to take notice, that in finding the Substylar Line, as above, to observe when the Shadow of the Extremity of the Rod falls upon the Plane, as soon as the Sun begins to shine thereon; as likewise to mind the Time very exactly when the Shadow of the Extremity of the Style comes again to touch the circular Arc; you may operate in this manner several Days successively, in order to see whether the Position of the Substylar Line has been found exactly.

When a Plane declines North-East or North-West, the Shadows of the Extremity of the Iron Rod fall above the Foot of the Style, and so the Center of the Dial must be downwards. Likewise the most proper Time for making these Operations is about 15 Days before or after the Solstices, for when the Sun is near the Equinoctial, his Declination is too sensible, and the Operations less exact. Nevertheless the Equinoctial Line may be drawn upon a Plane, when the Sun is in the Equinoctial Points, and by that means a vertical declining Dial constructed, by the following Method.

To draw the Equinoctial Line upon a vertical Plane by means of the Shadow of the Extremity of an Iron-Rod.

The most simple and easy Method to draw the Equinoctial Line upon a Wall or Plane, is at the Time when the Sun is in the Equinoctial, (though this may be done at any other Time

Time by more complicated Methods) for when the Sun defcribes the Equinoctial by his diurnal Motion, the Shadows of the Extremity of the Iron-Rod or Style, will all fall upon a Plane in a right Line, which is the common Section of the Equinoctial Circle of the Heavens and the Plane. Therefore if feveral Points, pricked down upon a Plane, made by the Shadow of the Extremity of the Rod, on the Day the Sun is in the Equator, be joined, the right Line joining them will be the Equinoctial Line, as the Line M N, in Figure 9. This being done, draw the right Line A H L thro' the Foot of the Style at Right Angles to the Equinoctial Line, and this will be the Subftylar Line: Moreover, draw the level Line D H C thro' the Foot H of the Style; this will be the Horizontal Line; and if H I be drawn equal to the Height of the right Style, and parallel to the Equinoctial Line and the Points K and L joined; and if A I be drawn at Right Angles to K I, then the Point A will be the Center of the Dial, and the upright Line A B the Meridian or Hour-Line of 12. The common Section of the Equinoctial and Horizontal Lines, will likewife be the Point thro' which the Hour-Line of 6 muft pafs, and confequently wherewith the Dial may be finifhed. *Note,* The Angle H F E will be the Plane's Declination.

To draw a Dial upon a Vertical Plane by means of the Shadow of the Extremity of an Iron-Rod or Style obferved upon the Plane at Noon.

A Style, as H I (*Vide* Figure 9.), being fet up on a Wall or Dial Plane, whofe Foot is H, and Extremity I; and if you know by any means when it is Noon, which may be known by a Meridian Line drawn upon a Horizontal Plane, as we fhall mention hereafter, note where the Extremity of the Shadow of the Style H I falls upon the Plane at Noon, which fuppofe in the Point N, and thro' this Point draw the Perpendicular A N B, which confequently will be the Meridian of the Place or Hour-Line of 12; then draw the level Line C H D, cutting the Meridian at Right Angles in the Point E; this will be the Horizontal Line. Again, Draw H F equal in Length to the right Style H I, and parallel to the Meridian; then take the Hypothenufe E F between your Compaffes, and lay it off upon the Horizontal Line from E to O, and make the Angle E O A equal to the Elevation of the Pole, *viz.* 49 Deg. and then the Point A will be the Center of the Dial.

Likewife make the Angle E O N, underneath the Horizontal Line, equal to the Complement of the Elevation of the Pole, *viz.* 41 Deg. and the Point N on the Meridian Line will be that thro' which the Equinoctial Line muft pafs. Then if the right Line A H K be drawn thro' the Center A, and the Foot of the Style H, this will be the Subftylar Line; and if a Perpendicular be drawn thro' the Point N to this Line, the faid Perpendicular will be the Equinoctial Line. Thus having found the principal Lines of the Dial, you may compleat it by the Methods before explained.

This Method of drawing a Dial at any Time of the Year, by means of the Shadow of the Extremity of the Style H I obferved at Noon, may ferve, when it is not poffible to find the Subftylar Line by the Obfervations of the Shadows of the Extremity of an Iron-Rod or Style, which happens when Planes decline confiderably Eaftwards or Weftwards.

There are feveral other Methods of drawing Vertical Dials on Walls or Planes: but thofe would take up too much time to mention in this fmall Treatife, wherein we have only laid down the moft fimple and eafy Methods of drawing Vertical Dials. And in order to draw Dials more exactly, we fhall hereafter lay down Rules for calculating the Angles the Hour-Lines make at the Centers; and fo the other Methods may be verified by thefe Rules.

The Conftruction of Non-declining inclining Dials.

The Inclinations of thefe Dials are the Angles that their Planes make with the Horizon, and fome of them face the Heavens, and others the Earth. There are likewife two Kinds of them with regard to the Pole; and two other Kinds with regard to the Equinoctial.

If a Plane facing the South hath an Inclination towards the North, this Inclination may be lefs or greater than the Elevation of the Pole; for if the Inclination be equal to the Elevation of the Pole, this Dial-Plane will be an upper or under Polar one, whofe Conftruction we have already laid down. Fig. 11, 12.

If the Inclination be lefs than the Elevation of the Pole, which at *Paris* is nearly 49 Deg. and you would make a Dial upon a Plane facing the South, having 30 Deg. of Inclination towards the North, fubftract 30 Deg. from 49 Deg. and the Remainder 19 Deg. will be the Height of the Axis or Style above the Plane. Then if a Horizontal Dial be made upon this Plane for the Latitude of 19 Deg. in the manner we have already laid down, we fhall have an Incliner of 30 Deg. drawn, becaufe the faid Plane thus inclined is parallel to the Horizon of thofe Places where the Pole is elevated 19 Deg. and confequently this muft be a Horizontal Dial for thofe Places. The Center of this Dial is downwards, underneath the Equinoctial Line, and the Morning Hour-Lines on the Left, and the Afternoon ones on the Right-hand of thofe looking at them.

The under oppofite Dial to this, which faces towards the North, is the fame as the upper one facing towards the South, excepting only that the Center is upwards above the Equinoctial

Equinoctial Line, and the Morning Hour-Lines on the Right, and the Afternoon ones on the Left-hand.

If the Inclination of the Plane be greater than the Elevation of the Pole, suppose at *Paris*, and it be 63 Deg. substract the Elevation of the Pole 49 Deg. from 63 Deg. and the Remainder will be 14 Deg. and then make an Horizontal Dial for this Elevation of 14 Deg. and you will have an Incliner of 63 Deg. the Center of the upper Plane facing towards the South, is upwards above the Equinoctial Line, the Morning Hour-Lines on the Left-hand, those of the Afternoon towards the Right; and in the opposite under Plane facing towards the North, the Center is downwards, the Morning Hours on the Right, and those of the Afternoon on the Left, as may be seen in Figure 11 and 12.

If the Plane faces the North, and inclines Southwards, the Inclination thereof may be less or greater than that of the Equinoctial; for if it be equal, we need only make an upper or under Equinoctial Dial thereon, which is a Circle divided into 24 equal Parts, as is above directed in speaking of Regular Dials.

If the Inclination be less than the Elevation of the Equinoctial, as, suppose a Plane at *Paris* inclines 30 Deg. Southwardly, add the 30 Deg. of Inclination to 49 Deg. the Height of the Pole, and make an Horizontal Dial for the Elevation of 79 Deg. and your Dial will be drawn: the Center of the upper Dial facing Northwardly, will be upwards, the Morning Hour-Lines on the Right-hand, the Afternoon ones on the Left; and on the opposite under Dial to this, the Center will be downwards, the Morning Hour-Lines on the Left, and the Afternoon ones on the Right-hand.

Finally, If the Inclination, which suppose 60 Deg. be greater than the Height of the Equinoctial, add the Complement of the Inclination, which is 30 Deg. to the Elevation of the Equinoctial, which is 41 Deg. at *Paris*, and the Sum is 71 Deg. and make an Horizontal Dial for this Elevation of the Pole. The Center of the upper one of these Dials is downwards, the Morning Hour-Lines on the Right-hand, and the Center of the opposite under Dial is upwards, and the Morning Hour-Lines on the Left-hand.

Note, The Meridian or Hour-Line of 12, is the Substylar Line of all Non-declining inclining Dials, passes thro' their Centers at right Angles to the Hour-Lines of 6, and may be drawn by means of the Shadow of a Plumb-Line passing thro' their Centers.

There ought to have been eight Figures to represent all these different Dials, *viz.* four for the upper ones, and four for the under ones; but since they are not difficult to be conceived or drawn, we have only represented two of them, with respect to the Dodecahedron on which we place them.

The Construction of Declining inclining Dials.

The Declination of a Dial is the Angle that the Plane thereof makes with the Prime Vertical; and it's Inclination is the Angle made by the Plane thereof with the Horizon: both of which we shall shew how to find hereafter.

Now suppose, for Example, that a Dial is to be drawn upon a Plane declining 36 Deg. South-eastwardly, and inclining 63 Deg. 26 Min. towards the Earth, as does the Plane C on the Dodecahedron of Figure 2.

But before we shew how to draw this Dial, you must first observe that the Horizontal Line, which passes thro' the Foot of the Style in Vertical Dials, must in no wise pass thro' it in inclining Dials; for in upper Incliners facing the Heavens, this Line must be drawn above the Foot of the Style, and in under Incliners, facing the Earth, below the Foot of the Style. Secondly, The Meridian or Hour-Line of 12, in inclining Dials, does not cut the Horizontal Line at right Angles, as it does in Vertical Dials, but must be drawn thro' two Points; one of which is found upon the Horizontal Line by means of the Angle of Declination, and the other upon a Vertical Line cutting the Horizontal one at right Angles.

This last Point in upper Incliners is called the Zenith Point, because if the Sun was in the Zenith of the Place for which the Dial is made, the Extremity of the Shadow of the Style would fall upon that Point, which consequently will be underneath the Style of these Dials. And in under Incliners, the said Point is called the Nadir Point, because if the Sun was in the Nadir, and the Earth transparent, the Extremity of the Shadow of the Style would touch that Point, which consequently will be above the Style, as in the proposed Dial.

Thirdly, The Center of the proposed under Dial which declines South-eastwardly must be upwards, the Substylar Line to the Left-hand of the Vertical Line, and the Meridian among the Morning Hour-Lines, and so on the Right of the Vertical Line. The Centers of upper Dials declining South-westwardly must be likewise upwards, the Substylar Line on the Right-hand of the Vertical one, and the Meridian among the Afternoon Hour-Lines; and the opposite upper Dials to these, have their Centers downwards, and are no other but these Dials inverted: and therefore one of these four Dials is enough to be drawn.

Fig. 13. In order for this, let it be required to draw a Dial upon a Plane of the abovesaid Declination and Inclination. First, Draw the two Lines A B, C D, cutting each other at right Angles in the Point E; then let C D be parallel to the Horizon, and upon it assume E F at

at pleafure, for the Length of the right Style, whofe Foot fhall be E, and Extremity F, and about the Center F defcribe the Arc G H, equal to the Plane's Inclination, *viz.* 63 Deg. 26 Min. and draw the right Line A F; likewife make the Angle G F I equal to the Complement of 63 Deg. 26 Min. *viz.* 26 Deg. 34 Min. This being done, the Point A will be the Nadir, and one Point of the Meridian Line, and if a right Line M L N be drawn thro' the Point L, parallel to C D, this will be the horizontal Line; and if the Diftance L F be taken between your Compaffes, and laid off from L to O, the Point O will be the Center thro' which Lines may be drawn dividing the horizontal Line. Again, About the Point O defcribe the Arc L P of 36 Deg. *viz.* the Plane's Declination, and draw the Line O P cutting the horizontal Line M L N in the Point 12; then if a right Line be drawn thro' the Nadir A and this Point 12, the faid Line A 12 will be the Meridian of the Dial or Hour-Line of 12: and moreover, if an Angle be made at the Point O on the Left-fide of the Line A B, equal to the Complement of the Plane's Declination, which here is 54 Deg. you will have a Point on the horizontal Line thro' which the Hour-Line of 6, as likewife the Equinoctial Line, muft pafs.

The next thing to be found is another Point, befides E the Foot of the Style, thro' which the fubftylar Line muft pafs; and in order for this, we need only find the Center of the Dial, after the following manner.

Draw the Line M R from the Point M, (thro' which the Hour-Line of 6 paffes) at right Angles to the Meridian A 12, lay off the Diftance O 12, from 12 to R, or elfe the Diftance A F from A to R, draw the occult Line 12 R, and about the Point R defcribe the Arc N K, of 49 Deg. *viz.* the Elevation of the Pole; then if R K be drawn cutting the Meridian in the Point K, this will be the Center of the Dial. After this, the Subftylar Line K E may be drawn; and if the Perpendicular M Q be drawn to this Line thro' the Point M, the faid M Q will be the Equinoctial Line. Moreover, the Point in the Meridian Line thro' which the Equinoctial Line muft pafs, may be found by making the Angle N R Q of 41 Deg. that is, the Complement of the Elevation of the Pole.

The Pofitions of the principal Lines being thus found, it will not now be difficult to find the Points on the horizontal or equinoctial Lines, thro' which the Hour-Lines muft be drawn; for if the Points are to be found upon the horizontal Line you muft apply the Center of a horizontal Dial to the Point O, in fuch manner, that the Hour-Line of 12 anfwers to the Line O 12, and the Hour-Line of 6 to the Line O 6: then the Points in the horizontal Line M N, thro' which the other Hour-Lines muft be drawn, may be determined eafily. And if the Points thro' which the Hour-Lines muft pafs on the equinoctial Line be to be found, you muft raife the Perpendicular E S on the Subftyle equal to E F, and draw the Axis S K; and afterwards take the Diftance T S between your Compaffes, and lay off on the Subftyle from T to V, then V will be the Center of the Equinoctial Circle, by means of which the Equinoctial Line may be divided, as we have directed in fpeaking of declining Dials, and the Hour-Lines drawn thro' the Center of the Dial K. Your Dial being thus made, you may draw a fair Draught thereof, wherein are only the principal Lines, and the Hour-Lines, as may be feen in the Pentagonal Figure marked 14.

By means of this Dial three others of the fame Declination and Inclination may be made. The two under ones declinining South-eaftwardly and South-weftwardly, have their Centers upwards; and the two upper ones, which decline North-eaftwardly and North-weftwardly, their Centers downwards, and are only the two former Dials inverted, as we have already mentioned.

The Dial of the Figure 15, reprefents that marked F in Figure 2, and is an upper Incliner of 63 Deg. 26 Min. declining South-eaftwardly 72 Deg. and may be drawn by the abovefaid Method. The Center of this Dial is upwards, and becaufe it has a great Declination, the Hour-Lines will fall very clofe to one another near the Subftylar Line; and therefore it ought to be drawn upon a large Plane, that fo the Part thereof next to the Center may be taken away, and the Style and the Hour-Lines terminated by two Parallels.

There is another way of drawing Mechanically any forts of Dials whatfoever, upon Polyhedrons or Bodies of different Faces or Superficies, without even knowing the Declinations or Inclinations of the Faces or Superficies, and that with as much exactnefs as by any other Methods whatfoever. In order to do this, you muft firft make an horizontal Dial upon one of the Planes or Faces that is to be fet parallel to the Horizon, and fet up the Style thereof upon the Hour-Line of 12, conformable to the Latitude of the Place. After this, the Subftylar Lines muft be drawn upon all the Planes or Faces of the Polyhedron that the Sun can fhine upon, that fo Brafs or Iron Styles, proportioned to the bigneffes of the Planes, or Faces, may be fixed upon them perpendicularly in fuch manner, that the Axes or upper Edges of the faid Styles be parallel to the Axis of the horizontal Dial. This may be done in filing them away in right Lines by Degrees, until their Axes, being compared with the Axis of a large Style fimilar to that of the horizontal Dial placed level, (or held up fo that its Bafe be parallel to the Horizon, by means of a Thread and Plummet hung to the Top of the Style) appear in a right Line with the Axis of the faid Style.

Things being thus ordered, fet your Polyhedron in the Sun, and turn it about, making the Shadow of the Axis of the horizontal Dial fall upon each Hour-Line thereof fucceffively,

 and

and if at each of the respective Times right Lines be drawn along the Shadows of the Axes of the Styles of the other Faces of the Body upon the said Faces, these will be the same Hour-Lines upon each of the Faces of the Body, that the Shadow of the Style of the Horizontal Dial fell upon, on the Horizontal Dial. For example; Suppose the Shadow of the Axis of the Horizontal Dial falls upon the Hour Line of 12; then at the same time draw Lines along the Shadows of the Styles upon the other Faces of the Body, and those Lines will be the Hour-Lines of 12 upon the said Faces: understand the same for others. This may be done likewise in the Night, by the Light of a Link moved about the Polyhedron.

There are great Stone Bodies cut into several Faces placed sometimes in Gardens having Dials drawn upon them, according to the abovesaid Method, And the Edges of the Stone which serve for Axes to some of these Dials, must be cut so as to be parallel to the Axis of the World.

The Arithmetical Construction of Dials by the Calculation of Angles.

This Method is a great help for verifying any Operations in Dialling, wherein there is great Exactness required, and chiefly when we are obliged to make a small Model for drawing a large Dial: for an Error almost insensible in the Model, will become very considerable in the long Hour-Lines to be drawn upon a large Plane.

In the Construction of Regular Dials, as of the Horizontal one of Figure 4, the Divisions of the Equinoctial Line LK, are the Tangents of the Angles of the Quadrant MH, and the dotted Lines are their Secants; and therefore they may be pricked down by means of a Scale or Sector, in supposing the Radius HB 100: for then the Tangent H 1 of 15 Deg. will be twenty-seven of the said Parts; H 2, the Tangent of 30 Deg. will be 58; H 3, the Tangent of 45 Deg. (equal to Radius) will be 100; H 4, the Tangent of 60 Deg. will be 173; and H 5, the Tangent of 75 Deg. will be 373 Parts. The Divisions on the other half of this Line for the Morning Hour-Lines are the same.

The Divisions for the halves and quarters of Hours may be found likewise upon the Equinoctial Line, by assuming the Tangents of the correspondent Arcs, which may be taken from printed Tables of natural Tangents, but from the Table of Secants we can deduce some Abreviations. For example, the Line B 4, which is the Secant of 60 Deg. being double to Radius, if twice BH be laid off from B 4, you will have the Point on the Equinoctial Line thro' which the Hour-Line of 4 must be drawn. The said Secant laid off from 4 to L, will give likewise the Point in the Equinoctial Line thro' which the Hour-Line of 5 must be drawn, *&c.*

The Points thro' which the half Hours must pass, may be found by means of the Secants of the odd Hours. For example, the Secant B 3, laid off at the Point 3 on the Equinoctial Line, will fall on one side upon the Point for half an Hour past 4, and on the other side, for half an Hour past 10; the Secant B 9, will give half an Hour past 7, and half an Hour past 1; B 11, will give half an Hour past 8, and half an Hour past 2; B 1, will give half an Hour past 3, and half an Hour past 9; B 7, will give half an Hour past 6, and half an Hour past 12; and lastly, B 5 will give half an Hour past 11, and half an Hour past 5.

The Division of the Equinoctial Line serves to make the Horizontal and Vertical Dials exactly, but chiefly the undeclining Regular Dials, *viz.* the Polar East and West ones: for there need nothing be added to the facility of constructing Equinoctial Dials, because the Angles that the Hour-Lines make at the Center of the Dials are all equal between themselves.

The Angles that the Hour-Lines of a horizontal Dial make with the Meridian in the Center of the Dial, may be found in the following manner by Trigonometry. As Radius is to the Sine of the Elevation of the Pole, So is the Tangent of the Distance of any Hour-Circle from the Meridian, to the Tangent of the Angle that the Hour-Line of that Hour makes with the Meridian or Hour-Line of 12, on the Horizontal Dial. For example; Suppose the Angle that the Hour-Lines of 1 and 11, make with the Meridian on a horizontal Dial for the Latitude of 49 Deg. be required: form a Rule of Proportion whose first Term let be the Radius 100000; the second, the Sine of 49 Deg. which is 75471; and the third, the Tangent of 15 Deg. (*viz.* the Tangent of the Distance of the Hour-Circles of 11 and 1 from the Meridian) which is 26597. Now having found the fourth Term 20222, seek it in the Tables of Tangents, and you will find 11 Deg. 26 Min. stand against it: therefore the Angle that the Hour-Lines of 1 or 11 make with the Meridian, is 11 Deg. 26 Min.

Thus may be found the Angles that all the Hour-Lines, and half Hour-Lines, *&c.* make with the Meridian in the Center of a horizontal Dial, *viz.* by as many Rules of Proportion, as there are Hour-Lines and half Hour-Lines, *&c.* to be drawn, whose two first Terms are standing, to wit, the Radius, and the Sine of the Elevation of the Pole; and so you have but the third Term to seek in the Tables; that is, the Tangent of the Hour-Circle's distance from the Meridian, in order to find the 4th Term. You may take the Logarithms of those Terms if you have a mind to it, which will save the trouble of multiplying and dividing.

The

The aforefaid Analogy may ferve likewife for vertical Dials, if the Sine Complement of the Elevation of the Pole, which is 41 Deg. about *Paris*, be made ufe of for the fecond Term; becaufe any vertical Dial at *Paris* may be confidered as an Horizontal one for the Latitude of 41 Deg.

Moreover, the aforefaid Analogy holds for undeclining Inclining-Dials, if the Sine of the Angle made by the Axis and Meridian-Line at the Center of the Dial be ufed for the fecond Term of the Analogy. For Example, Becaufe the Dial B on the Dodecahedron of Figure 2, inclines 63 Deg. 26 Min. you muft fubftract the Elevation of the Pole, which is 49 Deg. from 63 Deg. 26 Min. and then if you make an horizontal Dial for the Latitude of 14 Deg. 26 Min. in taking 14 Deg. 26 Min. for the fecond Term of the Analogy, you may calculate the Angles that all the Hour-Lines make with the Meridian or Hour-Line of 12.

A TABLE *of the Angles that the Hour-Lines make with the Meridian at the Center of an horizontal Dial.*

Latitude	*Hours.* I. *and* XI.		II. *and* X.		III. *and* IX.		IV. *and* VIII.		V. *and* VII.		VI. *and* VI.	
41 Deg.	9 D.	58 M.	20	45	33	16	48	39	67	47	90	00
49 Deg.	11	26	23	33	37	3	52	35	70	27	90	00

To draw the principal Lines upon a vertical Decliner by Trigonometrical Calculation.

This manner of Calculation confifts in the five following Rules.

The Declination of a Plane being given, to find the Angle that the fubftylar Line makes with the Meridian.

Rule I. As Radius is to the Sine of the Plane's Declination, So is the Tangent Complement of the Latitude, to the Tangent of the Angle made by the fubftylar Line and Meridian in the Center of a vertical Decliner. And the Angle that the fubftylar Line makes with the Horizon at the Foot of the right Style, is the Complement of this Angle. Alfo the Angle that the Equinoctial Line makes with the Horizon at the Point wherein the Hour-Line of 6 cuts it, is equal to the Angle made by the fubftylar Line and Meridian; and the Angle of the Equinoctial Line and Meridian is it's Complement.

Rule II. To find the Angle which the Axis of the Dial makes with the fubftylar Line, which may be called likewife the Height of the Pole above the vertical Plane; fay,

As Radius is to the Sine Complement of the Latitude, So is the Sine Complement of the Plane's Declination to the Sine of the Angle required. *Note*, The Angle that the Axis makes with the right Style, is the Complement of this Angle; and the Angle that the Radius of the Equinoctial Circle makes with the right Style, is equal to the Angle that the Axis makes with the Subftyle. Alfo the Angle made by the Radius of the Equinoctial Circle and the Subftyle, is the Complement thereof.

Rule III. To find the Arc of the Equinoctial or Angle between the fubftylar Line and the Meridian in declining Dials; that is, the Difference between the Meridian of the Place, and the Meridian of the Plane, for the fubftylar Line is the Meridian of the Plane; fay,

As Radius is to the Sine of the Latitude, So is the Tangent Complement of the Plane's Declination to the Tangent of an Arc, whofe Complement will be that required.

Rule IV. To find the Angle that the Hour-Line of 6 makes with the horizontal Line, and the Meridian in the Center of the Dial; fay,

As Radius is to the Sine of the Plane's Declination, So is the Tangent of the Latitude, to the Tangent of the Angle that the Hour-Line of 6 makes with the Horizon; the Complement of which, is that made by the Hour-Line of 6 and the Meridian.

Rule V. To find the Angles that the Hour-Lines make with the fubftylar Line; and by this means, the Angles that they make with the Meridian in the Center of a vertical Dial.

This Propofition is founded upon this Gnomonick Principle, *viz.* that any Plane may be parallel to fome Horizon, and confequently will be an horizontal Dial for that Latitude, the fubftylar Line being the Meridian, from which the proper Hour-Lines muft be laid off on both Sides.

But before this can be done, the Angle that the Subftyle makes with the Meridian muft be found, by *Rule* I. the Elevation of the Pole above the Plane, by *Rule* II. the Arc of the Equinoctial between the Subftyle and the Meridian, by *Rule* III. with the Difference or Degrees of the two firft Diftances from the Style; one being between the Subftyle and the Meridian, and the other between the Subftyle and the Hour-Line of 6. Thefe being found, fay,

As Radius is to the Sine of the Elevation of the Pole above the Plane, So is the Tangent of the Diftance of any Hour-Circle from the Meridian of the Plane or Subftylar Line to the Tangent of the Angle made by the Hour-Line of the propofed Hour-Circle and the fubftylar Line in the Center of the Dial.

Note,

Note, If the fubftylar Line happens to fall upon any half or whole Hour, then the two firft Diftances of the Hour-Circles from the fubftylar Line will be each 7 Deg. 30 Min. or 15 Deg. and in this Cafe, the Angles of the Hour-Lines of the Hour-Circles, equally diftant on both fides the Hour the fubftylar Line falls upon, will be equal on both fides the fubftylar Line.

The Application of the precedent Rules to a vertical Decliner of 45 *Deg. South-weftwardly, in the Latitude of* 49 *Deg.* (*Vide* Figure 9.)

The Angle made by the fubftylar Line and the Meridian, will be found by the firft Rule 31 Deg. 35 Min.

The Angle of the Axis and fubftylar Line, by *Rule* II. will be 27 Deg. 38 Min. and the Arc of the Equinoctial between the Meridian of the Place and the Meridian of the Plane, by *Rule* III. will be found 52 Deg. 58 Min. and confequently the fubftylar Line falls between the Hour-Lines of 3 and 4 in the Afternoon; and the Angle made by the Hour-Line of 6 and the Meridian, is 50 Deg. 52 Min.

The Arc of the Equinoctial 52 Deg. 58 Min. being found, fubftract 45 Deg. which is the Arc of the Equinoctial anfwering to the Hour of 3, from it, and the Remainder 7 Deg. 58 Min. will be the Arc of the Diftance of the Hour of 3 from the Subftyle, and confequently 7 Deg. 2 Min. is the Diftance of the Hour of 4 from the Subftyle.

Therefore to find the Angles that the Hour-Lines make with the Subftyle in the Center of the Dial, you muft begin with one of thefe Diftances, in faying, for Example, As Radius 100000 is to the Sine of the Elevation of the Pole above the declining Plane, which in this Example is 27 Deg. 38 Min. whofe Sine is 46381, So is the Tangent of 7 Deg. 2 Min. which is 12337, to a fourth Number, which fhall be found 5722, *viz.* the Tangent of 3 Deg. 16 Min. and confequently the Angle that the Hour-Line of 4 makes with the Subftyle, is 3 Deg. 16 Min. and to find the Angle that the Hour-Line of 5 makes with the fubftylar Line, you muft firft add 15 Deg. to 7 Deg. 2 Min. and feek the Tangent of the Sum 22 Deg. 2 Min. and then proceed, as before, and you will find the Angle made by the Hour-Line of 5 with the fubftylar Line will be 10 Deg. 38 Min. the Angle of the Hour-Line of 6 with the fame, will be 19 Deg. 17 Min. the Angle of the Hour-Line of 7, 30 Deg. 44 Min. and the Angle of the Hour-Line of 8 in the Evening, 47 Deg. 35 Min.

But if the Angles that the faid Hour-Lines make with the Meridian or Hour-Line of 12 be required, you muft add 31 Deg. 35 Min. to each of the aforefaid Angles; and confequently the Angle that the Hour-Line of 4 makes with the Meridian, will be 34 Deg. 51 Min. the Hour-Line of 5, 42 Deg. 13 Min. the Hour-Line of 6, 50 Deg. 52 Min. the Hour-Line of 7, 62 Deg. 19 Min. and the Hour-Line of 8, 79 Deg. 10 Min.

Having calculated, in the abovefaid manner, the Angles made by the Hour-Lines on the other fide the fubftylar Line, with the faid fubftylar Line, you will find the Angle of the Hour-Line of 3, 3 Deg. 45 Min. that of the Hour-Line of 2, 11 Deg. 7 Min. that of the Hour-Line of 1, 19 Deg. 54 Min. that of the Hour-Line of 12, 31 Deg. 35 Min. that of the Hour-Line of 11, 48 Deg. 54 Min. that of the Hour-Line of 10, 75 Deg. 7 Min. and that of the Hour-Line of 9, 106 Deg. 48 Min.

Now if 31 Deg. 35 Min. *viz.* the Subftyle's Diftance from the Meridian, be taken from each of thefe laft Angles, then the Angle that the Hour-Line of 9 makes with the Meridian, will be 75 Deg. 13 Min. that of the Hour-Line of 10, 43 Deg. 32 Min. that of the Hour-Line of 11, 17 Deg. 19 Min. and fo of others.

When the Declination of a Plane is very great, the Center of a Dial cannot then be pricked down conveniently thereon, fince the Hour-Lines will fall too near each other. And in this Cafe they may be drawn between two horizontal Lines; for the Angles that the Hour-Lines make with the faid horizontal Lines, are the Complements of the Angles that the refpective Hour-Lines make with the Meridian.

How to find the Declination of an upright or vertical Wall or Plane, by means of the Shadow of the Extremity of an Iron Rod or Style.

Becaufe the Exactnefs of Vertical Dials chiefly depend on the Knowledge of the Situations of the Walls on which they are to be made or fet up againft, with refpct to the Heavens, that is, their Declinations: therefore it is very neceffary that their Declinations be found with all poffible Exactnefs, which we fhall endeavour to do before we clofe this Chapter.

Preparations.

You muft firft fix an Iron Rod or Wire in the Wall obliquely, having it's Extremity fharp and pretty diftant from the Wall, as the Rod A I, whofe Extremity I is fharp. *Vide* Fig. 9.

Secondly, The Foot H of the Style muft be pricked down upon the Dial Plane. This Point is that wherein the Perpendicular H I drawn from the Extremity of the Rod or Style meets the Plane of the Dial. You muft likewife draw the vertical Line H F paffing thro' that Point, which reprefents the perpendicular Vertical to the Plane of the Dial, and alfo the horizontal Line D C cutting the faid vertical Line at right Angles, in the Foot of the Style

Style H. This being done, meaſure exactly the Length of the right Style HI or HF, its equal, that is, meaſure the Diſtance from the Foot of the Style to its Extremity, with ſome Scale divided into ſmall Parts. Then having obſerved where the Extremity of the Shadow of the Iron Rod falls upon the Wall at different Times in the ſame Day, as at the Points 2, 3, 4; you muſt meaſure the Diſtance of each Extremity of the Shadow from the horizontal Line with the Scale: as, for example, the Diſtance from the Point 2 to the Point Z in the horizontal Line; as likewiſe the Diſtance from the ſame Point to the Vertical Line paſſing thro' the Foot of the Style; as from the Point 2 to the Point X; and then you muſt ſet down the Numbers found orderly in a Memorial, that ſo they may be made uſe of in the following Analogies.

But to prick down upon the Wall nicely the Shadow of the Extremity of the Rod or Style, you muſt uſe the following Method, which I had from M. *de la Hire.* Faſten a little Tin-Plate, having a round hole therein, near the Extremity of the Rod, in ſuch manner, that the Extremity of the Iron Rod be exactly in the Center of the ſaid round hole, and the Plate expoſed directly to the Sun; then you will ſee a little Oval of Light upon the Wall in the Shadow of the Plate: and if you draw quickly with a Pencil, a light Track upon the Wall about the ſaid Oval of Light, which is moving continually; the Center of the ſaid Oval may be taken for the true Shadow of the Extremity of the Rod.

Having thus marked the Points 2, 3, 4, whereat the Extremity of the Shadow falls, you muſt find the Amplitude, and the Sun's Altitude anſwering to each of them, and ſet them down in the Memorial.

Note, The Amplitude that we mean here, is the Angle that the height of the Style or Rod makes with the Line drawn from each of the obſerved Extremities of the Shadow to the horizontal Line (for each of theſe Lines repreſents upon the Wall the vertical Circle the Sun is in at the Time of Obſervation). This Angle is marked H F Z in the Figure, and is the Amplitude correſpondent to the Point 2. Now to find this Angle, you muſt ſay, As the Height of the Rod or Style is to the Diſtance from the Extremity of the Shadow to the vertical Line, So is Radius to the Tangent of the Amplitude. And by making this Analogy for each Extremity of the Shadow of the Rod obſerved at different Times, the correſpondent Amplitudes will be had, and muſt be ſet down in one Column in the Memorial.

Then to find the Sun's Altitude above the Horizon, you muſt take the Complement of the Amplitude, and the Diſtance of each obſerved Extremity of the Shadow from the horizontal Line. This being done, ſay, As the Height of the Style is to the Sine Complement of the Amplitude, So is the Diſtance of the Extremity of the Shadow from the horizontal Line, to the Tangent of the Sun's Altitude above the Horizon, which being found for the Times of each Obſervation of the Shadow of the Iron Rod, ſet them down orderly in one Column.

Note, If the Extremity of the Shadow obſerved falls upon the vertical Line paſſing thro' the Foot of the Style, there will then be no Amplitude; and in this Caſe you will have the Sun's Altitude by one Rule only, in ſaying, As the Height of the Style is to the Diſtance of the Extremity of the Shadow from the Foot of the Style, So is Radius to the Tangent of the Sun's Altitude.

After this, you muſt find the Diſtance of each obſerved vertical or azimuth Line from the Meridian; and in order to do this, the Sun's Declination muſt be had for the Times wherein the Extremities of the Shadow were taken: if it be at the time of the Solſtices, the ſame Declination will ſerve for all the Extremities of the Shadow obſerved in one Day; but if the Sun be in the Equinoctial, you muſt have his Declination for each time of the Obſervation of the Extremity of the Shadow, in taking the Parts proportional.

Now the Sun's Declination being had, you muſt take the Complement thereof, as likewiſe the Complement of his Altitude, and the Complement of the Latitude, and add them all three together; and take half the Sum, and from this half Sum take the Complement of the Sun's Altitude, and the Remainder will be a firſt Difference: and moreover, if the Complement of the Latitude be taken from the ſaid half Sum, you will have a ſecond Difference. This being done, ſay, As the Sine Complement of the Latitude is to the Sine of the firſt Difference, ſo is the Sine of the ſecond Difference to a fourth Sine: and as the Sine Complement of the Sun's Altitude is to Radius, ſo is that fourth Sine found to another Sine; which being multiplied by Radius, and the Square Root of the Product, will be half the Diſtance of the Extremity of the Shadow obſerved, or of its vertical Line from the Meridian or Hour-Line of 12.

This Diſtance being found in Degrees and Minutes, we may have the Declination of any Wall, which here is the Angle H F E, by ſome one of the five following Caſes.

Firſt, If the Extremity of the Shadow of the Style is between the vertical Line paſſing thro' the Foot of the Style, and the Hour-Line of 12, as is the Point 2 in this Example, which was obſerved ſome time in the Afternoon; then you muſt add the Amplitude to the Diſtance of the vertical Line from the Meridian.

Secondly, If the Extremity of the Shadow falls beyond the vertical Line paſſing thro' the Foot of the Style, as here the Point 3 does, you muſt ſubſtract the Amplitude from the Diſtance of the vertical Line from the Meridian, to have the Declination of the Wall.

Thirdly, If the obſerved Extremity of the Shadow be found exactly upon the vertical Line paſſing thro' the Foot of the Style, then there will be no Amplitude, and its Diſtance from the Meridian will be the Wall's Declination.

Fourthly, If the Extremity of the Shadow is on this ſide of the Meridian, as here the Point 4 is, which was obſerved before Noon, the Amplitude will be greater than the Declination; to have which, you muſt ſubſtract from the Amplitude the Diſtance of the Vertical Line from the Meridian.

Fifthly, If the Extremity of the Shadow was obſerved preciſely at Noon, the Wall's Declination would be equal to the Amplitude; and ſince the Snn's Declination, and the Latitude is known, it will be eaſy to know whether the Altitude obſerved any Day be the greateſt for that Day, that is, whether it be the Sun's Meridian Altitude. *Note*, What we have ſaid is eaſily applicable to all Declinations, whether Eaſtwards or Weſtwards, if the Line of Midnight be uſed inſtead of that of Noon, when Walls decline North-Eaſt or North-Weſt.

An Example will make all this manifeſt: in order to which, let us ſuppoſe, that, in a Place where the North-Pole is elevated, or, which is all one, where the Latitude of the Place is 48 Deg. 50 Min. we have obſerved the Extremity of the Shadow of an Iron-Rod upon a very upright Wall about the time of the Summer Solſtice, whoſe Diſtance from the vertical Line paſſing thro' the Foot of the Style is 100 equal Parts of ſome Scale, and the Height of the Style 300 of the ſame Parts.

The Operation by Logarithms.

The Logarithm of 100 — — — —	20000000
The Logarithm of Radius — — — —	100000000
The Sum — — — — —	120000000
The Logarithm of 300 — — — —	24771212
The Remainder — — — —	95228788

This Number remaining is the Logarithm Tangent of 18 Deg. 26 Min. for the Amplitude of the obſerved Extremity of the Shadow, and the Complement thereof, is 71 Deg. 34 Min.

Then to find the Sun's Altitude, ſuppoſe the Diſtance from the Extremity of the Shadow obſerved to the horizontal Line be 600 of the aforeſaid equal Parts.

The Logarithm Sine of 71 Deg. 34 Min. — — —	99771253
The Logarithm of 600 — — — —	27781512
The Sum — — — —	127552765
The Logarithm of 300 — — — —	24771212
The Remainder — — — —	102781553

This remaining Number is the the Logarithm Tangent of 62 Deg. 13 Min. the Sun's Altitude.

	Deg.	*Min.*
Then ſuppoſe the Complement of the Latitude is — — —	41	10
The Complement of the Declination of the Sun — —	66	45
The Complement of the Height of the Sun — — —	27	45
The Sum — — —	135	42
Half of the Sum — — —	67	51
The Complement of the Latitude — —	41	10
The firſt Difference — — —	26	41
Again, taking from — — — — —	67	51
The Complement of the Sun's Altitude — —	27	47
We ſhall have the ſecond Difference — —	40	4

The firſt Analogy.

The Logarithm Sine of the firſt Difference 26 Deg. 41 Min. — —	96523035
The Logarithm Sine of the ſecond Difference 40 Deg. 4 Min. — —	98086690
The Sum — — — — —	194609725
The Logarithm-Sine of 41 Deg. 10 Min. ſubſtract — —	91883919
The fourth Sine remaining — — —	96425806

The

The second Analogy.

The Logarithm of Radius	10000000
The fourth Sine	96425806
The Sum	196425806
Substract the Logarithm Sine of 27 Deg. 47 Min.	96685064
The remaining Sine	99740742
The Log. Sine of Radius	100000000
The Sum	199740742
The half of this Number for the Square Root	998870371

This last Number is the Logarithm Sine of 76 Deg. 4 Min. which being doubled, makes 152 Deg. 8 Min. but since this Angle is obtuse, you must substract it from 180 Deg. and the Remainder 27 Deg. 52 Min. is the Distance of the observed vertical Circle or Line from the Meridian: and because the Extremity of the Shadow 2, for which the Calculation is supposed to be made, is between the vertical Line passing thro' the Foot of the Style, and the Hour-Line of 12; you must add the aforesaid 27 Deg. 52 Min. to the calculated Amplitude 18 Deg. 26 Min. to have the Declination 46 Deg. 18 Min.

The Declination of a Wall may be found by one Observation of the Extremity of the Shadow of a Style or Iron-Rod only; but it is better to make several Observations thereof in one Day, or in different Days, that so the Declination of the Wall may be calculated for each Observation, and the proportional Parts of the Differences arising may be taken: if, for Example, the Extremity of the Shadow of the Style hath been six times observed, you must take the one-sixth Part of the Differences produced by the Calculations, in order to have the true Declination of the Wall.

CHAP. II.

Of the Construction and Uses of the Declinatory.

THIS Instrument is made of a very even Plate of Brass or dry Wood, in figure of a Rectangle, about one Foot in Length, and seven or eight Inches in Breadth. We draw the Diameter of a Semi-Circle upon it parallel to one of the longest Sides of this Plate, *viz.* parallel to A B, and we divide this Semi-Circle into two Quadrants, containing 90 Degrees each, which we divide sometimes into half Degrees, the Degrees being both ways numbered from the Point H, as may be seen in the Figure of the Instrument. When this is done, we add an Index I to the said Plate, which turns about the Center G, by means of a turned headed Rivet. On the fiducial Line of this Index we screw a Compass, with the North-Side towards the Center G, and likewise sometimes a small horizontal Dial, whose Hour-Line of 12 turns to the Center G. I shall say no more as to the Construction of this Instrument, it being easy to understand, from what has been said elsewhere in this Treatise. Fig. 16.

The Use of this Instrument in taking the Declinations of Planes.

A Plane is said to decline, when it does not face directly one of the Cardinal Parts of the World, which are North, South, East, and West; and the Declination thereof is measured by an Arc of the Horizon comprehended between the Prime Vertical, and the vertical Circle parallel to the said Plane, if it be vertical, *viz.* perpendicular to the Horizon; for if a Plane be inclined, it can be parallel to no vertical Circle. And in this Case, the Arc of the Horizon comprehended between the Prime Vertical, and that vertical Circle that is parallel to the Base of the inclined Plane, or else the Arc of the Horizon computed between the Meridian of the Place, and the vertical Circle perpendicular to the Plane, is the Plane's Declination.

There are no Planes, unless vertical or inclined ones, that can decline; for a horizontal Plane cannot be said to decline, because the upper Surface thereof directly faces the Zenith, and it's Plane turns towards all the four Cardinal Parts of the World indifferently.

Now, in order to find the Declination of a Plane, whether vertical or inclined, you must draw first a level Line thereon, that is, a Line parallel to the Horizon, and lay the Side A B of the Instrument along this Line: then you must turn the Index and Compass 'till the Needle fixes itself directly over the Line of the Declination or Variation thereof on the Bottom of the Box. This being done, the Degrees of the Semi-Circle cut by the fiducial Line of the Index

Index gives the Plane's Declination towards that Coast shewn by the writing graved upon the Instrument. If, for example, the Index be found fixed upon the 45th Degree, between H and B, and the end of the Needle respecting the North be directly over the Point S of its Line of Declination; in this Case, the Plane declines 45 Deg. South-westwardly: but if in the same Situation of the Declinatory, the opposite end of the Needle, respecting the South, should have fixed itself over the Point S of the said Line of Declination, then the Plane would have declined 45 Deg. North-eastwardly.

Again, If the Index be found between A and H, and the North-end of the Needle over the Point S of its Line of Declination, then the Declination of the Plane will be South-eastwardly; but if in this Situation of the Index, the South-end of the Needle fixes itself over the said Point S, then the Plane will decline North-westwardly.

If the Sun shines upon the Wall or Plane whose Declination is sought, and the time of the Day be known exactly by some good Dial, as the Astronomic Ring Dial, we may find the Declination of the Wall or Plane by means of a small horizontal Dial fastened on the Index, which must be turned 'till the Style of the Dial shews the exact Time of the Day; and then the Degrees of one of the Quadrants cut by the Fiducial Line of the Index, will be the Wall or Plane's Declination: and by this means may be avoided the Errors caused by the Compass, as well on account of the Variation of the Needle, as because of Iron concealed near the Compass.

When the Sun shines upon a Wall, we may find likewise the Substyle or proper Meridian by means of observing two Extremities of the Shadow of an Iron-Rod, in the manner we have above mentioned, and afterwards the Declination; or else we may draw a meridian Line upon an horizontal Plane near the Wall, which being produced to the Wall, will be a means to find the Declination thereof, as also to find the Variation of the Needle. Now the manner of drawing a Meridian Line is thus:

Fig. M. Draw a Circle upon some level Plane, (suppose this to be represented by the Figure M) and in the Center thereof set up a sharp Style very upright, or else fix a crooked Style in some Place, as A, in such manner, that a Line drawn from its sharp end to the Center of the said Circle be perpendicular to the Plane of the Circle; which you may do by a Square. But before you draw the Circle, it is necessary to know the Length of the Shadow of the Style, that so the Circumference of the Circle may be drawn thro' the Extremity of the Shadow of the Style observed some time before Noon. Now the Circle being drawn, suppose the Extremity of the Shadow touches the Circumference of the Circle in the Morning at the Point G, and about as many Hours after Noon as when in the Morning you observed the Extremity of the said Shadow in G before Noon, you find the Extremity of the Shadow again to touch the Circumference of the Circle in F; then if the Arc F G be bisected in the Point C, and the Diameter B C be drawn, this Diameter will be a meridian Line.

If you have a mind to find a meridian Line when the Sun is in the equinoctial Line, there is no need of drawing a Circle, for all the Extremities of the Shadow of the Style will then be in a right Line, as E D, which is the common Section of the Equinoctial and the horizontal Plane; and so any right Line, as B C, cutting E D at right Angles, will be a meridian Line.

Thus having drawn a meridian Line, if the Hour-Line of 12 of a horizontal Dial be placed so as to coincide therewith, we may have the Time of the Day thereby: and therefore if at the same time the Index of the Declinatory be turned so, that the small horizontal Dial fastened thereon shews the same Hour or Part, then the Degrees of the Circumference of the Instrument cut by the Index, will shew the Declination of the Wall or Plane. Or else you may produce the abovesaid meridian Line 'till it cuts the declining Plane, for then it will make two unequal Angles with the horizontal Line drawn upon the Plane, *viz.* an acute and obtuse Angle, which being measured with all the Exactness possible, the Difference between either of these Angles and a right Angle, will be the Declination of the Plane. For Example, if the acute Angle be 50 Deg. and consequently the obtuse one 130 Deg. then the Difference between either of them and a right Angle, will be 40 Deg. for the Declination of the Plane.

If you have a mind to find the Variation of the Needle, apply one of the Sides of the square Box of the Compass along the meridian Line drawn on the Plane; and when the Needle is at rest, observe how many Degrees the North Point thereof is distant from the *Flower-de-luce* of the Card; and these Degrees will be the Needle's Declination or Variation; but this Variation will not last long, for it changes continually. *Note*, When the Declinations of Planes be taken with a Compass, you must have regard to the Variation of the Needle, in letting it rest over a Line shewing the Variation, which is drawn commonly on the Bottom of the Compass-Box.

The Use of the Declinatory in taking the Inclinations of Planes.

This Instrument serves to take the Inclinations of Planes, as well as their Declinations, that is, the Angles the Planes make with the Horizon, and for this end there is a little Hole in the Center G, having a Plumb-Line fastened therein.

The

The 17th Figure ſhews the manner of taking the Declinations aud Inclinations of Planes. The Plane A, of this Figure, whereon the Declinatory is applied, is a vertical Meridional undeclining Plane. The Plane B declines South North-weſtwardly 44 Degres. The Plane C, is a direct Weſt one. The Plane D, declines 45 Degrees North-weſtwardly. And the other Declinations are taken in the ſame manner, in applying the Side A B of the Declinatory to them, ſo that the Plane of the Semi-circle be parallel to the Horizon.

Now to meaſure the Angle of a Plane's Inclination, you muſt apply ſome one of the other Sides of the Inſtrument to the Plane or Wall, and keeping the Plane of the Semi-circle perpendicular to the Horizon, ſee what Number of Degrees of the Circumference thereof the Plumb-Line plays upon, for theſe will be the quantity of the ſaid Angle of Inclination.

If, for example, the Side CD be applied to the Plane E, and the Plumb-Line plays upon the Line G H, then the ſaid Plane will be parallel to the Horizon. But if the Side C A of the Inſtrument being applied on the Plane F, and the Plumb Line plays, as *per* Figure, this Plane inclines 45 Degrees upwards. Again, If the Inſtrument being applied to the Plane G, and the Plumb-Line plays upon the Diameter, then this Plane is vertical. And laſtly, If the Side A C, being applied on the Plane H, and the Plumb-Line plays as *per* Figure, then the Inclination thereof will be 45 Deg. downwards.

CHAP. III.

Of the Conſtruction and Uſes of Inſtruments, for drawing upon Dials the Arcs of the Signs, the Diurnal Arcs, the Babylonick *and* Italian *Hours, the Almacanters, and the Meridians of principal Cities.*

WE now proceed to deſcribe upon Dials certain Lines which the Shadow of the Extremity of the Style paſſes over, when the Sun enters into each of the 12 Signs of the Zodiack.

Of the Trigon of Signs.

The firſt Figure repreſents the Triangle or Trigon of Signs, made of Braſs or any other ſolid Matter of a bigneſs at pleaſure. The Conſtruction of this is thus: Firſt draw the Line *a b*, repreſenting the Axis of the World, and *a c* perpendicular thereto, repreſenting the Radius of the Equinoctial, and about the Point *a* deſcribe the circular Arc *d c e* at pleaſure. This being done, reckon 23 ½ Deg. both ways from the Point *c* upon the ſaid Arc, for the Sun's greateſt Declination, and draw the two Lines, *a d*, *a e*, for the Summer and Winter Tropicks; likewiſe draw the Line *d e*, which will be biſected by the Radius of the Equinoctial in the Point *o*, about which, as a Center, draw a Circle, whoſe Circumference paſſes thro' the Points *d* and *e* of the Tropicks, and divide the Circumference thereof in 12 equal Parts, beginning from the Point *d*. Then thro' each Point of Diviſion equally diſtant from *d* and *e*, draw occult Lines parallel to the Radius of the Equinoctial Circle. Theſe Lines will interſect the Arc *d c* in the Points thro' which and the Center *a* Lines being drawn, theſe Lines will repreſent the beginnings of the Signs of the Zodiack, at 30 Deg. diſtance from each other, *Plate* 23. *Fig.* 1.

But to divide the Signs into every 10th or 5th Degree, you muſt divide the Circumference of the Circle into 36 or 72 equal Parts. After this, we denote the Characters of the Signs upon each Line, as appears *per* Figure. And when the Trigon is divided into every 10th or 5th Degree, we place the Letter of the Month to the firſt 10 Degrees of each Sign agreeing therewith.

But the Trigon of Signs may be readier made by the means of a Table of the Sun's Declination; for having drawn the two Lines *a b* and *a c* at right Angles, lay the Center of a Protractor on the Point *a*, with its Limb towards the Point *c*; and keeping it fixed thus, count 23 ½ Deg. on both ſides the Radius *a c*, for the Tropicks of ♋ and ♑, 20 Deg. 12 Min. for the beginnings of the Signs ♌, ♊, ♐ and ♒, and 11 Deg. 30 Min for ♉, ♍ ♏ and ♓. And in this manner we divide the Spaces for each Sign into every 10th or 5th Deg. by means of the following Table of the Sun's Declination. *Note*, The Equinoctial Points of ♈ and ♎ are placed at the end of the Radius of the Equinoctial *a c*.

A TABLE *of the Sun's* Declination *for every Degree of the Ecliptick.*

Degrees of the Ecliptick.	*Signs.* ♈ ♎ D.	M.	*Signs.* ♉ ♏ D.	M.	*Signs.* ♊ ♐ D.	M.	*Degrees of the Ecliptick.*
1	0	24	11	51	20	25	29
2	0	48	12	12	20	36	28
3	1	12	12	32	20	48	27
4	1	36	12	53	21	0	26
5	2	0	13	13	21	11	25
6	2	23	13	33	21	21	24
7	2	47	13	53	21	32	23
8	3	11	14	12	21	42	22
9	3	35	14	32	21	51	21
10	3	58	14	51	22	00	20
11	4	22	15	9	22	8	19
12	4	45	15	28	22	17	18
13	5	9	15	47	22	24	17
14	5	32	16	5	22	32	16
15	5	55	16	22	22	39	15
16	6	19	16	40	22	46	14
17	6	42	16	57	22	52	13
18	7	5	17	14	22	57	12
19	7	28	17	30	23	2	11
20	7	50	17	47	23	7	10
21	8	13	18	3	23	11	9
22	8	35	18	16	23	15	8
23	8	58	18	34	23	18	7
24	9	20	18	49	23	21	6
25	9	42	19	3	23	24	5
26	10	4	19	18	23	26	4
27	10	26	19	32	23	27	3
28	10	47	19	46	23	28	2
29	11	9	19	59	23	29	1
30	11	30	20	12	23	30	0
	♓	♍	♒	♌	♑	♋	

By this Table we may know the Sun's Declination and Distance from the Equinoctial Points each Day at Noon, in every Degree of the Signs of the Zodiack, the greatest Declination being 23 Deg. 30 Min. tho' at present it is but about 23 Deg. 29 Min. but a Minute difference is of no consequence in the Use of Dials. The Degrees of the first Column to the Left-hand, are for the Signs set down upon the Top of the Table, and the Degrees in the last Column numbered upwards, are for the Signs set at the Bottom of the Table.

Of the Trigon of Diurnal Arcs.

The second Figure represents the Trigon of Diurnal and Nocturnal Arcs. These are drawn upon Sun-Dials by Curve-Lines, like the Arcs of the Signs, and by means of them the Shadow of the Style shews how many Hours the Sun is above the Horizon, in any given Day, that is, the Length of the Day, and consequently the Length of the Night too; for this is the Complement of that to 24 Hours.

The Trigon of Signs is the same for all Latitudes, since the Sun's Declination is the same for all the Earth: but the Diurnal Arcs are different for every particular Latitude, and we draw as many of these Arcs upon a Dial, as there are Hours of Difference between the longest and shortest Days of the Year.

Fig. 2. Now to construct the Trigon of Diurnal Arcs upon Brass or any other solid Matter, first draw the right Line R Z for the Radius of the Hour-Line of 12, or of the Equinoctial; and about the Point R, with any Opening of your Compasses taken at pleasure, describe the circular

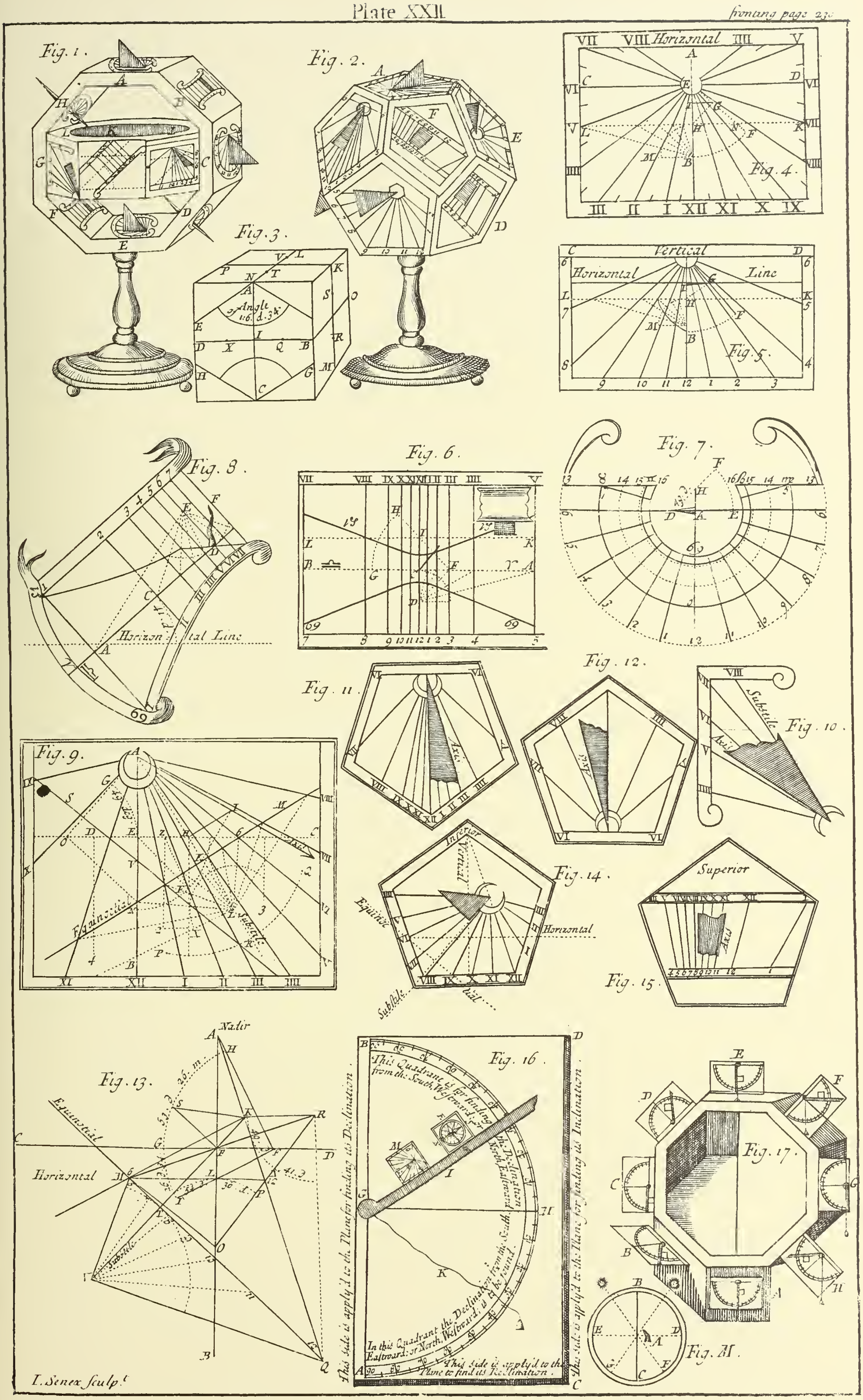
Fig. 1.
Fig. 2.
Fig. 3.
Angle of 16 d. 3
Fig. 4.
VII VIII Horizontal IIII V
III II I XII XI X IX
Fig. 5.
Vertical
Horizontal
Line
9 10 11 12 1 2 3
Fig. 6.
Fig. 7.
Fig. 8.
Horizontal Line
Fig. 9.
Equinoctial
Subtile
Axis
XI XII I II III IIII
Fig. 10.
Subtile
Axis
Fig. 11.
Axis
Fig. 12.
Axis
Fig. 13.
Nadir
Equinoctial
Horizontal
Subtile
Fig. 14.
Inferior
Vertical
Equinoctial
Horizontal
Subtile
Fig. 15.
Superior
Axis
Fig. 16.
This Quadrant is for finding the Declination from the South Westward or North Eastward
In this Quadrant the Declination from the South Eastward; or North Westward; is to be found.
This side is apply'd to the Plane to find its Reclination.
This side is apply'd to the Plane for finding its Declination.
This side is apply'd to the Plane for finding its Inclination.
Fig. 17.
Fig. M.
I. Senex sculp.

circular Arc T S V, and lay off both ways thereon from the Point S, two Arcs, each equal to the Complement of the Latitude. For Example, if the Latitude be 49 Deg. make the Arcs S V, and S T, of 41 Deg. each. This being done, draw the right Line T X V, and about the Point X, as a Center, defcribe the Circumference of a Circle T Z V Y, which divide into 48 equal Parts by dotted Lines, drawn parallel to the Radius of the Equinoctial R Z: then thefe Lines will cut the Diameter T X V in Points, thro' which and the Point R, you may draw the Radius's of the Hours. And fince the longeft Day at *Paris* is 16 Hours, and the fhorteft 8, you need but draw four Radius's on one Side the Line R Z, and a like Number on the other Side.

Moreover, the Angles that all the Radius's make at the Point R may be found Trigonometrically, by the following Analogy, *viz.* As Radius is to the Tangent Complement of the Latitude, So is the Tangent of the Difference between the Semidiurnal Arc at the time of the Equinox and the Arc propofed, to the Tangent of the Sun's requifite Declination. For Example; Suppofe it be required to draw upon the Trigon the diurnal Arc of 11 or 13 Hours, the Semidiurnal Arc is 5 ½ Hours, or 6 ½ Hours, and the Day of the Equinox the diurnal Arc is 12 Hours; and confequently the Semidiurnal Arc is 6 Hours, and the Difference is half an Hour: therefore Radius muft be put for the firft Term of the Analogy, the Tangent of 41 Deg. (*viz.* the Complement of the Latitude of *Paris*) for the fecond Term, and the Sine of 7 Deg. 30 Min. for the third Term. Now the fourth Term being found, the Sun's Declination is 6 Deg. 28 Min. South, when the Day at *Paris* is 11 Hours long; and 6 Deg. 28 Min. North, when the Day is 13 Hours; and making three other Analogies, you will find that the Declination of the diurnal Arc of 10 Hours and 14 Hours, is 12 Deg. 41 Min. of 9 Hours and 15 Hours, 18 Deg. 25 Min. and of 8 Hours and 16 Hours, 23 Deg. 30 Min.

Of the Trigon with an Index.

The third Figure reprefents the Trigon of Signs put upon a Rule or Index A, in order to draw the Arcs of the Signs upon great Dials. The diurnal Arcs may be drawn likewife upon this Trigon; but the Arcs of the Signs and diurnal Arcs too muft not be drawn upon one and the fame Dial, for avoiding Confufion. In the Center of the Index there is a little Hole thro' which is put a Pin, that fo the Inftrument may turn about the Center of a Dial. The Trigon flides along the Index, and may be fixed in any part thereof by means of the Screw B. The Arcs of the Signs with their Characters are round about the Circumference, and there is a fine Thread fixed in the Center thereof, in order to extend over the Radii quite to the Hour-Lines of a Dial, as we fhall by and by explain. Fig. 3.

The fourth Figure reprefents one half of a horizontal Dial, having the Morning Hour-Lines to 12 o'Clock thereon, and the Equinoctial Line C D. This being enough of the Dial, for explaining the Manner of drawing the Arcs of the Signs thereon, by means of Figure 5, which reprefents a Trigon of Signs drawn upon a Plate, on which the Hour-Lines of an horizontal Dial are adjufted in the following manner Fig. 4.

Take the Length of the Axis V R of the horizontal Dial between your Compaffes, and lay it off on the Axis of the Trigon from O to C; after this, take the Diftance from the Center V of the Dial to the Point C, wherein the Equinoctial Line cuts the Hour-Line of 12, and lay it off on the Trigon from C to *a*, and draw lightly the Line *c a* 12, cutting all the feven Lines of the Trigon. This being done, take upon this Line the Diftance from the Point *c* to the Interfection of the Summer Tropick, and lay it off from the Center V of the Dial on the Hour-Line of 12, and you will have one Point thro' which the Summer Tropick muft pafs; likewife take the Diftance from the Point *c* to the Interfection of the Parallel of ♊, and lay it off on the Hour-Line of 12, from the Center of the Dial, and you will have a Point on the faid Hour-Line thro' which the Parallel of ♊ muft pafs; likewife affume all the other Diftances on the Trigon, and lay them off fucceffively on the Hour-Line of 12 of the Dial, from the Center to the Point thro' which the Winter Tropick paffes, which muft be the moft diftant from the Center of the Dial, and you will have the Points in the Hour-Line of 12 thro' which each of the Parallels of the Signs muft pafs. And by proceeding in this manner with the other Hour-Lines, you will have Points in them thro' which the Parallels of the Signs muft pafs. For Example, Affume on the Hour-Line of 11 of the Dial, the Diftance from the Center thereof to the Point wherein the Equinoctial Line cuts it, and lay this Diftance off upon the Trigon from *c* towards *a*, and draw the right Line C 11; then take the Diftances from the Point *c* to the Interfection of each of the Parallels of the Signs, and lay them off from the Center of the Dial, on the Hour-Line of 11, to the Points 2 2, *&c.* and thofe will be Points in the Hour-Line of 11, thro' which the Parallels of the Signs muft pafs. Underftand the fame for others.

But becaufe the Hour-Line of 6 is parallel to the Equinoctial Line, make this likewife parallel to the Radius of the Equinoctial *o a* on the Trigon: and to prick down the Line for the Hour of Seven in the Evening, defcribe an Arc about the Point C, as a Center, from the Line for the Hour of 6 to that for the Hour of 5; and lay off that Arc on the other Side of the Line for the Hour of 6, and then you may draw the Hour-Line of 7, which will not meet the Summer Tropick. Finally, The Line for the Hour of 8 muft make the fame Angle with the Line of the Hour of 6, as the Line for the Hour of 4 does; but

but it is uſeleſs to draw this Line for the Latitude of 49 Deg. becauſe this Line being parallel to the Tropick of ♋, cannot cut any one Radius to the Signs. Now the Points thro' which the Arcs of the Signs muſt paſs, being found on the Hour-Lines of the Dial, you muſt join all thoſe that appertain to the ſame Sign with an even hand; and you will have the curved Arcs of the Signs, whoſe Characters muſt be marked upon the Dial, as *per* Figure. *Note*, We ſometimes ſet down the Names of the Months, and of ſome remarkable moveable Feaſts upon the Dial. The Arcs of the Signs are drawn upon vertical Dials in this manner; but here the Winter Tropick muſt be nigheſt to the Center of the Dial, and the Summer Tropick furtheſt diſtant from it.

If the Arcs of the Signs or diurnal Arcs are to be drawn upon a great Dial, the third Figure muſt be uſed in the following manner:

Fig. 6. Faſten the Rule or Index to the Center of the Dial by a Pin, ſo that it may be turned and fixed upon any Hour-Line, as may be ſeen in Figure 6: then having fixed the Center of the Trigon upon the Index, at a Diſtance from the Center of the Index equal to the Diſtance from the Center of the Dial to the Extremity of the Axis thereof, by means of the Screw R; take the Thread on one Hand, and with the other raiſe or lower the Inſtrument upon the Plane of the Dial, ſo that the Thread extended along the Radius of the Equinoctial of the Trigon, meets the Point wherein ſome Hour-Line cuts the Equinoctial Line of the Dial, and in this Situation fix the Index. This being done, extend the Thread along the Radius's of the Trigon, and prick down the Points upon each Hour-Line of the Dial, thro' which the Parallels of the Signs muſt paſs, both above and below the Equinoctial Line, as we have done on the Hour-Line of 12 of the Dial repreſented in Figure 6. And if you do thus on all the Hour-Lines ſucceſſively one after the other, and the Points marked thereon appertaining to the ſame Sign, be joined by an even Hand, you will have the Parallels of the Signs upon the Surface of the Dial. But to make the Points on the Hour-Line of 6, the Inſtrument muſt be turned ſo that the Fiducial Line of the Index be upon the Hour-Line of 12, and the Radius of the Equinoctial Circle of the Trigon parallel to the Hour-Line of 6. The Inſtrument being thus fixed, extend the Thread along the Radius's of the Signs, until it cuts the Hour-Line of 6, and the Points where it cuts the ſaid Hour-Line, will be thoſe thro' which the Parallels of the Signs muſt paſs in that Hour-Line.

When the Arcs of the Signs are drawn on one ſide of the Dial, for example, on the Morning Hour-Lines, you may lay off the ſame Diſtances from the Center on the Hour-Lines of the other ſide the Meridian; as the Points denoted on the Hour-Line of 11 muſt be laid off on the Hour-Line of 1, thoſe on the Hour-Line of 10 on the Hour-Line of 2; and ſo draw the Arcs of the Signs on the other ſide of the Meridian. *Note*, The Arcs of the Signs are drawn upon declining Dials in the ſame manner, if the Subſtylar Line be made uſe of inſtead of the Meridian, and the Diſtances from the Center be taken equal upon thoſe Hour-Lines equally diſtant on both ſides of the Subſtyle from it.

If the diurnal Arcs are to be pricked down upon a Dial inſtead of the Arcs of the Signs, that is, the Length of the Days, we may likewiſe put thereon the Hour of the Sun's riſing and ſetting, if the Length of the Day be divided into two equal Parts. For example, when the Day is 15 Hours long, the Sun ſets half an Hour paſt 7 in the Afternoon, and riſes half an Hour paſt 4 in the Morning; and ſo of others.

If the Arcs of the Signs are to be drawn upon Equinoctial Dials, as on that of Figure 7, *Plate* 22, take the length of the Axis of the Style A D, and lay it off upon the Axis of the Trigon (of Figure 5. *Plate* 23.) from O to P, and draw the Line P N parallel to the Radius of the Equinoctial; this ſhall cut the Summer Tropick and two other Parallels: then take the Diſtance from the Point P to the Interſection of the Tropick of ♋; and with that Diſtance about the Center A of the Dial draw a Circle, which ſhall repreſent the Tropick of ♋. Take likewiſe the two other Diſtances on the Parallel of the Trigon, and draw two other Circles about the Center of the Dial, the one for the Parallel of ♊ and ♌, and the other for that of ♉ and ♍, which may be drawn upon an upper Equinoctial Dial. But if this was an under Equinoctial Dial, then the above deſcribed Circles would repreſent the Parallels of ♏, ♐, ♑, ♒ and ♓: but as for the Parallels of ♈ and ♎, they cannot be drawn upon Equinoctial Dials, becauſe when the Sun is in the Plane of the Celeſtial Equator, his Rays fall parallel to the Surfaces of Equinoctial Dials, and the Shadows of their Styles are indefinitely protended.

The Horizontal Line is thus drawn: Firſt lay off the Style's length on the Hour-Line of 6, and about the Extremity D thereof, deſcribe the Arc E F (upwards for an upper Dial) equal to the Latitude, *viz.* 49 Deg. for *Paris*, and draw the Line D F, which ſhall cut the Meridian in the Point H, thro' which the horizontal Line muſt be drawn parallel to the Hour-Line of 6, as may be ſeen in Figure 7, *Plate* 22.

The Uſe of this Line is to ſhew the riſing and ſetting of the Sun at his entrance into the beginning of each Sign. For example, becauſe it cuts the Tropick of *Cancer* on the Dial, in Points thro' which the Hour-Line of 4 in the Morning, and 8 in the Evening paſs; therefore the Sun riſes the Day of the Solſtice at 4 in the Morning, and ſets at 8 in the Evening at *Paris.* Underſtand the ſame of others.

To draw the Arcs of the Signs upon Polar Dials.

The Dial being drawn (as appears in Fig. 6. Plate 22.), the dotted Radii of the Hours continued out 'till they meet the Equinoctial Line muſt be laid off ſucceſſively upon the Radius of the Equinoctial of the Trigon of Signs (Fig. 5. Plate 23.) for drawing as many Perpendiculars thereon as there are dotted Radii, *viz.* one for the Hour of 12, and the five others for the Hours of 1, 2, 3, 4, and 5, which ſhall cut the Radii of the Signs of the Trigon. This being done, take the Diſtances from the Radius of the Equinoctial of the Trigon upon the ſaid Perpendiculars, to the Radius's of the other Signs, and lay them off upon the Hour-Lines of the Dial on both ſides the Equinoctial Line A B. For Example; Take the Diſtance 12 ♑, and lay it off on the Dial from the Point C upon the Hour-Line of 12, and you will have two Points in the ſaid Line thro' which the Tropicks muſt paſs. Likewiſe take the Space on the Trigon upon the Line 5 ♑ or ♋, and lay it off upon the Hour-Lines of 5 and 7 on both ſides the Equinoctial Line of your Dial, and you will have Points in the Hour-Lines of 5 and 7, thro' which the Tropicks muſt paſs. And in this manner may Points be found in the other Hour-Lines thro' which the ſaid Tropicks muſt paſs; as alſo the Points in the Hour-Lines thro' which the Parallels of the other Signs muſt be drawn, which being found muſt be joined. *Note*, We have only drawn the two Tropicks in the Figure of this Dial for avoiding Confuſion. And the Parallels of the Northern Signs muſt be drawn underneath the Equinoctial Line, and the Southern Signs above it. Alſo the diurnal Arcs are drawn in the ſame manner as the Arcs of the Signs are.

How to draw the Arcs of the Signs upon Eaſt and Weſt Dials.

The Arcs of the Signs are drawn nearly in the ſame manner upon Eaſt and Weſt Dials as upon Polar ones: for Example, Let it be required to draw the Arcs of the Signs upon the Weſt Dial of Figure 8. Plate 22. the dotted Radii of the Hours produced to the Equinoctial Line C D, muſt be laid off upon the Trigon of Figure 1. (Plate 23.) from the Point *a* upon the Radius of the Equinoctial, that ſo Perpendiculars may be drawn upon the Trigon cutting the Radius's of the Signs; after this, you muſt take upon the ſaid Perpendiculars the Diſtances from the Radius of the Equinoctial to the Interſection of the Radii of the other Signs, and lay them off upon the Hour-Lines of the Dial, on both ſides the Equinoctial Line. For Example, Take the Space 6 ♑, or ♋, and lay it off on both ſides the Point D upon the Hour-Line of 6 on the Dial: Proceed in this manner for finding Points in the other Hour-Lines thro' which the Curve Parallels of the Signs muſt be drawn with an even Hand, ſo that the Northern ones be under the Equinoctial Line, and the Southern ones above it. *Note*, The diurnal Arcs are drawn in the ſame manner; and we have only drawn the two Tropicks thereon for avoiding Confuſion.

The Conſtruction of a horizontal Dial, having the Italian *and* Babylonian *Hours; as alſo the Almacanters and Meridians deſcribed upon it.*

Having already ſhewed the manner of pricking down the Aſtronomical Hours upon Sun-Dials, as alſo the Diurnal Arcs, and Arcs of the Signs, there may yet be ſeveral other Circles of the Sphere repreſented upon Dials, being pleaſant and uſeful, which the Shadow of the Extremity of the Style paſſes over; as the *Italian* and *Babylonian* Hours, the Azimuths, the Almacanters, and the Meridians of principal Cities.

The firſt Line of the *Italian* and *Babylonian* Hours is the Horizon, like as the firſt Line of the Aſtronomical Hours is the Meridian; for the *Italians* begin to reckon their Hours when the Center of the Sun touches the Horizon at his ſetting, and the *Babylonians* when he touches the Horizon at his riſing.

A general Method for drawing the Italian *and* Babylonian *Hours upon all kinds of Dials.*

The Aſtronomical Hour-Lines, and the Equinoctial Line being drawn, as alſo a diurnal Arc or Parallel of the Sun's riſing for any Hour, at pleaſure, as, for the Hour of 4 at *Paris*, which Arc will be the ſame as the Summer Tropick, you may find two Points (as we ſhall ſhew here) in each of the aforeſaid Lines, *viz.* one in the Equinoctial Line, and the other in the diurnal Arc drawn, by means of which it will not be difficult to prick down the *Italian* and *Babylonian* Hour-Lines; becauſe they being the common Sections of great Circles of the Sphere and a Dial-Plane, will be repreſented in right Lines thereon. Fig. 7.

Now ſuppoſe it be required to draw the firſt *Babylonian* Hour-Line upon the horizontal Dial of Figure 7, firſt conſider that when the Sun is in the Equinoctial he riſes at 6, and at 7 he has been up juſt an Hour; whence it follows, that the firſt *Babylonian* Hour-Line muſt paſs thro' the Point wherein the Aſtronomical Hour-Line of 7 cuts the Equinoctial Line; the ſecond thro' the Interſection of the Hour-Line of 8; the third thro' that of the Hour-Line of 9; and ſo of others.

But when the Sun riſes at 4 in the Morning, the Point in the Tropick of ♋, wherein the Hour-Line of 5 cuts it, is that thro' which the firſt *Babylonian* Hour-Line muſt paſs; the Interſection of the Hour-Line of 6 in the ſaid Tropick, that thro' which the ſecond *Babylonian* Hour-Line muſt paſs; the Interſection of the Hour-Line of 7 with the ſaid Tropick, that Point thro' which the third *Babylonian* Hour-Line muſt paſs; and ſo of others. Then if a

Ruler be laid to the Point wherein the Hour-Line of 5 cuts the Tropick of *Cancer*, and on the Point in the Equinoctial Line cut by the Hour-Line of 7, and you draw a right Line thro' them; this Line will repreſent the firſt *Babylonian* Hour-Line. Proceeding in this manner for the other *Babylonian* Hour-Lines, you will find that the 8th *Babylonian* Hour-Line will paſs thro' the Point the Tropick of *Cancer* is cut by the Aſtronomical Hour-Line of 12, and the Point in the Equinoctial cut by the Hour-Line of 12; and the 5th *Babylonian* Hour-Line thro' the Point in the ſaid Tropick cut by the Hour-Line of 7 in the Evening, and the Point in the Equinoctial Line cut by the Hour-Line of 5.

One of the *Babylonian* Hour-Lines being drawn, it is afterwards eaſy to draw all the others; becauſe they proceed orderly from one Aſtronomical Hour-Line to the other, on the Parallel and the Equinoctial Line, as appears *per* Figure. Finally, The Sun ſets at the 16th *Babylonian* Hour, when the Day is 16 Hours long: he ſets at the 12th when he is in the Equinoctial; and at the 8th when the Night is 16 Hours long, becauſe he always riſes at the 24th *Babylonian* Hour.

You muſt reaſon nearly in the ſame manner for pricking down the *Italian* Hour-Lines. Here we always reckon the Sun to ſet at the 24th Hour; and conſequently in Summer, when the Nights are but 8 Hours long, he riſes at the 8th *Italian* Hour; at the Time of the Equinox he riſes at the 12th *Italian* Hour; and in Winter, when the Nights are 16 Hours long, he riſes at the 16th *Italian* Hour: and therefore the Hour-Line of the 23d *Italian* Hour muſt paſs thro' the Interſection of the Aſtronomical Hour-Line of 7, and the Summer Tropick the Interſection of the Hour-Line of 5, and the Equinoctial Line, and the Interſection of the Hour-Line of 3, and the Winter Tropick. But two of the ſaid Points are ſufficient for drawing the ſaid *Italian* Hour-Line. The 22d *Italian* Hour-Line paſſes thro' the Interſection of the Hour-Line of 6 in the Evening, and Summer Tropick, the Interſection of the Hour-Line of 4, and the Equinoctial Line, and the Interſection of the Hour-Line of 2, and the Winter Tropick. Proceeding on thus, you will find that the 18th *Italian* Hour-Line paſſes thro' the Points of the 12th Equinoctial Hour, that is, at the Time of the Equinox, it is Noon at the 18th *Italian* Hour; whereas at the Time of the Summer Solſtice it is Noon at the 16th *Italian* Hour, and at the Winter Solſtice it is Noon at the 20th *Italian* Hour, in all Places where the Pole is elevated 49 Degrees, as may be ſeen in the following Table.

A TABLE *for drawing the* Babylonian *and* Italian *Hour-Lines upon Dials.*

Babylonian Hours.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
Paſſing in the Parallel of ♋ *thro'*	5.	6.	7.	8.	9.	10.	11.	12.	1.	2.	3.	4.	5.	6.	7.	8.
♈	7.	8.	9.	10.	11.	12.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
♑	9.	10.	11.	12.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Italian Hours.	23.	22.	21.	20.	19.	18.	17.	16.	15.	14.	13.	12.	11.	10.	9.	8.
Paſſing in the Parallel of ♋ *thro'*	7.	6.	5.	4.	3.	2.	1.	12.	11.	10.	9.	8.	7.	6.	5.	4.
♈	5.	4.	3.	2.	1.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.
♑	3.	2.	1.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.	12.

The Uſe of the *Italian* Hour-Lines upon a Dial may be to find the Time of the Sun's ſetting, in ſubſtracting the *Italian* Hour preſent from 24; and by the *Babylonian* Hours may be known the Time of the Sun's riſing.

How to draw the Almacanters, and the Azimuths.

Fig. 7 The Almacanters or Circles of Altitude are repreſented upon the horizontal Dial by concentrick Circles, and the Azimuths by right Lines terminating at the Foot of the Style B, which repreſents the Zenith, and is the common Center of all the Almacanters: and therefore you need but divide the Meridian B XII into Degrees, the Extremity of the Style C being the Center; and the Tangents of thoſe Degrees on the Meridian will be the Semidiameters of the Almacanters, which ſhall terminate at the two Tropicks. Now to find theſe Tangents, you may uſe a Quadrant like that of Figure 8, in this manner: Lay off the Length of the Style C B from A to H, and draw the Line H I parallel to the Side A C of the Quadrant; then will this Line be divided into a Line of Tangents by Radii drawn from the Center A to the Degrees of the Limb. And theſe Tangents may be taken between your Compaſſes, and laid off upon the Meridian Line B XII. in ſuch manner, that the 90th Degree anſwers to the Point B. But ſince this Dial is made for the Latitude of 49 Deg. and ſo conſequently the Sun in his greateſt Altitude there, is but 64 Deg. 30 Min. you need only prick down this greateſt Altitude, which will terminate at the Summer Tropick.

This being done, if one of the Circles of Altitude be divided into every 10th Deg. beginning from the Meridian B XII. which is the 90th Azimuth, and thro' these Points of Division right Lines are drawn to the Foot of the Style B: these right Lines will represent the Azimuths or vertical Circles. We have not drawn them upon the Dial, for avoiding Confusion, but they may be easily conceived.

Now the Use of the Almacanters is to shew the Sun's Altitude above the Horizon at any time, and of the Azimuths, to shew what Azimuth or vertical Circle the Sun is in: and this is known by observing what Circle of Altitude or Azimuth Line, the Shadow of the Extremity of the Style of the Dial falls upon.

How to draw the Meridians or Circles of Terrestrial Longitude upon the horizontal Dial.

About the Point D, the Center of the Equinoctial Circle, describe the Circumference of a Circle, and divide it into 360 equal Parts or Degrees, or only into 36 Parts, for every 10th Degree; then from the Hour-Line of 12, which represents the Meridian of the Place for which the Dial is made, *viz. Paris*, count 20 Deg. Westward for its Longitude, or Distance from the first Meridian passing thro' the Point G; on which having wrote the Number 360, prolong the Line G D to E, in the Equinoctial Line, and afterwards from the Center A draw the first Meridian thro' E, which passes thro' the Island *de Fer*, and so of others. But it will be easier to draw the Meridians eastwardly for every 5th or 10th Degree, and place those principal Cities upon them whose Longitudes you know: as, for example, *Rome* is 10 ' Deg. more eastwardly than *Paris*, *Vienna* 15 Deg. more eastwardly than the said City of *Paris*, and so of other eminent Cities, whose Differences of Meridians from that of *Paris*, are known by a good Globe, or Map, made according to the exact Observations of the Academy of the Sciences. Fig. 7.

The Use of these Meridians on the Dial, is, to tell at any time when the Sun shines thereon, what Hour then it is under any one of the said Meridians, in adding to the time of Day at *Paris*, (for which the Dial is made) as many Hours as there are times 15 Deg. of Difference between the Meridians, and 4 Min. of an Hour for every Degree.

For example; When it is Noon by this Dial at *Paris*, it will be One a-clock at *Vienna*, because this City is more to the East than *Paris* by 15 Deg. and consequently receives the Sun's Light sooner than *Paris* does. And at *Rome* it will be 42 Min. past 12, because it is 10 $\frac{1}{2}$ Deg. more eastward than *Paris*, and so of others. These Lines of Longitude represent the Meridians of the Places attributed to them; so that when the Shadow of the Style falls upon any one of them, it will be Noon under that Meridian.

CHAP. IV.

Of the Construction and Uses of Instruments for drawing Dials upon different Planes.

THE eighth Figure represents a Quadrant made of Brass or any solid Matter, of a bigness at pleasure, having the Limb divided into 90 Degrees. The Use of this Quadrant may be to find the Lengths of Tangents, and by this means to divide a right Line into Degrees, as we did the Meridian of the horizontal Dial (Fig. 7.) we may find likewise thereon the Divisions of the Equinoctial Line thro' which the Hour-Lines must pass, in regular Dials; as also in declining Dials, if the Substyle falls exactly upon a compleat Hour-Line, by laying off the Length of the Radius of the Equinoctial Circle, from the Center A to H or L, and drawing a right Line, as H I or L M, parallel to the Radius of the Quadrant A C. For example, the Length L 1 or 11, answering to 15 Deg. of the Quadrant, shall be the Tangent of the first Hour-Line's distance from the Meridian or Substyle of the Dial, which being laid off upon the Equinoctial Line, whose Radius is supposed equal to A L, will determine a Point therein thro' which the said Hour-Line must be drawn. L 12, answering to 30 Deg. of the Limb of the Quadrant, will be the Tangent of the second Hour-Line's distance from the Meridian or Substyle. L 3, the Tangent of 45 Deg. will be that of the third, and so on. Now if by this means you draw the Hour-Lines of three Hours successively on each side the Meridian or Substyle, which in all make six Hours successively; these are sufficient for finding the Hour-Lines of the other Hours, according to the Method before explained in speaking of declining Dials, and which may be even applied to all regular Dials. For example, If the Hour-Lines of six Hours successive be drawn upon an horizontal Dial, as, from 9 in the Morning to 3 in the Afternoon, you may draw all the other Hour-Lines of the Dial by the aforesaid Method; as the Hour-Lines of 7 and 8 in the Morning, and 4 and 5 in the Afternoon, whose Points in the Equinoctial Line are some- Fig. 8.

sometimes troublesome to be pricked down, and principally the Points of the Hour-Lines of 5 and 7, because of the Lengths of their Tangents.

The Hour-Lines found by the abovesaid Method, which we shall not here repeat, will serve for finding of others; and these which are last found being produced beyond the Center, will give the opposite ones.

The said Quadrant will serve moreover as a portable Dial, since the Hour-Lines may be drawn upon it by means of a Table of the Sun's Altitude above the Horizon of the Place for which the Dial is to be made. See more of this in the next Chapter.

The Construction of a moveable horizontal Dial.

Fig. 9. This Instrument is composed of two very smooth and even Plates of Brass, or other solid Matter, adjusted upon each other, and joined together by means of a round Rivet in the Center A. The undermost Plate is square, the Length of the Side thereof being from 6 to 8 Inches, and is divided into twice 90 Degrees; by means of which, the Declinations of Planes may be taken. The upper Plate is round, being about 8 Lines shorter in Diameter than is the Length of the Side of the under Plate, and having a little Index joined to the Hour-Line of 12, shewing the Degree of a Plane's Declination.

About the Center A is drawn an horizontal Dial upon the upper Plate, for the Latitude of the Place it is to be used in, and the Axis B is so adjusted, that the Point thereof terminates in the Center A, wherein a small Hole is made for a Thread to come thro'. There is also a Compass D fastened to this upper Plate, having a Line in the Bottom of the Box, shewing the Variation of the Needle.

The Use of the moveable horizontal Dial.

The Use of this Instrument is for drawing Dials upon any Planes, of whatsoever Situations (as on declining inclining Planes, or both) in the following manner:

First draw a Horizontal or level Line upon the proposed Plane; place that side of the Square along this Line, whereon is wrote *the Side applied to the Wall*, and turn the horizontal Dial 'till the Needle settles itself over the Line of Declination in the Bottom of the Box: then extend the Thread along the Axis of the Dial 'till it meets the Plane, and the Point wherein it meets the said Plane will be the Center of the Dial. This being done, extend the Thread along each of the Hour-Lines of the horizontal Dial that the Plane can receive, and mark the Points on the horizontal Line upon the Plane, cut by the Thread: then if Lines be drawn from the Center found on the Plane thro' each of those Points, those will be the respective Hour-Lines that the Thread was extended along on the horizontal Dial, and must have the same Figures set to them. *Note*, If the Dial be vertical, not having any Declination, the Hour-Line of 12 will be perpendicular to the horizontal Line of the Plane.

The substylar Line is drawn thro' the Center of the Plane, and the Angular Point of a Square, one Side whereof being laid along the horizontal Line, and the other Side touching the Style of the horizontal Dial.

Again, The Distance from the Side of the Square laid along the Plane to the Axis, is the Length of the right Style, which being laid along in the same Place at right Angles to the Substyle, you may draw the Axis from the Center to the Extremity thereof; which may be formed on the Plane by means of an Iron Rod, parallel to the Situation of the Thread extended along the Axis of the horizontal Dial, and must be sustained by a Prop planted in the Plane perpendicular to the Substyle.

If you have a mind to have a right Style only, some Point must be sought in the Substyle distant from the Center of the Dial, proportional to the Bigness of the Dial, and an Iron-Rod must be set up perpendicularly therein: but the Point of this Rod must touch the Thread extended along the Axis. Finally, You may give what Figure you please to the Dial, and produce the Hour-Lines as is necessary, according to the bigness of the Plane. If a great Dial is to be drawn, you may place the Instrument at a Distance from the Plane it is to be drawn on; but then you must take care that it be very level, and the Side thereof parallel to the Plane. And if North Dials are to be drawn, having first found the Declination of the Plane, for Example, 45 Deg. North-westwardly, place the Index of the Dial over the Degree of the opposite Declination on the square Plate, *viz.* over 45 Deg. South-eastwardly, then invert the whole Instrument, and extend the Thread along the Axis, that so the Center of the Dial may be found upon the Plane underneath the horizontal Line, on which having pricked down the Points thro' which the Hour-Lines must pass, you may draw them to the Center, and then proceed as before.

The Construction of the Sciaterra.

Fig. 10. This Instrument is composed of an Equinoctial Circle A, made of Brass or any other solid Matter, adjusted upon a Quadrant B. The Point of the Hour of 12 of this Equinoctial Circle is fastened to one end of the Quadrant, and a little Steel Cylinder about two Lines in Diameter, serving for an Axis, and going thro' the Center of the Equinoctial Circle, is so fixed to the other End C of the Quadrant, as to keep the said Equinoctial Circle fixed at right Angles to the Quadrant.

The

The Quadrant is divided into 90 Deg. and is made to ſlide on the Top of the Piece L, according to different Elevations of the Pole. The little Ball G is hung at the End of a Thread, whoſe other End is faſtened to the Top of an upright Line on the Piece L, and ſo by means of this, and the Ball and Socket H, the Inſtrument may be ſet upright. The Piece I is of Steel, and the End thereof is forced into a Wall or Plane, to ſupport the whole Inſtrument when it is to be uſed. The Figure D is the Trigon of Signs put on the Axis, and turns about the ſame by means of a Ferril. This Trigon has a Thread F faſtened to the Extremity thereof, and there is another Thread E faſtened to the Center of the Dial. But note, We do not place the Trigon upon the Axis, unleſs when the Arcs of the Signs are to be drawn upon Dials.

The Uſe of the Sciaterra.

You muſt firſt force the Steel Point I, into the Wall or Plane whereon a Dial is to be drawn, and place the Quadrant to the Degree of the Elevation of the Pole: then you muſt take a Square Compaſs, and lay the Side thereof along the Plane of the Quadrant, and turn the Inſtrument until the Needle fixes itſelf directly over the Line of Declination; or if you have not a Compaſs when the Sun ſhines, and the Hour of the Day is known, turn the Inſtrument 'till the Shadow of the Axis falls upon the Hour of the Day upon the Equinoctial Circle.

The Inſtrument being thus diſpoſed, extend the Thread E from the Center along the Axis 'till it meets the Wall or Plane propoſed, and there make a Point for the Center of the Dial: then extending the ſaid Thread over each Hour of the Equinoctial, note the Points wherein it meets the Wall or Plane, and draw Lines from the Center (before found) thro' them, and thoſe will be the Hour-Lines. After this, you may give the Dial what Figure you pleaſe, and ſet the ſame Figures upon the Hour-Lines as are upon the correſpondent Hours of the Equinoctial Circle. *Note*, The Style is ſet up in the manner we have mentioned in ſpeaking of the moveable horizontal Dial.

If the Arcs of the Signs, or diurnal Arcs, are to be drawn upon the Dial, you muſt put the Ferril at the End of the Trigon upon the Axis, and fix it over each Hour of the Equinoctial one after another by means of the Screw: then extending the Thread F along the Lines appertaining to each Sign, mark as many Points on each Hour-Line on the Wall or Plane, and join them by curve Lines, which ſhall form the Arcs of the Signs, whereon muſt be ſet their reſpective Characters.

The Arcs of the Signs may be otherwiſe drawn in the following manner: The Axis of the Dial being well fixed, chuſe a Point in the ſame for the Extremity of the right Style, repreſenting the Center of the Earth; and upon this Axis put the Ferril of the Trigon in ſuch manner, that the Extremity of the right Style exactly anſwers to the Vertex of the Trigon, repreſenting the Center of the Equinoctial and the World. Then having fixed the Trigon by means of the Screw preſſing againſt the Axis, turn it ſo that one of the Planes thereof (for the Signs ought to be drawn upon both ſides) falls exactly upon the Hour-Lines one after another, and extend the Thread F along the Radius's of the Signs on the Trigon, and by means thereof mark Points upon each Hour-Line of the Wall or Plane: and if theſe Points be joined, we ſhall have the Arcs of the Signs.

Proceed thus for drawing North Dials, as likewiſe inclining and declining Dials, in obſerving to invert the Inſtrument when the Centers of the Dials are downwards.

The Conſtruction of M. Pardie*'s Sciaterra.*

This Inſtrument, which is made of Braſs or other ſolid Matter, of a Bigneſs at pleaſure, conſiſts of four principal Pieces or Parts. The firſt is a very even ſquare Plate D, called the horizontal Plane, becauſe it is placed horizontal or level when uſing, having a round Hole E in the Middle, wherein is placed a Pivot, upon which turns the ſecond Piece, called the meridional Plane, in ſuch manner that the ſaid Piece is always at right Angles to the horizontal Plane. On the narrow Side C of this Piece is faſtened a Plumb-Line, whoſe uſe is for placing the Inſtrument level. The Top of this Piece is cut away into a concave Quadrant, both ſides of which are divided into 90 Deg. beginning from the Perpendicular anſwering to the Middle of the Pivot, and there is a pretty deep Slit made down the Middle of this Quadrant to receive a prominent Piece of a Semi-Circle H, which is the third principal part, that ſo the ſaid Semi-Circle may be in the ſame Plane as the ſecond Piece is, and likewiſe be raiſed or lowered according to different Elevations of the Pole. The Diameter of this Semi-Circle is called the Axis, and the Center thereof is ſimply called the Center of the Inſtrument, like as the Thread faſtened thereto is called the centeral Thread. The fourth Piece A is a very even Circle, both ſides thereof being divided into 24 equal Parts, for the 24 Hours of the Day; and this is fixed at right Angles to the Semi-Circle H, and ſo moves along with it. One of the Sides thereof is called the upper-ſide, and the other the under-ſide. The Trigon of Signs is drawn (in the manner before explained) upon both ſides of the Semi-Circle, having the Point A, the Extremity of the Diameter of the Equinoctial Circle, for the Vertex thereof. Fig. 11.

The Use of this Instrument.

Having first placed the Points of ♈ and ♎ of the Semi-Circle upon the Degree of the Elevation of the Pole in the Place for which you would draw a Dial, set the Instrument upon a fixed horizontal Plane, near to the Wall or Plane you are to draw a Dial on. Then turn the Meridional Plane 'till the Shadow of the Equinoctial Circle falls upon the Day of the Month or Degree of the Sign on the Axis the Sun is in. This being done, the Shadow of the said Axis or Diameter of the Semi-Circle H, will shew the Time of Day upon the Equinoctial Circle, and the whole Instrument will be well situated, the Meridional Plane answering to the Meridian of the Heavens, the Equinoctial Circle parallel to the Celestial Equinoctial Circle, and the Axis of the Dial parallel to the Axis of the World. This being done, extend the Thread F fastened to the Center, along the Axis to the Wall or Plane you are to draw a Dial on, and the Point wherein it meets the Wall will be the Center of the Dial. The said Thread thus extended will likewise give the Position of the Style or Axis of the Dial; for if an Iron Rod be placed in the said Point of Concourse, and in the same Situation as the Thread is, this will be the Style of the Dial: but if you have a mind to have a right Style only, you need but set up a Rod in the Wall or Plane, whose end touches the Thread extended along the Axis of the Instrument; and this Rod may have what Figure you please given to it, as a Serpent or Bird, provided the Extremity of the Bill thereof meets the said Thread.

Now to mark the Hour-Lines upon the Dial, extend the Thread from the Center over the Plane of the Equinoctial Circle along the Hour-Lines thereof one after another, until it meets the Wall: then if Lines be drawn from the Center of the Dial to the said Points of Concourse, these will be the Hour-Lines. But the Hour-Lines may be otherwise pricked down in the Night, by the light of a Link or Candle; for the centeral Thread being first extended along the Axis, and fastened to the Wall, afterwards move the Link 'till the Shadow of the Axis falls upon any given Hour upon the Equinoctial Circle, and then the Shadow of the said extended Thread upon the Wall will be the same Hour-Line; and by drawing a Line upon the Wall along the same with a Pencil, that will be the Hour-Line.

Proceed thus for drawing the other Hour-Lines. *Note*, This Method of drawing Dials is a very good one, particularly when a Surface is not flat and even, or when the Center of the Dial falls at a great Distance. You must observe likewise, that the Shadow of the Axis of the Instrument shews the Time of Day on the upper-side of the Equinoctial Circle from the 20th of *March* (N. S.) to the 22d of *September*, and on the under-side the other six Months; and the Side of the Equinoctial Circle that the Sun shines upon, must always but just touch the Center of the Semi-Circle.

CHAP. V.

Of the Construction and Uses of Portable Dials.

Of the Construction of a Globe.

Fig. 12. THIS Figure represents a Globe, whereon are drawn the Meridians or Hour-Circles. There are divers Sizes of them; the great ones are set up in Gardens, and are of Stone or Wood well painted, and the small ones are made of Brass, having Compasses belonging to them, and may be reckoned among the Number of Portable Dials.

The manner of turning round Balls of any Matter is well known, but if a large Stone-Ball is to be made, that cannot be turned because of it's Weight: first, you must nighly form it with a Chissel, and then take a wooden or brass Semi-Circle of the same Diameter as you design your Ball. This being done, turn the Semi-Circle about the Ball, and take away all the Superfluities with a Raspe, until the Semi-Circle every where and way just touches the Superficies thereof; afterwards make it smooth with a Pumice-Stone or Sea-Dog Fish's Skin, *&c.*

The Globe being well rounded and made smooth, you must take the Diameter thereof with a Pair of Spheric Compasses, *viz.* such whose Points are crooked, which suppose the right Line A B; this Line is divided into two equal Parts in E by the vertical Line Z N, the upper Point whereof Z, represents the Zenith, and the lower one N, the Nadir. Now set one Point of the Spheric Compasses in E, and extend the other to A, and draw the Meridian Circle A Z B N; likewise setting one Foot of your Compasses in Z, with the last Opening describe the Circle A E B, representing the Horizon; and from the Point B to C count 49 Deg. the Elevation of the Pole on the Meridian, and setting one Foot of your Compasses in the Point C, representing the North Pole, extend the other to 41 Deg. on the Meridian below the Point B, and draw the Equinoctial Circle; likewise setting one Foot of your

your Compaffes, opened to the fame Diftance as before, upon the Point in the Meridian cut by the Equinoctial, you may draw the Hour-Circle of 6 paffing thro' the Poles C and D. By this means the Equinoctial fhall be divided into four equal Parts by the Meridian and Hour-Circle of 6; and if each of thefe four Parts be divided into fix equal Parts, for the 24 Hours of a Natural Day, and about the Points of Divifion as Centers, with the extent of a Quadrant of the Globe, Circles be defcribed; thefe will all pafs thro' the Poles of the World C and D, and are the Hour-Circles. If you have a mind to have the half Hours or Quarters, each of the Divifions on the Equinoctial muft be divided into 2 or 4 equal Parts. The Hour-Circles are numbered round the Equinoctial both above and below it, as appears per Figure.

If the Parallels of the Signs are to be drawn upon the Globe, you muft count upon the Meridian both ways from the Equinoctial, the Declination for every Sign, according to the Table expreffed; as, for Example, For the two Tropicks you muft count 23 Deg. 30 Min. from the Equinoctial, and about the Poles C and D, draw Circles on the Globe. *Note*, The two Polar Circles muft be drawn at 23 Deg. 30 Min. from the Poles, or 66 Deg. 30 Min. from the Equinoctial.

The Globe thus ordered muft be placed upon a Pedeftal proportionable to the bignefs thereof in a Hole made in the Nadir N, diftant from the Pole the Complement of it's Elevation (*viz.* 41 Deg.) and fixed in a Garden, or elfewhere, well expofed to the Sun, fo as to be conformable to the Sphere of the World.

But if it be a fmall Portable Globe, we place a little Compafs upon the Pedeftal thereof, that fo the Globe may be fet North and South when the Hour of the Day is to be fhewn thereby, which is fhewn thereon without a Style, by the Shadow of the fame Globe: for the Shadow or Light thereon always occupies one half of the Globe's Convexity, when the Sun fhines upon it; and fo the Extremity of the Shadow or Light fhews the Hour in two oppofite Places. If, moreover, the different Countries on the Earth's Superficies, as likewife the principal Cities, are laid down upon the Globe according to their true Latitudes and Longitudes, you may difcover any Moment the Sun fhines upon the fame, by the illuminated part thereof, what Places of the Earth the Sun fhines upon, and what Places are in Darknefs. The Extremity of the Shadow fhews likewife what Places the Sun is rifing or fetting at; and what Places have long Days, and what have fhort Nights: you may likewife diftinguifh thereon the Places towards the Poles that have perpetual Night and Day. All this is eafy to be underftood by thofe who are acquainted with the Nature of the Sphere. *Note*, This Dial is the moft natural of all others, becaufe it refembles the Earth itfelf, and the Sun fhines thereon as he does on the Earth.

You may find the Hour of the Day otherwife, by means of a thin brafs Semi-Circle divided into twice 90 Deg. adjufted to the Poles or Extremes of the Axis, by help of two little Ferrils. This Semi-Circle being turned about the Globe with your Hand, until it only makes a perpendicular Shadow upon the Globe, reprefents the Hour-Circle wherein the Sun is, and confequently fhews the Hour of the Day, and alfo what Places of the Earth it is Noon at that Time. But in this Cafe the Number 12 muft be fet to the Meridian, and the Numbers 6 and 6 to the two Points wherein the Equinoctial cuts the Horizon: and this is the reafon why we commonly place two rows of Figures along the Equinoctial. The Shadow of the two ends of the Axis, if they are continued out far enough beyond the Poles, and the Hours are figured round the Polar Circles, will likewife fhew the Hour. *Note*, In order to make fmall Portable Globes univerfal, we adjuft Quadrants underneath them, that fo the Pedeftal may be flid according to the Elevation of the Pole. This is eafy to be underftood.

The Conftruction and Ufe of the Concave and Convex Semi-Cylinder.

Thefe Dials, which are made of different bigneffes, the fmall ones of Brafs, and the great ones of Stone or Wood, are very curious on account of their fhewing the Hour of the Day without a Style. Their Exactnefs confifts very much in being very round and even both within fide and without.

The 13th Figure reprefents one of thefe Dials, fet upon and faftened on it's Pedeftal, inclining to the Horizon under an Angle equal to the Elevation of the Pole, and directly facing the South: and therefore the Hour-Lines and the Edges A B, *a b*, ferving as a Style, are all parallel between themfelves, and to the Axis of the World. The whole Convex Cylinder is divided into 24 equal Parts, or twice 12 Hours, by parallel Lines; and the Concave Semi-Cylinder is divided into 6 equal Parts by Right Lines, which are the Hour-Lines from 6 in the Morning, to 6 in the Afternoon. Fig. 13.

Now when the Sun fhines upon this Dial, the Hour of the Day is fhewn on the Convex fide thereof, by the defect of Light, that is, by a right Line feparating the Light from the Shadow. But the Hour of the Day is fhewn in the Concave part of the Dial, by the Shadow of one of the Edges A B or *a b*; fo that when the Sun in the Morning is come to the Hour-Circle of 6, the Shadow of the eaft Edge *a b* will then fall upon the other Edge A B, which is the Hour-Line of 6: and as the Sun rifes higher above the Horizon, the Shadow of the faid Edge *a b* will defcend and fhew the Hour among the Hour-Lines. (*Note*, The

The Figures on the Top are for the Morning Hours, and those on the Bottom for the Afternoon ones.) When the Sun is come to the Meridian, he directly shines into the Dial, and then the Edges will cast no Shadow: but when the Sun has passed the Meridian, and descends westwards, the Shadow of the opposite Edge A B will shew the Hour from 12 to 6 in the Evening. If you have a mind to have the Halves and Quarters of Hours, you need but double or quadruple the Divisions.

Small Dials of this kind have Compasses belonging to them, that so the Dials may be set North and South.

The Construction and Use of the Vertical Cylinder.

This is a vertical Dial drawn upon the Superficies of a Cylinder by means of a Table of the Sun's Altitude above the Horizon at every Hour, when he enters into every 10th Degree of the Signs, according to the Latitude of the Place for which the Dial is to be drawn; and for this end the following Table is calculated for 49 Degrees of Latitude.

A TABLE *of the Sun's Altitudes for every Hour of the Day at his Entrance into every* 10*th Degree of the Signs, for the Latitude of* 49 *Degrees.*

Hours.		XII.		XI. I.		X. II.		IX. III.		VIII. IV.		VII. V.		VI. VI.		V. VII.	
Signs.		D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
30	♋	64	30	61	56	55	19	46	35	37	1	27	10	17	30	8	21
20	10	64	9	61	33	55	1	46	18	36	42	26	54	17	10	8	4
10	20	63	2	60	31	54	1	45	28	35	5	26	6	16	20	7	12
♊	♌	61	12	58	49	52	34	44	7	34	39	24	50	15	6	5	50
20	10	58	48	56	30	50	29	42	14	32	53	23	6	13	20	3	57
10	20	55	52	53	42	47	57	39	55	30	41	20	57	11	11	1	40
♉	♍	52	30	50	30	45	1	37	14	28	10	18	28	8	40		
20	10	58	51	46	48	41	44	34	13	25	19	15	43	5	44		
10	20	44	58	43	12	38	15	31	0	22	18	12	48	2	59		
♈	♎	41	0	39	20	34	37	27	38	19	9	9	47				
20	10	37	2	35	26	30	58	24	15	15	58	6	42				
10	20	33	9	31	40	27	24	20	55	12	51	3	44				
♓	♏	29	30	28	4	23	58	17	42	9	50	0	54				
20	10	26	8	24	46	20	51	14	45	7	6						
10	20	23	12	21	52	18	5	12	12	4	43						
♒	♐	20	48	19	30	15	48	10	3	2	42						
20	10	18	48	17	44	14	6	8	27	1	13						
10	20	17	52	16	38	13	3	7	27	0	19						
♑	30	17	30	15	15	12	42	7	8								

We now proceed to shew the Construction of the aforesaid Dial upon a Plane which afterwards may be made Cylindrical, or wrapped round a Cylinder; or this Dial may be made upon the Surface of a Cylinder itself, if the Lines be drawn thereon as upon a Plane.

Fig. 14. Describe the Right-angled Parallelogram A B C D upon a brass Plate or Sheet of Paper, whose Breadth A B or C D let be nearly equal to the Circumference of the Cylinder it is to be wrapped round, and prolong the Line A B, upon which assume A E for the Length of the Style, which shall determine the Length of the Cylinder. Then about the Point E, as a Center, with the Radius E A, make a circular Arc equal to the Sun's Meridian Altitude at his entrance into *Cancer*, and draw the occult Line E D, determining the Length or Height of the Cylinder A D; but if this length was given, and the Length of the Style required, you must describe an Arc about the Point D, equal to the Complement of the Sun's greatest Meridian Altitude, which, if the greatest Altitude be 64 Deg. 30 Min. will be 25 Deg. 30 Min. and draw the occult Line D E, which shall determine the Length of the Style E A, proportioned to the Length of the Cylinder.

This being done, divide the Arc A F into Degrees and Minutes, and draw occult Lines thro' each of the Points of Division, from the Center E to the Line A D, that so this Line may

may be made a Scale of Tangents. But this Line may be otherwise divided, by supposing the Radius A E 100 or 1000 equal Parts, according to the Length of the Cylinder, and taking the correspondent Tangents from printed Tables, and laying them off from A.

Things being thus ordered, divide the Sides A B, D C, into 6 equal Parts, and join the Points of Division by five parallel right Lines, which will represent the beginnings of the twelve Signs; then trisect each of these parallel Spaces for the 10th and 20th Degree of each Sign. Now by this means the Beginnings of the Months may be set upon your Dial, because there will be no sensible Error in fixing the Sun's Entrance into every Sign the 20th Day of every Month (N. S.). Then to prick down the Hour-Points upon all these Lines one after another, you must use the foregoing Table: for example, to prick down the Hour-Point of 10 in the Morning, or 2 in the Afternoon, upon the Line A D representing the Summer Tropick, you will find by the Table, that the Sun's Altitude at the Time of the Summer Solstice at the Hours of 10 or 2, is 55 Deg. 19 Min. therefore you must take the Tangent of 55 Deg. 19 Min. from your Scale of Altitudes A D, and lay off from the Side A B upon the said Tropick, and then you will have a Point therein thro' which the proposed Hour-Line must pass. Again, To prick down the Hour-Point of the said Hour of 2 upon another Parallel, suppose on that of the 1st Degree of *Leo* or *Gemini*, you will find by the Table that the Sun's Altitude will then be 52 Deg. 34 Min. and the Tangent of these Degrees being taken from the Scale of Altitudes A D, and laid off upon the said Parallel from A B, will give a Point therein thro' which the Hour-Line of 2 must pass. And if you proceed in this manner, and find Points in the other Parallels, and likewise on their Divisions of every 10th and 20th Degree; these Points joined will give the curved Hour-Line of 10 in the Morning, or 2 in the Afternoon.

And thus likewise may be found Points in the Parallels thro' which the other Hour-Lines must pass; which being done, you must join all those belonging to the same Hour by an even Hand, and mark the Characters of the Signs, the first Letters of the Months, as likewise the Hour-Figures, each in their respective Places, as *per* Figure, and your Dial will be finished; which afterwards must be wrapped about the Cylinder, or bent Cylindrically, so that the Lines representing the two Tropicks be parallel between themselves.

The Style is fastened to a Chapiter serving as an Ornament, and must be moveable on the Line A B, that so it may be placed at right Angles on the Degree of the Sign or Day of the Month. This Dial being placed upright, or hung by a Ring, turn it to the Sun, so that the Shadow of the Style may fall down right upon the Parallel of the Day you desire to know the Hour in, and then the Extremity thereof will shew the Hour or Part.

The Sun's Altitude may be shewn likewise by this Instrument thus: Put the Style upon the Scale of Altitudes, keeping the Cylinder suspended or horizontally placed, and turn it about so that the Style be towards the Sun; then the Shadow of the Extremity thereof shall shew the Sun's Altitude above the Horizon.

The abovesaid Parallelogram may serve likewise as a Dial, without being wrapped round a Cylinder, or turned up cylindrically, if the Style be so adjusted as to slide along the Line A B, that so it may be set over the Day of the Month, or Parallel of the Sign the Sun is in. This is easily done, in making a little Slit along the Top of the Plate, and flatting the Foot of the Style, so that it may slide in the said Slit without varying it's Length. Now if this Parallelogram be placed upright, and the Line A B level (which may be easily done by means of a Plumb-Line fastened to one of the Sides), and you hold it thus in your hand, or suspend it by a Ring, so that it be directly exposed to the Sun, and the Shadow of the Style falls upon the Parallel of the Sign or Month; then the Extremity of the Shadow of the said Style will fall upon the Hour.

The Construction and Use of a Dial drawn on a Quadrant.

This Figure represents a Portable Dial drawn on a Quadrant, whose Construction we have thought fit to lay down here, since it is made, as well as the Cylindrical Dial, by means of a Table of the Sun's Altitude calculated for the Latitude of the Place the Dial is made for. Fig. 8.

First, Divide the Limb B C of the Quadrant into Degrees, and about the Center A describe another Arc R S, representing the Tropick of ♋. Likewise divide the Radius A B nearly into 3 equal Parts, and with the Distance A D draw a circular Arc for the Tropick of ♑; divide the Space B D into 6 equal Parts, and describe the like Number of circular Arcs about the Center A, which shall represent the Parallels of the other Signs, as they are denoted on the Side A C of the Quadrant. The next thing to be done, is to draw the Hour-Lines. Let it be required (for Example) to find a Point in the Tropick of ♋ thro' which the Hour-Line of 12 must pass: By the above posited Table, the Sun's Altitude (at *Paris*) at the said Time is 64 Deg. 29 Min. therefore take a Thread, or Ruler fastened to the Center A, and extend it to that Number of Degrees and Minutes on the Limb of the Quadrant, and where the Thread or Edge of the Ruler cuts the Tropick of ♋, will be one Point thro' which the Hour-Line of 12 must be drawn. Then seek the Sun's

Sun's Altitude when he enters into ♊, which being found 61 Deg. 12 Min. lay the Thread over 61 Deg. 12 Min. on the Limb, and where it cuts the Parallel of ♊, make a Mark for a Point in the said Parallel thro' which the Hour-Line of 12 must pass. And if you proceed in this manner, Points may be found in the Parallels, or their Parts (if the Quadrant be big enough), thro' which the Hour-Line of 12 must pass, as likewise all the other Hour-Lines; and if the Points be joined, the curve Hour-Lines will be had, and the Dial will be finished, when there are two Sights fixed upon the Side A C.

The Use of this Quadrant.

Direct the Plane of the Instrument towards the Sun in such manner, that his Rays may pass thro' the Holes of the Sights G G, and then the Plumb-Line freely playing, will shew the Time of Day by intersecting the Parallel that the Sun is in. But if you put a little Bead or Pin's Head upon the Plumb-Line, then you may extend the Thread from the Center, and slide the Bead thereon, and fix it over the Degree of the Sign or Day of the Month, and holding up the Quadrant, as before, the Bead will fall upon the Hour of the Day.

The Construction and Use of a Particular right-lined Dial.

Fig. 15. This Dial, which we call Particular, because it serves but for one determinate Latitude, is made upon a very even Plate of Brass, or other Metal, about the bigness of a playing Card. The Construction thereof is thus: First, draw the two right Lines A B, C D, crossing one another at right Angles in the Point E, about which, as a Center, with the Radius E C describe the Circle C B D, and divide it into 24 equal Parts, beginning from the Point D; then thro' each two Divisions thereof equally distant from the Points C and D, draw parallel right Lines, which will be the Hour-Lines, whereof D R is that of 12, B E that of 6, and C M that of Midnight. This being done, form the right-angled Parallelogram P M Q R, and draw the occult Line D R, making an Angle with C D equal to the Elevation of the Pole, *viz.* 49 Deg. This Line shall represent the Radius of the Equinoctial, and by means thereof the Trigon of Signs must be formed, having D for it's Vertex. In order for this, produce the Hour-Line of the Sun's rising in the longest Day of Summer, which here is the Hour-Line of 4; as likewise the Hour-Line of 6, until it meets the Radius of the Equinoctial Circle D R; then the Point in the Radius of the Equinoctial cut by the Hour-Line of 6, will be the Center of a Circle, whose Diameter shall be perpendicular to the said Radius, and is terminated by the Intersection of the Hour-Line of 4 therewith. This Circle being described, divide the Circumference thereof into 12 equal Parts, in order to form the Trigon of Signs, as is before explained in the third Chapter of this Book. *Note*, The two Tropicks will be at the Extremities of the said Diameter, each making an Angle of 23 Deg. 30 Min. with the Radius of the Equinoctial, the Vertex being the Point D. Now the next thing to be done, must be to make a little Slit along the Radius of the Equinoctial, that so a little Slider or Cursor may slide along it, having a little Hole drilled thro' it for fastening a Thread and Plummet with a Bead or Pin's Head on the Thread. And after this, we place two Sights on the Extremities of the Line P Q.

The Use of this Dial.

Slide the Cursor, and fix the Hole in which the Thread is fastened over the Degree of the Sign the Sun is in, or the Day of the Month; then slip the Bead or Pin's-head along the Thread, until it be upon the Hour-Line of 12. This being done, hold up your Instrument, lifting it higher or lower 'till the Sun shines thro' the Holes of the Sights R and S, and the Thread freely plays upon the Plane thereof; then the Bead will fall upon the Hour of the Day.

The Construction of an Universal right-lined Dial.

Fig. 16. This right-lined Dial, which serves for all Latitudes, is made of a bigness at pleasure, upon a very even Plate of Brass or other solid Matter. The Construction of it is thus: Draw the Lines A B, C D, cutting each other at right Angles in the Point E, about which, as a Center, describe the Quadrant A F, which divide into 90 Deg. and with the Point E for the Vertex, make a Trigon of Signs according to the Method explained in Chap. 2. Divide each Sign into 3 Parts, each being 10 Deg. and set the first Letters of the Months to the Places corresponding to them, by supposing (as we have already) that the Sun's Entrance into every Sign is the 20th Day of the Month (N. S.); for Example, his Entrance into ♈ the 20th of *March*, his Entrance into ♉ the 20th of *April*, &c. This may be without any sensible Error in so small an Instrument. Now draw dotted Lines from the Center E thro' the Divisions of the Quadrant A F, to the Line A G, which will divide it into Points, from which Parallels must be drawn to the Line A B, which shall be the different Latitudes or Elevations of the Pole, which must be only marked between the two Tropicks, as you see in the Figure, wherein they are drawn to every 5th Deg. On both sides the Point B lay off upon the Line B H, the Divisions that the Radii of the Signs of the

the Trigon make on the Line *a a*, reprefenting the Latitude of 45 Deg. that fo the Reprefentation of another Zodiack may be made upon the Line B H.

Now the manner of drawing the Hour-Lines upon this Dial is thus: Draw Lines thro' every 15th Deg. of the Quadrant A F, parallel to E D, which is the Hour-Line of 6; and thefe Parallels will be the Hour-Lines from 6 in the Evening to 6 in the Morning, A L being the Hour-Line of Midnight. And if the parallel Spaces be laid off on the other fide of the Hour-Line of 6, you will have the Hour-Lines from 6 in the Morning to 6 in the Evening. And for drawing the half Hours, divide each 15th Deg. of the Quadrant A F into half, and draw Parallel Lines between the Hour-Lines.

The Hour-Lines may be yet otherwife drawn, by means of a Circle, whofe Diameter is the Line A B, and whofe Circumference is divided into 24 equal Parts for the 24 Hours of the Day, or into 48, for the Half-Hours. For then if right Lines be drawn thro' the oppofite Points of Divifion, parallel to E D, we fhall have the Hour-Lines, and thofe of the Half-Hours, as we have faid in the Conftruction of the former right-lined Dial.

About the Point I, as a Center, draw an occult Quadrant, which divide into 90 Deg. and laying a Ruler to the Center I, and on each Divifion mark the fame Degrees upon the Sides G Q, and G S of the Inftrument. *Note*, By means of thefe Divifions we may find the Sun's Altitude above the Horizon, as we fhall fhew by and by. R R are two Sights fixed on the Side G H. And the Piece K is a fmall Arm or Index, made of 3 Blades of Brafs, fo joined to each other by headed Rivets, that they may have a Motion either to the right or left: at the fharp end of this Arm is made a very little Hole, thro' which goes a Thread with a Plummet at the end thereof, and a little Bead or Pin's Head thereon. This little Arm is faftened to the Inftrument with a headed Rivet, that fo it may have a Motion at the Place K.

The Ufe of this Dial.

If the Hour of the Day be to be found by this Inftrument, you muft adjuft the End *a* of the Index on the Interfection of the Line of the Latitude of the Place, and the Degree of the Sign the Sun is in, or the Day of the Month; then extend the Thread, and flide the Bead to the fame Degree of the Sign in the little Zodiack, drawn on the Hour-Line of 12 B I. This being done, hold the Inftrument up until the Sun fhines thro' the Sights R R, and the Thread freely playing upon the Plane of the Inftrument, the Bead will fall upon the Hour of the Day.

If the Time of the Sun's rifing and fetting in all the Signs of the Zodiack for the Latitudes denoted upon the Inftrument be required, fix the End *a* of the Index on the Interfection of the Line of the Latitude of the Place, and the Degree of the Sign the Sun is in; then the Thread freely falling parallel to the Hour-Lines, will fhew the Hour of the rifing and fetting of the Sun. For Example, The End of the Index being fixed on the Interfection of the Sign of ♋, and the Line of the Latitude of 49 Deg. the Thread will fall along the Hour-Line of 4 in the Morning, or 8 in the Evening: and this fhews, that about the 20th of *June*, (N. S.) the Sun rifes at *Paris*, at 4 in the Morning, and fets at 8 in the Evening, and fo of others.

The Elevation of the Pole is found thus: Place the End of the Index on the Point I, and raife or lower the Inftrument until the Sun's Rays pafs thro' the Holes of the Sights; then the Thread freely playing, will fhew the Sun's Altitude upon the Degrees on the fide Q S or Q G.

All thefe kinds of Dials, that fhew the Hour of the Day by the Sun's Altitude, are convenient in this, that they fhew the Time of Day without a Compafs; but their common Imperfection is, that about Noon the Hour cannot be exactly determined by them, unlefs by feveral Obfervations to know whether the Sun increafes or decreafes in Altitude, and confequently whether it is before or after Noon.

The Conftruction of an horizontal Dial for feveral Latitudes.

This Dial, which is made upon a very even and fmooth Plate of Brafs, or other folid Matter, hath a little Piece of Brafs in form of a Bird, the lower part of which is adjufted in two little knuckles, that fo it may be rendered moveable, and lie down upon the Plane of the Dial. This Bird is kept upright by means of a Spring that is underneath the Dial-Plate, which going thro' a little fquare Hole in the Plate, keeps the Bird firm upon it's Foot. There is a Style or Axis going into the Bird, which is double, the lower End of which goes into a little Knuckle at the Center of the Dial, that fo the faid Style may be raifed or lowered, according to the Latitude. There is on the Style a circular Arc, whereon the Degrees are fet down from 35 or 40 to 60. There is a Slit made along this divided Arc, paffing by the Eye of the Bird, that fo it's Bill may be fet to the Degree of the Pole's Elevation, and fixed there. The Dial-Plate is hollowed in circular, that fo a Compafs may be added thereto, faftened underneath by two Screws. The Needle and the Glafs covering it, are placed in the fame manner as in other Compaffes, of which we have already fpoken. *Plate* 24. Fig. 1.

The

The Surface of this Dial is divided into 4 or 5 Circumferences for the like Number of different Latitudes, according to some one of the Methods before laid down for drawing of horizontal Dials, whereof that by the Calculation of Angles is most in use for such small Dials as these. They may be drawn also by means of a Plat-form, upon which are several Dials divided by the Rules before given. But this is well known to the Instrument-Maker.

The outmost Circumference, which is divided for 55 Deg. of Latitude, may well enough serve for those Places contained between the 58th and 53d Deg. of Latitude. The second, which is divided for 50 Deg. of Latitude, may serve for Places contained between the 53d and the 47th Deg. of Latitude. The third, which is divided for 45 Deg. may serve for Places between the 47th and 42d Deg. And the fourth, which is divided for 40 Deg. serves for Places contained between the 42d and 38th Deg. of Latitude.

When a 5th Dial is drawn upon the Plate for the Latitude of 35 Deg. this serves for all Places contained between the 37th and the 32d Deg. of Latitude. Now by means of a good Map of the World, or Globe, you may see at what Places these Dials will be in use; for that which is made for one Latitude, will serve for all Places round about the Earth, having the same North and South Latitude. We commonly grave underneath the Dial a Table of the principal Cities of the World with their Latitudes and Longitudes; that so the convenient Circumference on the Plate may be chose, and the Axis of the Dial raised to the proper Elevation of the Pole.

The Use of this Dial.

To find the Hour of the Day, raise or lower the Style, so that the End of the Bill of the little Bird may answer to the Degree of the Elevation of the Pole marked on the Style, as at *Paris* against the 49th Degree. The Style being thus raised, place the Dial parallel to the Horizon, that is, level, and turn it so to the Sun 'till the North Point of the Needle usually marked with a little Ring, fixes itself over the Line of Declination, whereon is a *Flower-de-luce*, and *North* is writ. Then the Shadow of the Style will shew the Hour of the Day upon the Circumference divided for the Latitude of the Place. You must take care not to set the Dial near Iron, for this changes the Direction of the Needle.

The Construction of a Ring Dial.

Fig. 2. Take a very round Ring of Brass, or other solid Matter, about two Inches in Diameter, four or five Lines in breadth, and of a convenient Thickness, and assume the Point A at pleasure thereon (whereat there is a little Hole), about which, as a Center, describe a Quadrant A D C, which divide into 90 Degrees. Then find the Sun's Altitudes in the foregoing Table at every Hour when he is in the Equinoctial for the Latitude of *Paris*, and laying a Ruler from the Center A thro' those Altitudes assumed on the Quadrant, you may draw Lines which will divide the concave Surface of the Ring into the Hour-Points. Now this Dial will be very good for the Times of the Equinox, it being suspended by the Ring B, so that the Line A D is upright.

But one of these Dials may be made for shewing the Hour of the Day at any other time of the Year, if the Hole A be made moveable. For doing which, make the Arcs A E, A I, 23 Deg. for the Signs ♉, ♍, ♏, and ♓; A F, A K, 40 Deg. 26 Min. for the Signs ♊, ♌, ♒, and ♐; and the Arcs A G, A L, 47 Deg. for the Signs ♋ and ♑. (The reason why we assume these Arcs double, is, because Angles at the Circumference are but half those at the Center.) Now by this means we shall have a Kind of Zodiack upon the convex Surface of the Ring, whereon must be marked the Signs in their proper Places, or else the first Letters of the Months, that so the Hole A may be put to the Degree of the Sign, or the Day of the Month.

You must describe likewise 7 Circles in the concave Surface of the Ring, whereof that in the Middle will be for the Equinoctial, and the others for the other Parallels. This being done, about the Points A, E, F, G, I, K, L, as so many Centers, describe Quadrants of 90 Deg. upon which Quadrants assume the Altitudes of the Sun every Hour when he is in every of the Signs, and produce the Radii drawn from the Centers to the Points of Assumption, until they cut the Circumferences in the concave Part of the Ring, and you will have Points thereon for the Hour-Lines, which must be joined.

Note, These Divisions may be separately drawn, and afterwards transferred on the Ring.

The Use of this Dial.

Place the moveable Hole at the Degree of the Sign wherein the Sun is; then holding the Ring suspended, turn it towards the Sun, so that his Rays passing thro' the Hole A, may fall upon the convenient Circumference of the Sign in the concave Part of the Ring, and then you will have the Hour of the Day shewn.

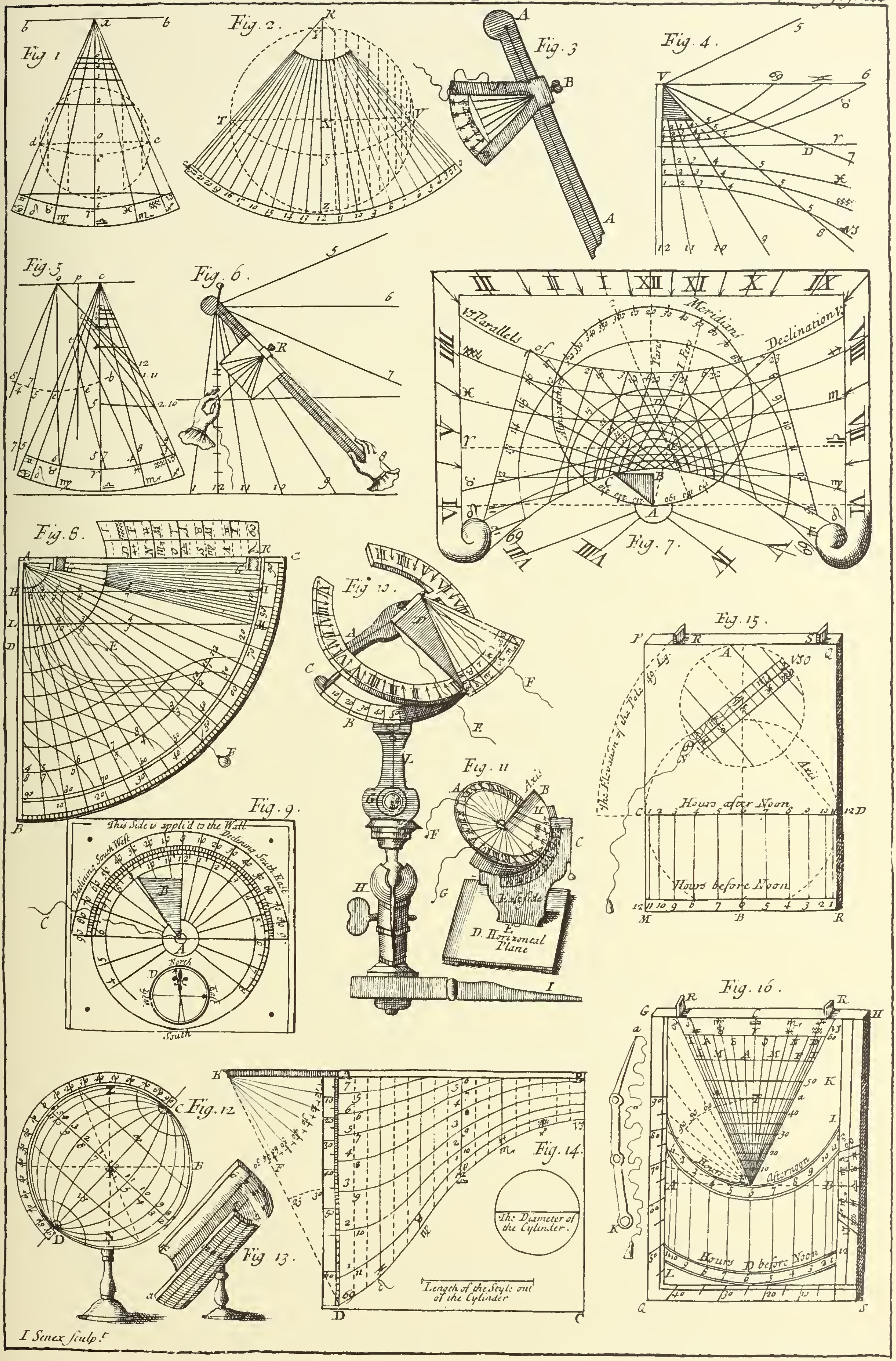
Fig. 1
Fig. 2.
Fig. 3
Fig. 4.
Fig. 5
Fig. 6.
Fig. 7.
Parallels of Declination
Meridians
Fig. 8.
Fig. 9.
This Side is appli'd to the Wall
Declining South West
Declining South East
North
South
Fig. 10.
Fig. 11
Axis
East side
D. Horizontal Plane
Fig. 12
Fig. 13.
Fig. 14.
The Diameter of the Cylinder.
Length of the Style out of the Cylinder
Fig. 15.
The Elevation of the Pole
Axis
Hours after Noon
Hours before Noon
Fig. 16.
Hours Afternoon
Hours before Noon
I Senex sculp.t

To deſcribe the Hour-Lines upon another ſort of Ring.

The fourth Figure repreſents this Ring compleat, and the Parallelogram A B C D, repreſents it laid open or ſtretched upon a Plane, that ſo the Hour-Lines may be pricked down thereon before it be turned up circularly. Fig. 3.

This Ring is made of a blade of Braſs, or other ſolid Matter, being in length proportionable to the Bigneſs you would have the Ring, and at leaſt 4 or 5 Lines broad, with a proportionable Thickneſs, and whoſe Extremes A C, B D, are cut at right Angles. About the Points C and D deſcribe two Quadrants A L, M B, and divide each of them into 9 equal Parts; and from each oppoſite Diviſion draw the Parallels of the Signs, whereof the Line C F D ſhall be for ♈ and ♎, A E B for the two Tropicks, and the others for the other Signs placed according to their order. Then biſect the Parallelogram A B C D by the Line E F, and draw the Line G H ſeparately equal to E B, that ſo a Scale may be made thereof, which muſt be divided into 9 equal Parts, each of which muſt be ſubdivided into 10 equal Parts more by little dots, and ſo the ſaid Scale will be divided into 90 equal Parts, anſwering to the 90 Deg. of a Quadrant. This being done, take the Degrees of the Sun's Altitude from the above poſited Table of Altitudes, at every Hour when the Sun is in the Equinox, and the Solſtices, for the Horizon of *Paris*. For example, When the Sun is in the 1ſt Deg. of ♋, his Meridian Altitude is 64 Deg. 29 Min. take 64 ½ equal Parts from the Scale G H between your Compaſſes, and lay them off upon the Braſs Blade both ways from E to the Points I and K, as likewiſe from the Point F to the Points L and M, and join the Points I L and K M, by right Lines: then take from the Table the Sun's Altitude at the Hours of 1 and 11, when he is in the Summer Solſtice, *viz.* 61 Deg. 54 Min. which here may be taken for 62 Deg. and opening your Compaſſes to the Extent of 62 equal Parts of the Scale, lay them off upon A B from K towards E, and you will have a Point of the Hour-Lines of 11 and 1; likewiſe take 41 equal Parts or Degrees, for the Sun's Meridian Altitude when he is in the Equinoctial, and lay them off from M to O, and from L to N, and the Points N and O are thoſe thro' which the two Hour-Lines of 12 muſt be drawn. Moreover, take 39 Deg. 20 Min. the Sun's Altitude when he is in the Equinox, at the Hours of 11 and 1, from the Scale, and lay them off from the ſaid Points M and L upon the ſaid Line C D, and you will have two Points in the Line C D, thro' which the Hour-Lines of 11 and 1 muſt be drawn. And in this manner may Points be found in this Line, thro' which the other Hour-Lines muſt paſs. Fig. 4.

But now to find Points in the Line A B, or Tropick of *Capricorn*, on this ſide the Point E, thro' which the Hour-Lines muſt be drawn, (for the Points of the ſame Line, on the other ſide of E, for the Tropick of *Cancer* may be found in the ſame manner as the Points for the Hour-Line of 11 and 1 was) you muſt take 17 ½ Degrees, or equal Parts from the Scale, *viz.* the Sun's Meridian Altitude, when he is in the Tropick of *Capricorn*, and lay them off from I to P, and P will be the Point thro' which the Hour-Line of 12 muſt paſs; and ſo may the Points be found thro' which the other Hour-Lines muſt be drawn. Now if the Points found in the Lines A B, and C D, thro' which the Hour-Lines paſs, be joined by right Lines; theſe right Lines will be the Hour-Lines.

But if you have a mind to be exacter, you may take the Degrees of the Sun's Altitudes at every Hour when he enters, and is in each 10th and 20th Degree of every Sign, and then find Points on the reſpective Parallels on the Dial thro' which the Hour-Lines muſt be drawn, which will not be right Lines but Curves; and in this caſe the Dial will be exacter.

Having drawn the Hour-Lines, you muſt Number them on both ſides the Lines A B, C D, and alſo ſet down the Characters of the Signs, and the firſt Letters of the Months, each in their proper Place. When this is done, you muſt drill two little Holes in the Points R and S (*viz.* the middles of the Lines I L, K M) in a conical Figure, the greater Baſes being outmoſt, that ſo the Sun's Rays may better come thro' them; afterwards round or turn up the ſaid Blade circularly, ſolder the Extremities A C, B D together, and place a Button, with a Ring in the middle of the Junction of the ſaid Extremities, ſo that the whole Inſtrument be *in equilibrio*; which that it may, you muſt turn the outſide thereof.

The Uſe of this Inſtrument.

Hold the Ring ſuſpended, and turn the Hole proper for the Time of Year towards the Sun, ſo that his Rays may fall upon the Parallel of the Sign he is in, the Day wherein you uſe the Inſtrument; and then the Hour of the Day will be ſhewn thereon by a bright Spot or Point of Light.

Note, The Hole S is in uſe from the 20th of *March*, (N. S.) to the 22d of *September*, and the Hole R for the other ſix Months. We likewiſe write upon the convex Superficies of the Ring near the little Holes, the 20th of *March*, and the 22d of *September*, as appears in Figure 3, and, laſtly, obſerve that theſe two laſt Dials are proper but for one Latitude.

The Construction and Use of the universal Astronomical Ring-Dial.

Fig. 5. This Instrument, whose Use is to find the Hour of the Day in any part of the Earth, by a bright Spot of the Sun's Light, is made of Brass or other Metal, and consists of two Rings, or flat Circles turned both within side and without. The Diameter of these Rings, which ought to be broad and thick proportionable to their bignesses, are from two to six Inches. The outward Ring A represents the Meridian of any Place wherein one is, and there are two Divisions of 90 Degrees thereon, which are diametrically opposite to each other, one whereof serves from our North Pole to the Equator, and the other from the Equator to the South Pole.

The innermost Ring represents the Equator, and ought to turn very exactly within the outward one, by means of two Pivots or Pins put into Holes made diametrically opposite in the two Rings at the Points of the Hour of 12.

There is a thin Riglet (called a Bridge) with a Cursor marked C, composed of two little Pieces that slide in an Aperture made along the Middle of the said Bridge, and which are kept together by two small Screws. Thro' the Middle of this Cursor is a very little Hole drilled, that so the Sun may shine thro' it. Now the Middle of the said Bridge may be considered as the Axis of the World, and the Extremities as the Poles of the World; and there are drawn on one side thereof the Signs of the Zodiack with their Characters, and on the other side the Days and Names of the Months, or only their first Letters, being placed according to the respect they have to the Signs. The Signs are divided into every 10th or 5th Degree, according to their Declination, by means of a Trigon already divided, the Vertex of which, or Extremity of the Radius of the Equinoctial, being within side the Equinoctial Circle, as at the Point F. The two Pieces D D which are screwed to the outermost Ring, serve to support the Bridge or Axis which is moveable round, and are so ordered as that the innermost Ring may lie exactly within the outermost, and they both make as it were but one. The two Pieces E are also screwed on the outermost Ring, and serve as Props to keep the Equinoctial Circle or inward Ring at right Angles to the Meridian or outermost Ring.

We shall not here repeat the manner of dividing the two Quadrants into Degrees, and the Equinoctial Circle into Hours, Halves, and Quarters, having sufficiently spoken of this elsewhere. We shall only add, that all the Divisions of the Equinoctial Circle must be drawn upon the concave Side thereof, which may be done by means of a Piece of Steel turned up square, according to the Curvature of the Circle.

Near the outward Edges, on each of the two flat Sides of the Meridian, is made a Groove for the Piece G to slide therein, the Middle of which is bent inwards, that so it may go into the said Grooves. The two Sides of this Piece, which must be well hammered that they may have a good Spring, are made flat, in order to press against the convex Surface of the Meridian, that thereby the Piece G may be held fast on any Degree of Division of the Meridian. The Button thro' which the Ring of Suspension H goes, is riveted to the Middle of the Piece G, so that it may turn round very freely, and by this means the Instrument be very perpendicularly suspended by the Ring H: for this is one of the principal things in which the Exactness of the Instrument consists.

The Use of the Astronomical Ring-Dial.

Place the short Line *a* on the Middle of the hanging Piece G over the Degree of the Latitude of the Place you are in upon the Meridian Circle, for example, over the 49th Deg. at *Paris*; and then put the Line crossing the little Hole of the Cursor on the Bridge to the Degree of the Sign, or the Day of the Month you desire to know the Hour of the Day in. This being done, open the Instrument so that the two Rings or Circles be at right Angles to each other, and suspend it by the Ring H, so that the Axis of the Dial represented by the Middle of the Bridge be parallel to the Axis of the World.

Turn the flat Side of the Bridge towards the Sun, so that his Rays coming thro' the little Hole in the Middle of the Cursor, fall exactly on a Line drawn round the Middle of the concave Surface of the Equinoctial Circle, or innermost Ring; and then the bright Spot or luminous Point shews the Hour of the Day in the said concave Surface of the Ring.

Note, The Hour of 12 cannot be shewn by this Dial, because the outermost Circle or Ring being then in the Plane of the Meridian, it hinders the Sun's Rays from falling upon the innermost or Equinoctial Circle. You must observe likewise, that when the Sun is in the Equinoctial, you cannot then tell the Hour of the Day by this Dial, because his Rays fall parallel to the Plane of the said Equinoctial Circle. But this is but about one Hour every Day, and four Days in the Year.

The Construction and Use of a Ring-Dial with three Rings.

Fig 6. This Instrument differs from the precedent one in nothing but only a third Ring or Circle, carrying the Sun's Declination. The Ring A represents the Meridian of the Place you would use the Dial in; the Ring B represents the Equinoctial Circle; and the Ring D, which turns exactly within the said Equinoctial Circle, produces the same Effect, as the Bridge

Bridge reprefenting the Axis of the World in the precedent Inftrument. The two Extremities of the Diameter of this laft Ring, or the two Points of the Circumference thereof, whereat it is faftened to the Meridian, anfwer to the two Poles of the World. On the oppofite Parts D D of the Circumference of this Circle, is denoted a double Trigon of Signs, whofe Center is the Vertex wherein all the Radius's reunite, the Arcs of each of which are fubdivided into every 10th or 5th Degree, to which may be likewife fubjoined the Days of the correfpondent Months.

The Index E is faftened to the Center of the innermoft Ring; having two Sights riveted to the Extremities thereof, each having a fmall Hole drilled therein, for the Sun's Rays to pafs thro'. *Note*, Dials compofed in this manner fhew the Hour of 12, becaufe the Index is without the Plane of the Meridian Circle: and when we make them large, as 9 or 10 Inches in Diameter, we divide the Equinoctial Circle into every 5th or every 2d Minute.

This Dial hath a Piece F like as the former Dial has, going into a Groove made on each fide the Meridian, to be flid to the Latitude of the Place. We fometimes fet thefe Dials upon Pedeftals, nearly like thofe of Spheres, which are flid to the Latitude; and in this Cafe they are placed upon an horizontal Plain; we likewife add Compaffes to them, by which means the Variation of the Needle may be exactly known.

The Ufe of this Dial.

Place the little Line in the middle of the hanging Piece F to the Latitude of the Place wherein you have a mind to know the Hour of the Day, and the fiducial Line of the Index on the Day of the Month, or Degree of the Sign the Sun is in. Then open the Equinoctial Circle at right Angles to the Meridian, and holding the Inftrument fufpended, raife or lower the innermoft Circle, fo that the Sun's Rays may go thro' the Holes of the two Sights; then the Line which is drawn along the middle of the Convexity of the faid Circle, will fhew the Hour or Part drawn in the middle of the Concavity of the Equinoctial Circle, even at all times of the Day.

This may likewife be done fomething more convenient, when the Inftrument is placed horizontally upon it's Pedeftal.

The Conftruction of an univerfal inclined Horizontal, and an Equinoctial Dial.

This Inftrument confifts of two Plates of Brafs, or other folid Matter, whereof the underone A is hollowed in about the Middle, to receive a Compafs faftened underneath with Screws. The Plate B is moveable by means of a ftrong Joint at the Plate C. Upon this Plate is drawn a horizontal Dial for fome Latitude greater than any one of thofe the Dial is to be ufed in, and having a Style thereon proportionable to that Latitude; for when the faid Plane B is raifed by means of the Quadrant, the horizontal Plane muft always have a lefs Latitude than that the Dial is made for, or otherwife the Axis of the Style will have an Elevation too little. Fig. 7.

Inftead of the Quadrant D we generally place but only an Arc from the Equator to 60 Degrees, which are numbered downwards, 60 being at the Bottom, and for this Latitude of 60 Deg. we commonly draw the aforefaid horizontal Dial. That Arc of 60 Deg. is faftened by two fmall Tenons, and may be laid down upon the Plate A, as likewife may the Style upon the Plate B, and both of thefe are kept upright by means of little Springs underneath the Plates. What remains of the Conftruction of this Dial, may be fupplied from the Figure thereof.

The Ufe of the inclined horizontal Dial.

Raife the upper Plate B to the Degree of Latitude or Elevation of the Pole of the Place wherein you are, by means of the Graduations on the Quadrant D. Then if the Plane A be fet horizontal, fo that the Needle of the Compafs fettles itfelf over it's Line of Declination, the Shadow of the Axis will fhew the Hour of the Day. *Note*, We grave the Names of feveral principal Cities, as likewife their Latitudes and Longitudes, underneath the two Plates, in order to avoid the Trouble of feeking them in Maps.

After the abovefaid manner, equinoctial Dials are made Univerfal throughout the whole Earth; but here we muft have a whole Quadrant. The upper Plate is commonly in form of a hollowed Circle, which we divide into 24 equal Parts, for the Hours, each of which we fubdivide into 4 equal Parts, for the Quarters; all thefe being drawn in the Concavity of the Circle.

There is a Piece that goes thro' the Circle, carrying the right Style, which is kept faft in the middle of the Circle by means of a little Spring faftened underneath the Circle; and by this means the right Style may be raifed above the faid Circle, and lowered underneath it. And when the equinoctial Dial is drawn, we ufe the little Piece F for a Style, placed in the Center of the Circle. *Note*, The upper part of the Dial fhews the Hour of the Day from the 22d of *March*, (N. S.) to the 22d of *September*, and the under part thereof the Hour of the Day, the other 6 Months of the Year.

The

The Use of the equinoctial Dial.

You must place the Edge of the equinoctial Circle to the Degree of the Elevation of the Pole, by means of the Quadrant; then if the Dial be set North and South by means of the Compass, the Shadow of the Style will shew the Hour of the Day at all times of the Year, even when the Sun is in the Equinoctial, because the Circle is hollowed in.

The Construction of an Azimuth Dial.

Fig. 8. This Dial, which is commonly made in the Bottom of a Compass, is called an Azimuth Dial, because it is made by means of the Azimuth's or Sun's Vertical Circles, upon a Plate of Brass, or other solid Matter, parallel to the Horizon. First, draw the Line A B, representing the Meridian, upon which describe a Circle at pleasure, half of which we shall only use here for drawing the Morning Hour-Lines, because those of the Afternoon are drawn after the same way. Divide this Circle into Degrees, beginning from the Point A, representing the North Pole. Then trisect the Semi-Diameter A C, and take A D equal to two thirds thereof, which must be divided into 6 Parts, thro' each Point of Division; about the Center C must be drawn concentrick Arcs, representing the Parallels of the Signs, the Arc H being the Summer Tropick, that nearest to the Center C the Winter Tropick, and each of the others for two Signs equally distant from the Tropicks, as appears *per* Figure.

The Parallels of the Signs may moreover be drawn, in describing a Semi-Circle upon the Line H D, which Semi-Circle being divided into 6 equal Parts, you must let fall dotted Parallels upon the Line H D; these Parallels will divide the said Line into unequal Parts, and if thro' the Points of Divisions Arcs be described about the Center C, these Arcs will be the Parallels of the Signs at unequal Distances from each other.

Now for drawing the Hour-Lines, the following Table of the Sun's Azimuths must be used; for example, to prick down a Point in the Tropick of *Cancer*, thro' which the Hour-Line of 11 in the Morning must be drawn, you will find the Sun's Azimuth will then be 30 Deg. 17 Min. and when he is in the first Degree of ♊, or last of ♌, his Azimuth at the same Hour is 27 Deg. 58 Min. and so of others. Therefore if a Ruler be laid on the Center C, and on the 30th Deg. and 27 Min. of the outward divided Limb, the Edge of the Ruler will cut the Parallel of ♋, in a Point thro' which the Hour-Line of 11 must pass: then keeping the Ruler to the Center, move it, and lay it over the 27th Deg. and 58th Min. of the outmost Limb, and you will have a Point in the Parallel of ♊ and ♌ thro' which the Hour-Line of 11 must pass; and in this manner may Points be found in the other Parallels thro' which the Hour-Line of 11 must pass; and also Points in all the Parallels thro' which the other Morning Hour-Lines must pass: each of which Points belonging to the same Hours being joined, you will have the curved Hour-Lines on one side of the Meridian. And to find the Points thro' which the Afternoon Hour-Lines must pass, take the Distances of each Point in the Parallels from the Meridian, and transfer them on the same Parallels continued out on the other side of the Meridian, because the Sun's Azimuth at any two Hours equally distant on each side the Meridian, is the same.

The Use of the Azimuth Dial.

Turn the Side B towards the Sun, so that the Shadow of the right Style planted in a Point without the Compass, and parallel to the Line of Noon, may fall along the Meridian Line: then the Needle pointing exactly North and South, will shew the Hour of the Day in the Intersection thereof with the Parallel of the Sign the Sun is in, upon condition that the Needle has no Variation. But since the Needle varies now above 12 Degrees at *Paris*, you must place the Style in the Point E over the Line of Declination or Variation K I, and adjust the Shadow of the Style along the said Line of Variation, and by this means the Error arising from the Needle's Variation will be avoided.

A TABLE *of the Sun's Azimuth or Distance from the Meridian every Hour of the Day, for the Latitude of* 49 *Degrees.*

Hours.	XI. I.		X. II.		IX. III.		VIII. IV.		VII. V.		VI. VI.		V. VII.		IV. VIII.	
Signs.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
♋	30	17	53	40	70	30	83	57	95	20	105	56	116	28	127	26
♌ ♊	27	58	50	33	67	34	81	6	92	45	103	35	114	56		
♍ ♉	23	30	43	52	60	29	74	17	86	21	97	36				
♎ ♈	19	33	37	25	52	58	66	57	78	34						
♏ ♓	16	42	32	25	46	30	59	28	71	12						
♐ ♒	14	56	29	11	42	23	54	26								
♑	14	19	28	2	40	48										

The Construction and Use of the Analemmatick or Ecliptick Horizontal Dial.

This is called an Analemmatick Dial, becaufe it is made by means of the Analemma, which is the Projection or Reprefentation of the principal Circles of the Sphere upon a Plane. The 9th Figure is the Analemma; and the 10th Figure reprefents the Dial compleat, which fhews the Hour of the Day without a Compafs.

Now to project the Analemma; upon a very even fmooth Plate of Brafs, draw the Lines A B and C D, cutting each other at right Angles in the Point E, about which, as a Center, defcribe the Circle A C B D, reprefenting the Meridian, it's Diameter C D, the Horizon, and A B the prime Vertical. Then affume the Arc D F equal to the Elevation of the Pole, which here is 49 Deg. and draw the Line E F reprefenting the Axis of the World; likewife affume the Arc C G equal to the Height of the Equinoctial 41 Degrees, and draw the Line G E for the Equinoctial. Affume the Arcs G H, G I, each of 23 Deg. 30 Min. for the Sun's greateft Declination, and draw the Line H I cutting the Equinoctial in the Point Y, about which, as a Center, defcribe the Circle H L I K, or only half of it, which divide into 6 equal Parts, and thro' each Point of Divifion draw Parallels to the Equinoctial, which continue out to the Horizon; then from the Sections made by the faid Parallels on the Meridian, let fall the Parallels M, N, O, and P to the Horizon, and from the Sections made by the faid Parallels on the Axis, let fall the indefinite Perpendiculars S *c*, R *b*, Q *a* to the Horizon. This being done, take the Diftance E M between your Compaffes, with which fetting one Foot in N, with the other make a fmall Arc upon the Line Q *a*, and with one Foot in O cut the Line R *b* with the other; then, continually keeping the Compaffes opened to the Extent E M, fet one Foot in P, and cut the Line S *c* in the Point C. Fig. 9.

Now to conftruct the little Zodiack, take the Diftance *t* C, and lay off from E towards A and B for the Tropicks of ♋ and ♑; again, lay off the Diftance 46, from the Point E on one fide, for the Parallel of ♊, and on the other fide for the Parallel of ♒; and finally, take the Diftance X *a*, for marking the Parallel of ♉ on one fide, and that of ♓ on the other, and then the little Zodiack may be formed, as per Figure. Now to prick down the Hour-Points, you muft defcribe the Circle M T Z V about the Center E, with the Diftance E M, and divide the Circumference thereof into 24 equal Parts, as likewife the Circumference of the Meridian A C B D, and from each oppofite Point of Divifion in the Meridian draw ftrait Lines parallel to A B, and in the Circle M T Z V, ftrait Lines parallel to C D, and thro' the Interfections of thefe Lines that are neareft to the Meridian, draw lightly an Ellipfis from Point to Point, as you fee in the Figure. Thefe Points of Section will be the Hour-Points, thofe for the Morning being on the left, and thofe for the Afternoon on the right; and to have the half and quarter Hour-Points, the two Circles A C B D, M T Z V, muft be divided into 96 equal Parts.

Things being thus prepared, transfer all the Hour-Points on another Brafs Plate, and form the Ellipfis B thereon, by lightly drawing Lines from Point to Point, and grave the proper Numbers upon it, as they are marked in the 10th Figure. Likewife transfer the Trigon of Signs upon the faid Plate, taking each of the Diftances between your Compaffes, the one after the other, fo that the Signs ♈ and ♎ be in the Line of the Hour of 6, and place the Characters of the Signs thereon, as alfo the firft Letters of the Months, each one in their order. When this is done, you muft adjuft a Curfor C fo as to flide along the middle of the Trigon. This Curfor carries the right Style D, which rifes and falls by means of two fmall Knuckles. Fig. 10.

On the other part of this Plate, is drawn an horizontal Dial according to the common Rules, for the fame Latitude the Analemma is made for, and we place the Style or Axis E thereon upon the Hour-Line of 12, which rifes, falls, and is kept upright by means of a fmall Spring underneath the Plate.

The Use of this Dial.

Set the Dial parallel to the Horizon, and put the Curfor with it's right Style upon the Day of the Month, or Sign the Sun is in; then turn the Inftrument until the fame Hour be fhewn upon the two Dials, which will be the Hour of the Day. If, for example, the Shadow of the Extremity of the right Style falls upon the 11th Hour on the Analemmatick Dial, and at the fame time the Shadow of the Style of the horizontal Dial falls likewife upon the 11th Hour, on the horizontal Dial; then the true Hour of the Day will be that of 11. The Conveniency of this Dial confifts in this, that the Hour of the Day may be found thereby without a meridian Line, or Compafs; but then it muft be pretty large, to fhew the Hour exactly.

The Construction of an universal Polar, East, and West Dial.

This Inftrument confifts of a very ftrait and fmooth circular Piece of Brafs, or other Metal, pretty thick, that fo it may preferve it's perpendicular Weight, as likewife that a Groove may be made round the Limb thereof, for a hanging-Piece to flide about the fame, like that on the aftronomical Ring. Fig. 11.

About the Center of the said circular Piece describe the Circumference of a Circle, which divide into twice 90 Degrees. Then draw a right Line from the 90th Degree thro' the Center, representing the Equinoctial, near the Top of which assume a Point at pleasure, thro' which draw a right Line perpendicular to the Equinoctial-Line, which shall be the Hour-Line of 6. Then to have the other Hour-Lines, you must lay off the answerable Tangents upon the Equinoctial-Line both ways from the Point therein of the Hour-Line of 6; as the Tangent of 15 Deg. for the Hour-Points of 5 and 7; the Tangent of 30 Deg. for 4 and 8; the Tangent of 45 Deg. for 3 and 9, &c. and if Lines be drawn thro' these Points parallel to the Hour-Line of 6, these will be the Hour-Lines; and the Length of the right Style must be equal to the Radius or Tangent of 45 Deg. and must be placed upright upon the Hour-Line of 6, at the Point wherein it cuts the Equinoctial-Line.

At the Points C C, on the Hour-Line of 9 in the Morning, and 3 in the Afternoon, are adjusted two small Knuckles, in which is placed the Piece V, which may lie down upon the circular Piece, and likewise stand at right Angles to it. Upon this Piece are pricked down the Hour-Lines of a Polar Dial, from 9 in the Morning to 12, and from 12 to 3 in the Afternoon. We shall not here repeat the manner of drawing these Hour-Lines, for we have sufficiently spoken of this already, as likewise how to draw the Arcs of the Signs; only observe, that the Parallels of the Signs are divided into every 10th Deg. and the first Letters of the Names of the Months are set down in their proper Place.

The Style B is adjusted to the circular Piece with a Joint, that so it may be raised or lie flat upon the said Piece; but it must be raised so that the Extremity thereof may be exactly over the Point in the Equinoctial-Line cut by the Hour-Line of 6, and the Distance of the said Extremity from this Point equal to the Distance from the Hour-Line of 9 to the Hour-Line of 6.

The Use of the said Dial.

If you have a mind to find the Hour of the Day before Noon, place the little Line on the middle of the hanging Piece L upon the Latitude of the Place; on that Quadrant on the Right-hand of the Style B, raise the Style so that the Extremity thereof be directly over the Intersection of the Equinoctial and the Hour-Line of 6, and it's Distance from that Point of Intersection equal to the Distance from the Hour-Line of 9 to the Hour-Line of 6. Then holding the Dial suspended by it's Ring, expose it to the Sun, so that the Shadow of the Extremity of the Style falls upon the Day of the Month; and you will have the Hour of the Day upon the East or Polar Dial. But if the Hour of the Day be required in the Afternoon, you must put the hanging Piece on the Latitude of the Place upon the Quadrant on the left side of the Style, and turn the Dial to the Sun so that the Shadow of the Extremity of the Style falls on the Degree of the Sign or Day of the Month. Then you will have the Hour of the Day as before.

Thus have I laid down the Construction and Uses of Portable Dials, chiefly in use, which may be set North and South, without a Compass or Meridian Line. But before I close this Chapter, I shall briefly describe some other Portable Dials, which are curious enough, but are something difficult to make.

The first of these is a horizontal Dial of 2 or 3 Inches square, which we make of Brass or any other solid Metal, for a given Latitude, and whose Axis shewing the Hour, is a Thread fastened at one end to the Center of the said Dial, and the other end of which is hung to the top of a pretty thick Brass Blade, placed at the Extremity of the Dial near the Hour-Line of 12. This Blade may lie down upon the Plane of the Dial, and is kept upright by means of a Spring underneath the Dial; and the Height of the Notch wherein the Thread lies above the Plane of the Dial, is equal to the Tangent of the Latitude.

About a quarter of the Height of the said Blade is adjusted thereon a Circle or Ring, proportioned to the bigness of the Dial-Plate. This Ring is moveable by means of a Joint, and so may lie down upon the Blade, and the Blade upon the horizontal Dial-Plane; and when the Instrument is using, there is a Prop to keep this Ring at the Height of the Equinoctial, *viz.* 41 Deg. but when the Thread serving for an Axis is extended, it must exactly pass thro' the Center of this Ring.

The Concavity of the Ring is divided into Hours, Halves, and Quarters, as the equinoctial Ring of the universal Ring-Dial is; and there is a Bead or Pin's Head put upon the Thread, that so it may be moved to the Sign the Sun is in, and serve as a Cursor to shew the Hour of the Day in the middle of the Concavity of the Ring or Equinoctial.

Now to place the Bead to the Sign or proper Month, you must have a separate Brass Riglet, having the Signs of the Zodiack, as also the Days of the Months drawn thereon in the manner they were drawn upon the Bridge of the universal Ring-Dial; and having placed the said Riglet from the Center of the horizontal Dial along the Thread or Axis, slide the Bead to the Degree of the Sign the Sun is in, and then take away the Riglet, and so will the Bead be placed for shewing the Hour of the Day.

On the backside of the Blade is drawn an upright Line for a Plumb-Line to play on, that so the Dial may be set level. *Note*, This Dial may be rendered universal, if an Arc of a Circle divided into Degrees be adjusted behind the Blade by means of a Joint, so as it may

may-lie upon the Blade, and the Point whereon the Plumb-Line is hung by the Center of the ſaid Arc; for then the Dial may be ſet to the Latitude, by making the Plumb-Line fall upon the proper Degree on the circular Arc. It is proper alſo to obſerve, that the Hours from eight in the Evening to four in the Morning may be taken away from the Equinoctial Ring, that ſo this Dial may be of uſe at the Time of the Equinox.

The Uſe of the aforeſaid Dial.

Having placed the Bead to the Degree of the Sign the Sun is in, or Day of the Month, as before directed, expoſe the Dial to the Sun, and turn it to the right or left until the Shadow of the Bead falls upon the ſame Hour or Part, on the middle of the Concavity of the equinoctial Ring, as the Shadow of the Thread or Axis does on the horizontal Dial; and then that will be the true Time of the Day.

We make ſeveral other portable Dials, as horizontal Aſtrolabes, being Projections of the Sphere upon the Plane of the Horizon; other Aſtrolabes vertically uſed by means of a Plumb-Line; horizontal Dials made by means of the Sun's Altitudes, which are likewiſe ſet North and South without a Compaſs, and whereon the Signs are drawn by right Lines iſſuing from the ſame Center, and the Hour-Lines, curve Lines; as likewiſe other portable Dials, which are curious enough, whoſe Conſtruction and Figures we reſerve for another time.

Horizontal Dials whereon are drawn the Signs, as that of *Fig.* 7. *Plate* 23. may likewiſe be ſet North and South without a Compaſs, if the Dial be ſo placed in the Sun, that the Shadow of the Extremity of the right Style falls upon the Degree of the Sign the Sun is in, or Day of the Month. But here there is this Inconveniency, that the Diſtance of the Parallel of *Cancer* from the adjacent Parallels is ſo ſmall, that the Space of 10 Days there cannot be diſtinguiſhed. So that when we have done all we can, it is ſcare poſſible to make a portable Dial that can ſet North and South without a Compaſs or Meridian Line, without falling into one of theſe Inconveniencies, either of having the Hour-Lines near Noon too nigh each other, or not exactly ſhewing the Hour of the Day at the Time of the Solſtices, becauſe of the ſmall Difference that there is in the Sun's Elevations and Declination at thoſe times.

CHAP. VI.

Of the Conſtruction and Uſes of a Moon-Dial, and a Nocturnal or Star-Dial.

Of the Conſtruction of an horizontal Dial for ſhewing the Hour of the Night by the Moon.

THIS is called a Moon-Dial, becauſe by it you may tell in the Night by the Shadow of the Moon, what Hour-Circle the Sun is in. It conſiſts of two Pieces or Plates of Braſs, or other ſolid Matter, of a bigneſs at pleaſure. The under-Plate H, is in figure of a Parallelogram, and the upper one A is circular, turns about the ſhadowed Space L, and the Center B, and has a horizontal Dial drawn upon it for the Latitude of the Place, according to the Rules before preſcribed for drawing horizontal Dials. The under Plate hath a Circle thereon divided into 30 unequal Parts, for the Days of a Lunar Month. Theſe Diviſions are made thus; let D E be the equinoctial Line by which the horizontal Dial was drawn, and F the Center of the equinoctial Circle (or the Center by which the equinoctial Line is divided.) About this Center deſcribe a dotted Circle, and divide it into 30 equal Parts, or half of it into 15, and having laid the Edge of a Ruler on the Center F, lay it over each Point of the Diviſions of the ſaid Circle one after another, and prick down Points upon the equinoctial Line; then lay the Ruler to the Center B, and on each Point of Diviſion of the equinoctial Line, and divide the Circle H; and when you have divided half of it, transfer the ſame Diviſions on the other Semi-Circle, and by this means the whole Circle will be divided into 30 unequal Parts for the 30 Days of a Lunar Month, about which Numbers muſt be graved, as they appear per Figure. This being done, place the Axis B C anſwering to the Elevation of the Pole, and diſpoſe it ſo that when it is ſet up it may not hinder the Hour-Plate from turning about the Center B. Fig. 12.

The Uſe of this Dial.

The Moon's Age muſt be found by an Ephemeris, or by the Epact, that ſo the Point of the Hour-Line of 12 on the horizontal Dial may be applied to the Day of her Age in the Circle H of the under Plate.

But before we go any further, you muſt obſerve, that the Moon by her proper Motion recedes Eaſtwards from the Sun every Day about 48 Minutes of an Hour, that is, if the Moon is in Conjunction with the Sun on any Day upon the Meridian, the next Day ſhe will

will cross the Meridian about three quarters of an Hour and some Minutes later than the Sun: and this is the Reason that the Lunar Days are longer than the Solar ones; a Lunar Day being that Space of Time elapsed between her Passage over the Meridian, and her next Passage over the same; and these Days are very unequal on account of the Irregularities of the Moon's Motion.

Now when the Moon is come to be in Opposition to the Sun, she will again be found in the same Hour-Circle as the Sun is; so that if, for example, the Sun should be then in the Meridian of our Antipodes, the Moon would be in our Meridian, and consequently would shew the same Hour on our Sun-Dials as the Sun would, if it was above the Horizon. But this Conformity would be of small duration, because of the Moon's retardation of about two Minutes every Hour. If moreover the Sun, at the Time of the Opposition, be just setting above our Horizon, the Moon being diametrically opposite to it will he just rising, *&c.* and therefore to remedy the said Retardation, we have divided the Circle H into 30 Parts.

Now the Point of the Hour-Line of 12 on the horizontal Dial being put to the Moon's Age, as above directed, and the under-Plate set North and South by means of a Compass or meridian Line, the Shadow of the Style will shew the Hour of the Night; but to have the Hour more exact, you must know whether it is the first, second or third Quarter of the Moon's Day that you seek the Hour in, that so the Point of the Hour-Line of 12 may be set against a proportionable part of one of the 30 Spaces or Lunar Days of the Circle H.

The Table on the under-Plate H, is used for finding the Hour of the Night by the Shadow of the Moon upon an ordinary Dial. To make this Table, draw 4 Parallel right Lines or Curves of any length, and divide the Space I I into twelve equal Parts for 12 Hours, and the two other Spaces K K into 15, for the 30 Lunar Days.

The Use of this Table.

First observe what Hour the Shadow of the Moon shews upon a Sun-Dial; then find the Moon's Age, and seek the Hour correspondent thereto in the Table, and add the Hour shewn by the Sun-Dial thereto; then their Sum, if it be less than 12, or else it's excess above 12, will be the true Hour of the Night. For example; Suppose the Hour shewn upon the Sun-Dial by the Moon, be the 6th, and her Age be 5 or 20 Days, against either of these Numbers in the Table you will find 4, which added to 6 makes 10, and so the Hour of the Night will be 10. Again, Suppose the Moon shews the Hour of 9 upon the Sun-Dial, when she is 10 or 25 Days old, against 10 and 25 in the Table you will find 8, which added to 9, makes 17, from which 12 being taken, the Remainder 5 will be the true Hour sought. And so of others.

To find the Moon's Age, you must first find the Golden Number; and this is done by adding 1 to the given Year, and dividing the Sum by 19, and the Remainder will be the Golden Number. Then you must find the Epact, by means of the Golden Number; and this is done thus: Divide the Golden Number by 3, and each Unit remaining being called 10, will be the Epact, if the Sum be less than 30; but if above 30, 30 being taken from it, and the Remainder added to the Golden Number will be the Epact. The Epact being found, the Moon's Age may be had after this manner: If the Moon's Age be sough. in *January*, add 0 to the Epact; in *February*, 2; in *March*, 1; in *April*, 2; in *May*, 3; in *June*, 4; in *July*, 5; in *August*, 6; in *September*, 8; in *October*, 8; in *November*, 10; and in *December*, 10: and the Sum, if it be less than 30, or the Excess above 30, added to the Day of the given Month (rejecting 30 if need be) will be the Moon's Age that Day. For example, To find the Moon's Age the 14th Day of *March*, in the Year 1716, (O. S.) the Golden Number is 7, and the Epact 17; therefore adding 1 for *March* to 17, and the Sum will be 18; and if to this 18 be added 14 for the Day of the Month, the Sum will be 32, from which 30 being taken, and the Remainder 2 will be the Moon's Age. *Note*, This way of finding the Moon's Age is not so exact as we have it by the Ephemeris. Likewise observe, that vertical Moon-Dials may be made in the manner as the horizontal ones are, but the Divisions of 30 Parts upon equinoctial Dials must be equal, and the moveable Circle divided into 24 equal Parts, *&c.*

The Construction of a Nocturnal or Star-Dial.

The 13th Figure shews the Disposition of the chief Stars composing the Constellation of *Ursa Major*, and *Ursa Minor*, about the Pole and the Pole-Star.

The Nocturnal we are going to mention, is made by the Consideration of the diurnal Motion, that the two Stars of *Ursa Major*, called his Guards, or the bright Star of *Ursa Minor*, make about the Pole, or the Pole Star, which at present is but about 2 Deg. distant from the North Pole.

Now to construct this Instrument, you must first know the right Ascension of the said Stars, or in what Days of the Year they are found in the same Hour Circle as the Sun is. This may be found, by Calculation, on a Globe, or a Celestial Planisphere, by placing the Star in question under the Meridian, and examining what Degree of the Ecliptick will be found at the same time under the Meridian. By this Method you will find that the bright

bright Star or Guard of the Little Bear, was found twice in one Year with the Sun under the Meridian, *viz.* in the Year 1715, once the 8th of *May*, (N. S.) above the Pole, and again the 8th of *November* below the Pole. Therefore in the ſaid two Days of the Year, the abovementioned Star will be in all the Hour-Circles at the ſame Time as the Sun is; and conſequently will ſhew the ſame Hour. You will find alſo, that the two Guards of *Urſa Major* were found two other Days of the Year under the ſame Meridian or Hour-Circle as the Sun, *viz.* the firſt Day of *September* below the Pole, and the firſt Day of *March* above it. And in theſe two Days the ſaid Stars will ſhew the ſame Hours as the Sun does; but becauſe the fixed Stars return to the Meridian every Day about 1 Deg. ſooner than the Sun, or four Minutes of an Hour, which is two Hours *per* Month, it is this, which is to be obſerved for having the Hour of the Sun, which is the Meaſure of our Days.

Theſe things being premiſed, it will not be difficult to make a Nocturnal or Star-Dial, in the following manner:

The Inſtrument is compoſed of two circular Plates applied on each other; the greater of which, having a Handle for holding up the Inſtrument when uſing, is about two Inches and a half in Diameter, and is divided into twelve Parts for the twelve Months of the Year, and each Month divided into every 5th Day; ſo that the middle of the Handle exactly anſwers to the Day of the Year wherein that Star which is uſed has the ſame right Aſcenſion as the Sun has. If, for example, This Inſtrument be made for the two Guards of *Urſa Major*, the firſt Day of *September* muſt be againſt the middle of the Handle; and if it be made for the bright Star of *Urſa Minor*, the 8th Day of *November* muſt be againſt the middle of the Handle. Therefore if you will have the Inſtrument ſerve for both theſe Stars, the Handle muſt be made moveable about the ſaid circular Plate, that ſo it may be fixed according to neceſſity; and this is eaſy to do by means of two little Screws. Fig. 14.

This being done, the upper leſſer Circle muſt be divided into 24 equal Parts, or twice 12 Hours, for the 24 Hours of the Day, and each Hour into Quarters, according to the Order appearing in the Figure. Theſe 24 Hours are diſtinguiſhed by a like Number of Teeth, whereof thoſe whereat the Hours of 12 are marked are longer than the others, that ſo the Hours may be counted in the Night without a Light.

In the Center of the two circular Plates is adjuſted a long Index A, moveable about the ſame upon the upper Plate. Theſe three Pieces, *viz.* the two Circles and the Index, are joined together by means of a headed Rivet, and pierced ſo, that there is a round Hole thro' the Center about two Inches diameter, for eaſy ſeeing the Pole-Star thro' it. *Note*, The Motions of the upper-Plate and Index ought to be pretty ſtiff, that ſo they may remain where they are placed when the Inſtrument is uſing.

The Uſe of this Inſtrument.

Turn the upper circular Plate 'till the longeſt Tooth whereat is marked 12 be againſt the Day of the Month on the under Plate; then bringing the Inſtrument near your Eyes, hold it up by the Handle, ſo that it leans neither to the Right or Left, with it's Plane as near parallel to the Equinoctial as you can; and looking at the Pole-Star thro' the Hole in the Center of the Inſtrument, turn the Index about, 'till, by the Edge coming from the Center, you can ſee the bright Star, or Guard of the Little Bear, if the Inſtrument be adapted for that Star, and that Tooth of the upper Circle that is under the Edge of the Index, is at the Hour of the Night upon the Edge of the Hour-Circle; which may be known without a Light, by accounting the Teeth from the longeſt, which is for the Hour of 12.

You muſt proceed in this manner for finding the Hour of the Night, when the Inſtrument is made for the Guards of *Urſa Major*, which Stars are nearly in a right Line with the Pole-Star, are of the ſame Magnitude, and are very uſeful for finding the Pole-Star.

CHAP. VII.

Of the Conſtruction of a Water-Clock.

THIS Clock is compoſed of a Metalline well ſoldered Cylinder, or round Box B, wherein is a certain Quantity of prepared Water, and ſeveral little Cells, which communicate with each other by Holes near the Circumference, and which let no more Water run thro' them than is neceſſary for making the Cylinder deſcend ſlowly by it's proper Weight. This Cylinder is hung to the Points A A by two fine Cords of equal Thickneſs, which are wound about the Iron Axle-Tree D D, which Axle-Tree goes thro' the exact middle of Fig. 15.

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the Cylinder at right Angles to the Bases, and as it descends shews the Hour marked upon a vertical Plane on both sides of the Cylinder. The Divisions on this Plane are made thus: Having wound up the Cylinder to the top of the Plane from whence you would begin the Hour-Divisions, let it descend 12 Hours, reckoned by a Clock or good Sun-Dial, and note the Place where the Axle-Tree is come to at the End of that Time, and divide the Space the Axle-Tree has moved thro' in 12 equal Parts, each of which set Numbers to, for the Hours.

We make likewise Clocks of this kind, that shew the Hour by a Hand turning about a Dial-Plate, as appears in the same Figure. This is done by means of a Pulley four or five Inches in diameter, fastened behind the Dial-Plate on a Brass or Steel Rod, going thro' the Center thereof; one End of this Rod goes into a little Hole for supporting it, and at the other End is fixed the Hand shewing the Hour.

The said Hand turns by means of a Cord put about the Pulley, one end of which supports the Axle-Tree at the Place H, and at the other end is hung a small Weight F; then as the Cylinder slowly descends, it causes the Pulley to turn about, and consequently the Hand, which by this means shews the Hour.

The Circumference of the Pulley must be equal to the Length the Axle-Tree of the Cylinder moves thro' during twelve Hours; and for this End you must take that Length exactly with a String, and then make the Circumference of the Pulley equal to the Length of the String; and so the Pulley and Hand will go once round in twelve Hours. When the Cylinder descends a little too swift, and consequently the Hand moves too fast, then the Weight F must be made heavier; and when it descends too slow, it must be made lighter.

The Construction of the Cylinder or Round Box.

Fig. 16. This Cylinder is sometimes made of beaten Silver, but commonly with Tin. The Diameter of each Base thereof is about 5 Inches, and the Height 2.

The Inside of this Cylinder is divided into seven little Cells (and sometimes into five), as the Figure shews. These little Cells are made by soldering seven Silver or Tin inclined Planes to each Base, and the concave Circumference of the Cylinder; each of which are about 2 Inches long, as BF, AL, EI, DH, CG. These Cells have such an Inclination when they turn about, that they receive the Water thro' a little Hole in each Plane near the Circumference, and by this means let it run from one Cell to the other; so that as the Cylinder rolls, it descends, and shews the Hour upon a vertical Plane by the Extremity of the Axle-Tree, which (as we have said) goes thro' the square Hole M in the middle of the Cylinder. *Note*, In a Cylinder of the abovesaid bigness we usually pour seven or eight Ounces of distilled Water. But before the Water be poured in, you must take great care to well solder the inclined Planes to the Bases and Circumference. After this, the Water must be poured thro' two Holes posited on one and the same Diameter, equally distant from the Center M; then these Holes must be well stopped with soldering, that so the Air may not get in, or the Water run out while the Cylinder is turning about.

You may perceive, by the Figure, that the inclined Planes within the Cylinder do not join each other, but end in G, H, I, L, F, that so when the Cylinder is winding up, the Water may run swiftly from one Cell to the other, and the Cylinder remain at any Height one pleases; because that at every Motion we give it when winding up, the Water running in a great Quantity thro' the Openings, the Cylinder will presently assume it's *Equilibrium*, which would not happen if the Cells were absolutely inclosed: for the little Holes in the inclined Planes, are not sufficient for letting the Water run thro' them so swift as it ought, it going through them but by drops.

It is manifest, if this Cylinder was suspended by the Center of Gravity thereof, as would happen if the Surface of the Axle-Tree should exactly pass thro' the Center of the said Cylinder, it would remain at rest; and the Cause of it's Motion is, that it is suspended without the Center of Gravity by the Cord's going about the Axle-Tree, which ought not to be, with regard to the bigness of the Cylinder, and the Quantity of Water in it, but about one Line, or one Line and a half, in thickness.

From what has been said it is evident, that the Swiftness or Slowness of the Motion of the Cylinder depends upon the Thickness of the Axle-Tree; for the thicker the Axle-Tree is, the slower will the Cylinder descend, and contrariwise, because it has more or less Excentricity, and consequently the Water will run more or less swift from one Cell to another; by which means the Force of it's Motion will be more or less ballanced by the Weight of the Water contained in the opposite Cell.

If you have a mind to see the Circulation of the Water in one of these Cylinders, you may have one made that shall have a Glass Base; but then it will be difficult to find a Matter that shall make the inclined Planes stick firm to this Glass Case, and this to the Circumference of the Cylinder.

When the Cylinder is nearly descended to the Bottom of the Cords, you must raise it up with your Hand, making it turn at the same time, so that the Cords may equally roll all along the Axle-Tree, and that it be hung horizontally.

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I have hinted before, that the Water poured into the Cylinder muſt be diſtilled, otherwiſe it muſt be often changed, becauſe it makes a Slime about the ſmall Holes thro' which it runs, which hinders it's running as it ſhould do.

CHAP. VIII.

Of the Conſtruction of an Inſtrument, ſhewing on what Point of the Compaſs the Wind blows, without going out of one's Room.

YOU muſt affix to the Ceiling, Mantle-Tree, or Wall of a Room, a Circle divided into 32 equal Parts, for the 32 Points of the Compaſs, ſo that the North and South Points thereof exactly anſwer to the meridian Line, which may be eaſily done by a Compaſs. Then there muſt be a Hand made moveable about the ſaid Circle, and this Hand muſt be turned about by an upright Axle-Tree, which may be turned round by the leaſt Wind blowing againſt the Fane at the Top thereof, above the Roof of the Houſe.

But to explain this more fully, conſult *Fig.* 17. The Wind turning the Fane A B (which ought to be of Iron), fixed to the Top of the Axle-Tree C D, turns this Axle-Tree, which is placed upright, and ſuſtained towards the Top by the horizontal Plane E F, which is a Piece of Iron faſtened to ſome convenient Place for holding up the Axle-Tree. And at the Bottom of the ſaid Axle-Tree is placed a Steel Square G H, having a ſhallow ſmall Hole D made therein for the Point of the Axle-Tree, which ought to be of tempered Steel for the Axle-Tree to ſtand in, and move with the leaſt Wind. The Pinion I K muſt have 8 equal Teeth for the 8 principal Winds. The Teeth of this Pinion take into the Teeth of the Wheel M L, whoſe Number are 16 or 32, according to the Points denoted upon the Circle Y Z; and ſo this Wheel is turned about by the Fane, as alſo it's Axis P Q, which being placed horizontally, goes thro' the Wall T at right Angles to it, as alſo to the Circle of Winds Y Z, fixed to the Wall. The Hand R ſhewing which way the Wind blows, is fixed to the End of this Axle-Tree P Q, and turns along with it; and the Names of the Winds muſt be diſtinguiſhed by Capital Letters, as on Compaſs Cards. Fig. 17.

By the Diſpoſition of the whole Inſtrument it is eaſy to perceive, that when the Wind turns the Fane A B, this likewiſe turns the Axle-Tree C D, which at the ſame Time turns the Pinion I K, and the Pinion I K the Wheel L M, and this the Axis Q P, and Q P the Hand. And ſo you may ſee which way the Wind blows, without going out of the Room.

A ſhort Deſcription of the principal Tools uſed in making of Mathematical Inſtruments.

THE chief and moſt neceſſary Tool is a large Vice, ſerving to hold Work while it is filing, *&c.* It is neceſſary that this Tool be well filed, that the Chops meet each other exactly, that they be cut like a File, be in good temperature, that the Screw be adjuſted as it ſhould be in it's Box; and that the whole Tool be well fixed to a Bench. There are alſo Hand-Vices of different bigneſſes, according to the Work to be filed.

The Anvil, which ſerves for hammering Work upon, ought to be very ſmooth and of tempered Steel, and placed upon a great wooden Billot, ſo that it may not give way when it is working upon.

There are alſo Bench-Anvils for ſtrengthening and rivetting ſmall Work; ſome of theſe, which are called Bec's, and ſerve to make Ferrils upon, *&c.* have one ſide conical, and the other in figure of a ſquare Pyramid.

Hand-Saws are made ſo as to have Branches drawing the Blades (which are of different bigneſſes) ſtraight by means of Screws and Nuts.

It is neceſſary to have good Files. The rough ones made in *Germany* are the beſt; and the ſmooth and baſtard Files of *England* are very good. There are alſo ſmall rough and ſmooth Files, for filing Work Triangular, Square, Circular, Semi-Circular, *&c.* Raſps for faſhioning

faſhioning of Wood; ſeveral ſorts of Hammers for ſtraightening, ſmoothing, rivetting, *&c.* of Work; Tapes and Plates for making Screws.

Pincers and Knippers of ſeveral kinds. Sciſſars of ſeveral Sizes for cutting of Metals. Burniſhing-Sticks for poliſhing Work. Steel-Drills of divers bigneſſes for making of Holes thro' Work, having one end filed like a Cat's Tongue, and the other ſharp. Theſe Drills are uſed different ways; for ſome of them are placed in a drilling Leath, which is compoſed of a ſmall ſquare Iron-Bar, and two little Poupets or Heads carrying a Pulley, wherein is placed the Drill in a ſquare Hole going thro' it, which is turned by means of a little Cat-gut Bow. *Note*, This Tool is placed in a Vice when it is uſing. Braſs or Wood may be drilled alſo by putting it firſt into the Vice, and the Drill in a Pulley. Then if the end of the Drill be put into a ſhallow Cavity made in a Piece of Braſs or Iron, placed againſt your Breaſt, and the Point thereof be put to the thing you would make a Hole thro'; by turning the Drill ſwiftly about by means of the Bow, and at the ſame time preſſing it with your Breaſt againſt the thing to be drilled, you will ſoon make a Hole thro' it.

The Leath is alſo of great uſe; the moſt ſimple of them is made of two Braſs or Iron Poupets or Heads ſliding along a ſquare Iron-Bar, and a Support which alſo ſlides along the ſaid Bar, upon which the Tools are laid when they are uſing. At the Top of the Poupets are two Screws of tempered Steel going thro' them, which are fixed by means of Nuts. When this Leath is to be uſed, it muſt be placed in a Vice, and the thing to be turned, between the two Points of the Screws; and if you have a mind to turn with your Hand, you muſt uſe a Cat-gut Bow.

Great Leaths for turning with one's Foot are compoſed of two wooden Poupets, and two wooden ſide Beams, of a Length and Breadth proportional to the bigneſs of the Leath, which are ſuſtained by two Pieces of Wood called the Feet of the Leath. Theſe ſide Beams are placed level, about two or three Inches diſtant from each other, according to the bigneſs of the Poupets put between them, and the ends of them are adjuſted upon the Feet, which are about four Foot high, and they are likewiſe joined underneath by two or three croſs pieces of Wood, for rendering the Machine more ſtable and ſolid.

The Poupets, which are two pieces of Wood of equal Length and Thickneſs, have one part of each cut ſo as to go in between the ſide Beams; and the other part, being the Head, is cut ſquare, and ſolidly poſited upon the ſide Beams; and that they may be very firm, there are Clefts of Wood drove with a Mallet into Mortice-Holes at the Bottom of the Poupets underneath the ſide Beams.

In the Head of each Poupet is a tempered ſteel Point ſtrongly incloſed in the Wood; ſo that when theſe two Points are brought to each other, they may exactly touch. There is likewiſe a wooden Bar going all along, which is ſuſtained by the Arms of the Poupets, which may be lengthened and ſhortened at pleaſure; and this ſerves as a Reſt for the Tools, when they are uſing.

Againſt the Ceiling, over the Leath, is fixed an Elaſtick wooden Rod, having at the End thereof a Cord faſtened, which comes down to the Ground, and is fixed to the End of a piece of Wood, called the Treader.

Now when you have a mind to work, the Cord muſt be put about the Piece to be turned, or about a Mandril adjuſted to it; and preſſing your Foot upon the Treader, you will turn the Work by means of the Rod which ſprings; then with proper Tools laid upon the Support, and againſt the Piece which is turning, you muſt firſt faſhion it with coarſe Tools, and finiſh it with fine ones.

Becauſe all Work cannot be turned between two Points, one of the Poupets muſt be taken away, and inſtead thereof muſt be placed a Piece of Wood furniſhed with Iron, adjuſted between the ſide Beams as the Poupets are, and inſtead of having a Steel Point has a very round Hole therein, in which goes the Colet of an Iron-Arbor, whoſe other End is ſuſtained by the Steel Point of the other Poupet.

The ſaid Arbor is fifteen or eighteen Inches long, and is compoſed thus: at the End, which is ſupported againſt the aforeſaid Piece of Wood, is a Screw of a very large Thread made round the Arbor, upon which are ſcrewed on divers Braſs Boxes, in which are held faſt the Pieces of Wood, which ſerve to place the ſeveral Works to be turned. And at the other End of the ſaid Arbor are made ſeveral Threads of Screws of different bigneſſes, that ſo Screws may be turned.

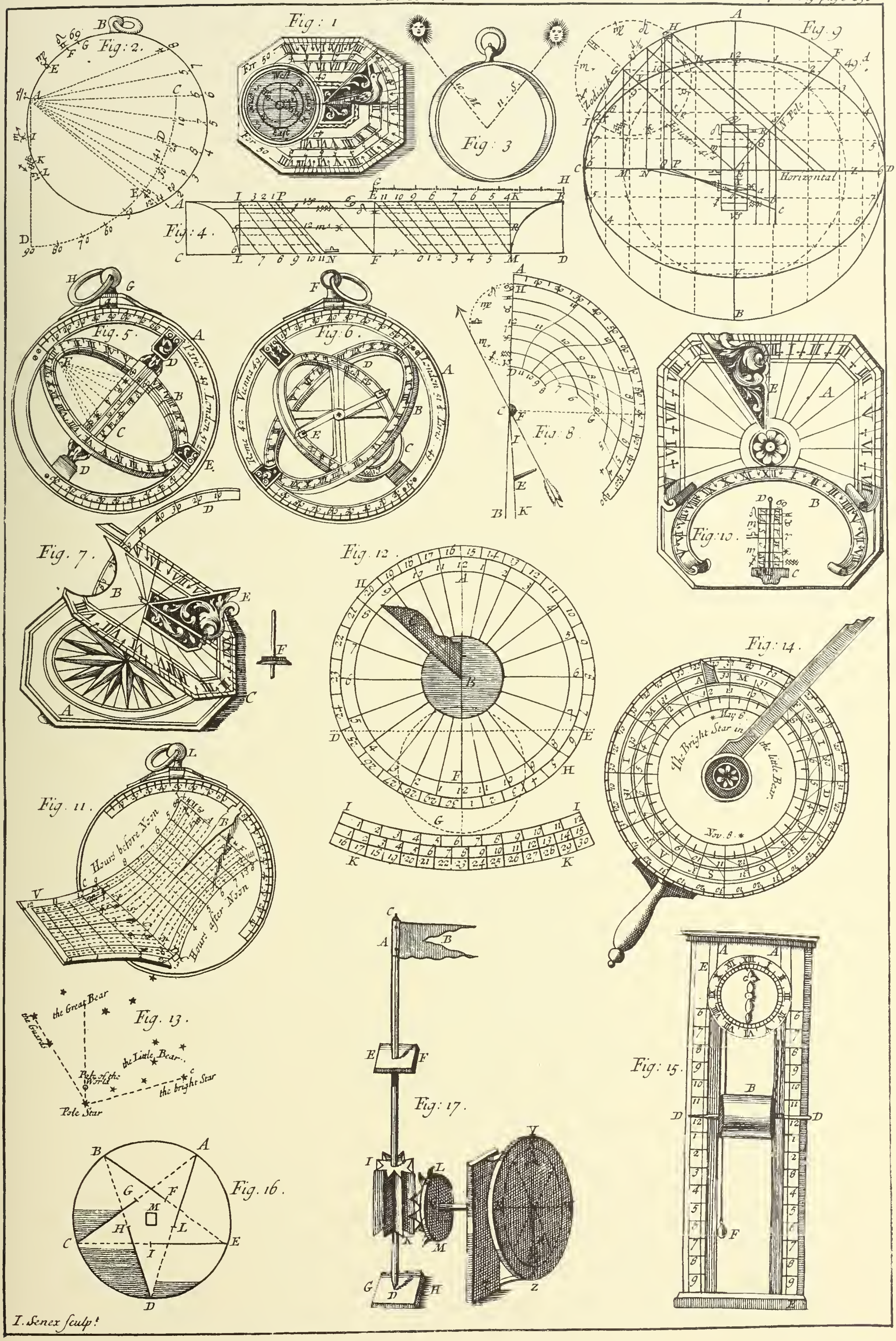
Near the middle of the ſaid Arbor, is placed a Mandril or wooden Pulley, about which goes a Cord. There may be ſeveral other Pieces adjuſted on this Arbor, for turning irregular Figures, as Ovals, Hearts, Roſes, wreathed Pillars, *&c.* All theſe Pieces are filed into the Figures that one would have them make, and have ſquare Holes in the middle of them, which are adjuſted to a Square near the End of the Arbor.

When the Pieces are diſpoſed on the Arbor, the pointed End thereof is placed in a little Hole in the Steel Point of the Poupet, and the other End in the aboveſaid wooden Piece (placed inſtead of a ſecond Poupet), which is made ſo, that there are two Pieces which ſpring, and puſh the Figure backwards and forwards, and by this means move the Arbor backwards and forwards, more or leſs, according to the Figure; and this is the

Cauſe

Plate XXIV

fronting page 256



I. Senex sculp.

Cause that the Tool gives the proper Figure to the Work, which moves to it, or recedes from it, according to the Motion of the Arbor; for the Tool must always be held fast upon the Support. But since these kinds of Figures are seldom used for Mathematical Instruments, I shall say no more as to this way of turning.

The principal Use of the said Arbor, serves for turning of Rings, making of Grooves in Compasses, and other the like things. And this may be done, in placing the Pieces to be turned upon the Wood belonging to the Boxes (of which we have already spoken), which are adjusted on the Leath for receiving the said Pieces. *Note*, The Rests or Supports of the Tools are likewise placed according as the Work requires; some before, and some side-ways.

Male and Female Screws are formed, by putting the proper Thread on the Arbor into a piece of Wood hollowed into a Screw of the same Thread, which is placed at the Poupet carrying the End of the Arbor. And the other End of the Arbor, where is a Colet of the same Thickness, is put exactly into the Hole of the abovementioned piece of Wood; then if the Treader be put in motion by your Foot, the Work will move backwards and forwards, so as that you may form a Screw or a Nut, with toothed Tools made on purpose, according to the Threads marked upon the Arbor. *Note*, For turning of Wood, Googes, Chissels, *&c.* are used. But for Brass and other Metals, smaller Tools of tempered Steel must be used, as Graving-Tools, *&c.*

Thus have I here, and in the Body of this Work, given a short Account of the Tools commonly used in making of Mathematical Instruments. The others may be easily supplied according to Necessity. But since they are usually made by those that use them, I shall here shew how to chuse the best Metal for their Construction.

The best Steel comes from *Germany*. This ought to be without Flaws, Black-Veins, or Iron-Furrows. You may know this by breaking of it, and seeing whether the Grain be very fine and equal.

In forging of Tools, or any thing else of Steel, you must take care of over-heating them, and perform it as soon as possible; for the longer they are hot, the more will they be spoiled.

When the Tools are forged and filed, and you have a mind to temper them, you must heat them red-hot 'till their Colour be something redder than a Cherry, and then they must be tempered in Spring or Well-Water: the colder the Water is, the better. And when they are cold, they must be taken out of the Water, and laid presently upon a Piece of hot Iron, so long, 'till the Colour they have contracted by tempering is lost, and they become yellowish; and then they must be thrown again into the Water, without staying 'till they become blue, because they will lose their Force.

To temper Bundles of Files, or other Pisces of Iron, you must take Chimney-Soot, the oldest and grossest being the best, and having finely powdered it, temper it with Piss and Vinegar, putting a little melted Salt therein, until the whole be as a liquid Paste. The Soot being tempered, the Tools must be covered over with it, and this covered with Earth, and the whole Bundle thrown into a strong Charcoal-fire; and when it is become something redder than a Cherry, it must be taken out and thrown into a Vessel full of very cold Water, and then the Files will be sufficiently hard.

We have already shewed the manner of soldering Brass or Silver to each other; and we would have it here observed, that Iron may be soldered to Iron, by putting thin Brass upon the Piece to be soldered, and the Powder of Borax, and then covering it all round with Charcoal, and heating it until we perceive the Brass melts and runs.

Note, Brass cannot be hammered when it is hot, for it will break; but Copper is hammered cold or hot: but this is seldom used in making of Mathematical Instruments, because Brass is finer and more convenient. Brass is made with red Copper and Calamin, which is a Stone giving a yellow Tincture to the Metal, and is found in the Country of *Liege*, and in *France*.

Gold and Silver may be hammered cold or hot, and may be melted also nearly as Brass is; and Mathematical Instruments are made with Gold and Silver in the same manner as with Brass.

The End of the Translation.

The Use of the Sector *in the Construction of Solar Eclipses.*

DEFINITION I.

THE Path of a *Vertex*, is that Circle of the Earth which any Place or *Vertex* on it's Superficies describes, in the Space of twenty-four Hours, by the Earth's diurnal Revolution. Whence the Paths of *Vertices* are Circles parallel to the Equator.

DEFINITION II.

If a Plane be conceived to touch the Moon's Orbit in that Point, wherein a Line connecting the Centers of the Earth and Sun intersects the said Orbit, and stands at right Angles to the aforenamed Line: And if an infinite Number of right Lines be supposed to pass from the Center of the Sun, thro' this Plane to the Periphery of the Earth, to it's Axis, as likewise to the Axis of the Ecliptick, and the Path of any *Vertex*; the said Lines will orthographically project the Earth's Disk, it's Axis, the Axis of the Ecliptick, and the Path of the *Vertex*, on the aforesaid Plane: and this is the Projection we are to delineate. This being presupposed, it will follow;

1. That when the Sun is in ♋, ♌, ♍, ♎, ♏, ♐, the Northern half of the Earth's Axis projected on the aforesaid Plane, viewed on that Side next to the Earth, lies to the Right-Hand from the Axis of the Ecliptick: But if the Longitude of the Sun be in any of the six opposite Signs, it lies to the Left-Hand from the Axis of the Ecliptick.

2. When the Sun's apparent Place happens to be either in ♈, ♉, ♊, ♋, ♌, ♍, the North Pole lies in the *illuminate*, or visible part of the Disk; but otherways in the *obscure*.

3. When the Sun's Place in the Ecliptick is 90 Degrees distant from either Pole; that is, when the Sun is in the Equator, *the Paths of the Vertices*, or all Circles of the Earth parallel to the Equator, will be projected in right Lines upon the said Plane: but if the Sun's Place be lesser than 90 Degrees, the said Paths will be projected in Ellipses upon the said Plane, whose conjugate Diameters will be so much the lesser, as the Place of the Sun is lesser.

4. *The transverse Diameter of the Ellipses* representing any Path, is equal to double the right Sine of the Distance of the said *Vertex* from the *Pole*; that is, equal to twice the Co-Sine of the Latitude of the Place or *Vertex*: but the *Conjugate*, to the Difference of the right Sines of the Sum, and Difference of the Distances of the Path and Sun from the Pole; that is, equal to the Sine of the Complement of the Sun's Declination added to the Co-Latitude of the Place, less the right Sine of the Difference of the Complement of the Sun's Declination, and the Co-Latitude of the Place.

5. The transverse Diameter lies at right Angles to the Earth's Axis, and the conjugate co-incides therewith.

SECTION.

SECTION I.

To repreſent in Plano, the Path of a Vertex in the Earth's Diſk, whoſe Diſtance from the North Pole is 38 Deg 32 Min. the Sun's Place being in 10 Deg. 40 Min. 30 Sec. of Gemini, *ſemblable to that which will be projected on a Plane, touching the Earth's Orbit in that Point, by ſtrait Lines produced from the Sun to the Earth.*

HAVING drawn the Semi-Circle H E R, let it repreſent the Northern half of the Earth's illuminate Diſk (becauſe the Sun is in *Gemini*) projected upon the ſaid Plane, the Sun it's Center, the Point therein oppoſite to the Sun, H ⊙ R an Arc of the Ecliptick paſſing through it. Upon ⊙ raiſe ⊙ E, perpendicular to the Ecliptick H R, and the Point E wherein it interſects the Limb of the Diſk, will be the Pole of the Ecliptick, and ⊙ E it's Axis. *Plate 25. Fig. 1.*

Again; Make ⊙ E equal to the Radius of a Line of Chords (by *Uſe* III. *of the Line of Chords*), from which taking the Chord of 23 Deg. 30 Min. (the conſtant Diſtance of the two Poles) ſet it off both ways from E to B and C, draw the Line B C, in which the Northern Pole of the World ſhall be found.

Make B A equal to A C, the half of this Line, the Radius of a Line of Sines, and therein ſet off the Sine of the Sun's Diſtance from the ſolſtitial Colure 19 Deg. 19 Min. 30 Sec. from A to P, on the Left-Hand of the Axis of the Ecliptick (becauſe the Sun is in *Gemini*), and draw the Line ⊙ P, which will be the Axis of the Earth, and P the Place of the North-Pole in the illuminate Hemiſphere of the Diſk.

Or the Angle E ⊙ I, which the Axis of the Earth and Ecliptick make with each other, may be more accurately determined by Calculation. For,

	Deg.	*Min.*	*ſec.*	
As Radius——to the Sine of the Sun's Diſtance from the ſolſtitial Colure - - - - - - - - -	90 19	00 19	00 30	10,000000 9,519731
So is the Tangent of the Sun's greateſt Declination to the Tangent of the Inclination of the Axis - - - -	23 8	30 10	00 54	9,637956 9,157687

Count the ſaid 8 Deg. 10 Min. 54 Sec. in the Limb of the Diſk from E to I, on the Left-hand, and draw the Line ⊙ I, this ſhall be the Axis; and the Point P wherein it interſects the Line B C, the Place of the Pole in the illuminate Diſk.

The next thing required will be the Sun's Diſtance from the Pole, or the Complement of his Declination, which will be found 67 Deg. 57 Min. 48 Sec. this added to the Diſtance of the Vertex from the Pole 38 Deg. 32 Min. makes 106 Deg. 29 Min. 48 Sec. and the ſame 38 Deg. 32 Min. taken from 67 Deg. 51 Min. 48 Sec. gives 29 Deg. 25 Min. 48 Sec. the Meridional Diſtance of the Sun from the Vertex.

Make ⊙ E the Radius of the Diſk, to be the Radius of a Line of Sines, from which take the Sine of 73 Deg. 30 Min. 12 Sec. (the Complement of 106 Deg. 29 Min. 48 Sec. to a Semi-Circle) and ſet it off in the Axis from ⊙ to 12; it there gives the Meridional Interſection of the Nocturnal Arc of the Path with the Axis.

Take the Sine of 29 Deg. 25 Min. 48 Sec. from the ſame Line of Sines, and ſet it off the ſame way from ⊙ to M, and it there gives the Interſection of the diurnal Arc of the Path with the Meridian. Whence M 12 will be the conjugate Diameter of the Path, it being the Difference of the Sines of 70 Deg. 30 Min. 12 Sec. and 29 Deg. 25 Min. 48 Sec. that is, the Difference of the Sines of the Sum, and Difference of the Diſtances of the Path and Sun from the Pole, which will be the conjugate Diameter of any Path.

Biſect 12 M in C, and through it draw 6 C 6 at right Angles to the Axis of the Globe; and then taking the Sine of 38 Deg. 32 Min. the Diſtance of the Pole from the Vertex, ſet it off from C both ways to 6 and 6; then the Line 6 6 will be the Tranverſe-Diameter of the Path, and C 6 the Semi-Diameter.

Making C 6 equal to the Radius of a Line of Sines, if from the ſame you take the right Sines of 15, 30, 45, 60, 75 Degrees, and ſet them off ſeverally both ways from C in the Tranverſe-Diameter, and from the Points ſo found erect Perpendiculars, *a* 11, *a* 1, *a* 10, *a* 2, *&c.* equal to the Co-Sines of the ſaid Arcs, taken from a Line of Sines, whoſe Radius ſhall be C 12, equal to C M, you will have twenty-four Points given, through which the Ellipſis repreſenting the Path ſhall paſs, which ſhall alſo ſhew the Place of the Vertex at every Hour of the Day. In the ſame manner may the Parts of an Hour be pricked down in the Path, in laying off the Sine of the Degrees and Minutes correſponding thereto from C towards 6, and then raiſing Perpendiculars from the Points ſo found in the Semi-Tranſverſe, and ſetting off from the ſaid Semi-Tranſverſe each way upon the Perpendiculars, the Sines of the Complements of the Degrees and Minutes correſponding to the aforeſaid Parts of an Hour. As, for Example; To denote half an Hour paſt 11 and 12, take the Sine of 7 Deg. 30 Min. and lay it off on both ſides from C to *b* and *b*; then take the Co-ſine of 7 Deg. 30 Min. and having raiſed the Perpendiculars *b* ', lay off the ſaid Sine-Complement from *b* to $\frac{1}{2}$, and you will have the Points in the Periphery of the Ellipſis, for half an Hour paſt 11, and half an Hour paſt 12; and in this manner may the Path be divided into Minutes, if the Ellipſis be large enough.

Take this for another Example; Suppoſe I would repreſent upon the Plane of the Earth's Diſk, the Path of *Gibraltar*, whoſe Latitude is 35 Deg. 32 Min. North, and the Sun's Place is in 15 Deg. 45 Min. of *Leo*.

Fig. 2. Having, as before, drawn the Semi-Circle H E R, for the Northern half of the Earth's illuminate Diſk, and drawn ☉ E perpendicular to R H, as alſo drawn the Line C B, which is always equal to twice the Chord of the Sun's greateſt Declination, 23 Deg. 30 Min. you muſt next make A B equal to a Radius of a Line of Sines, and then lay off from A to P, on the Right-hand of the Axis of the Ecliptick (becauſe the Sun is in *Leo*), the Sine of the Sun's Diſtance from the ſolſtitial Colure 45 Deg. 45 Min. or, the Angle E ☉ I may be more nicely determined by Calculation, as was before directed, and then ☉ P I, will be the Axis of the World.

Now the Sun's Diſtance from the Pole, or the Complement of his Declination is 73 Deg. 51 Min. which being added to the Complement of the Latitude 54 Deg. 28 Min. the Sum will be 128 Deg. 19 Min. and this taken from 180 Deg. the Remainder will be 51 Deg. 41 Min. alſo if 54 Deg. 28 Min. be taken from 73 Deg. 51 Min. the Difference will be 19 Deg. 23 Min.

Then if you make the Semi-Diameter of the Diſk the Radius of a Line of Sines, and lay off from the Center ☉ to 12, the Sine of 51 Deg. 41 Min. the Point 12 in the Axis will be the Meridional Interſection of the Nocturnal Arc of the Path with the Axis; and if again you lay off the Sine of 19 Deg. 23 Min. from ☉ to M, you will have the Meridional Interſection of the Diurnal Arc of the Path with the Axis; whence M 12 will be the conjugate Diameter of the Elliptical Path.

And if you biſect M 12 in C, and draw the Line 6 C 6 at right Angles to the Axis ☉ I; and then lay off the Sine Complement of the Latitude 54 Deg. 28 Min. from C to 6, on each ſide the Axis, you will have the Tranſverſe-diameter of the Path, which may be drawn and divided, as before directed, for that of *Fig.* 1.

Note, When the elevated Pole is in the obſcure Hemiſphere of the Earth, the diurnal Arc, or illuminated Part of the Path, is in that Part of the Ellipſis that lies neareſt to the ſaid Pole, but otherways in the more remote; and where the Ellipſis cuts the Limb of the Diſk, are the Points on it from which the Sun appears to riſe and ſet, *&c.* And becauſe theſe Points are neceſſary to be found, when an Eclipſe happens near Sun-riſing or Sun-ſetting, they may be determined in the following manner:

Fig 1. Lay off the Sun's Declination 22 Deg. 2 Min. upon the Limb of the Diſk from R to N, as alſo the Complement of the Latitude of 38 Deg. 32 Min. from R to P; then draw the Line ☉ N, and from the Point P let fall upon the Diameter R H, the Perpendicular P Q, cutting the Line ☉ N in L. This being done, take the Extent ☉ L, between your Compaſſes, and lay it off upon the Axis ☉ I from ☉ to K; then draw a Line both ways from the Point K, parallel to the tranſverſe Axis C 6 of the Path, and the ſaid Line will cut the Limb of the Diſk in the Points *q p* of the Sun's riſing and ſetting.

Or the Arc I *p* may be more accurately determined by Calculation; for in the Triangle ☉ Q L, right-angled at Q, are given the Angle Q L ☉, equal to the Sun's Diſtance from the Pole; and the Side Q ☉ equal to the Sine of the Latitude. To find the Side O L, which is equal to the Sine Complement of the Arc I *p*, the Canon is, As the Sine of the Sun's Diſtance from the Pole, is to Radius; So is the Sine of the Latitude to the Sine Complement of the Arc I *p*, or I *q*.

SECTION II.

HAVING in the foregoing Section ſhewn how to draw the Path of any Vertex upon the Earth's Diſk, as likewiſe to divide it, the next things neceſſary to be given, in order to conſtruct the Phaſes of a Solar Eclipſe in any given Place on the Earth's Superficies, are;

I. The apparent Time of the neareſt Approach of the Moon to the Center of the Diſk, or the Time of the Middle of the Eclipſe.

II. The neareſt Diſtance of the Moon's Center from the Center of the Diſk in her Paſſage over it; which is equal to her Latitude at the Time of the Conjunction.

III. The Semi-Diameter of the Diſk at the Time of the Conjunction.

IV. The Moon's Semi-Diameter at the ſame Time.

V. The Sun's Semi-Diameter.

VI. The Semi-Diameter of the Penumbra.

VII. The

Fig. 1.

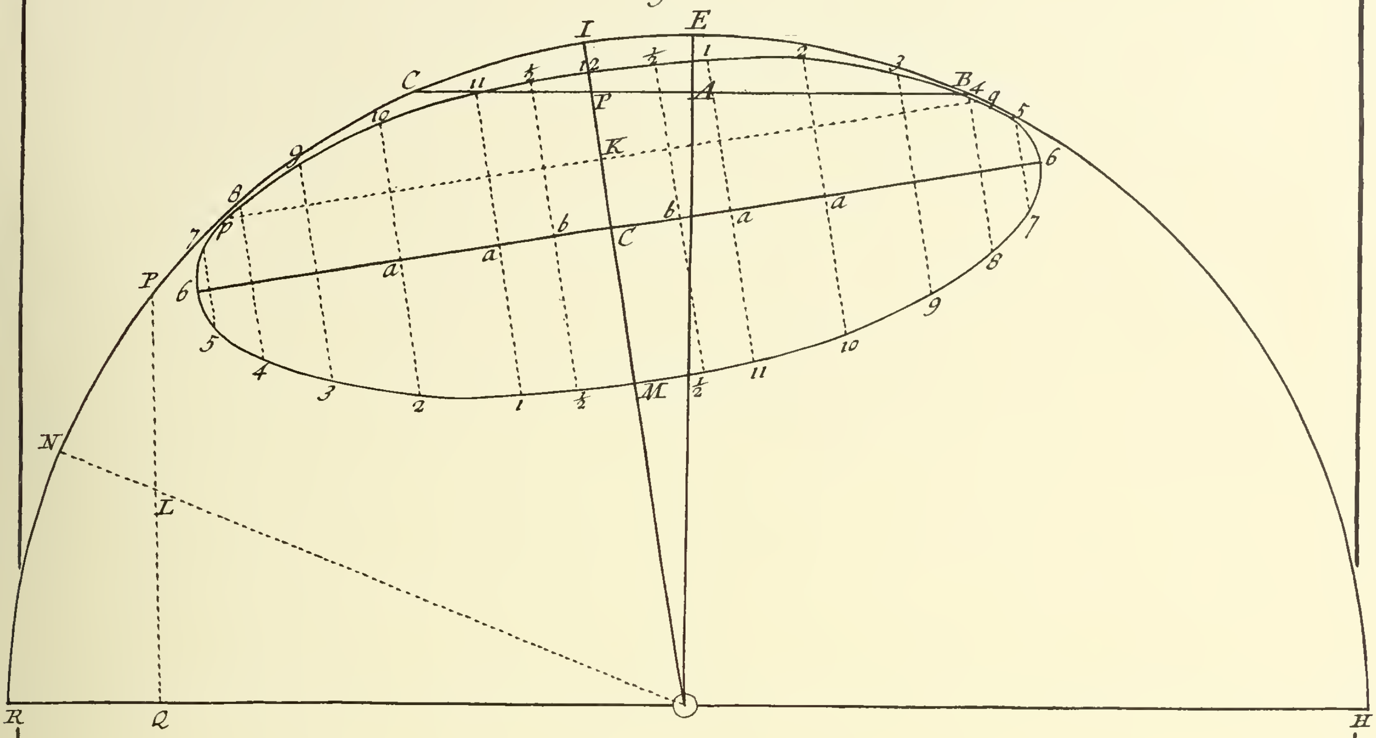
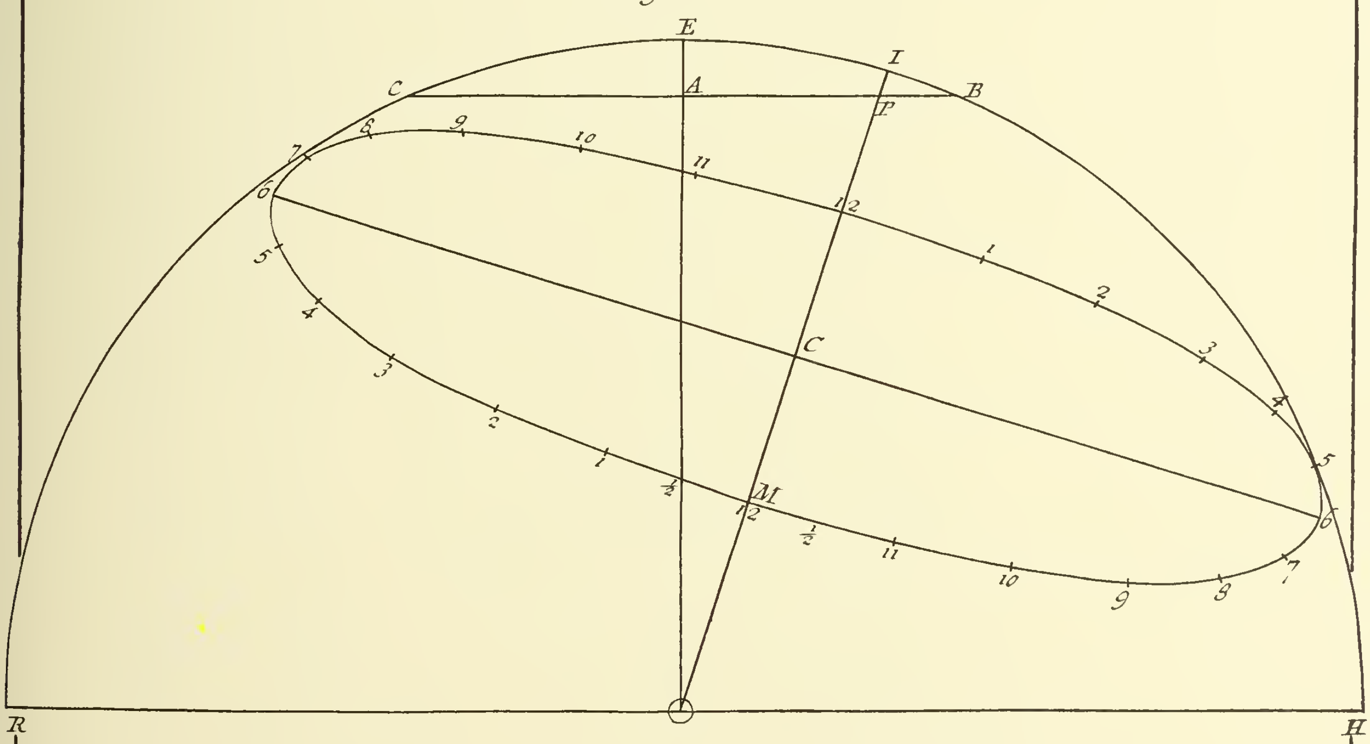


Fig. 2.



I Senex sculp^t

VII. The Angle of the Moon's Way with the Ecliptick, which is equal to the Angle that the Perpendicular to the Moon's Way forms with the Axis of the Ecliptick; and if the Argument of Latitude be more than 9 Sines, or less than 3, the said Perpendicular lies to the Left-hand; if more, to the Right, from the Axis of the Ecliptick.

VIII. The hourly Motion of the Moon from the Sun at the Time of the Conjunction.

Note, The Semi-Diameter of the Disk is always equal to the Difference of the Sun and Moon's horizontal Parallaxes.

All these for the Solar Eclipse of *May* 11, 1724, will be as follows:

	Hours.	*Min.*	*sec.*	
The apparent Time of the nearest Approach of the Moon to the Center of the Disk, will be - - - - - - -	5	12	0	Afternoon.
The nearest Distance of the Moon's Center from the Center of the Disk - - - - - - - - - - - -	0	32	14	
The Semi-Diameter of the Disk - - - - - -	0	61	38	
The Moon's Semi-Diameter - - - - - - - -	0	16	42	
The Sun's Semi-Diameter - - - - - - - -	0	15	53	
The Semi-Diameter of the Penumbra - - - - -	0	32	35	
The Angle of the Moon's Way with the Ecliptick - -	0	5 deg.	37 min.	
The hourly Motion of the Moon from the Sun - - -	0	35	18	

These being found from Astronomical Tables and Calculations, I shall shew how to draw the Line of the Moon's Way, or Path of the Penumbra, upon the Plane of the Earth's Disk, as it falls at the Time of the Conjunction of *May* 11, 1724, and the manner of dividing the same, for *London*, *Genoa*, and *Rome*.

Having drawn the Semi-Circle H E R of the Earth's Disk, and the Paths of *London*, *Genoa*, and *Rome*, by the Directions of the last Section, the Sun's Place being 61 Deg. 38 Min. 45 Sec. and the Latitude of *London* 51 Deg. 30 Min. that of *Genoa* 44 Deg. 27 Min. and that of *Rome* 41 Deg. 51 Min. you must next draw the Perpendicular to the Moon's Way; which is done thus: Take the Semi-Diameter ⊙ H of the Disk between your Compasses, and open your Sector so, that the Distance from 60 to 60 of Chords be equal to that Extent; then taking 5 Deg. 37 Min. parallel-wise from the Lines of Chords (which is the Angle of the Moon's Way with the Ecliptick, or the Angle that a Perpendicular to her Way makes with the Axis E ⊙ of the same Ecliptick), lay them off upon the Limb of the Disk from E to F, on the Right-hand of the Axis of the Ecliptick, because the Argument of Latitude is more than three Sines, and the Line ⊙ F being drawn, will be the Perpendicular to the Moon's Way at the Time of the general Conjunction, *May* 11, 1724. *Plate* 26. Fig. 1.

Again: Take the Semi-Diameter of the Disk between your Compasses, and open the Sector so, that the Distance from $61\frac{38}{60}$, the Semi-Diameter of the Disk, on each Line of Lines be equal to that Extent; then the Sector remaining thus opened, take between your Compasses the parallel Extent of $32\frac{14}{60}$, the nearest Approach of the Moon to the Center of the Disk, and lay it off from ⊙ to M, upon the Perpendicular to the Moon's Way; then, if upon the Point M, a Perpendicular, as M G, be drawn both ways, this will be the Line of the Moon's Way, or Path of the Penumbra.

Now to divide the said Path into it's proper Hours, which let be for *London*. The middle of the general Eclipse, or the Time when the Moon's Center will be at M, happens at 12 Minutes past 5 in the Afternoon: say, As 1 Hour or 60 Minutes is to 35 Min. 18 Sec. the hourly Motion of the Moon from the Sun; So is 12 Minutes the Time more than 5 in the Afternoon, to 7 Min. 3 Sec. the Motion from 5 a-Clock to the middle.

Your Sector remaining opened to the last Angle it was set to, take the Extent from $7\frac{3}{60}$ to $7\frac{3}{60}$ on each Line of Lines, and setting one Foot of your Compasses upon M, with the other make a Point on the Moon's Way to the Right-hand; and this shall be the Place of the Penumbra at 5 a-Clock in the Afternoon at *London*; which therefore denote with the Number V.

The hourly Motion of the Moon from the Sun is 35 Min. 18 Sec. therefore take the parallel Extent of $35\frac{18}{60}$, on the Line of Lines, between your Compasses, and setting one Foot upon V, with the other make Points on each side V, these shall shew the Place of the Moon's Center at the Hours of IV and VI; and if from these Points you farther set off the said Extent in the said Line, you may thereby find the Place of the Moon's Center for every Hour, whilst the Penumbra shall touch the Disk: and if the Space between every Hour be divided into 60 equal Parts, you shall have the Place of the Moon's Center in the Line of her Way, to every single Minute of Time.

Or, you may take the Semi-Diameter of the Disk between your Compasses, and make a Scale thereof, in dividing it, by means of the Sector, in the following manner: Open the Sector so, that the Distance between $61\frac{38}{60}$, the Semi-Diameter of the Disk, and $61\frac{38}{60}$ on the Line of Lines, be equal to the Semi-Diameter of the Disk. This Distance lay off from

A to B: then your Sector remaining thus opened, take between your Compasses successively, the parallel Distances of each Division to 61 $\frac{35}{60}$, and lay them off from A towards B, every 5th of which Number, and your Scale will be divided into Minutes. And by the same Method you may divide each Minute into Parts, serving for Seconds, if your Scale be long enough. Now your Scale being divided, you may make use thereof, for drawing and dividing the Path of the Penumbra, without the Sector: For 32 $\frac{14}{60}$ of these Parts of the Scale, give you the nearest Distance of the Moon's Center to the Center of the Disk. Also 7 $\frac{3}{60}$ Parts of the said Scale, will be the Distance of the Center of the Penumbra from the Point M, at five a-Clock; and 35 $\frac{18}{60}$ of the Parts of the Scale, will be the Distance from Hour to Hour, on the Path of the Penumbra.

Now to fix Numbers upon the said Path of the Penumbra, representing the Hours when the Moon's Center will be at the said Hours, at *Rome* and *Genoa*, we must have the Difference of Longitude between *London* and the said two Places given; as also, whether they are to the East or West from *London*; the Difference of Longitude between *London* and *Rome*, is 12 Deg. 37 Min. and between *London* and *Genoa*, is 9 Deg. 37 Min. they being both to the East from *London*. Each of these being reduced to Time, the former will be 50 Minutes, and the latter 38 Minutes, wherefore 5 a-Clock for *Rome* on the Moon's Way, must be at 10 Min. past 4, for *London*; and 6 a-Clock at 10 Minutes past five, *&c.* Understand the same for other Hours and Minutes. I have noted the Hours for *Rome* under the Line of the Moon's Way, with *Roman* Characters. Again, 5 a-Clock on the Moon's Way for *Genoa*, must be set at 22 Minutes past 5 for *London*; and 6 a-Clock, at 22 Minutes past 6, *&c.* I have noted the Hours for *Genoa* with small Figures over the Line of the Moon's Way.

Note, The 10 Minutes, and 22, are each of them the Complement of 50 Minutes, and 38 Minutes to 60 Minutes.

SECTION III.

To determine the apparent Time of the Beginning or End of a Solar Eclipse, the Time when the Sun shall be eclipsed to any possible Number of Digits, the Inclination of the Cusps of the Eclipse, and the Time of the visible Conjunction of the Luminaries, in any given Latitude.

THE Paths of *London*, *Rome*, and *Genoa*, as also the Path of the Penumbra being drawn and divided, as directed in the two last Sections for the great Eclipse of 1724, which will be a very proper Example for sufficiently explaining this Method, take between your Compasses the Semi-Diameter of the Penumbra 32 $\frac{35}{60}$, from the Line of Lines on the Sector, it being first opened to the Semi-Diameter of the Disk 61 $\frac{33}{60}$; or you may take it from your Scale, which being done, carry one Foot of your Compasses along the Line of the Moon's Way, from the Right-hand to the Left; wherein find such a Point, that if the said Foot be set, the other Foot shall cut the same Hour or Minute, in the Path of the Vertex of any given Place; then the Points in the Paths upon which either of the Feet of your Compasses stand, will shew the Time of the Beginning of the Eclipse at that Place.

For example; If you carry the Semi-Diameter of the Disk along the Line of the Moon's Way, you will find that one Foot of the Compasses being set at *a*, on the Moon's Way, which is 41 Min. past 5 in the Afternoon for *London*, the other Foot will fall on the Point *b* on the Path of *London*, which is likewise 41 Min. past 5 in the Afternoon; wherefore the Beginning of the Eclipse at *London* will be at 41 Min. past 5 in the Afternoon.

Again: If you carry still on the Foot of your Compasses, they remaining yet opened to the Semi-Diameter of the Disk, and find another Point on the Moon's Way, whereon if you fix one Point of your Compasses, the other shall cut the Path of the Vertex at the same Hour or Minute, which this stands upon in the Line of the Moon's Way, the Points whereon your Compasses stand in either Path, shall shew the Minute the Eclipse ends.

For example: One Foot of the Compasses being set to *g* in the Path of the Vertex, which is 29 Min. past 7 in the Afternoon, the other Foot will fall upon the Line of the Moon's Way, at the same Hour and Minute, *viz.* 29 Min. past 7; therefore the Eclipse ends at *London* 29 Min. past 7: but take notice, that the Line of the Moon's Way should be continued further out beyond 7 a-Clock, that so the Point of the Compasses may fall upon the proper Minute, to wit, 29.

Moreover: If one side of a Square be applied to the Ecliptick H R, and so moved backwards or forwards, until the other side of the said Square cuts the same Hour or Minute in the Path of the Vertex, and Line of the Moon's Way; this same Hour or Minute will be the Time of the visible Conjunction of the Luminaries at the given Place.

For example; When the perpendicular side of the Square cuts the Path of the Moon's Way at *e*, which is 37 Min. past 6, the said side will likewise cut the Path of the Vertex for *London* at *c*, which is 37 Min. past 6; therefore the Time of the visible Conjunction of the Luminaries at *London* will be 37 Min. after 6.

Draw

Draw the Line *a b*, as also the Line ⊙ *b*; this shall represent the vertical Circle, and the Angle ⊙ *b a* will be the Angle that the vertical Circle makes with the Line connecting the Centers of the Sun and Moon, at the Beginning of the Eclipse at *London*.

Draw the Line *g m*; to wit, join the Points in the Path of the Vertex, and the Path of the Moon's Way, which shews the End of the Eclipse at *London*; and the Line ⊙ *g*; then the Angle ⊙ *g m*, will be that which the vertical Circle forms with the Line joining the Centers of the Luminaries.

Take the Semi-Diameter of the Sun, *viz.* $15\frac{53}{60}$ between your Compasses, either from your Sector, opened as before directed, to the Semi-Diameter of the Disk, or from your Scale, and with that upon the Center *c* (to wit, the Minute in the Path of *London*, whereat the Time of the visible Conjunction happens) describe a Circle; this Circle shall represent the Sun.

Again; Take the Moon's Semi-Diameter $16\frac{42}{60}$ from your Sector (remaining opened as before), or your Scale, and upon the Center *e* (to wit, the Minute in the Path of the Moon's Way, whereat the true Conjunction happens at *London*) describe another Circle. This shall cut off from the former Circle so much as the Sun will be eclipsed, at the Time of the visible Conjunction.

From ⊙ draw the Line ⊙ *c v*: This shall represent the vertical Circle, and *v* the vertical Point in the Sun's Limb, whereby the Position of the Cusps of the Eclipse, in respect of the Perpendicular passing thro' the Sun's Center, are plainly and easily had.

Produce *d c* 'till it interfects the Moon's Limb in *p*, then shall *p q* be the part of the Sun's Diameter eclipsed, at the Time of the greatest Obscuration at *London*: And if the Sun's Diameter be divided into 12 equal Parts, or Digits, you will find *p q* to be $11\frac{66}{100}$ of those Parts or Digits.

Whence at *London*,

	H.	M. Aftern.
The Beginning of the Eclipse, *May* 11, 1724, at - - -	05	41
The visible Conjunction of the Luminaries - - - -	06	37
	Digits then $11\frac{66}{100}$	
The End - - - - - - - - - - -	07	29
After the same manner, the Beginning of the Eclipse at *Genoa* will be	06	27
Visible Conjunction, or middle of the Eclipse - - - -	07	20

The Sun will there set eclipsed, and the Eclipse will be Total.

And the Beginning of the Eclipse at *Rome* is - - - -	06	42

The visible Conjunction, or Middle, will there be when the Sun is set, and consequently also the End.

I have, as you see in the Figure, also drawn a fourth Path for *Edinburgh*, whose Latitude is 55 Deg. 56 Min. and Longitude about 3 Deg. to the West from *London*. Wherefore for each Hour in the Moon's Way for *London*, you must account 12 Min. more for the same Hour at *Edinburgh*; that is, for example, 5 a-Clock on the Line of the Moon's Way for *Edinburgh*, must stand at 12 Min. past 5 at *London*. Understand the same for other Hours, *&c.*

And by proceeding according to the Directions before given, you will find,

At *Edinburgh*,	H.	M. Aftern.
The Beginning of the Eclipse at - - - - - - -	05	22
The Middle - - - - - - - - - -	06	20
	Dig. then 11.	
The End - - - - - - - - - - -	07	14

Note, The Path of the Moon's Way ought to be continued out further to the Left-hand, in order to determine the Time of the End of the Eclipse at *Edinburgh*.

If you have a mind to know at what Time any possible Number of Digits or Minutes shall be eclipsed at any Place in the Sun's antecedent or consequent Limb; divide the Sun's Diameter into Digits or Minutes, and cut off the Parts required to be eclipsed from the Semi-Diameter of the Penumbra; then take the remaining part of it between your Compasses, and carrying it along the Line of the Moon's Way, find the first Point in it, in which placing one Foot, the other will cut the same Hour in the Path of the Place that the fixed Foot stands upon; then the Hour and Minute in either Path upon which the Feet of your Compasses stand, will be the Time of that Obscuration.

As, for example; Suppose it was required to find at what Time 6 Digits or $\frac{1}{2}$ of the Sun's Diameter shall be eclipsed in his antecedent Limb at *London*: Cut off $\frac{1}{2}$ of the Sun's Semi-Diameter from the Semi-Diameter of the Penumbra, and carrying the Remainder, as directed, you will find, that if one Point of your Compasses be set at 6 Hours 9 Minutes in the Afternoon, on the Path of the Moon's Way, the other Point will also fall upon the same Hour and Minute in the Path of *London*; and therefore the Time when the Sun's antecedent Limb at *London* will be half eclipſed, will be at 9 Minutes paſt 6; and when it's conſequent Limb will be half eclipſed, will be at 5 Minutes paſt 7.

Fig. 2. Now to determine the Poſition of the Cuſps of the Eclipſe, for example, at *London*: Draw a Circle A D B E, repreſenting the Sun's Body, and the right Line A C B, repreſenting his vertical Diameter. This being done, lay off the Angle ⊙ *b a* upon the Sun's Limb from A to D, draw the Diameter E C D, and the Point D will be the firſt Point of the Sun's Limb obſcured by the Moon at the Beginning of the Eclipſe.

Fig. 3. Again; To determine the Poſition and Appearance of the Eclipſe at the Time of the middle, or greateſt Obſcuration, take the Sun's Semi-Diameter between your Compaſſes, and upon the Point C, deſcribe a Circle; then draw the vertical Diameter A C B, and make the Angle A C D equal to the Angle *v c p*, and draw the Diameter D C F. This being done, take the Moon's Semi-Diameter between your Compaſſes, and having laid off from the Center C to E, the Diſtance *c e* in the firſt Figure; upon the Point E, as a Center, deſcribe an Arc cutting the Sun's Limb, and the Poſition and Appearance of the Eclipſe at the Time of the greateſt Obſcuration, or the middle, at *London*, will be as you ſee in the Figure.

Laſtly, To determine the Poſition of the End of the Eclipſe, draw a Circle (as in the 4th Figure), and croſs it with the vertical Diameter A C B; then make the Angle A C E equal to the Angle ⊙ *g m*, and draw the Diameter E D; then will the Point E on the Limb of the Sun, be that which is laſt obſcured, or whereat the Eclipſe ends.

If you have a mind to find the Continuation of total Darkneſs at any Place where the Sun will be totally eclipſed, cut off the Semi-Diameter of the Sun, from the Semi-Diameter of the Penumbra, and taking the Remainder between your Compaſſes, carry it along the Line of the Moon's Way, and find the firſt Point in it; on which placing one Foot, the other will cut the ſame Hour in the Path of the Place, which Hour note down. Again; Carrying on further the ſame Extent of your Compaſſes, find two Points on the Paths of the Vertex and Moon's Way, which ſhall ſhew the ſame Hour and Minute on them both. This Time alſo note down; then ſubſtract the Time before found from this Time, and the Difference will be the Time of Continuance of total Darkneſs.

THE SUP-

A SUPPLEMENT TO THE *English* TRANSLATION OF Mr *Bion*'s CONSTRUCTION and USE OF Mathematical Inſtruments:

Containing a further

ACCOUNT

Of ſome of the moſt uſeful

MATHEMATICAL INSTRUMENTS, both Antient and Modern, as now improved.

By *EDMUND STONE.*

LONDON:

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M.DCC.LVIII.

ADVERTISEMENT.

IT is now almoſt forty Years ſince I tranſlated Mr *Bion*'s *French* Book of Mathematical Inſtruments into *Engliſh*. I did it with Reluctance, at the Deſire of Friends, and a little for the Sake of Intereſt, the Subject becoming ſomewhat unpleaſant to me, at that Time, by Uſe and long Acquaintance; for having at firſt moſtly applied myſelf, even from twelve Years of Age, in the Knowledge of Mathematical Inſtruments, I began to be tired and ſatiated, as I may ſay, with them, when I undertook this Work, although they are generally ſo pleaſing and uſeful, and betook myſelf to the more refined and difficult Branches of the Mathematicks. I was reading, and trying to underſtand Sir *Iſaac Newton*'s *Mathematical Principles of Natural Philoſophy*, at the Time I was prevailed upon to tranſlate Mr *Bion*'s Book, which I liked to do much better. The deſire of travelling further, and acquiring new Knowledge in this amiable Science is very natural to one who loves it. But having ſet about the Buſineſs, I ſoon perceived that many *French* Inſtruments of Mr *Bion*'s were excelled by ſome of ours, of the ſame kind in Contrivance; and as to Workmanſhip, I never did ſee one *French* Inſtrument ſo well framed and divided, as ſome of ours have been; for Example, Mr *Sutton*'s Quadrants, made above one hundred Years ago, are the fineſt divided Inſtruments in the World; and the Regularity and Exactneſs of the vaſt Number of Circles drawn upon them is highly delightful to behold. The mural Quadrant at the *Royal Obſervatory* at *Greenwich*, far exceeds that of the *Royal Obſervatory* at *Paris*. Alſo the Theodolites of Mr *Siſſon* and *Heath*. The Clocks and Watches of Mr *Graham*, *Tompion*, and *Quare*. The Orreries of Mr *Graham* and Mr *Wright*. And many more curiouſly contrived, and well executed Mathematical Inſtruments which I could mention, far exceed thoſe of the *French*, or indeed any other Nation in the World. The making good Mathematical Inſtruments is almoſt peculiar to the *Engliſh*, as well as their Skill in all Branches of the Mathematicks and Natural Philoſophy has been generally ſuperior to that of other Nations.

I therefore thought of adding ſome *Engliſh* Inſtruments to thoſe of Mr *Bion*, and accordingly made choice of ſuch which I thought were then preferable to his of the ſame Kind, and moſt pleaſing to me.

This firſt Impreſſion being publiſhed, imperfect as it was, rather for want of Inclination in me, than Abilities to make it better, was ſoon ſold off; and the Bookſeller and others, at Times for many Years laſt paſt, were putting me in mind of preparing a ſecond Impreſſion with Additions. But being at thoſe Times always immerged in other more delightful Branches of the Mathematicks, eſpecially pure Geometry and Mechanicks, I declined doing it. But having at laſt been prevailed upon, through ſeveral Motives, this ſecond Impreſſion is here publiſhed, with a Supplement or Appendix, containing the Deſcriptions, and ſome of the Uſes of ſuch Inſtruments as were before omitted, or have been made better, as I thought ſince, for the Purpoſes deſigned by them, and wherein there are ſome other Inſtruments, though juſt touched upon and deſcribed, yet there is given ſo much of their Nature and Uſe, as may ſatisfy the Curioſity of ſome; and for thoſe who deſire more ample and compleat Deſcriptions of them, I have ſo far ſatisfied their Wiſhes, as to point out ſome of the ſeveral Writings where they are treated of more at large.

In a Word, this Treatiſe here publiſhed is a ſufficient Explanation of the main Body of all the Inſtruments of the moſt Value, and in the greateſt Eſteem, that are generally made, and ſold in the Shops of the Mathematical Inſtrument-Makers, and which have been contrived to anſwer the various uſeful Purpoſes of practical Mathematicks. And if a better Book on this Subject has not yet been compiled, it is becauſe the able Geometricians, who are the moſt capable of making one, have the leaſt Inclination for it, becauſe of the ſmall Honour and Reputation they would acquire by it; and thus it has happened that there never has yet been a very good Book upon this Subject.

THE

SUPPLEMENT,

OR

APPENDIX.

CHAP. I.

Of Instruments for drawing or copying of Draughts, and making the Pictures of Objects.

I. *One of the best Kind of Parallelograms for drawing.*

THIS Instrument (Plate I. Fig. 1.) usually made of Brass or Wood, consists of four Rulers; the longest one A B being about 20 Inches in Length, so fastened together at the Places A, F, D, E, that the Parallelogram A F D E may vary it's Species, while the Sides remain the same, that is, that any two adjacent Sides may make all variety of Angles from 180° to no Degrees. Upon the opposite Sides, A E, D F are fixed two sliding Sockets *e*, *f*; in these Sockets likewise goes the Ruler G C, a little longer than A B; these Sockets may be fastened to any parts of the Sides A E, F D, by means of Screws underneath, and the Ruler G C may be fastened in them by two Screws a top, so as to be always parallel to the Sides A F, E D; which opposite Sides A F, E D are each in Length equal to E B. But the Sides A E, F D, are each a little longer, *viz.* by twice the width of the Sockets, so that when the Sockets are shoved home to the Side A F, the Distance from the Middle of the Socket *e*, to the Middle of the End E of the Ruler D E may be equal to the Middle of the End E from the Point B: and the Length of the Ruler G C is such, that when the Point C is drawn out as far as possible, so that the Point L near the Middle of it falls upon the Middle of the Side D F; the Part C L is equal to the Part E B, and the three Points C, D, B, are in the same streight Line, which they must always be, when the Instrument is fit for use. Upon each of the opposite Sides A E, F D, are three Scales of Divisions into 100 Parts, beginning at the Width of the Socket's Distance from the farther Edge of the Side D E, and ending at the same Distance on the hither Side of the opposite Side F A; the outermost Scale of Divisions is of 100 equal Parts, the middlemost is made by extracting the Square Root of each of the Numbers between 1 and 100, and multiplying that Root by 10, as to find against what division of the equal Parts 25 must stand on the middle Scale. It's Square Root 5 multiplied by 10, is 50; wherefore 25 must stand against 50 on the equal Parts, and so of others. Just after the same manner is the inner Scale Fig. 1.

made, *viz.* by extracting the Cube Root of each Number between 1 and 100, and multiplying the Root by 100. The same Scales of Divisions are likewise upon the part, or half C L, of the long Rule G C, beginning near the Point C. The first of these Divisions of equal Parts are for fixing the long Rule C G, and the Sockets, so as to readily describe a Figure, whose Circuit shall have a given Proportion to the Circuit of a given similar Figure. The second, for the Description of a Figure whose Area shall have a given Proportion, to the Area of a given similar Figure. And the third, for the Description of the representation of a Solid that shall have a given Proportion to a given Solid.

D *d* is a turned Brass Pillar about $\frac{3}{4}$ of an Inch long, with a Worm at the End to go into a very even Table; the Sides F D, D E of the Parallelogram are fastened to the Top D of this, so as to be any ways moveable horizontally. And at the Ends B, C, are equal round Holes wherein are put a Steel Point C *c* for moving over every part of a given Figure or Draught, and a Pencil Point B *b* for describing a Figure or Draught similar to a given one. Lastly, The turned Brass Pillar A *a* has a free horizontal Motion about the Point A, and a little Wheel *a* upon the Bottom of it to run upon the Table, like one of the Legs of a Child's *Go-Cart*. The Lengths of these Pillars and the two Points C *c*, B *b* must be such, that in all Positions of the Parallelogram the Points *d*, *c*, *b*, *a*, may be in one Plane, *viz.* upon the Surface of the Table, or of a Paper laid upon it.

The Use of this Instrument is contained in the Description, sufficiently to be apprehended by any intelligent Person.

II. *Of the Camera Obscura.*

This is a Room darkened in the Day-time, and chiefly when the Sun shines, having a Convex-Glass or Lens placed in an Hole in the Window-shutter, through which Glass the Light only passes into the Room; which Glass if properly placed, will project the Pictures of Objects from without Doors, inverted upon any white upright Surface within the Room, placed at it's focal Distance from the Glass. And here it is to be noted, first, that the Pictures will appear more bright and lively in the Morning, if the Window and Objects be to the West, and contrariwise in the Afternoon, if the Window and Objects be to the East, and about Noon if the Window and Objects be Northwardly.

For the Convenience of directing the Axis of the Glass towards any Object, the Lens is placed in a large cylindrical Hole boared through the middle of a wooden Ball, called a Sky Optick-ball, which is easily moved about it's Centre, within a hollow wooden Zone, and fastened to the Window-shutter: This Zone consists of two half Zones screwed together in the middle after the Ball was let in; and the Concavity of the Zone hinders the Light from passing between it and the Ball.

When the focal Distance of the Lens is 8 or 10 Feet, the Pictures of the Objects may be received upon a large Screen covered with white Paper, or Linnen, or painted white, and to have it moveable upon small Wheels, that so it may be brought to a proper Distance from the Lens.

The Pictures of the Objects will be so much the larger as the focal Distance of the Lens is longer, and so much the brighter, all things else the same, as it's Aperture is larger.

The inverted Pictures may be viewed upright by reflecting them downwards upon a Table by means of a Looking-Glass, whose Surface is set at an half right Angle, or thereabouts, to the Horizon.

To draw Copies of Prints, Paintings, &c. *by tracing their Pictures formed by the Lens.*

Place the Original at a proper Distance without Doors, and let it's Picture within the dark Room be received upon a Sheet of Paper, or upon a large plane Glass, not polished on one side; this Glass being placed upright with it's rough side turned from the Window, you may easily trace upon it with a Black-lead Pencil, the Out-Lines of the Picture. And if a Sheet of fine Paper be strained over the Glass, the Strokes of the Pencil will appear through it when held against the Sky-light; and thus the Picture may be drawn upon the Paper. But note, The easiest way of making the Image distinct upon the Glass, when fixed, is to place the Lens in a Tube that shall slide within another short Tube fixed in the Window-Shutter.

To do this without the Trouble of making two Draughts.

Having strained the drawing Paper upon a smooth Board, lay it upon a firm Table, to be placed under the Lens in the Window-shutter, and let an inclined Looking-Glass *g h i k*,
Fig. 2. N. I to reflect the Picture upon that Paper, be fixed over the Table as thus, *a b* and *c d* (Fig. 2. N. I.) are two Boards fixed upright upon the Table on each hand of the drawing Paper; *p q* and *e f* is a third Board equal in Length to the Distance between the upright Boards having a round Pin at each End of it; when this Board is laid over the back-side of a Looking-Glass, and screwed to the Frame of it, the Pins must be lodged in two Slits at the Tops of the upright Boards; and by two Nuts that screw upon the Pins. The Glass may be stayed at a proper Inclination for throwing the Picture directly down upon the Paper below, and

and then it may be made diftinct by moving the Tube that holds the Lens inwards or outwards.

III. *Of the portable Camera Obfcura.*

This Inftrument when carrying or not ufed, is like a wooden Box, and fome of them are more like very large Folio Books; the reafon of the whole Contrivance is this. The Rays of Light that come from the Object P Q R, after paffing through the Lens E, are tending to form an Image *p q r*, but being reflected upwards by the Looking-Glafs A B C, they form an horizontal Image *o x z*, upon a Glafs Plane, whofe unpolifhed fide lies uppermoft, upon which a Copy of the Picture may be fketched out by a Black-lead Pencil; and to the Spectator facing the Object the Picture appears upright, there is only reprefented in this Figure a Section of the Inftrument through the Axis of the Tube that holds the Lens, and through the middle of the Box and Looking-Glafs within it: The Section of the Side oppofite to the Tube is not here reprefented, it is only a Door opening fide ways; the Edges of the rough Glafs at the Top are placed in two Grooves upon the Sides of the Box, and being taken off, it is placed in a Drawer *e f*, at the Bottom of the Box; the Looking-Glafs A B C may alfo be drawn out of the Grooves in the Sides of the Box and lodged in the fame Drawer. The fquare wooden Tube confifts of three Parts; the innermoft that carries the Lens draws inwards or outwards, to make the Pictures diftinct; the Parts *g h*, *i k*, being fixed together, and to the Box, with fmall Bolts, may be taken afunder and put into the Box; then the Lid *a t* at the Top, and the Door at the End, being both fhut and fixed, the Inftrument becomes more commodious for carriage; the infide of the Lid whofe Section is *a t* (Fig. 3.) has two Wings that open at right Angles on each Side of it, and reft upon the Sides of the Box, to darken as much as poffible the Image upon the rough Glafs. Fig. 2. N. II.

IV. *Of another portable Camera Obfcura for drawing.*

Into the middle of the top of a fquare Box (Fig. 4.) there is put an upright Tube, or rather a piece of a fquare Pyramid; in the top of which there flides a fhort fquare Tube, having a broad Object-Glafs fixed in a Hole at the top of it; the focal Diftance of the Glafs being fomewhat lefs than it's utmoft Height above the Bottom of the Box, where the Pictures of Objects are to be formed by it: upon one edge of the Top of this Tube there turns a Lid, having a plane Looking-Glafs fixed flat upon the infide of it, and thereby capable of being ftayed at any inclination proper for reflecting the Rays that come from an Object directly downwards through the Object-Glafs, to the drawing Paper fixed upon the Bottom of the Box, where the Picture of the Object will be diftinctly painted, when the Object-Glafs is fet to a proper Height: the Box being quite clofed and dark within, this Picture is viewed through a fmall Hole in the upper Edge of the Box floped off from the Side oppofite to the Object where the Draught's-Man ftands, and puts his Hand and Pencil through a Hole made in this fide; or rather in a long piece that flides in it horizontally, according as he has occafion to move his Hand. The fquare Box is *a b c d e f*, the fixed Tube is *g*, the fliding Tube is *h*, the Object-Glafs is *o*, the inclined Looking-Glafs is *i*, the Hole for the Eye is *k*, that for the Hand in the Slider *m n* is *l*. Fig. 4.

There are other portable *Camera Obfcura*'s that have been made and are defcribed in Books, only differing from thefe either in bignefs or fome lefs effential Parts. There are two of them to be feen at the End of Mr *s'Gravefande*'s *Effay on Perfpective*; a large one and a fmall one; the former being almoft like a common Chair, or Sedan, with a folding Table in it, to lay the drawing Paper upon, and in which may be put a fmall Stool for the Draught's-Man to fit upon. The Convex Lens and inclined Looking-Glafs is a Top over the horizontal Table: where he gives fome ufeful Cautions. 1. There muft be ufed but one Convex-Glafs, fince when there are two or more the true Appearance of the Object is loft: the fame inconvenience will alfo lie in the way, when a Concave Looking-Glafs is ufed in the Conftruction of the Machine. 2. When more than two plane Looking-Glaffes are ufed, the Rays after a triple Reflexion become fo weak as to obfcure the Appearances of the Objects; and when two Looking-Glaffes are ufed they ought to be well polifhed. 3. The Looking-Glaffes muft not be put into the Machine, becaufe in fuch a clofe Place the Draught's-Man's Breath will fully the Glaffes, which cannot happen to the Convex-Glafs, by reafon of it's being inclofed within a Tube. He fays alfo, that the Faces of Perfons may be drawn in Minature, when the Picture of the Perfon upon the drawing Paper is not above half an Inch, but when it is bigger it is not eafy to trace out the exact Refemblance, becaufe each Point of the Face has not the fame Focus.

CHAP. II.

Of some more modern Instruments used at Sea, in taking the Degrees of the Altitude of the Sun or Stars, or the Degrees of their Distances.

BESIDES the Instruments for this Purpose already spoken of in our Treatise of Mathematical Instruments, there have been of late Years several others actually made, described, and used for this Purpose, indeed not essentially differing from one another, all of them being designed to be used where the Motion of the Objects, or Observer, or any thing occasioning an unsteadiness in the other Instruments when using, make the Observations difficult or subject to Error.

The first of these Instruments for taking the Moon's Distance from the fixed Stars was invented long ago by Sir *Isaac Newton*, as appears in a Paper of Sir *Isaac Newton*'s own Hand-Writing, found amongst those of the late Dr *Halley*, and the very Instrument itself that Sir *Isaac Newton* either made himself, or caused to be made so long ago as when Dr *Halley* went about making the Catalogue of the fixed Stars in the *South-Seas*, which was in the Year 1672, was not long ago to be seen at Mr *Heath*'s the Mathematical Instrument-Maker in the *Strand*. See the *Philosophical Transactions*, Number 465, for the Year 1742.

Many Years after this, Instruments for doing the same thing, were published in the *Philosophical Transactions*, Number 420, and 425, under the Name of Mr *Hadley*'s Octants or Quadrants, not much differing from Sir *Isaac*'s. Two or three more of the like, not essentially varying from the Original one of Sir *Isaac Newton*, have also been made and published under the Name of Mr *Caleb Smith*, with Engravings of two of them described in a printed Sheet of Paper, entituled *The Use of the new Instrument, or Sea Quadrant, for taking Altitudes of the Sun, Moon, and Stars, from the visible Horizon, by an Observation either forwards or backwards, as well as any other angular Distances, without Impediment or Interruption from the Ship's Motion, whereby the Latitude at Sea may be obtained with greater Certainty and more frequently, than by any other Instrument commonly used for that purpose.* Sold by Mr *Heath*, Mathematical Instrument-Maker in the *Strand*, and several other Mathematical Instrument-Makers, and by some Booksellers too.

But as Sir *Isaac Newton* has now been found to be the first Inventor of this Instrument, and as one of those not much different from his, called by the Name of Mr *Hadley*'s Sea-Octant or Quadrant, is now most in vogue and used at Sea, I shall therefore in this Chapter give only the Description and Use of Sir *Isaac Newton*'s Quadrant of this Kind, and that other called Mr *Hadley*'s, leaving Mr *Smith*'s to be seen in the Paper above described.

I. *The Description and Use of Sir* Isaac Newton's *Sea Instrument or Quadrant.*

Fig. 5. The Figure P Q R S denotes a Brass Plate accurately divided in the Limb D Q into half Degrees half Minutes, and $\frac{1}{12}$ Minutes by a Diagonal Scale; and the $\frac{1}{2}$ Degrees, and $\frac{1}{2}$ Minutes, and $\frac{1}{12}$ Minutes, counted for Degrees, Minutes, and $\frac{1}{6}$ Minutes. A B is a Telescope three or four Feet long, fixed on the Edge of that Brass Plate. G is a Speculum fixed on the said Brass Plate perpendicularly, so as to be inclined 45 Degrees to the Axis of the Telescope, and intercept half the Light which would otherwise come from the Telescope to the Eye. C D is a moveable Index turning about the Centre C, and with it's fiducial Edge shewing the Degrees, Minutes, and $\frac{1}{6}$ Minutes on the Limb of the Brass Plate P Q; the Centre C must be over against the middle of the Speculum G. H is another Speculum parallel to the former, when the fiducial Edge of the Index shewing the Degrees falls upon 00 Deg. 00 Min. 00 Seconds; so that the same Star may then appear through the Telescope in one and the same place, both by the direct Rays, and by the reflected ones; but if the Index be turned, the Star shall also appear in two places, whose Distance is shewed on the Brass Limb by the Index.

The Use of this Instrument.

By this Instrument the Distance of the Moon from any fixed Star is thus observed: view the Star through the Telescope by the direct Light, and the Moon by the reflected Light, or the contrary, and turn the Index 'till the Star appears to touch the Edge of the Moon, and the Index will shew upon the Brass graduated Arch of the Instrument, the Distance of the Star from the Limb of the Moon; and though the Instrument shakes by the Motion of your Ship at Sea, yet the Moon and Star will move together as if they did really touch one

another

another in the Heavens, so that an Observation may be made as exactly at Sea as at Land; and by the same Instrument may be observed exactly the Altitudes of the Moon and Stars by bringing them to the Horizon, and thereby the Latitude and Times of Observations may be determined more exactly than by the Ways now in Use. *Note*, In the Time of the Observation, if the Instrument moves angularly about the Axis of the Telescope, the Star will move in a Tangent of the Moon's Limb, or of the Horizon; but the Observation may notwithstanding be made exactly, by noting when the Line described by the Star is a Tangent to the Moon's Limb, or to the Horizon; and to make the Instrument useful, the Telescope ought to take in a large Angle; and to make the Observation true, the Star must touch the Moon's Limb, not on the outside of the Limb, but on the inside: thus far Sir *Isaac Newton*.

The Description and Use of Mr Hadley's *Instrument for taking the Latitude, or other Altitudes at Sea.*

The Instrument A B C (Fig. 6.) is an Octant having it's Arch or Limb C B but 45 Degrees diagonally divided into 90 equal Parts, and these again subdivided into as many equal Parts as possible with distinction; each of the first mentioned equal Parts being Degrees, and the subdivisions Parts of a Degree, as serve to take any Altitude from the Horizon to the Zenith. The Octant is generally made of Mahogany Wood, and sometimes of Ebony, or Brass, the Radius of the Instrument being generally 20 Inches; the Thickness is half an Inch; the graduated Arch is usually of Box-Wood, and sometimes Brass about the Thickness of a Half-penny. About the Centre of the Instrument there is an Index D, freely but stiffly moving on a Pin, shewing the Degrees upon the Arch. Upon this Index, near the Centre, is fixed a plane Speculum E, or piece of Looking-Glass Quick-silvered on one side, having it's middle E directly over the Centre of the Octant, which Speculum is perpendicular to the Plane of the Index, and whose Plane coincides with the fiducial Line E D drawn along the middle of the Index: though it may make some Angle with that Line in other Instruments of this kind. This Speculum is about 2 Inches by $2\frac{1}{2}$. This Glass is to receive the first Image of the Sun, or other Object to be observed by, and to reflect it to another lesser plane Speculum, or Looking-Glass F, $\frac{3}{4}$ of an Inch square, fixed perpendicularly on a piece of Wood fastened to the Side A B, or Limb of the Octant, and having it's Surface so situated, that when the Index is brought to mark the Beginning of the Divisions on the Limb (that is $0°$), it may be exactly parallel to the Surface of the other Speculum. That part of this Speculum is Quick-silvered which is next to the Plane of the Instrument, and so set in it's Brass Work, that it may at any time be set perpendicular to the Plane of the Instrument, if by any accident it should be removed from that posture. When it is in the Perpendicular, it can also be turned round, keeping still perpendicular, so as at pleasure to bring it to it's true position with regard to the Glass fixed on the Index. This Glass serves for forward Observations with the Instrument, and from it the Eye receives the Image of the Object by a second Reflection. There is another such Speculum G placed on the side of the Octant, but farther from the Centre, and with the same Sort of Brass Work to set it perpendicular, and to it's true Position. Fig. 6.

There are two dark or smoaked Glasses H, set in Frames of Brass fixed to a Pin, the one lighter, and the other darker, so as to be turned at pleasure either of them, or both together, as the Sun's brightness may require between the Speculum on the Index, and either of the two on the side of the Instrument, according as the fore or back Observation is used. For which reason there are two Holes on the side of the Instrument, that they accordingly may be shifted from the one to the other. I being the Hole for them in the back Observation. Each of the Observations has a sight Vane K, of Box-Wood, about $2\frac{1}{2}$ Inches from the Centre. In that for the fore Observation are made two Holes to direct the placing the Eye, the one being exactly as high above the Plane of the Instrument as is the middle of the unquickfilvered Part of the lesser Glass; the other the Height of the Edge of the Quick-silver itself, or a little lower. That for the back Observation requires but one Hole, which is placed exactly at the Height of the middle of the clear Part of that Glass. The Altitude of the Sun or Star above the Horizon, taken by this Instrument, is determined by the Inclination of the Planes of the two Glasses (F and E for the fore Observation, and F and G for the back Observation) to each other, when the Sun or Star appears in the Horizon. In the fore Observation, the double of this Angle of Observation is the Altitude sought, and is marked by the Index, on the Arch of the Instrument divided into half Degrees. But in the back Observation, twice the Difference of this Inclination from a right Angle gives the same Altitude, and is marked by the Index in the same manner, the same Scale of Degrees serving for both; so that when the Index stands at the beginning of the Scale, the Surface of the lesser Glass F used for the fore Observation, is parallel to the Surface of that one the Index, *viz.* E, but the Surface of the other Glass G perpendicular to it.

The Use of this reflecting Octant as thus described, in taking the Altitude of the Sun, or a Star at Sea.

I. *To adjust the Glass for a fore Observation.*

Set the Index D exactly to the Beginning of the Scale C, that is, close up to the Button on the Side of the Limb A C. Then holding the Instrument with your Left-hand, by the Index and Limb together, near the Arch B C, or over the Button, as upright as you can with the Arch downwards, and keeping the Index all the while touching the Button, look thro' the lower Hole in the Sight-Vane, that is, that next to the Instrument, and see the Edge of the Sea (taken for the Horizon), through the Part of the Glass which does not reflect, and mark whether the Line of the Sea's Edge, that is, the Horizon thus seen by direct vision coincides, or is the same, with the same Edge seen by the Reflection of the Quick-silvered part of that Glass. If not, turn that Glass by the Handle on the back side of the Instrument, 'till they do thus join or coincide. This being done,

II. *To take an Altitude by a fore Observation.*

Set the Index to the Altitude as near as you can judge, if it be within 10 or 15 Degrees the Matter is not much. Hold the Instrument upright, as near as possible, with it's Plane in a vertical Circle passing through the Zenith and Sun, or other Object, with the Arch downwards. And supposing the Object to be the Sun, placing your Eye at the upper Hole of the Sight-Vane K. Look at the Edge of the Sea (taken for the Horizon), just under the Sun by direct view, through the outer Part of the Speculum on the Limb not Quick-silvered over. Then by moving the Index, the Image of the Sun must be brought to appear as if it were really joined to the Edge of the Sea, that is, as if it were brought down to the Horizon, touching it where the vertical Circle passing through the Zenith and Centre of the Sun cuts the Horizon, and the Degrees and Minutes of the Arch marked by the Index will denote the Sun's Altitude.

Note, If the Reflection be too bright for the Eye, turn up one or both the coloured Glasses H, according as you see occasion. But if the Image be too faint to appear on this Part of the Glass, or it be any other Object than the Sun, or a bright Moon, then look through the lower Hole of the Vane, and see it on the Quick-silvered part of the Speculum, and then you must judge when the Line of the Horizon, seen directly through the unquick-silvered part if produced, would pass through it.

Note also the Line of the Direction of the Sight, or the Line in which you see the Image when directly looking at it, must be kept as near as possible parallel to the Plane of the Instrument, in order to be very exact. Wherefore when you look through the upper Hole, take the Observation about the middle of the unquick-silvered Part of the Glass, and not too near the Edge of the Quick-silver, nor the outer Edge of the Glass; but, if you use the under Hole, bring the Image near the Edge of the Quick-silver, lean the Instrument a little awry to right and left by turns, by which the Image will seem to swing to and fro, and move the Index 'till you have brought the Image just to brush the Horizon in the lowest Part of it's swing.

Note, For common use it will be well enough to observe by the Sun's Centre. But this way is not so exact, as to do it by the Sun's upper or under Edge; if the under Edge be used, in the fore Observation, you must add 16 Minutes to those pointed by the Index; if the upper, substract 16 Minutes.

To adjust the Glasses for the back Observation.

Set the Index off so much before the beginning of the Scale of Degrees as is the double of the Dip of the Horizon, or Edge of the Sea (*viz.* 8 Minutes, for 44 Feet above the Surface of the Sea; 7′ for 34 Feet; 6′ for 25; 5′ for 17 Feet; 4′ for 11 Feet; 3′ for 6 Feet; and 2′ for 3 Feet). Look through the Hole of that Sight-Vane holding the Instrument upright, and see that the Edge of the Sea behind you, appearing by reflection either from the clear Part of that Glass, if it be discernable there, or else from the Quick-silver on both sides; join all along with that before you, seen through the same clear Part of the Glass. If those two Edges cross one another, the Instrument is not held upright, and the Adjustment will not be so certain.

Note, The Edge of the Sea seen by reflection will be inverted; the Water appearing above, and the Sky below.

To take an Altitude by a back Observation.

Having set the Index to the Altitude as near as you can guess, look through the Hole in the Sight-Vane, and the middle of the clear Part of that Glass between the Quick-silvered Parts on both Edges, just over your own Shadow, and move the Index 'till you bring the Image

Image of the Sun, *&c.* exactly to the Horizon; and the Degrees and Minutes marked on the graduated Arch will be the Altitude sought.

Note, To know whether you hold the Instrument upright, carry it to Right and Left by turns, keeping your Arms steady, and you will see the Image slide along the Edge of the Sea, if it be held upright; otherwise it will run in a Line cutting it. In this Observation, if you use the apparent under Edge of the Sun, substract 16 Minutes, otherwise add 16 Minutes, contrary to what is to be done in the fore Observation.

Note, The Dip of the Horizon set down above, in the back Observation, is to be added to the Degrees of Altitude marked upon the Arch, and not substracted as in the fore Observation.

To observe the Altitude of a Star, by the fore Observation.

Look directly up at it first with the Index standing close to the Button, then move the Index forwards, and cause the Star to slide down to the Horizon, that you may not mistake it for another; and the Degrees marked by the Index will be that Star's Altitude; and thus is the Altitude of a Star taken by the fore Observation. But when the Horizon is bright, and the Star faint, it may be best to take the Altitude by a back Observation; in which case you are to look directly up at the Star, and bring the Horizon, or Edge of the Sea, behind you to touch the Star by reflection. As the Stars are used in the Night, Moonlight Nights are best for the Observation, either fore or back.

Note, The back Observation is not so easy as the fore one, and is useful in taking small Altitudes of the Sun when it's Light may be troublesome to the Eye in the fore Observation. It is also useful in verifying the Truth of an Altitude of the same Object, taken at the same time, or nearly so, by a fore Observation with the same Instrument. That the back and fore Observation may agree accurately, in a lofty Ship especially, the Index must not stand exactly at the beginning of the Scale in the Adjustment for the fore Observation, but must be set off so much before the beginning of the Scale, as has been already said, as is the double of the Angle of the Dip of the visible Horizon, or Edge of the Sea, below the true Horizon, for which reason there is a few Minutes graduated before the beginning of the Arch.

The chief Advantage of this Instrument, says Mr *Hadley*, consists in this; that whereas by taking an Altitude by other Instruments now in use, a certain exact Posture of them is required. But the Motion of the Ship continually disturbs the Observer, by putting them out of that true Posture, even in moderate Weather, and in hollow Seas to that degree, that Observations taken by them cannot oftentimes be accurate enough to be depended upon. But with this Instrument, though the Ship rolls ever so much, provided the Instrument be kept in, or near, an upright Posture, though it be leaned forewards or backwards therein, yet the Image of any Object, when once brought by sliding the Index to appear on the Edge of the Sea, will there remain absolutely immovable, as long as the Index continues in the same Place, without being stirred, and the Observer has the same Advantage of making the Observation as if he took it in smooth Water, and the Instrument was held still without Motion. Thus far Mr *Hadley*.

Hence it is evident from what has been said, that whatever Improvement as to the Use and Exactness of these Instruments others may have found out, whether by the Matter, Figure, Weight, alteration of some Parts, *&c.* Sir *Isaac Newton* most certainly must have the Honour of the Invention. All the Praise due to others, can only consist in the Degree of Facility of the Use, and the Exactness, whereby the Instrument, as contrived by them, exceeds that described by Sir *Isaac Newton*; and, indeed, whatever this be, Sir *Isaac Newton* at least must have been the original Cause; for had he not shewn this his Invention to Dr *Halley*, it is probable that Nobody else might have thought of such a Contrivance, and consequently there would have been no such Instrument, and accordingly no Improvement upon it. Or supposing somebody else to have thought of such a Contrivance by himself, as well as Sir *Isaac Newton*; yet as Sir *Isaac* appears to have been the first Inventor, this other Inventor is allowed to have no share of the Honour of the Invention, and much less should those who only alter or improve some particular Part or Parts of the original Invention; perhaps (more to disguise and distinguish it from that of others, thereby the better to appropriate the whole to themselves) than render the original Invention more perfect and better suiting the Design of the original Thing itself. He who alters the Situation of a Door, Window, Chimney, or other lesser Part in a commodious well designed Building, has but a small Title to any of the Reputation acquired by the Architect of that Building. *Columbus*, who first discovered the *West-Indies*, is justly allowed by all to have the Honour of the Discovery, although *Americus Vesputius*, who failed there after him, had the Reputation to have that Country called by his Name. So Mr *Hadley*'s and Mr *Smith*'s Quadrants, though they do not differ from that of Sir *Isaac Newton* as to the Main of the Invention, yet these Instruments are called by their Names.

The Inconveniences in the Use of this Instrument seem to be, 1st, For him who wants to find the Altitude of the Sun by that Instrument, first to guess at that Altitude without such Instrument, at least within 10 or 15 Degrees, which is not easy to do by one who is not much

much used to the taking of Altitudes. Secondly, The too great Obscurity of the Objects when seen by a double Reflection, especially in hazy Weather. Thirdly, The uncertainty of seeing the true Horizon in hazy Weather or at Nights. Fourthly, The uncertainty of bringing a Star to the true Horizon in the Night, even when the Moon shines. The Horizon being then not easily to be guessed at, there being Vapours oftentimes on the Surface of the Sea causing the Refraction to be sometimes more and sometimes less according to the Weather, Time of the Year, *&c.* so unconstantly various, that no Rule, or Tables, by any one hitherto given for determining it's Quantity, can be entirely depended upon. By which means the true Horizon, as to Situation, will sometimes appear higher or lower through an Instrument looking at it, that is, at the Line seperating the apparent Surface of the Sea from the Sky. And that according as the Medium that he looks at it through, is more or less dense, or the Mercury in the Barometer is higher or lower. For all which Reasons I am apt to think that this *Newtonian Quadrant* or *Octant*, is subject to uncertainty with regard to the Practice, as well as all the others that have been hitherto invented. But how far this Instrument exceeds any of them, it is not in my Power to say. Experience, and that a good deal, sufficient Judgment, and proper Caution, are the best Proofs. This may have the Advantage of some of them in some Particulars, and they of this in others. This I am certain, when the Sun is bright and the visible Horizon clear, the Altitude, all things else alike, will be best taken by this Instrument. But in the Night when the Moon does not shine, the meridian Altitude of a noted Star, as the Pole Star, which is chiefly wanted in order to find the Latitude, cannot be taken so exactly by this Instrument, because the Horizon cannot be distinctly seen, as it can by a Brass Astrolabe, or Ring, properly poised by a dexterous Person, who has been used to all the various Motions of a Ship, and where the sensible Horizon has nothing to do with the Operation. Besides, the Divisions of the Degrees upon this Astrolabe, are twice as big as those upon one of these Octants of the same Radius, which must certainly be some advantage. Moreover, as there are but few who use this Octant who know the Reason why it gives the Altitude that they are finding by it, but, on the contrary, every body who understands the first Rudiments of Astronomy and Geography, immediately sees the Reason of his Operation by the Astrolabe; the latter has some advantage over the former upon this account, *viz.* with respect to simplicity. And this simplicity may be a Means to bring this Astrolabe more in use than it is at present, and I think should stir up some impartial, ingenious Persons to compare the Uses of the two Instruments, which are best, and if the Astrolabe be deficient herein, to render it more perfect by some additional Contrivances, which I think it is capable of. Telescopick Sights to the Index might be of use; and if the Wind should be so high as to disturbe the true poising of the Instrument, this may also be remedied. In short, of all the Instruments for taking the Altitudes at Sea, this most ingenious Octant, as to contrivance, and that most simple one the Astrolabe, I take to be the best of any others.

The Foundation of the Contrivance of the *Newtonian Octant*, is chiefly built upon the following *Catoptrical* Principles: 1. The Angle of Reflection is equal to the Angle of Incidence. 2. The Place of the Image of an Object, seen by the Reflection of a plane Speculum, will be behind the Speculum in a Perpendicular let fall from that Object, and as far behind the Speculum as that Object is before the Speculum (the Objects are supposed to be Points for the ease of Imagination). 3dly, The Image of an Object seen by the successive Reflections of one or more plane Speculums, will be seen in the second reflected Ray, by an Eye any where placed in that Ray, provided the Speculums themselves do not hinder the Sight. Hence,

Fig. 7. If two plane Speculums *b* B, *c* C (Fig. 7.) inclined to one another under the Angle H R I, be at right Angles to some third Plane M A N, as suppose that of the Horizon M A N. And if a Ray of Light A F emitted from any Point A of an Object, be successively reflected by those two plane Speculums (that is, by the Speculum *b* B from F to G, and by the Speculum *c* C from G through L, the Angle A F B being equal to *b* F G, and the Angle C G F equal to the Angle *c* G L); the first Image of A being at M, as far behind the Continuation of the common Section R H of the Speculum *b* B and the Horizon, as A is before it; and the second Image N made by the Reflection of the second Speculum *c* C, as far beyond the Continuation R I of the common Section of the Plane of the second Speculum *c* C, as the Image M is on this Side of it. I say, the three Points R, M, N, will be at the same Distance from the common Section R of the Planes of the two Speculums with the horizontal Plane. And the Angle A R N contained under the two right Lines A R, N R, drawn from the Object A, and it's second Image N, to the said common Section R, will be the double of the Angle H R I of the Inclination of the Planes of the two Speculums *b* B, *c* C, *viz.* their common Sections with the Plane of the Horizon.

For draw R A, join A M, M N, cutting R H, R I, in the Points P and S. Also let N G, A F, continued out, meet in L; then since all Reflection is made in the same Plane, with the incident Rays, the Points R, L, M, A, N, F, G, will be all in the same Plane, *viz.* that of the Horizon, now because P M = P A, and P F = P F, and the Angle A F H = R F G = M F H; the Triangles F M P, F A P will be equal to one another; wherefore F M = F A, and the Angle M F A will be twice the Angle A F H = M F H, and so the

the Arch A M = 2 Arch A H, or = 2 Arch M H. In like manner, ſince the Angle M G C = Angle L G R = N G I, and M S = S N, and G S = G S, therefore will the Triangles G M S, G N S be equal; wherefore G M = G N, and the Angle M G N = 2 Angle N G I = I G M, therefore the Arch M N = 2 I M = 2 I N. Hence the Triangles R M F, R A F will be equal, and ſo R M = R A; alſo the Triangles R M G, R N G will be equal to one another, wherefore R M = R A = R N = R I.

Again, make the Angle I R Q = I R H, or the Arch I G, I Q = I H, then will Q H = 2 I Q. But Q H = A N, for ſince N Q = M H = A H. If A Q be added to N Q, and A H be added to N Q, there will be had A N = Q H; wherefore A N = 2 H I, or the Angle A R N = 2 H R I.

Alſo the Angle N L A will be equal to the Angle N R A, and ſo the four Points A, N, R, L, will be all in one and the ſame Circle. For ſince the Angles A L N + G F L = F G N, and ſo A L N = F G N — G F L. But F G N = 2 F G I, and G F L = 2 G F R, and conſequently their Difference is = 2 F R G = 2 H R I, wherefore the Points A, N, R, L will be in the Circumference of the ſame Circle, the Angle N L A being equal to the Angle N R A. Hence, if the Eye be placed at L, in the ſecond reflected Ray N G L, the Image N of the Point A of a viſible Object will appear to be at N, ſuppoſing the Part *c* G of the Speculum *c* C to be away, *viz.* not hindering the Sight, or contrariwiſe, if N be a viſible Point of an Object, the Eye at L will ſee it's Image at A; if the Part *b* F of the firſt Speculum be away, ſo as not to hinder the direct Progreſs of the Rays A F L coming from A to L. Since the Radius A R of the Horizon M A N (or, indeed, of any great Circle in the Heavens), is infinite, the Points R, L, G, will coincide, or fall into one another; and in this Caſe, the Arch A N of the Horizon contained between the viſible Point A, and it's ſecond Image N, will be exactly equal to one half the Arch H I of the Angle of the Inclination of the two Speculums *b* B, *c* C, to one another. Hence, if M A N be the Meridian, and A be a Point where that Meridian cuts the Horizon, and if N be ſuppoſed to be the Sun, in the Meridian, elevated above the Horizon by the angular Diſtance, or Arch A R N. If theſe two Speculums *b* B, *c* C be ſo inclined to one another, that the Image of the Sun N, appears to the Eye at R or L, by the Reflection of the Speculums to be caſt upon the Point A of the Horizon. Twice the Angle of the Inclination of the Speculums will be equal to the Sun's Meridian Altitude A N; and this is the main Foundation of the whole Contrivance of all theſe new Sorts of reflecting Sea Octants for the fore Obſervation. See a more particular Account of this Theory by Mr *Hadley* himſelf, in the *Philoſophical Tranſactions*, Numb. 420, 425; as alſo in Dr *Smith*'s *Opticks*.

And the Reaſon of the Operation for a back Obſervation is derived from this *Theorem*, viz. twice the Difference of this Angle of Inclination of the Planes of the two Speculums, from a right Angle, is the Angle A R N of Altitude.

Mr *Hadley* tells us, that upon Trial of one of theſe Inſtruments, three Obſervations made at Sea of the Diſtance between two Stars with a Braſs Octant of this Kind, differed from Mr *Flamſtead*'s at Land, only about a Minute; and a dozen Obſervations of the Sun's Altitudes taken by one of theſe wooden Inſtruments, differed but about half a Minute at a Medium from one another, when the Ship lay at Anchor; and when the Ship was under Sail, another dozen differed at a Medium about a Minute. He tells us moreover, the Obſervations made of this Inſtrument at Sea, were by the Help of ſome curious Watches, with Second Hands, and from the Time by the Watch corrected, noted at the Inſtant of each Obſervation, the true apparent Altitude of the Sun was computed, the Refraction, and Obſerver's Elevation being allowed for, and the Difference between the Altitude thus found by Computation, and what was given by the Inſtrument at the ſame Time, gives the Error of the Altitude found by the Inſtrument. And he afterwards ſays, that a Meridian Altitude may be taken at Sea to a Minute, by this Inſtrument, which I much doubt; for as a Minute of the Arch of the wooden Octant of 22 Inches Radius, is but about 5 Parts of 200 of an Inch, which is but a very ſmall Interval, and as theſe ſmall Diviſions by Workmen are oftentimes not made equal between themſelves, I think it is too much to ſay, an Angle can be taken to one Minute, much leſs to half a Minute by ſuch an Inſtrument, even ſuppoſing there were no other material Cauſes of Error in the Way. If I ſhould aſſert that an Altitude could not be taken truly at Sea to 2 Minutes by the Inſtrument, I do not know whether I might not be right. For Mr *Hadley*'s Computation of the Altitude is ſubject to ſome Error, becauſe, as he himſelf owns, an Error of ſeven Seconds in the Time, will cauſe an Error of one Minute in the Altitude, alſo an Error of the Sun's Declination at the Time of Obſervation, as alſo one in the Latitude of the Place, for both theſe muſt be given, together with the exact Time from Noon. In order to compute the Altitude from Spherical Trigonometry by the Tables of Sines and Tangents, that is, to find one Side of a Spherical Triangle when the Angle oppoſite to it (*viz.* the Time, and the other two Sides, including that Angle, the one being the Complement of the Latitude, and the other the Complement of the Sun's Declination), are all three given. I ſay, an Error in either of theſe, will cauſe one in the Altitude thus found by Trigonometrical Calculation, for which reaſon I am not certain, whether the Goodneſs of one of theſe Octants might not be better tried by a number of Perſons in the ſame Ship, near the Shore, each with one of theſe Inſtruments taking the Sun's Altitude at the same Time, and other Persons on the Shore, near at Hand at the same Time, by some Signal, taking that same Altitude with some proper Instruments; then by comparing the Sea Observations of that Altitude, with the Land ones taken at the same Time, their agreement, or difference at a Medium, after all proper Allowances are made, will shew the Exactness of the Octant in taking Altitudes of the Sun at Sea. And for greater Certainty of Exactness, that Altitude may be calculated too, by Spherical Trigonometry. By having given the Latitude, and Sun's Declination, as also the Sun's *Azimuth* found at the Time of Observation, by a good Azimuth Compass, rather than the Time from Noon, given by a Watch; an Error in the Azimuth causing less Error in the computed Altitude. Then if the Land Altitude, Sea Altitude, and computed Altitude all agree, to a Trifle, we may be assured of the Certainty of the Altitude taken at Sea. But when the Land Altitude and computed one agree, and the Sea Altitude differs too much from them, there is an Error in the Sea Altitude taken.

CHAP. III.

The Description of another modern Quadrant of Mr Elton's *for taking Altitudes without an Horizon, either at Sea or Land.*

Fig. 8. THIS Instrument (Fig. 8.) consists of four principal Parts, *viz.* the Frame A B C D E F, the Index G H, a Label I K, and a Shield or Ray Plate *d f g*; and these consist of several Parts: the Frame has two Parts, the one a graduated Arch D E, of 30 Degrees, each Degree subdivided into 6 equal Parts; the other B C, a Chord of an Arch of 60 Degrees, divided into two equal Parts. At the Extremities, and in the Middle of which are three Holes, or Stops, *a*, *b*, *c*, for the Label. The Index G H turns about the Centre of the Frame, through the whole Extent of the Arch, and has three Parts, a *Nonius* Plate *n*, Eye-Vane *v*, and Glass Tube *t*, as a Spirit Level; the *Nonius* Plate moves with the Index, and subdivides each of the small Divisions into ten equal Parts or Minutes; the Eye-Vane *v*, is to look through in a fore Observation, and the Glass Tube *t* is to shew by the Bubble of Air when the Index is horizontal; the Label I K moves about the Centre of the Frame, the whole Extent of the Chord B C of the Arch of 60 Degrees; having three fixed Stations thereon at 30 Degrees, 60 Degrees, and 90 Degrees, and contains two principal Parts, *viz.* a Lens *l*, and a Stool *o*, for a Lanthorn Q; the Lens is to form the Sun's Image upon the Shield *d f g*, the Lanthorn is necessary in nocturnal Observations, the Shield is fixed in the Centre of the Frame, and has three Parts; an Azimuth Tube *z*, an Axis *x*, and an horizontal Tube *t*; or in backward Observations a ray Plate; the Hole in the Shield is to receive the Sun's Image, the Azimuth Tube is to direct the Plane of the Instrument perpendicular to the Horizon, the horizontal Tube is to shew when the Label is level, the Axis is to cut the Objects in forward Observations.

The Use of the Instrument, for either a backward or forward Observation.

If the Altitude does not exceed 30 Degrees, the Label must be placed at the Station on the Radius or longest Limb of the Quadrant. If the Altitude be between 30 and 60 Degrees, at the middle Station; and if the Altitude exceeds 60 Degrees, at the uppermost Station.

To take the Sun's Altitude by a back Observation.

In doing this, the Sight-Vane, or horizontal Tube, on the Shield is not used. Hold the Quadrant with both Hands so as to keep it as steady as possible, with the back Arch turned towards the Sun. When the Bubble of the Azimuth Tube is brought under the Hole in the Shield, cause the Sun's Image to fall on the Hole in the Shield, so that it may rest in the Centre of the Sun's Image; the Instant the Azimuth Tube and the Sun's Image are thus regulated, see if the Bubble in the horizontal Tube on the Index (which 'till then is disregarded) leaves the open End of the Tube, or stops any where clear of the Ends of the Tube. If these happen at the same Time, the Altitude is then truly taken. But if the Bubble had remained in the enclosed End of the Tube, when the Azimuth Bubble and Sun's Image were regulated, the Index must have been slid up; and if it staid in the open End, moved down 'till the horizontal Bubble on the Index quits the open End of the Tube, or stopt between the Ends, as was before observed; and then is the Quadrant set. In continuing the

the Obfervation for a Meridian Altitude, the Quadrant being fet, as the Sun rifes, the horizontal Bubble on the Index will not quit the open End of the Tube, or ftop between the Ends, but hang there, or leave it after the Azimuth Bubble and Sun's Image have been regulated, which will require the Index to be continually moved down in order to keep the Quadrant fet; when the Sun is up, or on the Meridian, the Quadrant will remain fet for fome time; and on the Sun's falling, the horizontal Bubble will have a reverfe Tendency, inclining or running wholly to the enclofed End of the Tube.

To take the Altitude of the Sun or Stars by a forward Obfervation.

In this Obfervation the Lens and Tube, on the Index are not ufed. Hold the Quadrant upright, and looking through the Eye-Vane direct the Axis, or upper End of the Shield, to the Sun or Star. If the Axis cuts the Sun or Star at the fame Inftant that the Bubble on the horizontal Tube on the Shield quits the open End, the Altitude is then truely taken, and the Quadrant fet. But if it fhould leave the open End of the Tube before the Axis, or upper Edge of the Shield, cuts the Sun or Star, then the Eye-Vane, or the Index, muft be flid down; and if it remains at the open End, or quits it when the Axis is above, the Sun or Star moved up until the Quadrant is fet. In continuing the Obfervation for a meridian Altitude, as the Sun or Star rifes, the Bubble in the horizontal Tube will always quit the open End of the Tube before the Axis cuts the Horizon; fo that to keep the Quadrant fet, the Eye-Vane muft on every fuch alteration be conftantly moved down; while the Sun or Star is upon the Meridian, the Quadrant will remain fet; and when the Sun or Star falls, the Bubble will act contrary to what it did in the rifing, refting wholly in the open End of the Tube.

To take the Sun's Altitude by means of the Horizon.

Turn the back of the Arch towards the Sun, and caufe the Sun's Image to fall on the Hole in the Shield, at the fame Time looking through the Eye-Vane, to cut the Horizon with the Axis.

Note, In taking the Altitude of the Stars, a fmall Light muft be fixed in the Lanthorn; the lefs the better. It will be beft in forward Obfervations of the Sun, to take the Altitude of the upper Limb, and then fubftract; and when the Sun is very clear, take his Altitude by a backward Obfervation, the forward way being chiefly intended for nocturnal Obfervations, and when the Sun is too much obfcured to give any Shade or Image.

In the *Philofophical Tranfactions*, at Number 423, pag. 273, from whence the aforementioned Defcription and Ufe of this Quadrant is taken, you have an Account how near Altitudes of the Sun taken at Sea by this Inftrument, in a Voyage to *Maryland*, agreed with the Altitudes of the Sun taken at the fame Time by *Davis*'s Quadrant. Sometimes they differed only one Minute, fometimes 5′ or 6′, fometimes 16′, and once their greateft Difference was 21 Minutes. In the fame Place, it is faid, that Obfervations of the Altitudes of the fixed Stars taken by both the Inftruments, and the Latitude found thereby, commonly differed about 4′ or 5′, and the greateft Difference once arofe to 13′. Moreover, the Captain of the Veffel that went to *Maryland* declares, that he obferved with his Quadrant of Mr *Elton*'s both by the Sun and Stars, in all the various Sorts of Weather he met with in his Voyage to and from *Maryland*, without regarding the Horizon, with as great Exactnefs as with *Davis*'s Quadrant when the Sun and Horizon were clear. Laftly, It is faid that another Sea Captain computed the Latitude in *Leghorn* Road, and feveral of the Ports of *Spain*, from Obfervations by this Quadrant, exactly agreeing with the known Latitudes of thofe Places. He adds, that he made feveral Obfervations by the Inftrument in his Paffage home in hard Gales and a great Sea, and when it was fo hazy that the common Quadrant was of no ufe for want of the Horizon.

Hence we may briefly obferve upon this Inftrument, that it is no better than *Davis*'s Quadrant, at leaft thefe two Captains thought fo, or elfe they would not have ufed *Davis*'s as the Standard of Exactnefs, whereby to compare the others in their Voyages. At moft it can only exceed *Davis*'s in hazy Weather, and Obfervations of the fixed Stars at Night, when the Horizon cannot be feen.

CHAP. IV.

Some Account of the great Mural Quadrant at the Royal Obfervatory in Greenwich *Park.*

THIS Quadrant chiefly confifts of ftreight Iron Bars, crofs-ways joined together (as appears in the Figure) flat-ways and edge-ways, the Breadth of each of which is 2 Inches and 9 Tenths, and the Thicknefs $\frac{1}{10}$ and $\frac{1}{4}$ nearly; all thofe that are edge-ways being *Plate* 2. Fig. 9.

behind the others. They are so placed to strengthen them the more, and are besides farther strengthened by a great Number of short Iron Plates bent to a right-Angle, and placed behind the Quadrant in the Angles made by the Bars, and riveted to them both. Behind at the Circumference of the Quadrant there is also a Bar placed edge-ways, bent circular, and fastened all along the middle of the flat Arch or Limb of the Quadrant, by a sufficient Number of little Iron Plates bent at right Angles.

The Arch or Limb of the Quadrant consists of two (every ways) equal quadrantal Arches laid upon one another; an Iron one behind, and a Brass one before; the Breadth of each being 3 Inches and 4 Tenths; and the common Part of their Breadths where they lie doubled one over the other and are riveted together, is 2 Inches and 2 Tenths; the Brass Limb being remoter from the Center, than the Iron one by 1 Inch and 2 Tenths. This Limb is reduced to a true Plane, by first placing the flat of the Quadrant very firm upon a Level or Horizontal Plane, and erecting perpendicularly over it's Center an Iron Axis, to the Bottom of which Axis is fixed at right Angles, an Iron Arm, equal in Length to the Radius of the Quadrant, and to the other End of this Arm is fixed an Iron Scraper directly over the Brass Limb, and being firmly supported by the Arm and it's Braces, was turned about the aforesaid Axis, 'till by scraping the Brass, the Surface of it was reduced to a perfect Plane; the Edge of the Scraper being exactly perpendicular to the Axis of it's Motion.

Upon the Brass Limb, there are two Arches struck, one with a Radius of 96.85 Inches, and the other with a Radius of 95.8 Inches, by a Beam-Compass, secured from bending by several Braces fastened to it; the inner Arch is divided into Degrees, and 12th Parts of a Degree; and the outward Arch into 96 equal Parts, each of which are subdivided into 16 equal Parts. These Divisions were made by bisecting every one of the equal Arches of 30 Degrees of the Quadrant, whereby the same became divided into 6 equal Parts, containing 15 Degrees apiece, and each of these 15 Degrees being trisected by trials, Arches of 5 Degrees were obtained; and the fifth Part of these was found by trials; and the Subdivisions of the Degrees into 12 Parts, or every 5 Minutes were made by Bisections and Trisections: the outward Quadrantal Arch was divided into 96 equal Parts by no other method than that of Bisection, 'till 60 Degrees of the Quadrant, or two thirds of it, became divided into 64, and the remaining third into 32 equal Parts, which make 96 in the whole; and every one of these were also divided into 16 equal Parts by continual Bisections. There is no occasion to direct skilful Workmen how they shall make these Divisions with the greatest accuracy; they know more ways than one how to do it of themselves. These two Sorts of Divisions are a check upon each other, being in effect two different Quadrants; and the Divisions in one being reduced into the Divisions of the other, by a Table made for that purpose, they are never found to differ above five or six Seconds in any place of the Limb, and when they do the Preference ought to be given to the bisected Divisions, as being determined by a simpler Operation. One of the 96 equal Parts of the Quadrant is $\frac{15}{16}$ of a Degree, or $56\frac{1}{4}$ Minutes, and one of the 16 equal Subdivisions of every one of those, will be $3\frac{33}{64}$ Minutes.

To avoid the trouble of subdividing the Quadrantal Arch into smaller parts, the Telescope belonging to it, and moving about it's Centre, carries a small Brass Nonius Plate which slides upon the Limb.

The 10th Figure represents a Degree A B of the upper Arch of the Quadrant divided into 12 equal Parts, contains five Minutes in each. And C D in the same Figure is $\frac{1}{96}$ Part of the Quadrant divided into 16 equal Parts, each being as already said $3\frac{33}{64}$ Minutes; and E F the *Nonius*, or subdividing Plate fixed to the Telescope and sliding with it in the Space between the Arches A B, C D: the Degrees and Minutes, and also those 96 Parts of the Quadrant are numbered from the Left-hand to the Right, beginning from the Intersections of the vertical Radius, in order to measure the Distances of Objects from the Zenith. But the Parts upon the Nonius are numbered the contrary way, beginning from the Line oo, called the Index; which is drawn perpendicular to the Sides of the Nonius at the End next the Right-hand, and the Line of Sight through the Telescope is so adjusted by the Cross Hairs in it's Focus, as to be parallel to the Index oo produced through the Centre of the Quadrant.

The Length of the upper Arch of the Nonius is equal to 11 of the 12 equal Parts that every Degree of the upper Quadrantal Arch is subdivided into; and this Arch of the Nonius is divided into 10 equal Parts, so that one of these Parts is $\frac{11}{120}$ of a Degree of the Arch of the Quadrant, or $5\frac{1}{2}$ Minutes. Consequently the Difference between one of those equal Parts of the Arch of the Quadrant, *viz.* 5′, and one of these 10 equal Parts of the Nonius will be half a Minute, or 30″.

The Length of the lower Arch of the Nonius is equal to 17 of the 16 equal Parts that each of the 96 equal Parts of the opposite Quadrantal Arch is subdivided into; and this Arch of the Nonius is divided into 16 equal Parts, wherefore the Length of this Arch of the Nonius will be about $59\frac{1}{2}$ Minutes; and the Arch of one of the Divisions of this Nonius will be about $3\frac{15}{16}$ Minutes.

This Quadrant is fixed (it's Centre, and one Side of it being even with the Top of the Wall) to the Eastern Side of a Free-stone Wall, built on purpose in the Plane of the

Meridian;

Meridian; the whole Weight of the Quadrant is ſupported by two ſtrong Iron Pins fixed to the Wall, and projecting through two Holes in ſquare Plates of Iron riveted to the Quadrant at *a* and *b*; the Pin at *a*, which bears the greateſt Part of the Weight, is immoveably fixed in the Wall; but the Pin at *b* is moveable up or down by a ſtrong Screw, in order to bring one ſide of the Quadrant into an Horizontal, and the other to a vertical Poſition.

That the Motion of the Teleſcope about the Centre of the Quadrant may be free and eaſy, and that this may be obtained by counterpoiſing the Teleſcope, and eaſing the Center of the Quadrant of as much of it's Weight as poſſible, there is the following Contrivance: In Fig. 9. *a b* repreſents an Iron Axis laid acroſs the Top of the Wall, having two Braſs Plates fixed perpendicularly to the Ends of it, with Notches or Holes cut in them for this Axis to turn in, which points to the Centre of the Quadrant at right Angles to it's Plane: to that End of this Axis next to the Quadrant, an Iron Arm *c d* is fixed, having two Braſs Plates *c e*, *d f*, almoſt perpendicular to it; to them are riveted two ſlender Slips of Fir, whoſe other Ends meet at *g*, near the Eye-Glaſs, being held together in a Braſs Cap or Socket. Through a ſmall Plate fixed to one ſide of a Collar embracing this lower End of the Teleſcope, there paſſes a ſcrew Pin at *g*, parallel to the Teleſcope; which Pin being ſcrewed into the Cap at the End of the Slips, holds up the Teleſcope right againſt the centre Work; the Slips are ſtrengthened by 5 or 6 croſs Braces of the ſame Wood, as is repreſented in the Figure to the other End of the Axis *a b* another Arm *h i* is fixed parallel to the Teleſcope, and in a contrary Direction, carrying a Weight *i* to counterpoiſe the Weight of the Teleſcope, and make it reſt in any Poſition; and for the greater eaſe and freedom of it's Motion, two ſmall Braſs Rollers are fixed to each Side of it, at *k* and *l*, which are held tight to the Plane of the Limb by a Plate ſpringing againſt it's back-ſide, which Plate has alſo a Roller at each End of it. Fig. 9.

When the Teleſcope is pretty nearly directed to an Object whoſe Altitude is to be taken, a Plate *m n* which is carried by the Teleſcope along the Limb, and lies croſs it, may be fixed to it by a Screw, then by twiſting the Head *o* of a long Screw *o p*, which is parallel to the Limb, and which works through a female Screw, annexed to the Plate *m n*, and whoſe Neck at *p* turns round in a Collar annexed to the Teleſcope; a very gradual Motion is given to the Teleſcope for bringing the croſs Hairs exactly to cover the Object.

The Quadrant is ſet into the Plane of the Meridian by Hold-faſts, ſo as that Radius or Side of it which terminates 90 Degrees, is exactly placed vertical, with a Plumb-Line of fine Silver Wire, ſo ſuſpended as to play exactly over the middle of the central Point *o*, and alſo over the Stroke at 90 Degrees upon the Limb below; this Poſition of the Quadrant being once found, another Plumb-Line was ſuſpended by the Side of the Quadrant, quite clear of the centre Work, ſo as to play exactly over the Middle of a fine Point made in the Limb, in order to examine afterwards, with more expedition, whether the Quadrant has kept it's Place.

In order to help to determine the Degrees, Minutes, and Parts of a Minute, of the Meridian Altitude of an Object by the Index upon the Limb, it may be obſerved, that in the Scheme of Fig. 10. the Nonius E F is ſo ſituated that the upper End of the Index *oo* is not oppoſite to any one ſtroke of the adjoining Arch A B, repreſenting a Degree of the Limb divided into 12 equal Parts, or 5 Minutes, but to ſome unknown Point of a 12th Part of a Degree, intercepted between 50 and 55 Minutes; and to find the Overpluſs above 50, I obſerve by looking back from the Index, that a Stroke of the Nonius, which lies between the Numbers 3 and 4, is directly oppoſite to a Stroke upon the adjoining Arch, which ſhews that 3 Minutes and a half is to be added to the 50 Minutes aforeſaid. Fig. 10.

But when it happens that no one Stroke upon the Limb is directly oppoſite to a Stroke upon the Nonius, then look for that ſingle Part of the Limb which is ſo oppoſed to a ſingle Part upon the Nonius, as to be exceeded by it at both Ends, as is repreſented in the Parts G and H. Then, if by Eſtimation of the Eye, this part of the Nonius exceeds the Part of the Limb equally at each End, allow 15″ more, than if they had coincided at their Ends next the Index: and according as the Exceſs next the Index, is judged to be one third, one half, double, or treble of the other Exceſs, allow $7\frac{1}{2}''$, 10″, 20″, $22\frac{1}{2}''$, reſpectively; for ſince the Sum of the two Extreams is always the ſame, *viz.* 30″, the Number of Seconds to be added will always be to 30″, as the Exceſs next the Index is to the Sum of the two Exceſſes.

Again, ſince, as has been ſaid, the lower Arch of the Nonius is divided into 16 equal Parts, and is equal in Length to 17 equal Parts upon the oppoſite Arch of the Quadrant C D; it will by conſequence determine 16th Parts of any one of them. In this Scheme, the oppoſite Strokes of the Nonius and the lower Arch are ſuppoſed to coincide at the End of the 9th Part upon the Nonius, which ſhews that the Index cuts off 9 Sixteenths of the oppoſite Part of the Arch; and ſo the Length of the Arch from the Beginning C of a 96th Part C D of the Quadrant is thus denoted 15,9, the lower Points being paſt the 15th Stroke; and becauſe the Arch C D is $\frac{90}{96}$, or $\frac{15}{16}$ of a Degree, or $56\frac{1}{4}$ Minutes; and ſo $\frac{1}{16}$ Part of the Arch C D is 3 Minutes and $30\frac{15}{16}$ Seconds. Therefore the Length of the ſaid

Arch will be 15 Times 3 Minutes, and 30 $\frac{15}{16}$ Seconds, together with $\frac{9}{16}$ of 3 Minutes and 30 $\frac{15}{16}$ Seconds.

It is agreed amongſt Aſtronomers, that a large Mural Meridian Quadrant, ſuch as this is which we have been deſcribing, is by far the moſt accurate, expeditious, and convenient Inſtrument of all others for the chief Purpoſes of Aſtronomy. For ſince the Meaſure of Time by Pendulum-Clocks, and conſequently the apparent Motion of the Heavens, is now brought to the utmoſt Perfection; if by obſerving the Time by one of theſe Clocks when any Object in the Heavens comes to the Plane of the Meridian, we ſhall have their right Aſcenſions; and by having given the Latitude of the Place, we have alſo their Declinations; and thence their Places in the Heavens. And ſo a Catalogue of the Places of the fixed Stars may be made in leſs than a tenth Part of the Time than by the beſt moveable Quadrant, or Sextant; not to mention the ſaving great Labour in Trigonometrical Calculations.

Note, This ſhort Deſcription of this Quadrant, may be enough for Gentlemen. As to Workmen, they may have a full Deſcription of it in the 7th Chapter of the 3d Book of the Second Volume of Dr *Smith*'s Opticks.

This Quadrant is certainly a moſt curious, elaborate, and ſkilful Piece of Workmanſhip, not, perhaps, equalled by any of the Kind that has ever yet been made: the Contrivance and Direction of the whole being that of the late Mr *George Graham*, Watch-Maker in *Fleet-Street*, one of the greateſt Maſters of Mechanicks in the World; who did himſelf actually perform the Diviſions of the Arch, and all the nicer Parts of the Work. But, for all this, I think the Inſtrument is too complex, and redundant in Contrivance, both as to Strength and Exactneſs. I mean, a Quadrant of that Kind might have been made as laſting, perfect, and fit for the Purpoſes deſigned by it, and having as much real Exactneſs (not apparent, which is often times the Caſe), with much leſs Art, Trouble, and Expence. Nor with all it's Exactneſs, do I think an Angle of Altitude can really be taken by it (or any other the moſt perfect Inſtrument of the ſame, or a leſs Radius), to $\frac{1}{6}$th of a Minute, or 10″. However, ſome People may deceive themſelves, and imagine the contrary: for ſince this Radius is not quit 8 Feet, one Minute of the Arch of the Quadrant will be but about 28 equal Parts of 1000 that one Inch is divided into; and 10 Seconds will be but about $\frac{1}{214}$ Parts of an Inch, which is ſuch a ſhort Length, as not to be diſtinguiſhable by the naked Eye. Certainly not by mine, and much leſs can any leſſer Number of Parts be defined, even by the Aſſiſtance of Diagonals, and Nonius's of what Kind ſo ever.

I have oftentimes thought that Angles of the Sun's Altitude may be taken to greater Exactneſs by more natural, ſimple, and leſs expenſive Contrivances, than by any of theſe, and ſuch like, very artificial Quadrants. Chiefly by means of a perpendicular Gnomon, many Feet high, or the Top of an high Building or Mountain, and the Shadow of the Extremity thereof caſt from the Sun upon an horizontal Plane: for by having given the Heighth of that Gnomon, and the Length of it's Shadow upon the horizontal Plane at any Period of Time, the Angle of the Sun's Altitude at that Time may be found to Seconds, by Trigonometrical Calculation; or a long Line may be horizontally extended from the Foot of that Gnomon upon the horizontal Plane, being firſt divided into a Line of Tangents by means of ſome of the Tables of Tangents to be found in Books, denoting every ten Seconds (as in *Benjamin Urſinus*'s Trigonometry), the Length of the Gnomon being the Radius, and the Parts of that Line which the Extremity of the Shadow of the Gnomon falls upon, will be the Degrees and Minutes, *&c.* of the Altitude. But when the Altitude is but ſmall, *viz.* under 30 Degrees, or leſs, and ſo the Length of the horizontal Shadow is too great, another perpendicular Gnomon, or ſome other high Object that will caſt a Shadow of equal Altitude with the other, and diſtant from it as much, or more, than by it's Height; and from the Top of this laſt Gnomon there muſt be extended the ſame Tangent-Line-String which was before Horizontal, as a Plumb-Line; then that part of the Tangent-Line, upon which the Shadow of the Extremity of the other Gnomon falls, will give the Degrees, *&c.* of the Sun's Altitude.

If the Penumbra be an Objection to the Accuracy of this Method, a Teleſcope with Croſs-Hairs, Wires, or Silken-threads in the Focus, may be uſed, or ſome other Contrivance, like a Camera Obſcura, to caſt the Sun's Image upon the Tangent-Line: it is true, theſe are but rude and imperfect Hints. But this I am certain, that ſomething may be done in this way, that will exceed any of the diminutive Quadrants, or other complicated, and expenſive Inſtruments that have hitherto been made, in meaſuring Angles to great Exactneſs. The Ancients uſed Gnomons, and ſo did ſome of the Moderns in the laſt Century, as is related in *Ricciolus*'s *Aſtronomia Reformata, & Geographia Reformata.* He ſays, That *Ulugh Beigh*, a King of *Pathia* and *India*, a Kinſman of *Tamerlane* the Great, uſed a Gnomon about 180 Feet in Heighth, about the Year 1437. *Ignatius Dante* errected one in the Church of St *Petronius* at *Bononia*, 67 Feet high, in the Year 1576. In the Year 1655, the celebrated *Caſſini* had another in the ſame Place 20 Feet high. Father *Heinrich*, the Jeſuit, had one of 35 Feet high erected in the Year 1705, at *Utrecht*. There have been others who have uſed Gnomons for taking Altitudes, of no great Moment to mention here.

Note,

Note, In the *Philosophical Transactions*, Numb. 483, there is a new Method of making a Mural Quadrant by one Mr *Gersten*; being a Mural Arch furnished with a Micrometer, free from some of the Inconveniences he would have to happen to those commonly fixed to a Wall; but this Contrivance appears to me, to be too artificial and over nice: rather tending to rectify and avoid imaginary Defects, than really supply any real and considerable Faults, if any such, in the Mural Quadrants already constructed.

CHAP. V.

Of Perspective Glasses, and Refracting Telescopes.

I. THE least Kind of perspective Glasses, are about 4, 5, or 6 Inches long, having only two Lenses within the Tube; the Object Lens or Glass, being a double Convex, or Plano-Convex, being a Segment of a greater Sphere: and the Eye-Glass, a double Concave, being the Segment of a lesser Sphere, placed before the Focus of the Object Lens, by the Distance of the virtual Focus of the concave one.

The Objects appear through these Perspective Glasses distinct, erect, and magnified in Diameter, in the Ratio of the focal Distance of the Object-Glass, to the focal Distance of the Eye-Glass: and for purblind Persons to see the Object distinctly, the Eye-Glass must be brought nearer to the Object-Glass. There are few of these Perspective Glasses now made that exceed 18 Inches long, because you see through them a part of the Object so much the lesser, as it's Diameter is the more magnified. Formerly, indeed, there were Telescopes made with only a double convex Object-Glass, and a concave Eye-Glass, from 18 Inches long, to about 3 Feet; which was the Length of *Galileo*'s best Telescope, so famous for the great Discoveries he made with it. *Hevelius* says, a good Telescope of this sort may be had when the double convex Object-Glass is 4 *Danzick* Feet in Diameter, and the double concave Eye-Glass in Diameter is 4 $\frac{1}{2}$ Inches. So likewise will it be, when the former is about 5, 8, 10, or 12 Feet, and the latter about 5 $\frac{1}{2}$ Inches. But, as I said before, the Field which these Telescopes take in at one View, is too narrow when they exceed 3 or 4 Feet in Length; they have long since been disused, excepting the Perspectives of 4 or 5 Inches in Length; where Mr *Huygens* recommends the Ratio of the Semi-diameter of the Object-Glass, to that of the Eye-Glass, to be quadruple, or not more than double.

II. There are Perspectives from 18 Inches, to 4 or 5 Feet in Length, chiefly for viewing Objects at Sea or Land, consisting of four convex Lenses or Glasses, *viz.* an Object-Lens A, and three equal Eye-Lenses, C, D, E; so placed in the common Axis A F, that B is the common Focus of the Object-Glass A, and the Eye-Glass C; the Eye-Glass D is so far from C, as that the Distance C D is equal to twice the focal Distance C B of the Eye-Glass C; and the Eye-Glass E is as much distant from D as C is from D; and the Eye must be placed beyond this last Eye-Glass E, by the Distance B C. *Plate* 2. Fig. 11.

By these four Lenses, and such a Disposition of them, very remote Objects will be distinctly perceived, erect, and magnified in the Proportion of A B, the focal Distance of the Object-Lens, to B C the focal Distance of either of the equal Eye-Lenses; there is a small Annulus, or Ring, placed within the Tube of this Perspective, either at the common Focus of the two Lenses D and E, or at B, the common Focus of the Object-Lens A, and the Eye-Lens C next to it; by which contrivance is cut off those irregular Rays which are not collected near enough to the two common Foci of the Lenses, and so are not by means of the succeeding Lenses sent parallel to the Eye: also the Colours near the Margins of the Lenses are taken away, which otherwise cannot well be avoided.

This is reckoned one of the best Compositions of the four Lenses of a Perspective, and is thought to have been invented by *Campani*, a famous practical Optician, who lived at *Rome* many Years ago. There are also Telescopes to view the Celestial Bodies through of this sort, having four Glasses, and of greater Lengths. But a Telescope of two Glasses only, is sufficient for this purpose.

There may be other Compositions of other Species of Lenses than all four double convex ones of equal Convexities, and three of them equal, that may do as well as these; and the best way to find them out is, by Trial: for Example, Suppose the Object-Glass a Plano-Convex, and the three Eye-Lenses equal Plano-Convex.

There have been also Perspectives with three Lenses, which invert the Object, and make it appear coloured. But these are not much esteemed.

There are also Perspectives and Telescopes, made with six Glasses, shewing Objects very distinct, enlightened and magnified. Such a Telescope may consist of five equal

equal Eye-Lenses of equal Convexities, placed in the Axis of the Tube equally distant from each other, by twice the focal Distance of either of them; and the outermost Eye-Lens next to the double convex Object-Lens is distant from the Object-Lens by the Sum of the focal Distances of the Object-Lens, and one of the Eye-Lenses; for by this Disposition of the Lenses, the Rays that come from a very remote Object that fall upon the Object-Lens, being as it were parallel, will become again parallel after their Refraction by the fifth Eye-Glass; so they will again, after their Refraction by the third Eye-Lens, and so will they once more after their Refraction by the first Eye-Lens: therefore since the Rays coming from the Object fall parallel upon the Eye, the Object will be distinctly seen. I am not certain whether these six Glass Telescopes are actually made with such Lenses, so disposed, as I have said; for I have not seen their Structure; but I know that according to the Theory, at least, such a Telescope may be so made, although, perhaps, there may be Inconveniences in this Method not to be discovered, unless by actual Practice: for in the Construction of Telescopes, though the Theory and Practice should always attend, and support each other; yet the Practice most conveniently takes the Lead, is the best Guide, and most to be depended upon.

Traber, in his *Opticks* says, That *Anthony Maria de Reita*, a *Roman* Optician, made a Tube with five Convex-Lenses, which was reported to be an exceeding good one: he likewise mentions one *Fontana*, another *Roman* Artist of this Kind, who made a Tube with 8 Convex-Glasses, clearly exhibiting the most minute Object, at the Distance of a *German* Mile, which was bought by Cardinal *Nepos*, for 800 Crowns, and presented by him to the Great Duke of *Florence*; but, says he, by reason of the strong and too violent Refraction, the Objects viewed through it appeared coloured, so that the Tube could not be much used without hurting the Eyes. He, moreover, says, that he was told one *Eustachio*, a *Neapolitan*, made a Telescope having 19 Convex-Lenses, in Tubes altogether of 19 Cubits long, that would shew the Pictures of Objects less coloured, by reason of the Interposition of some of the nicer Glasses (neither magnifying, nor diminishing the Objects), which took away the Colourings of the Objects. Hence (says he) we may see what a great Mystery there is in the apt Composition of Lenses.

But how true this is, what are their particular Constructions and Effects, I know not: for unless the Composition of the Glass, wherewith the Lenses are made, be exceeding fine and pure, free from Veins and Spots, and the Lenses be most exquisitely figured, and polished, and besides, very thin, such a multiplicity of them must hinder the Passage of so many of the Rays that come from the Object to the Eye, as will cause a very obscure View of the Object.

It is as easy in Theory to contrive a Disposition of the Lenses for an 8, 10, or 12, *&c.* Glass-Telescope; as for one of four Lenses. It is but taking as many Equal Eye-Lenses, except 1, as the Telescope is to have Glasses, and placing them in the Axis of the Object-Lens (being a double Segment of a greater Sphere, which is in all Cases supposed), at equal Distances from one another, each equal to twice the focal Distance of either: the last Eye-Lens next to the Object-Lens, being so far from the Object-Lens as is the Sum of the focal Distances of the Object and Eye-Lens.

III. The Astronomical Telescope, *viz.* one through which the Heavenly Bodies are viewed, consists of but two double Convex-Lenses, the Object-Lens and the Eye-Lens; these shew the Object inverted, and magnified in the Ratio of the focal Distance of the Object-Lens to that of the Eye-Lens; the focal Distance of the Object-Lens being always much greater than that of the Eye-Lens; these two Lenses being so placed in the common Axis, as that their Distance is equal to the Sum of their focal Distances, or, which is the same thing, that their Foci unite in one Point.

Telescopes have been made with other Species of Lenses, besides double convex ones, and *Honoratus Fabri* in his *Synopsis Optica*, says, That *Eustachius Divini*, a famous *Roman* Optick-Glass maker, made the Eye-Lens of his Telescopes to consist of two equal, narrow Plano-Convex-Lenses, touching one another's Convexities in the Axis, and so placed, that the Centre of that Plano-Convex-Lens next to the Object-Lens, was in the Focus of the Object-Lens; by which means the Rays that came parallel from the Object, would fall parallel upon the Eye: and, says *Fabri*, some of the Advantages of this Telescope are, that the Colours of the Rain-Bow are excluded from it. The Angle of Sight is augmented. A greater Field is taken in at one View. The Object appears more lively and bright. Lastly, He would have Water included in the Vacuity between the Convexities of the two touching Plano-Convex Eye-Lenses. See a more particular Account of all this in the 46th Proposition of *Fabri*'s Opticks.

If two equal Lenses be joined together so as to touch, the Focus will be removed to a Distance double to that of one of them. *Dechales* takes Notice, that an Object-Glass of 2 $\frac{1}{4}$ Feet focal Distance, fits an Eye-Glass of 1 $\frac{1}{4}$ Inch. *Eustachius Divini* joins an Object-Glass of 8 Feet, to an Eye-Glass of 4 Inches. Mr *Huygens*'s Telescope, by which he first observed the true Phases of *Saturn*, and one of his Satellites, consisted of an Object-Glass of 12 Feet focal Distance, and an Eye-Glass a little lesser than 3 Inches. Afterwards he observed the same Phenomena by a Telescope of 23 Feet, with two Eye-Glasses of 1 $\frac{1}{4}$ Inch Diameter,

Diameter, touching one another, and producing the same Effect as one only, *viz.* collecting the parallel Rays at about 3 Inches Distance. He likewise observed the same with an Object-Glass of 30 Feet joined to an Eye-Glass of 3 $\frac{3}{10}$ Inches; and in his *Dioptricks* gives the following Table for the Construction of Telescopes, though here contracted.

TABLE *for* TELESCOPES.

The focal Distance of the Object Lens, or the Length of the Telescope. *Rhinland Feet.*	The Diameter of the Aperture of the Object Lens. *Inches, and Decimals.*	The focal Distance of the ocular Lens. *Inches, and Decimals.*	The Proportion of magnifying, considered as to Diameter.
1.	0,55.	0,61.	20.
2.	0,77.	0,85.	28.
3.	0,95.	1,05.	34.
4.	1,09.	1,20.	40.
5.	1,23.	1,35.	44.
6.	1,34.	1,47.	49.
7.	1,45.	1,60.	53.
8.	1,55.	1,71.	56.
9.	1,64.	1,80.	60.
10.	1,73.	1,90.	63.
13.	1,97.	2,17.	72.
15.	2,12.	2,33.	77.
20.	2,45.	2,70.	89.
25.	2,74.	3,01.	100.
30.	3,00.	3,30.	109.
35.	3,24.	3,56.	118.
40.	3,46.	3,81.	126.
45.	3,67.	4,04.	133.
50.	3,87.	4,26.	141.
55.	4,06.	4,47.	148.
60.	4,24.	4,66.	154.
65.	4,42.	4,86.	161.
70.	4,58.	5,04.	166.
75.	4,74.	5,21.	172.
80.	4,90.	5,39.	178.
85.	5,05.	5,56.	183.
90.	5,20.	5,72.	189.
95.	5,34.	5,87.	194.
100.	5,48.	6,03.	199.

Which Table he thus constructs; he multiplies the Number of Feet in the focal Distance of the Object-Lens by 3000, and the square Root of the Product will give the Diameter of the Aperture in Inches and Decimal Parts: and the same augmented by a tenth Part of itself, will be the focal Distance of the Eye-Lens; and the apparent Breadths of the Objects are as the Diameters of the Apertures.

Because two Telescopes where the Ratio of the focal Distance of the Object-Lens of the one to that of it's Eye-Lens, is equal to the Ratio of the focal Distance of the Object-Lens of the other to that of it's Eye-Lens, do equally magnify the same Object; some may be apt to think, that the Trouble of figuring and polishing the Object-Glass of a long Telescope may be spared; and that two such Telescopes are equally good: but in this they are mistaken, for the Object in the short Telescope will appear more confused, dark, and indistinct than in the long one, all things else alike. What doth it avail to magnify an Object very much, if it cannot be seen distinctly; this is all we want, and long Telescopes will always do it best.

Sir *Isaac Newton* has proved in his Opticks, that the Perfection of Telescopes is impeded by the different Refrangibility of the Rays of Light, and not by the spherical Figure of the

Glass, the Diameter of the little Circle or Focus, through which the Rays are scattered by the unequal Refrangibility, being about the 55th Part of the Aperture of an Object-Glass, whose Length was 12 Feet, and Aperture 4 Inches: so that the Error arising from the spherical Figure of the Glass, will be to the Error arising from the different Refrangibility of the Rays, as 1 to 5449, which being so little in comparison to the other, deserves not to be considered. He says also, If the Theory of making Telescopes could at length be fully brought into Practice, yet there would be certain Bounds, beyond which Telescopes could not perform, by reason of the perpetual Tremor of the Air, through which we look at the Sun or Stars; even though long Telescopes may cause Objects to appear larger and brighter than short ones can do, yet they cannot be so formed, as to take away that confusion of the Rays which arises from the Tremors of the Atmosphere; the only Remedy is, a most serene and quiet Air, such as may, perhaps, be found on the Tops of the highest Mountains, above the grosser Clouds.

Since the first Invention of Telescopes, which is now almost 150 Years ago, there have been many made of all Lengths and Goodnesses, in *Italy*, *England*, *France*, and *Germany*. Several ingenious Persons having imployed their utmost Care and Pains to procure the best and clearest Glass possible, and used their greatest Skill and Dexterity in figuring and polishing the spherical Lenses. Sir *Isaac Newton* and Mr *Huygens* took to this Business, but the former made no great Progress in it, rather for want of Inclination to continue the Pursuit, than Abilities to excel in it, if he had. But Mr *Huygens* went further, and made very good and long Telescopes of 23, 25, 30, and 123 Feet, which last has been in *England* many Years; the late Doctors *Derham* and *Pound*, made Observations with it at *Wansted* in *Essex*. *Hevelius* of *Dantzick*, had a Telescope of 140 Feet; and *Eustachius Divini*, and *Joseph Campani* at *Rome* (long since deceased), made Telescopes of various Lengths, *viz.* of 23, 24, 30, 33, 45, 50, 60, 70, *Roman* Palms; the longest of *Campani*'s was 150 *Roman* Palms, and *Francis Bianchini*, the Pope's domestick Prelate, made Observations with it, in the Year 1725. The Telescopes made by these two *Italians* have been generally esteemed the best in the World. *Cassini* too had very good Telescopes, the longest being 136 Feet; and the longest that ever I heard off, is one of *Artouquellius*'s (see the *Philosophical Transactions*, Numb. 92, 181.), of 220 Feet. There are very good Telescopes of Mr *Petroni*'s made in *Italy*.

Mr *Huygens* wrote a Treatise about grinding Glasses for Telescopes, wherein he supposes the Glasses to be double Convex ones of equal Convexities, and the focal Distance to be given; then the Radius of the spherical Surface will be found, by taking it in proportion to that focal Distance, as 12 is to 11; the focal Distance being given, the Aperture of the Glass is also given by the Table above. When the Radius of the Convexity of the Glass is very great, and so the said Convexity cannot be described from a Centre, he finds Points through which that spherical Convex is to pass, by laying off in Effect, upon a Tangent to the Pole of the Convex, some of the first Figures of the several Sines of very small Arches (near one another), taken from the Tables of Sines, &c. and that by means of a Pair of Compasses, and an accurate Diagonal Scale of Inches, subdivided into centesimal Parts; and afterwards taking the same Number of the first Sines of the several correspondent versed Sines of those Arches, and laying them off respectively upon Perpendiculars drawn from the Extremities of those Sines that are upon the Tangent-Line; the Radius of the Tables being reduced by cutting off the Figures to the Inches expressed in the given Radius of the Convexity. Mr *Huygens*'s Concave-Tool, or Plate, or Dish, in which an Object-Glass is ground, was Copper, or cast Brass fastened upon a cylindrical Stone, with a Composition of Pitch and Ashes; and these Tools were applied to a turning Lathe, in order to turn the concave Surface exactly spherical; then he polishes the Tool by an Incrustation of Pitch and Emery, and afterwards with blue Hones. In the Choice of Glass, he says that is best which looks somewhat yellow, red, or green when looked through against the Sky-Light, or laid upon a Sheet of white Paper: that which is perfectly white, though it transmits more Light, is generally fuller of Veins, and is often subject to grow moist in the Air, which in Time destroys it's polish; that such Glass was pretty good wherewith Drinking-Glasses was made. That he ordered Pieces of Glass to be made for his Use, of half an Inch, or three Quarters thick, in the same Manner as they make Looking-Glasses, *viz.* by cutting off the Top and Bottom of the round Globe they blow up, and by slitting the Side of it, and then by flattening it upon the Hearth of the Furnace, and afterwards it was ground to an equal Thickness by a Stone-Cutter, by the Contrivance wherewith he polishes Marble, or ground to an equal Thickness, and polished a little by a common Glass-Grinder. Then having found how free from Veins this Glass was, by looking very obliquely against a small Light in a Room otherwise dark, or by Reflexion with a Candle in that Room, and made choice of the best for his Purpose; he rounds his Glass-Plate, by first describing upon the two Surfaces two Circles directly over each other, with a Diamond pointed Compass, representing the Circumference of the Object-Glass; and also two more concentrick Arches with a Radius about a tenth or twelfth Part of an Inch bigger. This done, he seperates the longer Part of the Glass from the outward Circle by a red-hot Iron, or by a strong broad Vice, opened exactly to the Thickness of the Glass. The remaining Inequalities may be taken off

* by

by a Grind-Stone; then having warmed the Glass and cemented a wooden Handle to it; and in a common deep Tool for Eye-Glasses, with white clear Sand and Water, he ground the Circumference of the Glass exactly true to the innermost Circle on each side of it. The Glass being made in figure of a Cylinder, it is ground upon the Tool with Emery (some use common, clean, fine, white Sand), and made fit for polishing; then it is polished (with a Mixture of fine Tripoly and blue Vitriol wetted with Vinegar), by means of a polishing Machine, which he describes. This is but an imperfect, scanty Account of what Mr *Huygens* says about the figuring and polishing Glasses, yet it may be sufficient for a Hint to satisfy those who are desirous only to be acquainted how the thing is done in general. See the Account at large at the beginning of the Second Volume of Dr *Smith*'s Opticks.

As spherical Figures for Optick-Glasses will not collect all the Rays coming from one Point of an Object into another Point or Focus, *Des Cartes*, and some other of his Geometrical-Followers, about the middle of the last Century, recommended hyperbolick and elliptick Glasses, instead of spherical ones; according to which, if the Glasses were figured, these would collect parallel Rays into one Point or Focus, and by that means Telescopes with such Glasses would be better than those with spherical Glasses; this is all fully explained by *Des Cartes* in his Dioptricks, where he gives an Instrument for describing hyperbolick Glasses; but the Difficulty of the figuring such Glasses deterred some, and the Objection of Sir *Isaac Newton* (who has given a long Proposition in his Opticks, proving to them that fully understand it, who I believe are but few, that it is not the spherical Figure of the Glasses, but the different Refrangibility of the Rays that hinders the Perfection of Telescopes), discouraged others from trying to make Optick-Glasses of hyperbolick and elliptick Figures. There is certainly great Respect to be paid to Sir *Isaac Newton* in this Matter, who was, doubtless, the greatest Optician in the World; but yet I do not take his Proof to be absolute Conviction, and that such hyperbolick Glasses would not be better than spherical ones: nay, he himself in his Optical Lectures, Section I. says, that Glasses cannot perform more than twice as well as Spherical ones, though they were formed into Figures, the best that could be devised for that end, and both figured and polished exactly alike. From whence one may certainly infer, Sir *Isaac* thought that it was possible for some Glasses not spherical, to be as good again as spherical ones; therefore, if hyperbolick Glasses will double the present Perfection of Telescopes, it will be worth while to try to make them. I am sure it may be done by those who have a sufficient Skill in Geometry and Mechanicks.

The Tubes of perspective Glasses are of several Sorts, and made of various Matter. Some are made of stiff Paper glued together, covered with Parchment or Shagreen; some consist of single Tubes, made of light dry Wood; others of several lesser Tubes, sliding one within another. The Glasses are fastened in the Tubes to wooden Rings, yet so as to be easily taken out and put in again, by means of Screws various ways, in order to wipe the Dust or Moisture off from them, with which they are more or less continually covered; and this should always be done before the Perspective is used. At the End of every one of the inward Tubes is fitted a wooden Ring, to hinder the lateral spurious Rays from coming to the Eye, which is found by Experience to be of more Use than could be thought; these Rings are generally furnished with female Screws in those Places whereat the Glasses are fited. The Perspectives of three or four Feet, which have but one wooden Tube, are most handy, especially, if at each End, instead of Covers that screw and unscrew, they have thin sliding Pieces of Brass for such.

Short Telescopes may have one single Tube, or several Sliding-Tubes; but if the Telescope is more than 20 Feet in Length, these will be too heavy, and apt to bend when the Telescope is using. When the Telescopes are long, there have been many Contrivances to manage and use them, as may be seen in *Hevelius*'s *Machina Cœlestis*; Father *Cherubin*'s Works; *de la Hire*'s, and others; all very expensive, complicated, and troublesome; but Mr *Huygens* happily freed Astronomy from this expensive Lumber, in his *Astroscopia Compendiaria Tubi Optici Molimine Liberata*, printed at the *Hague*, in 1684; who placing the Object-Glass upon a long upright Pole, contrived to direct it's Axis towards any Object, by a fine Silk-Line coming down from the Glass above to the Eye-Glass below; which Invention was succesfully practised both by himself and others; particularly the late Dr *Pound*, and Mr *Bradley*, with Mr *Huygens*'s own Object-Glass of 123 Feet focal Distance, presented with it's Aparatus by Mr *Huygens* to our Royal Society. His contrivance is this:

a b (Fig. 12. Plate 2. of the Appendix) is the Mast or Pole set upright in an open Place, nearly as long as the Telescope itself, having one side of it made flat almost all it's whole Length, upon which is nailed two Slips of Wood forming a long Dove-tail Channel; in which is fitted a moveable Board *c d*. On the Top of the Pole there is placed a Pully *a*, over which goes an endless Rope, near half an Inch thick, and as long again as the Pole. All way up the Pole, are fixed triangular wooden Steps to go up to the Top upon Occasion, not marked in the Scheme. This Rope serves to draw up or down the moveable Board *c d*, with a steady easy Motion, and to the Middle of this Board *c d*, there is fixed a wooden Arm *e*, extended a Foot from the Pole; and the Middle of another Board *f f* a Foot and a half long, is laid horizontal, and at right Angles, over the End of

Fig. 12. *Plate 2.*

of that Arm, and fixed to it; the Object-Glass must be placed upon one End of this cross Board, and the whole must be lifted up and down by the Rope above-mentioned, the Ends of it being tied to the Top and Bottom of the sliding Board *c d*, and the whole must be counterpoised by a leaden Weight *h*, of a conical Form, fixed to the Rope on the other Side of the Pully in such a Place, that the Weight may be at the Top when the Object-Glass is at the Bottom, and contrariwise.

The Object-Glass is fixed within a Tin or Brass Tube *i*, four Inches long, and to the Outside of this Tube is fixed a strait Stick *k l*, about an Inch thick, at the Distance of 8 or 12 Inches from the End of the Tube; the whole is supported by a Brass Ball *m*, as big as a Marble, fixed to the said Stick by a short Neck, lodged in a hollow Socket, in which it may play very freely without dropping out; let the Socket and it's cylindrical Pedestal be slit into two Leaves, and held together by a Screw passing through both, but not so close as to pinch the Ball. By this means the Object-Glass and the Stick annexed are moveable every way, and to keep them in Equilibrio, an equal Counterpoise of Lead *n* is fixed to the under Part of the Stick *k l*, by a stiff Brass Wire *l*; so that by bending this Wire to and fro, the common Centre of Gravity of the Weight, the Lens and Parts annexed may be easily placed in the Centre of the Brass Ball, and then the Compound will be moveable with the least Touch, and will rest in any given Position; and in this consists the Judgment of the whole Invention. Having stuck the Pedestal of the Ball and Socket *m* into a Hole in the End of the cross Board *f f*, a Silk Line is tied to the Bottom of the Stick *k l*, annexed to the Object-Glass, whose Length is longer than that of the Telescope, as that the other End of it may be brought to the Eye-Glass; so that when the Object-Glass is raised up to the Top of the Pole by gently drawing this Thread, while you are moving round the Pole, the Object-Glass will readily obey it's Motion, and be directly opposed to what Star you please. Which could never be performed without placing it in the State of Liberation abovementioned: now since it is absolutely necessary that the Stick *k l* should be parallel to the Silk Line *l n*, a short Brass Wire is placed at the Tail of the Stick, and is bent downwards so far 'till the End of it, where the Silk is tied, be as much below the Stick as the Centre of the Ball and Socket is.

The Eye-Glass is included in a short Tube *o*, joined to a Stick *p*, which may also have a Ball to rest upon, or rather a little transverse Axis *q*, and a Weight below the Stick to balance the Tube and Eye-Glass. The Observer takes hold of a Handle *r*, fixed to the transverse Axis, and holds the lower Stick *p*, directed towards the upper one *k l*, by means of the Line that connects them, and winds about a Peg or Spool *t*, fixed in the lower Stick *p*; so that by pulling gently to extend the Line, the two Glasses will become parallel to each other; the lower Part of the String comes through a small Hole made with a Wire, at the farther End of the lower Stick *p*, like the Pin of a Fiddle, shortening or lengthening the String at pleasure, 'till the Distance between the Glasses be brought to a just Length requisite for distinct Vision.

Note, At *u* are the Pins stuck cross each other to make a Hole for the Line to pass through.

In order to keep the Eye-Glass steady, the Observer, whether standing or setting, should support his Arms upon a wooden Rest *x*, with only two Feet, holding the Eye-Glass in one of his Hands, which is a readier and more commodious Way than to fix the Eye-Glass upon a three-footed Rest.

When the Nights are dark, and a Star is to be found in the Telescope, a Lanthorn *y* is used, which collects the Light into a Stream, either by Transmition through a convex Lens, or by Reflection by a concave Speculum; for by directing this Stream of Light 'till it falls upon the Object-Glass, and makes it visible, it is easy for the Observer to change his place, 'till he finds the Star is covered by the Middle of the Object-Glass, and then to apply his Eye-Glass, which is sooner done than by a Telescope with a long Tube. By Moon-Light the Object-Glass is visible without a Lanthorn. But in viewing the Moon through the Telescope, a Paste-board Umbrella should be put about the Object-Glass, of such a Diameter as to cover a Space in the Sky above twice as broad as the Moon, to intercept that Light from coming to the Eye, which would pass by the Sides of the Object-Glass, and by mixing with the Light that comes through the Telescope, and would dilate the Appearance of the Lights and Shades in the Face of the Moon.

This is the Substance of that excellent Contrivance of Mr *Huygens* for viewing Objects without long Tubes; those who want the full Account, may consult Mr *Huygens*'s little Treatise abovementioned, or Dr *Smith*'s English Translation of it, in the Second Volume of his Opticks.

Of all the Instruments invented by Mankind, there are none that ever exceeded Perspectives and the Telescope, for the vast amusive Pleasure, and real Use afforded by them. How is a Person delighted with clearly viewing distant terrestrial Objects through these Instruments, but faintly, or not at all, appearing with the naked Eye, by reason of their great Distance? And how great is their real Use upon many Occasions for Instance, in discovering the Enemy at Sea or Land, in time of War, to which all the good Consequences derived from such Discovery, are to be attributed? How poor, scanty, obscure, and imperfect, was the State of Astronomy, in it's physical Part, before the Use of the Telescope, which wiped off from it the Dust of Obscurity, cleared away the Rubbish of Error, and opened and widened the narrow clogged up Passage of Sight, leading into the bright, and delightful Mansions of the heavenly Bodies, so vastly distant placed, and imperfectly and minutely seen by the naked Eye. By this they are distinctly seen with Wonder, and pleasing Amazement, enlarged in Bigness and Distinction; and many things are discovered, which now are become familiar to us, that mortal Eyes never before saw, or Men could ever have thought of. This is the Instrument which has brought us into a perfect Acquaintance with those surprising Parts of the Creation, far separated from our Globe of Earth, and with which we are allowed no other Commerce but looking and observing, and admiring the delightful Mechanism of the Works of the Creator, thereby giving us the greatest and most respectful Notions of his Wisdom and Power; and the first Inventors of this Instrument, whoever they were, ought always to be highly esteemed, as well as looked upon to be great Promoters of Human Happiness.

But enough of this. Some of the particular Discoveries of the Telescope are these.

I. *Spots in the Sun* first discovered by *Galilæo* and *Scheiner*, a Jesuit, about the Year 1611, of various Shapes, Bignesses, and Durations, they all consist of a black Part in the Middle, of some irregular Figure, encompassed with a cloudy Border of a colour less Dark. *Hevelius* says it sometimes happens, that after the gradual Decay and disappearing of the black Part, it's place seems brighter than the rest of the Sun, and continuing so for two or three Days, but Mr *Huygens* could not observe this. *Scheiner*, in his Book called *Rosa Ursina*, tells you he made 2000 Observations of solar Spots for 20 Years together, and sometimes saw above 50 at once; but betwixt the Years 1650, and 1670, he saw none; sometimes they have been seen with the naked Eye. I myself saw one Spot at *London* in the middle of *November*, some Years ago. There are many Observations of solar Spots to be found amongst Authors, such as *Galilæo*, *Scheiner*, *Hevelius*, *Huygens*, *Cassini*, and in the *Transactions* of the Royal Societies of *London*, *Paris*, *Berlin*, in my Opinion not worth particularizing. By means of these Spots it is found the Sun revolves about his Axis in about 25 or 26 Days. 2. *Galilæo*, with his little Telescope, first discovered the Phases of *Venus*; he first saw *Venus* perfectly round, neat, and distinctly terminated, but very small; after which her roundness decreased, 'till she appeared semi-circular, and then horned, less and less, 'till they became so thin as to vanish at her Occultation in the Beams of the Sun, imitating all the Phases of the Moon. Mr *de la Hire* says, that he never failed of seeing the Transits of *Venus* through the moveable Telescope belonging to the Mural Quadrant at the Royal Observatory at *Paris*, though within two Degrees of the Sun. He observed her Transit in *August* 1700, with a sixteen Foot Tube, which magnified her Diameter of one Minute about 90 Times, when she appeared in the Form of a fine slender Crescent, with her Horns in an horizontal Line, and her Back upwards; that in the interior Arch of the Crescent he saw some Inequalities more considerable than those of the New Moon, by which he judged she might have Spots upon her, like those of the other Planets. In *November* 1691, he saw her at Noon, very near her superior Conjunction with the Sun, appearing round and very small. Mr *Cassini* several Times observed two Spots on *Venus*, and in the Year 1666, on the third of *March* at *Bologna*, with a Telescope of 16 $\frac{1}{2}$ Feet he saw four Spots, and on *February* 24th he discovered two others larger, which were seen at *Rome* with a 35 Foot Telescope by Mr *Campani*. Mr *Blanchini*, in the Year 1726, at *Rome*, with *Campani*'s Glasses of 70 and 100 *Roman* Palms focal Distance, discovered several dark Spots on the Disk of *Venus*, from whence he concludes that a Revolution of *Venus*, about her Axis, was not finished in 23 Hours, 20 Minutes, as *Cassini* imagined, but in 24 $\frac{1}{3}$ Days.

The Phases of *Mercury* are exactly like those of *Venus*, or the Moon; but the Telescopes must be pretty good, and properly managed, to observe them well, by reason of the excessive Brightness, and short Digression from the Sun of this Planet. *Kepler* once took a large Spot in the Sun for *Mercury*; and *Gassendus* at *Paris* took *Mercury* in the Sun to be a Spot: on *October* 29, 1723, there was a Conjunction of *Mercury* and the Sun, when the Diameter of *Mercury* was then observed by a very good Micrometer, applied to the *Huygenian* Telescope of 123 Feet, to be 10″ $\frac{1}{4}$. On the 24th of *November* 1629, our *Horox* saw *Venus* in the Sun, by projecting the Sun's Image upon a white Paper in a dark Room, whereon she appeared to be a fine round dark Spot. The Particulars about the Phases of *Mercury* and *Venus*, are to be found in *Gassendus*'s *Mercurius in Sole visus*, & *Venus invisa*; and in *Hevelius*'s *Mercurius in Sole visus*. See the Treatise of this last, entituled, *Selenographia*. Also see the *Memoirs* of the Royal Academy of Sciences at *Paris* for the Year 1700, &c. There is nobody that ever could discover Spots in *Mercury*.

Dr *Halley* tells us, that *Mercury* in 46 Years, makes 191 Revolutions about the Sun; that he appeared the 28th of *October*, in the Year 1631, on the Sun's Disk; seen by *Gassendus* at *Paris* afterwards, on the 28th of *October* 1677; seen by himself at *St Helena*. He says also, that *Mercury* appeared in the Sun on the 23d of *April*, 1661. On the 26th of *April*, 1674. On the 24th of *April*, 1707. And on the last Day of *October* 1726. The Doctor also says, that on the 26th of *May* (O. S.), in the Year 1761, near Six o'Clock in the Morning at *London*, *Venus* will appear in the Sun's Disk, not above four Minutes South of its Centre; and observes, if proper Observations be made upon the then transit of *Venus*, with good Clocks and Telescopes, in different distant Parts of the World, the Sun's Parallax may be determined, and Distance from the Earth to the Exactness of the 100th Part of the whole, (see the *Philosophical Transactions*, Numb. 348.) whereas by the best Observations hitherto made, says he, we are not absolutely certain of those Quantities to less than about the 7th Part of them.

That we are not certain of these Quantities I always thought, and ever shall; the Sun's Parallax is too small a Matter to be obtained by Observation to any Exactness that can be depended upon. The Doctor himself owns it to be exceeding small, but says, upon many repeated trials Dr *Pound* and Dr *Bradley*, by a Micromoter apply'd to the Focus of a Telescope, found it to be not more than 12″, and less than 9″. Now, setting aside the Inaccuracy in the Structure, and Smallness of the Radius, *&c.* of the Instruments used for this Purpose, the Error in defect arising from the Refraction alone may be so great, as even to exceed 9″. If so, what becomes of this slender Parallax, and how immensely farther off must the Sun be distant?

Indeed Dr *Halley*, in the *Philosophical Transactions*, at Numb. 368. would have the Air's Refraction to be so little, that none but nice Instruments can perceive it's Effects; and that Sir *Isaac Newton* was the first who ascertained it, and made a true Table thereof, which is set down in that Transaction. But the Air's Refraction is so unconstant as not to be ascertained, and is always greater than many able Astronomers are willing to allow and suppose. Sir *Isaac Newton*'s Table makes the horizontal Refraction to be 33′. 45″. *viz.* fix'd, when nevertheless it is known to be very inconstant at different Times, and in different Places. The *Dutch* formerly wintering at *Nova Zembla*, I found there the horizontal Refraction to be 4 Degrees. Sir *Isaac*'s Table takes the Refraction at Altitudes above 75 Degrees to be so small as not to be worth Notice. But *De la Hire*'s Table *of Refractions* is extended to 89 Degrees of Altitude, making it there to be one second. And the famous Mr *Cassini* says, it extends quit up to the Zenith. Dr *Halley*, in the Transaction before mentioned, says, all Distances of Stars are contracted by Refraction, when they are parallel to the Horizon, by the same constant Quantity, be they high or low, *viz.* one second to a Degree; because (says he) the Chords of the real and visible Arches, are in the constant Ratio of the Sine of the Angle of Incidence to that of the refracted Angle. But herein, I think, lies a Fallacy, for he takes the refractive Power of the Air at all Times to be constant at equal Altitudes above the Horizon. Whereas this is known to be otherwise. For in Mr *Hawkesby*'s *Physico-Mechanical Experim.* pag. 175. it is shewn, that the refractive Power of the Air is proportional to the Air's Density, and since this Density is found to be variable, that refractive Power must by consequence be so too. Moreover, Sir *Isaac Newton*, in the tenth Proposition of his second *Book* of *Optics*, says, that the refractive Power of Air, and all Bodies, seems to be proportional to their Densities, or very nearly, excepting so far as they partake of sulphureous oily Particles, and thereby have their refractive Powers made greater or less; whence, says he, it seems rational to attribute the refractive Power of all Bodies chiefly, if not wholly, to the sulphureous Parts with which they abound. Consequently, as I said above, Dr *Halley*'s taking the Refraction of the Air at equal Altitudes to be always the same cannot be true, because the Air's Density at those Altitudes varies, as is found by the Barometer. Whence the denser the Air is, the greater is the Refraction at a given Altitude; and it will be still the more so, as it abounds with sulphureous oily Particles. Hence it should seem, that in dry Weather, when the Mercury in the Barometer is highest, and the Air is the fullest of such Particles, the Refraction will be the greater at a given Altitude. But whether the Air in Summer has not more such Particles in it than in the Winter, in the same parallel of Latitude; or whether in very cold frosty Weather there are not more than in warmer Weather, are Things that should be more inquired into than the Astronomers have hitherto done, in order to determine and ascertain the true Quantity of the Refraction of the Light coming from an observed celestial Object, at a given Time, Place, and Altitude.

But to observe a little farther on this Subject. Dr *Nettleton*, in the *Philosophical Transactions*, Numb. 388. says, in taking the Altitudes of some Hills, the Observations were so disturbed by the Refractions that he could come to no Certainty. Having measured the Height of one Hill in a clear Day, he found it, by repeating the Operation on a cloudy Day, when the Air was somewhat gross and hazy, to be different, and the small Angles to be so much augmented by the Refraction, as to make the Hill much higher than before, though they were carefully taken each Time by good Instruments. That pointing the Quadrant to the Tops of some Mountains, they would appear higher in the Morning before Sun-rising, and also late in the Evening, than at Noon in a clear Day. That one Morning in *December*, when the Vapours lay condensed in the Vallies, and the Air above was pure, the Top of a Mountain appeared more elevated by above 30′ than it did in *September* at Noon on a very clear Day. Hence, says he, Refraction is sometimes greater than at others, but, probably, it is always very considerable, and there being no certain Rule to make allowance for it, all Observations made on very high Hills, view'd at a Distance under very small Angles are uncertain, and scarcely to be depended upon, generally erring in making the Heights greater than they

they really are. Thus far Dr. *Nettleton.* Mr. *Huygens* long since, in his *Treatise of Light* observes, that the Refraction of the Air changes every Hour, though indeed his Experiments were made upon very small Altitudes on Objects at Land. And father *Laval* (as appears in the *French History* of the *Royal Academy*, for the Year 1710.) observed the Sun's meridian Altitude on the 22d of *June* (N. S.) 1710, to be 70°. 25′. 50″. And on the 23d of *June*, at the Distance of 36 Hours from the Time of the Solstice, he found the meridian Altitude to be 70°. 26′. 00″. greater by 10″, when it ought to be less. And having oftentimes observed such before, he suspected the Refraction varied according to the Difference of the Quarters from whence the Wind blew. *Tycho Brache* makes the Refractions of the Sun, Moon, and fixed Stars, to be different, and would have that of the Sun to end at 46 Degrees of Altitude, that of the Moon at 45, and that of the fixed Stars at 20 Degrees; he makes the Sun's horizontal Parallax to be 34′, the Moon's 33′, and the fix'd Stars 30′, which are all too little. But *de la Hire* and *Cassini* makes it to be 32′ in every Star. He makes that of the Sun at 33° to be 55″, and at 52°. to be 58″. See his Table in his *Progymnosm. Lib.* I. pag. 79, 124, 280.

Hence amidst so much Difference and Uncertainty about the Refraction of the Air, one would be almost led to be doubtful of the Truth of many astronomical Conclusions, entirely depending upon and deduced from Observations, which Doubts must last 'till the Nature and Quantity of the Air's Refraction at all Times and Places, are better known and settled. How can one believe the apparent Diameters of the Planets, which are so very small, are such as the Astronomers make them, when the true Refraction, at the Time and Place of Observation, is not well known; and how can any great Degree of Belief be fixed upon what they say, as to one of these very small apparent Diameter's being less at one Time of the Year than at another. Indeed, if the Observations were made near or under the Line, where the Refraction is by all allowed to be the least, especially of the meridian Altitudes, these would be the most to be depended upon, and accordingly so would the Consequences deduced from them. The late Dr *Halley* had too slight an Opinion of the Air's Influence upon the Light moving through it, just as he had of it's Influence upon the Motion of Projectiles, which he would have to move nearly in a Parabola, though now it is really found by Experience they do not. All this by the by. But to return from this seemingly out of the way Digression to the Telescopick Discoveries in the Moon.

It is discovered by the Telescope, that the Moon's Surface is rough, like our Earth, distinguished into innumerable Mountains, Caves, and Vallies; because at the new Moon the Line joining her Horns, and passing between the bright and dark Parts of the visible Hemisphere appears to be composed of many crooked Turnings and Windings. That many small bright Spots appear in the dark Portion, standing out at several small Distances from that common Boundary. And at some Time after they grow sensibly larger, and apparently nearer to, and at last unite with, the bright Portion of the Hemisphere, just as the Rays of the rising Sun shine first upon the Tops of our high Mountains, then descend gradually to their Bases, and at last into the Vallies. There are many small Spots observed interspersed all over the bright Portion of the Disk, some of which have their dark Sides next to the Sun, and their opposite Sides very bright and circular, which shews them to be round Cavities, whose Shadows fall within them; and some of these are surrounded with Ridges of Mountains. That those larger and less luminous Tracts, which are visible to the naked Eye, appear smoother in the Telescope, and more depressed than the ambient brighter Regions; as is evident by a greater Regularity and Evenness of the part of the Boundary of Light and Shade which passes through them at certain Times, and by it's Protuberances at both it's Extremities; and these Tracts are not quite free from smaller Inequalities, especially of Light and Shade.

Now these darker Regions may be compared to the Receptacles of our Seas emptied of their Water, for that they contain none appears from those permanent bright Spots observed in them by short Telescopes; and because larger Telescopes plainly not only discover small Eminencies, but Cavities within them, which seems quite repugnant to the Nature of Seas, and the Obscurity of their Colour may proceed from some sort of Matter that reflects less Light than the other Regions do. There may be Soils in the Moon very different from any we are acquainted with on the Earth.

The Magnitude of the Moon's Mountains is found to be much higher than any of the Earth's. When only the Tops of them are enlightened by the Sun, they appear as bright Specks in the dark Part of the Moon's Disk; and the apparent Distance of several such Specks from the Limit between the dark and bright Parts of the new Moon, has been found to be about the 20th Part of the Moon's apparent Diameter, and from thence it has been concluded, that the Height of those Mountains is above five Miles. *Ricciolus* reckons the Height of one of the Moon's Mountains called *St Catherine*, to be nine Miles.

It could never be discover'd by the best Telescopes, that there were any visible Changes in the Colours, Shapes, and Situation of the Moon's Spots; their Appearances being always the same, Allowances being made for their different Lights and Shades in different Ages of the Moon; for the different Goodnesses of Telescopes, and the Clearness of our Atmosphere; nor can it be fairly concluded, from any Observations hitherto made, that the Moon has an Atmosphere, although many Astronomers would make one believe it has. *Cassini*, in the *Memoirs*

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moirs of the *French Academy*, for the Year 1706, fays, he has often obferved the fpherical Figure of *Saturn*, *Jupiter*, and the fixed Stars, at their Occultations near the enlightened or obfcure Limb of the Moon, to be changed into an oval Figure; and at other Occultations he has difcovered no fuch Alteration in their Figure, juft as the Sun and Moon in a vapourous Horizon appear to be ovular at their rifing or fetting. *Hevelius*, in his *Cometographia*, Page 363, fays, that he has obferved feveral Times in the moft ferene Air, wherein he could difcover Stars of the fixth and feventh Magnitude, when the Moon had the fame Altitude and Diftance from the Earth, and with an excellent Telefcope, that the Moon and her Spots did not appear equally bright and diftinct, but at fometimes did fo much more than at others. Sometimes in a clear Air, when Stars of the fixth or feventh Magnitude were feen, the Moon has quite difappeared, fo as not to be feen by the beft Telefcopes; fuch a thing was feen by *Kepler* in the Years 1580, 1583, and 1620; as alfo by *Hevelius* and by *Ricciolus*, the 14th of *April* 1642. See *Kepler*'s *Optical Aftronomy*, Pag. 227, 297. his *Epitom. Aftron. Copernican.* Lib. v. Pag. 825. *Hevelius*'s *Selenogr.* Pag. 117, and *Ricciolus*'s *Almag. Nov.* Lib. iv. Cap. 6. Pag. 203.

It has been difcovered by the Telefcope, that the Hemifphere of the Moon vifible to us, is not at all Times quite the fame; at one full Moon we fee a fmall Gore, or Segment, in the Margin of her Difk that was quite hid at another; fo that her Body appears to us as if it librated to and fro; fometimes eaftward or weftward, fometimes northward or fouthward, and fometimes in a Direction between both. See concerning this in *Hevelius*'s Book *de Motu Lunæ Libratorio.*

Langrenus and *Hevelius* were the firft who publifhed Maps of the Moon's Spots, as they appear through a Telefcope, with Names of the moft remarkable Spots and Regions, as well for the more immediate aftronomical Purpofes, as for Pofterity to obferve what Changes may happen in them.

Now we are upon the Moon, I believe the following fhort Digreffion, concerning her Light, will not be difagreeable. Dr *Smith*, in the firft Volume of his Opticks, Pag. 29, 30, fays Day-Light is about 90000 Times greater than Moon-Light. And in his *Remarks*, Vol. II. pag. 16. tells us, one Mr *Bouguer*, in a Treatife of his entituled, *Effai dioptrique fur la gradation de la Lumiere*, finds the Light of the full Moon to be about 300000 Times weaker than that of the Sun, at a Medium of feveral Trials: and the Doctor fays, juft after that, he himfelf found it by Theory, not much above 90000 Times: the Difference may arife chiefly (continues he) from the lofs of Light in the Moon's Body, which could not be allowed for in the Theory. But, I neither take the Sun's Light to exceed that of the Full-Moon near fo much, as either of thefe Gentlemen here make it, nor the Experiments and Theory upon which they have proceeded, and deduced thefe their Conclufions, to be depended upon, and free from Error: for 1. the Doctor's Proof is upon the Suppofition that all Day-Light is much the fame, *i. e.* equal, he fays it, at Pag. 30, Vol. I. which is moft evidently not fo; for the Day-Light of a cloudy Day in Winter is moft certainly many Times lefs than that of a clear Day in the Summer. 2. The Doctor would have Moon-Light to be no greater than the Light of a white Cloud, which I take to be another Miftake. 3dly, Mr *Bouguer* fuppofes two Lights to be equal, when he thinks he fees them fo, by looking at them; which at beft is but a fort of guefs-work, liable to great Uncertainty: the feeing minute Objects equally diftant from two Lights with the fame Diftinction, would be better than this, though not certain neither. 4thly, That Light decreafes as the Square of the Diftance, I am doubtful of, and have been fo many Years; there is no Proof of it by actual Experiments as I know: indeed, it has been long made out by Theory to be fo; but the practical Proof of thefe things is beft, and moft to be relied upon; and I have often thought, that Light in fome cafes, as well as Heat, may decreafe, rather as the Cubes of the Diftances, than as the Squares; be this as it will, I am fure neither the Doctor nor Mr *Bouguer* have made out their Affertions to my Satisfaction. I could fay a great deal more about this, but at prefent have not Inclination. If any body has a Mind to defend thefe Gentlemen in this Point, I fhall then be urged on to proceed to farther Particulars. But to return to the

Telefcopick Difcoveries.

The Diameter and Parallax of *Mars* are faid to be above five Times greater in his Oppofition to the Sun than in his Conjunction. In both thefe Pofitions his enlightened Hemifphere is fully expofed to the Earth, as well as the Sun: but not fo fully in his Quadratures with the Sun, where he appears through a Telefcope to be a little gibbous, like the Moon about three Days from the Full. Dr *Hook* and Mr *Caffini* firft difcovered Spots upon *Mars* of determinate Figures, by which it has been found that he revolves about his Axis in twenty-four Hours and forty Minutes. One of thefe Spots appeared like a Joyner's Square, or obtufe-angular Bevel; thefe Spots change their Colours, Shapes, Situations, and vanifh at different Times. Mr *Miraldi* fays, that one of the bright Spots upon *Mars* appeared for near 60 Years, and that it was the only parmanent Spot upon his Body. Mr *Caffini* fays, he obferved a fixed Star in the Water of *Aquarius*, which at the Diftance of fix Minutes from the Difk of *Mars*, became fo faint before it's Occultation, that it could not be feen

feen with the naked Eye, nor with a three Foot Telefcope. The like Diminution of it's Light after it's Occultation, was obferved by Mr *Reaumeur* at *Paris*, who could not fee that Star with a large Telefcope in a very clear Air, 'till it's Diftance from *Mars* became equal to $\frac{2}{3}$ of it's Diameter, and yet Stars of that Magnitude are plainly vifible, even in Contact with the Moon: by a Comparifon of feveral Obfervations then made upon that Star, it was judged that it varied it's Diftance from the neighbouring Stars; and by fome it has been therefore concluded, that *Mars* has a very extenfive Atmofphere. See more particularly about thefe Spots in *du Hamel*'s *Hiftory of the* French *Royal Society*, pag. 97, 107. Edit. 2.

In the *Philofophical Tranfactions*, at Numb. 128, Dr *Halley* fhews how to find from three given Diftances of *Mars* from the Sun, his mean Diftance and Excentricity, which in effect amounts to the Conftruction of a Geometrical Problem, *viz.* one of the Foci of an Ellipfis, paffing through three given Points being given: to defcribe fuch an Ellipfis, thence find the other Focus, as alfo the two Axes. Now the Doctor's Conftruction is by the Interfection of two Hyperboles, which will be in the other Focus required; but this is only a plain Problem, conftructable by right Lines and Circles, without the actual Defcription of two Hyperbolas, although, indeed, the doing it by two Hyperbolas is the moft fhort and natural way poffible; the plain Problem in effect is, to find the Centre of a Circle (which will be the other Focus), that fhall touch three given Circles: and a more elegant Computation of this than what the Doctor has given in that *Tranfaction*, may be obtained from *Lemma* 16. Sect. 4. Lib. i. *Princip. Mathem. Philof. Naturalis* of Sir *Ifaac Newton*.

By the Telefcope it is difcovered, that on *Jupiter* there are two Belts lighter than the reft of his Difk terminated by parallel Lines, which are fometimes broader, fometimes narrower, and not always fituated on the fame Part of the Difk. And Mr *Huygens* fays (*In his Syftem of* Saturn, pag. 7.), that he faw in *March*, in the Year 1656, a much broader though obfcurer Belt, in the Middle of the Difk of *Mars*.

About the End of *November*, in the Year 1609, *Simon Marius* (fee his Preface *Ad Mundum Jovialem)*, firft Mathematician to the Elector of *Brandenburgh*, firft difcovered three Stars moving about *Jupiter*, and attending him; and in *February* 1610, he faw a fourth: and in *Italy*, *Galileo* faw the fame Stars the 7th of *January*, in the Year 1610, and publifhed his Obfervations upon them in a Treatife, entituled, *Nuncius Sidereus*. Thefe four Stars, called Satellites, moving about *Jupiter*, are at different Diftances from him, and the periodical Times of the Motions of the firft, is one Day and about 18 $\frac{1}{2}$ Hours; of the fecond, three Days, and about 13 Hours; of the third, feven Days, and almoft 14 Hours; and of the fourth, fixteen Days, and about 16 $\frac{1}{2}$ Hours. Mr *Caffini*, and others, difcovered Changes in *Jupiter*'s Belts. Spots upon the Satellites. *Jupiter*'s Figure to be fpheroidical. The Ratio of his Axis to his equatorial Diameter being according to *Caffini* as 14 is to 15; and according to the late Dr *Pound*, as 12 is to 13. Tranfits of the Body and Shadows of the Satellites, in Form of black Spots over the Difk of *Jupiter*. There are a great many Particulars about thefe Spots, to be found in aftronomical Writings, efpecially in the *Tranfactions* of the *French* Royal Society, fcarcely worth mentioning here, as I think.

Tables of the Motions of *Jupiter*'s Satellites were firft publifhed by Mr *Caffini*, then Mr *Flamftead*, and by Dr *Halley*, and afterwards by Dr *Pound*. There are others too who have given Tables of them. But thefe may be looked upon as little more than bare Tranfcribers. Dr *Pound* particularly has endeavoured to rectify the Motion of the firft Satellite next to *Jupiter*, and to make the Calculation eafy of it's Eclipfes in the Shadow of *Jupiter*, thirteen of which happen in a Month, thereby affording very frequent Opportunities of determining the Longitude of Places, efpecially at Land, by Time-keepers, and Telefcopes of proper Lengths, *viz.* thofe of 12, 16, or 20 Feet at leaft; it being found, that with fhorter Telefcopes the Times of the Emerfions cannot be fo well determined as with longer.

This Way of finding the Longitude of Places at Land is practicable, and fufficiently exact, but at Sea it has been found to be ineffectual, by reafon of the Length of the Telefcope, which cannot be managed with fufficient Steadinefs and Dexterity in a Ship, during it's various Agitations by the Winds and Waters. The Way of finding the Longitude of Places, by the Obfervations of the Times of the Eclipfes of the Satellites of *Jupiter* is certainly ingenious, but liable to fome Difadvantages; for were the Telefcope, by which *Jupiter* and his Satellites is obferved, never fo good, and well managed, there are none but Aftronomers, and thofe who have been a good while accuftomed to celeftial Obfervations that could do this Bufinefs without being liable to miftakes. Though almoft every Body knows *Jupiter*, and can diftinguifh him from the reft of the Stars near him, a Perfon not ufed to Obfervation may take one Satellite for another, or fixed Stars for Satellites, as did *Antonius Maria Schyrleus de Rheitá*, a *Capuchin* of *Cologne*, who thought he had difcovered five more Satellites moving about *Jupiter* on the 29th Day of *December*, 1642, which for the Honour of *Pope Urban* the VIIIth he call'd by the Name of *Sidera Urbana Octaviana*. See *Gaffendus*'s Letter to *Gabr. Naudæum de novem Stellis circa Jovem vifis*, which turn'd out to be only five fix'd Stars. Befides all this, the Influence that the Satellites have upon each other are obferved to difturb their Motion in fome meafure; and that of the firft is faid to be liable to Inequalities that will caufe

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Effects not easily reduced to any Rule, but from a long Series of Observations. See the *Philosophical Transactions*, Numb. 394. Also in these Transactions, are to be seen Observations of Eclipses of the Satellites of *Jupiter*, in many Parts of the World.

One Mr *Reaumeur*, a *foreign Astronomer*, many Years ago, from the Difference between the calculated Time of one of these Eclipses, and the observed Time, will have it that Light is about ten Minutes in its Passage from *Jupiter* to us. Sir *Isaac Newton* and his Followers will have it to be in but seven or eight Minutes. Others have said, the whole is doubtful. See the *Memoirs* of the *French* Royal Society for the Years 1707, and 1729. of which last Number I myself am one. For I think no Man ever knew enough of the Nature and Motion of Light, to be certain how it really moves and acts. Dr *Barrow* was a very great Man, as well as Sir *Isaac Newton*. He says in his *Opticks*, that the Arguments for and against the direct Propagation of projected Light, and it's Propagation by the small Impulses of some elastick fluid Medium were so equal, that he durst not venture to assert which was right; they might be partly both, says the *Doctor*.

I have lately thought, that the Sun's Light and Heat too is not only greatest where it is most reflected and refracted, but in the vast Space between the Sun and the Planets, where there is little or no reflective or refractive Matter, the Sun's Light and Heat is least, whether the Distances be nearer or farther from the Sun. That his Heat is most in Vallies, and low sandy Situations, is well known by Experience; and that on the Tops of many Mountains, even near the Line there is Snow to be found at all Times of the Year. (See *Varenius*'s Geography.) And I take many of our upper Clouds to be chiefly Snow or Hail, both Summer and Winter. Hence, at a small Distance from the Earth's Surface the Sun's Heat must needs be considerably lesser than at the Surface itself, and at greater Distances upwards towards the Sun, it is not unreasonable to suppose the Sun's Heat to be still less, and that it may decrease to certain Limits, either towards the Sun or from it. Consequently, if this be the Case of the Sun's Heat, I should think it must be so of his Light too, which is greatest at the Surface of the Planets, by reason of the Refraction of their Atmospheres (supposing them to have such) and the Reflections at their Surfaces, *&c.* and therefore from these, and many other Considerations, I am doubtful whether the Sun's Light at the several Planets, be the greatest, the nearer the Planet is to the Sun; and whether, if a Person were a thousand Miles, for Instance, above the Earth, towards the Sun, the Light that he would perceive there would not be very weak and faint. These foreign, short, and rude Suppositions, may perhaps serve as a Bar in the Way to hinder the taking Conjectures for Certainty, and making Conclusions founded upon the imaginary Nature of Light, which no Body truly knows (nor perhaps never will) be taken as indubitable Truths. Sir *Isaac Newton*'s Proposition, about the Time of the Motion of Light from the Sun to the Earth is very short. It is *Prob.* XI. of *Part* 3. *Book* 2. of his *Opticks*. He says, when the Earth is between the Sun and *Jupiter*, the Eclipses of the Satellites happen above seven or eight Minutes sooner than they ought to do by the Tables; and when the Earth is beyond the Sun, they happen about seven or eight Minutes later than they ought to do, the Reason being, that the Light of the Satellites has farther to go in the latter Case than in the former, by the Diameter of the Earth's Orbit, *&c.*

In the *Philosophical Transactions*, Numb. 406. there is a curious Account of Dr *Bradley*'s to prove, that Light is eight Minutes coming from *Jupiter* to us, chiefly from Observations of the Declinations of the fixed Stars being different some Times of the Year, from what they are at other Times, by some Seconds, though never exceeding 40″ or 41″. The Particulars of his Observations are there set down for two Years, *viz.* 1727, and 1728. Upon the whole of which I shall briefly observe, 1. The taking Angles to Seconds are very nice Things, oftentimes subject to Uncertainty, especially by reason of the Difference of the Refraction in the Air at the Time of the Observations, not yet settled as it should be, by any Rules or Tables that ever yet has appeared in publick. 2dly. There is besides another Doubt with me, *viz.* whether upon the increasing or lessening the Magnitude of the Pupil of the Eye, the Magnitude of the Optick-Angle will not be accordingly increased or decreased. And if this be the Case, since the Pupil increases as the Light falling upon it decreases, whether small Distances and apparent Diameters of Objects in the Heavens, though these Objects be equally distant from the Eye, will not at one Time appear more or less by some Seconds, than at another Time, *viz.* generally less in the Summer, when the Light is greatest; and greater in the Winter, when the Light is least. And whether the apparent Diameter of the same distant Object, at the same Distance, will not appear greater, if by any Cause it becomes less bright, or more obscure, than when it is more shining or white. 3dly. It seems to require several Years Observations, at several Places, by several People, to fix the Truth of this ingenious Gentleman's great Conclusions beyond Dispute. He is certainly a fine Observer, and great Astronomer; but the Love of Truth, and that only, has made me advance these Doubts, not out of any Ill-nature, or Disrespect, or with any View of acquiring the least Praise thereby. If I am but right, that is all I care for. And no Man can justly be blamed for reasonable Suspicions in Science especially, which should always be clothed in Truth.

There

There is another ingenious Discourse of the Doctor's in the *Philosophical Transactions*, Numb. 485, for the Year 1747, on the apparent Motion of some of the fixed Stars.

Saturn has the most extraordinary Appearance through a Telescope, of any of the rest of the Planets. *Galileo*, with a bad Telescope, took him for three Globes, a larger between two smaller, almost contiguous to one another. Sometimes the middle Globe was left quite alone. Then they seemed to stick to the middle Globe; to put on various Shapes, appearing sometimes round, sometimes oblong, like Acorns, sometimes semicircular, then lunar, with Horns pointing towards the Globe in the middle, and growing by degree so long, and so wide, as to encompass it with an oval Ring. These monstrous Appearances puzzled the Astronomers at first to account for them. *Hevelius*, in his little Tract, *De nativa facie Saturni*, reckons *Saturn*, 1. To be Mono-spherical. 2. Tri-spherical. 3. Two-handled-spherical. 4. Elliptical-handled. 5. Spherical-horned. And these Phases he has again sub-distinguished. But Mr *Huygens*, in his *System of Saturn*, evidently made it appear, that these wonderful imperfect Appearances were owing to the want of good Telescopes. For with better ones he discovered a Ring about *Saturn*, which was the true Cause of all those surprizing Phænomena; and besides that, a Satellite revolving round *Saturn* every fixteen Days, or thereabouts, in the Plane of the Ring produced. This Satellite was first discovered by him in *March*, 1655, by Telescopes of 12 and 23 Feet; and afterwards *Cassini*, at different Times, found out four more Satellites, *viz.* the two nearest to *Saturn*, with the Telescopes of *Campani* of 100 and 136 Feet, in *March*, 1684. The third in *December*, 1672. with one of *Campani*'s Telescopes of 35 Feet; and the fifth (*viz.* that of *Huygens*'s) in *October*, 1671. by a Telescope of 17 Feet. Afterwards he saw the two inward ones, with Telescopes of *Campani* of 47 and 34 Feet; and by Telescopes of *Borelli* of 40 and 70 Feet; and at length, by Telescopes of *Artouquellius*'s of 80, 155, and 220 Feet. See the *Philosophical Transactions*, Numb. 92, and 181. Mr *Huygens*, with a 12 Feet Tube, saw *Saturn* with two Arms oppositely extended from his Globe, and a darkish Zone or Line passing straight over the Disk, between the upper Sides of the Arms was visible. He saw *Saturn* quite round sometimes, and that Zone was parallel to our Equator. He makes the Space between the Globe and the Ring to be equal to, or rather bigger than, the Breadth of the Ring, and the greatest Diameter of the Ring is to the Diameter of the Globe, as nine is to four, but Dr *Pound* makes it to be as seven is to three. That there is empty Space between the Ring and the Body is evident, from seeing the fixed Stars sometimes through it. The broad side of the Ring has sometimes appeared bisected quite round by a dark elliptick Line, divided as it were into two Rings, of which the inner one appeared brighter than the outer. The Ring continues invisible all the Time that it's Plane produced passes between the Sun and the Earth. Mr *Cassini* says, he could see all the five Satellites of *Saturn* with a 34 Feet Tube; whereas Mr *Hadley* owns, he never could have the Fortune to see them all five with his Reflecting Telescope. The periodical Times of the Satellites are of the first, one Day and about 21 Hours; of the 2d two Days and about 17 Hours; of the 3d four Days and about 12 Hours; of the 4th fifteen Days and about 22 Hours; and of the 5th seventy-nine Days and about 7 Hours. There are many other curious particular Phænomena of *Saturn* and his Satellites, observed by *Huygens*, *Cassini*, *Muraldi*, Dr *Pound*, &c. with the Telescope. See the Second Vol. of Dr *Smith*'s *Opticks*, from page 438, to 447. and our *Philosophical Transactions*, as well as those of the *French*. I shall only add here, that it might somewhat strengthen the Truth of the Assertion already mentioned, *viz.* that Light is about seven or eight Minutes in it's Passage from *Jupiter*'s Satellites to us. If from the like Observations of the Eclipses of *Saturn*'s Satellites, the Difference of the observed Times, from the Times by the Tables, were found to be proportional to that seven or eight Minutes, as *Saturn*'s mean Distance is to *Jupiter*'s. But this I leave to Astronomers, having neither Inclination nor Abilities to attempt it myself. It may, perhaps, be impossible, though indeed, at present, I do not see that it is.

The best Telescopes do not at all magnify the apparent Diameters of the fixed Stars, nor can it be discovered that the fixed Stars have any sensible Parallax. Mr *Cassini*, in the *French* Memoirs for the Year 1717, says, that he saw *Sirius* under an Angle of 5″ or 6″, with a 36 Foot Telescope, and then with a 3 Foot Telescope. But Dr *Halley*, in the *Philosophical Transactions*, Numb. 369, has shewn the Fallacy of this his Assertion. The Radius of *Cassini*'s Instrument of 3 Feet being too small for such a nice Observation, and the Difference of the Refractions of the Air, which may be 7″ or 8″, being greater than this whole Parallax, 5″ or 6″ supposed to be observed.

The best Telescopes have discovered nothing extraordinary in the fixed Stars; you only see vast Multitudes of them hid from the naked Eye, excepting indeed, that Mr *Huygens*, in his *System of Saturn*, tells us, he saw three Stars in the middle of Orion's Sword (marked Θ by *Bayer*) together with four more, shine out, as it were, through a whitish Cloud, much brighter than the ambient Sky, which being very black and serene, caused the lucid Parts to appear like an Aperture that gave a Prospect into a bright Region. He found it to continue in the same Place, and not altered in Shape. See another of these in our *Philosophical Transactions*, Numb. 347.

CHAP.

CHAP. VI.

Of Dr James Gregory's *Reflecting Telescope, as improved by Mr* Hadley.

Plate 3. Fig. 13. AT the Bottom of a large brafs Tube, Fig. 13. (expreffed by a ftrong black Line) is fix'd a concave metaline Speculum B B, with a round Hole C C in the middle thereof, oppofite to which Hole is placed a fmall Speculum of the fame Metal F F, concave towards the great Metal, and fo fix'd to a crooked Arm, that it may be brought towards or carried from the great Speculum, keeping it's Axis ftill in the fame Line (*viz.* the common Axis of both the Speculums); and by that means parallel Rays, or Rays from the Points of a very diftant Object, coming to the great Speculum in the Lines O O, P P, *&c.* and falling upon the great Speculum between B and C will be fo united at it's Focus, as to form there the Image G G of the Object O P, fuppofed to be at a vaft Diftance. The Rays diverging again from their refpective Points of the Images, go on diverging, and fall upon the little Concave F F, whofe focal Length is D I, and from it's Surface are reflected nearly parallel to their refpective Axes (not quite fo, becaufe D G is greater than D I) and with all the Axes, or principal Rays, move parallel to the convex Axis through the Hole in the great Speculum, in the Direction D A; fo entering into the fmall Tube N M M N, which is fixed to the great Tube behind the Speculum, fall upon the plano-convex Eye-Lens N N, and paffing through it form a fecond Image at *g g*, whofe Bignefs is limited by the Hole of a perforated opake Circle placed at R R. That fecond or erect Image of the firft inverted Image of an erect Object, is feen large by the Eye at E, which fees it through the fmall Hole in the Plate M M, and the laft Eye-Glafs S S, which is a *Menifcus.* For the Eye will fee it under the Angle S E S, made by the Axes of thofe Pencils of Rays which came from the Extremities of the vifible Object; and the Rays belonging to each Pencil will be parallel to their refpective Axes, and the fpurious Rays will be all cut off by the Plate M M, which makes the Vifion diftinct. This Telefcope is not only good for common Eyes, but the Rays that enter the Eye will be made to converge a little for old Eyes, or to diverge a little for fhort-fighted Eyes, by means of a Screw fixed to the Arm of the little Concave to remove, or to bring it forwards upon Occafion.

Fig. 14. The Figure of this Telefcope, upon a Support, is feen at Fig. 14. This Support is contrived thus, the Bafe of the Pedeftal *a b* is a thick Board refting upon four brafs Feet underneath it, one of which being a Pin *p*, that fcrews through the Board, will make it fteady upon any uneven Plane. *b* is a fmall upright Pillar about a Foot long, fixed in the Board *a*, and *c d* is a brafs Arm that fcrews into it. *d e* is a fhort brafs Piece, that turns round upon the End of the Arm *c d*, and is tightened and ftayed by the Screw *d*. *c* is a hollow Socket, having a round brafs Ball in it, moveable any way, and tightened and ftayed by a Screw or two; the Neck of this Ball is fixed to the middle of a long brafs Piece *f g*, which is fixed along the Side of the Tube *h i*, by the Screws *f g*, and may be taken off at pleafure. Thus the Tube is fupported, and made capable of being gradually moved, and ftayed in any Pofition. The larger concave Metal having a Hole in the middle of it, is lodged in the Bottom of the larger Tube *h i k*, and the fmaller concave Metal is held in the Axis of the Tube, near the Mouth of it, by a fmall brafs Arm coming through a Slit in the Tube at *h*. The long iron Wire *h i k* on the outfide of the Tube fkrews through a Hole in the Arm at *h*, and is confined from moving Length-ways, by two Shoulders on each fide of a fmall Hole in a fixed Plate at *i*; and being turned round itfelf by the Knob at *k*, it draws the Arm and little Concave backward or forward, in order to procure diftinct Vifion of Objects, at various Diftances, and for Eyes of different Sorts; while the Obferver is looking in at the End *l* of a fhort flender Tube that is fkrewed into the End of the larger, and carries the Eye-Glaffes. When this Telefcope is ufed at home, the Pedeftal *a b* may be placed upon a Table near a Window, or upon a Window board; but when it is ufed abroad, the Pedeftal may be left at home. For having tapped a Hole in the Side of a Tree, or any piece of Wood, with the Hand-Augre *m*, the wooden Screw at the End of the Arm *c d*, may be prefently fcrewed into it; the Augre *m* being put through the Hole *c*, to give Power to the Hand in turning the Screw.

This reflecting Telefcope made about 16 Inches long, is equivalent to a common dioptrick Telefcope of above 15 or 16 Feet long; that is, magnifies as much, and is faid to fhew Objects as diftinctly too; but of this laft I am not fo certain. And the Proportion of the magnifying Power of any one of them, will be as the Square of the focal Diftance of the larger Concave is to the Rectangle under the focal Diftance of the leffer Concave and the Eye-Glaffes.

In Dr *Defagulier*'s *Appendix* to the Second Edition of Dr *David Gregory's Elements* of *Catopricks* and *Dioptricks*, there is not only a more particular Defcription of the feveral Parts of

this

this Reflecting Telefcope, for the fake of thofe who have a Mind to try at making it, but alfo two Tables for their Conftruction of other Dimenfions, which the Doctor received from Mr *Hadley*, which being curious and ufeful, and the Book fcarce, I fhall here fubjoin,

The following Dimenfions in Fig. 15. are each a fourth Part of a Telefcope of 12 Inches focal Length. B B is a large concave Speculum, and A G is it's focal Length. F F is a fmaller concave Speculum, it's focal Length is I D. The Breadth F F is about $\frac{1}{50}$ of an Inch wider than the Hole C C in the larger Speculum. N, the firft Eye-Glafs, is Plano-Convex. S, the fecond Eye-Glafs, is Plano-Convex, or rather a *Menifcus*. M is a Plate with a fmall Hole in it to exclude all foreign Light, $\frac{1}{18}$ of an Inch. R R is the limiting Circle or *Diaphragm*, as it is fometimes called. The Arrows are the fucceffive Images of any Object. Fig. 15.

TABLE I.

If A G be 3 Inches, or 12 Inches, or 18 Inches, or 27 Inches; and the Charge or Power of magnifying is 12 $\frac{2}{3}$, or 13; or 36, 49, 66, all correfpondently taken. Then will B B be 0.7, or 2, or 2.7, or 3.7. I D, 0.82, or 2.32, or 3.22, or 4.22. F F, 0.315, or 0.56, or 0.7, or 0.88. C C, 0.295, or 0.54, or 0.68, or 0.86. The focal Diftance of N, 1.48, or 3.27, or 3.97, or 4.91. The focal Length of S, 0.7, or 1.3, or 1.54, or 1.85. A D, 3.96, or 14.66, or 21.69, or 31.9. A N, 0.5, or 0.7, or 0.75, or 0.8. N S, 1.4, or 2.6, or 3.08, or 3.7. S M, 0.45, or 0.76, or 0.88, or 1.0. R R, 0.2, or 0.37, or 0.44, or 0.53.

Note, Thefe Expreffions of Inches and Decimal Parts are for the Day, where Objects are to be magnified but a little, in proportion to what the heavenly Bodies may be at Night; for which the following Table gives the Proportions.

TABLE II.

If A G be either 12 Inches, or 18 Inches, or 27 Inches; and the Charge be 70, or 95, or 128 Inches. Then muft B D be 2, or 2.7, or 3.75. I D, 1.74, or 2.36, or 3.22. F F, 0.4, or 0.47, or 0.56. C C, 0.38, or 0.45, or 0.54. N, 2.29, or 2.79, or 3.47. The Focus of S, 0.87, or 1.03, or 1.25. A D, 13.95, or 20.64, or 30.56. A N, 0.7, or 0.75, or 0.8. N S, 1.74, or 2.06, or 2.5. S M, 0.47, or 0.56, or 0.7. R R, 0.25, or 0.29, or 0.36.

Note, The Breadth in the Hole of the Plate M muft be $\frac{1}{35}$ of an Inch. And the varying the Length of A N, the Diftance of the firft Eye-Glafs behind the fore Surface of the Speculum B B, alters the other Proportions but a little; fo that if the Thicknefs of the Speculum, or other Circumftances require it, there is no need to keep exactly to the Numbers here fet down for it.

The Proportions of the feveral Parts of Gregory's *Reflecting Telefcope as computed by Mr* Hadley, *Algebraically expreffed; with numerical Examples, when the focal Length is either* 40 *Inches*, 60 *Inches*, *or* 90 *Inches.*

Let (Fig. 16.) A D reprefent the common Axis of the Telefcope, and two concave Speculums B B, F F; fuppofe A G the focal Length of the Speculum B B, whofe proper Aperture B B, and Charge, are likewife known. Let C C be the Breadth of the Perforation. F F the Breadth of the fmaller Speculum, equal to, or a little greater than C C. I it's Focus. N the Eye-Glafs. N A it's focal Length, and M a Plate with a fmall Hole to exclude all foreign Light; and let it be required to take in at one View fo much of the Object as may appear through the Telefcope, under a given Angle, *viz.* C N C. To do this with the Lofs of the feweft Rays of Light near the Axis; the Proportions fhould be as follow. Fig. 16.

Call A G, a; B B, b, the Power or Charge m; the Ratio of twice the Semi-tangent of the apparent Angle of Comprehenfion required C N C to the Radius; that is, $\frac{CC}{AN} = n$. Then H H, the Breadth of the Image of fo much of the Object as is feen at once, will be $= \frac{n}{m} a$.

Note, If inftead of $\frac{n}{m} a$, you fubftitute c for H H, the algebraick Expreffions become fomething more fimple; the Breadth of the Perforation C C, and the great Speculum $= \frac{na + \sqrt{na} \times \sqrt{na + mb}}{m}$, or $c + \sqrt{bc + cc}$. the focal Length of the fmall Concave

I D $\frac{a \times \sqrt{\sqrt{na} \times \sqrt{na} + \sqrt{na + mb}},}{\overline{na + mb} + 2\sqrt{na} \times \overline{na + mb},}$ or $\frac{a \times \overline{c + \sqrt{bc + cc}}}{b + c + 2\sqrt{bc + cc}}$.

 The

The Diſtances of the Specula, *viz.* $AD = a + \frac{a\sqrt{na}}{\sqrt{na} + mb}$, or $\frac{a + a\sqrt{c}}{\sqrt{b} + c}$.

The focal Length of the Eye-Glaſs, and it's Diſtance from A, that is $AN = \frac{a}{m} + \frac{\sqrt{naa + mab}}{m\sqrt{n}}$, or $\frac{c + \sqrt{bc + cc}}{n}$.

The Diſtance of the Plate M behind the Eye-Glaſs $NM = \frac{DN \times AN}{DA}$.

The Breadth of the Hole in $M = \frac{NM \times CC}{DN}$.

If a double Eye-Glaſs be uſed with this Teleſcope to prevent the Objects being coloured near the Edges of the Area, the Image of the Object muſt be thrown back by the ſmaller Concave, ſo far behind the great Speculum, that there may be Room enough to place the firſt Eye-Glaſs N at a ſufficient Diſtance before it, and then the Algebraick Expreſſions of the ſeveral Parts become much more complex, and ſo are omitted; and I have added the following numerical Proportions for the following Sizes.

For the Night.

If A G be either 40 Inches, or 60 Inches, or 90 Inches, and the anſwerable Powers be either 172, or 234, or 317, then will

the Focal length	be	or	or
B B	4. 9.	6. $\frac{3}{7}$.	9.
I D	4. 28.	5. 88.	8. 01.
F F	0. 67.	0. 81.	0. 97.
C C	0. 66.	0. 80.	0. 96.
N	4. 23.	5. 15.	6. 29.
S	1. 52.	1. 82.	2. 2.
A D	44. 72.	66. 4.	98. 68.
A N	0. 9.	1. 1.	1. $\frac{1}{8}$.
N S	3. 04.	3. 64.	4. 4.
S M	0. 8.	0. 93.	1. 13.
R R	0. 43.	0. 52.	0. 63.

For the Day.

If A G be 40 Inches, and the Power 86, then will

	be
B B	4. 9.
I D	5. 95.
F F	1. 0.
C C	0. 99.
N	6. 02.
S	2. 22.

and

A D	6. 74.
A N	0. 9.
N S	4. 44.
S M	1. 15.
R R	0. 63.

In the Remarks of Dr *Smith*'s Opticks, *pag.* 104, 105. are to be found two Tables of the Dimenſions, and magnifying Powers of ſome of the *Gregorean* Reflecting Teleſcopes, calculated by the *Doctor*, and founded upon the following Dimenſions of one of Mr *Short*'s beſt Reflecting Teleſcopes of this kind, *viz.*

	Inches.
The focal Diſtance of the larger Speculum — — —	9.6.
It's Breadth or Aperture — — — — — —	2.3.
Focal Diſtance of the leſſer Speculum — — — —	1.5.
It's Breadth — — — — — — — —	0.6.
Breadth of the Hole in the larger Speculum — — —	0.5.
Diſtance between the leſſer Speculum and the next Eye-Glaſs —	14.2.
Diſtance between the two Eye-Glaſſes — — — —	2.4.
Focal Diſtance of the Eye-Glaſs next the Metals — — —	3.8.
Focal Diſtance of the Eye-Glaſs next the Eye — — —	1.1.

The firſt of which is as follows:

TABLE

TABLE I.

CT	C x	T c.	ct	CA	ca=CB	xl	Mag. Power	r
5.65.	2.987.	1.131.	1.106.	0.773	0.155	1.223.	39.69.	8.5509
9.60.	4.923.	1.653	1.5.	0.15	0.198	1.565	60.	9.7840
15.50	7.948.	2.343	2.148.	1.652	0.250	1.973	86.46	11.0090
36.	4.	3.724	3.432.	3.132	0.324	2.561	165.02	11.7408
60.	6.	5.391.	5.012.	4.605	0.414	3.271	242.94	13.2426

Note. In the firſt Table, C T at the Head of the firſt Column is the focal Diſtance of the great Speculum. C x is the Diſtance of the ſecond Image of the Object from the great Speculum. T c is the Diſtance of the leſſer Speculum from the Focus of the greater. $c\,t$ is the focal Diſtance of the leſſer Speculum. C A half the Breadth of the greater Speculum, *viz.* the Chord of half it's Arch. $c\,a$ or C B the Breadth of half the leſſer Speculum, which is equal to half the Breadth of the Hole in the greater Speculum. $x\,l$ is the focal Diſtance of a ſingle Eye-Glaſs, that will magnify as much as the two Eye-Glaſſes, ſuppoſe m, n, of the Teleſcope. r is the Ratio of the Diſtance of the Place of the ſecond Image from the leſſer Speculum to T c.

The other Tables, with two more for Mr *Caſſegrain*'s Reflecting Teleſcope, I ſhall not inſert here.

If ſeveral Teleſcopes of the ſame Kind have nearly the ſame Length, or the ſame magnifying Powers, though of different Kinds; thoſe are the beſt of their Kind by which you can read the ſame print at the greateſt Diſtance, when that print is equally enlightened.

The beſt of theſe Kinds of Teleſcopes that have been hitherto made has been by Mr *Short*. He has made ſix with glaſs Speculums quickſilver'd behind, which ſhew the Image very diſtinctly, as the late Mr *Maclaurin* ſays, three of 15 Inches focal Diſtance, and three of 9 Inches. The preſent Duke of *Argyle* had one of the 15 Inch ones, by which it is eaſy to read in the *Philoſophical Tranſactions* at the Diſtance of 230 Feet; and with one of the 9 Inch ones at the Diſtance of 138 Feet. With another of theſe, at another Time, and with a ſmaller Print, you may read at the Diſtance of 125 Feet. He has alſo made theſe Teleſcopes with metaline Speculums, which are very good. He has made ſome of 2 Inches and 6 Tenths focal Diſtance; others of 4, 6, 9, and 15 Inches. By thoſe of 4 Inches the Satellites of *Jupiter* may be ſeen very well, and the *Philoſophical Tranſactions* may be read at 125 Feet Diſtance. By thoſe of 6 Inches focal Diſtance you may read at 160 Feet Diſtance. By thoſe of 9 Inches at 120 Feet. By thoſe of 15 Inches focal Diſtance you may read the ſaid *Tranſactions* at the Diſtance of 500 Feet, and by it the five Satellites of *Saturn* have been ſeen together, which Mr *Caſſini* had ſometimes ſeen with a 17 Foot Refracting Teleſcope. See more particularly Mr *Maclaurin*'s Letter to Dr *Smith*, dated at *Edinburgh*, *Dec.* 28. 1734, to be ſeen at *Page* 80. of the Remarks, at the End of Dr *Smith*'s ſecond Volume of *Opticks*.

The firſt Thought we find in print concerning a Reflecting Teleſcope, is that of Dr *James Gregory*, in his *Optica Promota*, publiſhed in the Year 1663; who at *Page* 93, 94, propoſes a Cata-Dioptrical Teleſcope, in order to make Teleſcopes ſhorter, and more handy; and this is the Deſcription of it by himſelf.

Let A D C E (Fig. 17.) be a parabolick concave Speculum moſt exquiſitely poliſhed, in whoſe Focus C is placed a ſmall elliptick concave Speculum, having a common Focus, and common Axis with the concave parabolick Speculum, and let it be fixed in that Situation. Now the ſaid Focus of that elliptick Speculum muſt be very near to it's vertex, and the other Focus of it muſt be very far from the ſame at F, in the common Axis produced beyond the parabolick Speculum; and through the Vertex of the parabolick Speculum muſt be made a round Hole M N, in which Hole muſt be placed a Tube, having the ſame Axis with the Speculums, and big enough to receive the Rays of a viſible Point or Object, reflected from the elliptick concave Speculum; and let it be continued to L very near to F, and at L let a Lens of cryſtal Convex towards the Speculums be placed, with the Convexity of a Conoid, and the Denſity of the Cryſtalline (of the Eye), whoſe exterior Focus muſt be at F, and which muſt be plane towards the Eye, and likewiſe have the ſame common Axis as the Speculums and the Tube. This will be the Way to make an excellent Teleſcope for purblind Eyes; for diſtant Objects ſeen through the Tube, will appear diſtinctly magnified very near in the Ratio of the Diſtances of the Vertexes from the common Focus, and enlightened in the ſame Manner as a viſible Object would be, when ſeen under ſuch an Angle; provided the Diameter of what produces the laſt Image be big enough to ſuffer the *Uvea* of the Eye to be filled with the Rays. And how that may done, we have taught in the *Scholium* of the 51ſt of this Book, ſays the *Doctor*. But it is plain, by what Dr *Gregory* has here ſaid, and by the Date of his Book, *viz.* in 1663, that he moſt certainly was the firſt Hinter and Founder of ſuch an Inſtrument, thereby giving Occaſion to others to purſue and improve upon his Original, Fig. 17.

Original, with such Alterations or Additions, as would best make the Thing succeed in actual Practice.

Sir *Isaac Newton* himself tells us, in the *Philosophical Transactions*, Numb. 80. that he had considered this Telescope of Dr *Gregory*'s, as above described in the *Optica Promota*; and this was not 'till the Year 1668. That he resolved, before he attempted any thing in the Practice, to alter Dr *Gregory*'s Design, and place the Eye-Glass at the side of the Tube, rather than in the middle.

Dr *Gregory* never brought his Telescope to any Degree of Perfection, but owning his Want of Skill in Mechanicks, only proposes it for others to execute; he had, indeed, an Object-Speculum of Metal ground to a Segment of a Sphere, and a little Concave, as well as a little Convex-Speculum, ground by *Rives* and *Cox*, two Optick Glass-Grinders of *London*, famous in those Days. But because the Object-Metal was not well polished, he only made some imperfect Trials, not so much as even fitting the Speculums and Eye-Glass into a Tube; and being discouraged, as much because he could not have the Object-Metal ground into a parabolick Concave, as because that which he tried was not well polished, he gave over the Thoughts of bringing such Telescopes into Use. See his Letter to Mr *Collins*, written from St *Andrews*, *Sept.* 23, 1672, to be found in Dr *Desaguliers*'s Appendix to Dr *David Gregory*'s *Opticks*, at Numb. 4.

Of late Years several Persons have endeavoured to make large Telescopes of this Kind, but I never heard they ever succeeded; and I must confess, that these Instruments, though they have the Advantage of magnifying, and being more manageable, yet do not shew the Objects so distinctly and bright (which are the two chief Things for which all Telescopes are used, and valuable), as refracting Telescopes do, which are longer; besides the Trouble of making these Instruments, hazard of spoiling them, and their Expence, are Considerations of some Weight; nor can they be used, in some astronomical Observations, where a Micrometer is necessary, because a Micrometer cannot easily be applied to one of them; tho', indeed, there is a Contrivance to do it in the *Philosophical Transactions*. Numb. 475.

CHAP. VII.

Of Sir Isaac Newton's *Reflecting Telescope, as improved by Mr* Hadley, *and presented by him to the* Royal Society.

Fig. 18. THE Instrument (Fig. 18.) consists of a metaline Speculum, about six Inches in Diameter; the Radius of the Sphere on which it's concave Surface was ground is 10 Feet $5\frac{1}{4}$ Inches, and it's focal Length is $62\frac{5}{8}$ Inches. The back has a hollow Screw made at it's Centre to receive the End of a Handle which is screwed on wherever the Metal is to be moved, in order to avoid sullying it's polished Surface by handling. The Object-Metal A is placed in one End of an octangular Tube B B about 6 Feet long, dyed black on the Inside; about 6 or 7 Inches in Length of the three uppermost Sides of the Tube C towards that End at which the Metal is placed, are seperated from the rest, and open with two Hinges to make room for the Metal to be put in, or taken out; the End of the Tube is closed by an octangular Piece of Board D, which has an opening about $\frac{2}{5}$ of an Inch broad, from the Top, down to a little below the Center, to give room for the before-mentioned Handle, when the Object-Metal is lifted into, or out of, the Tube; at other Times it is closed with a sliding Shutter; the Metal is placed so as to have it's Axis coincide with that of the Tube, by the Means of three small Buttons fixed to the Inside of the Tube, having their hinder Ends all in the same Plane to which this Axis is Perpendicular; two of these appear at *a a*, the third being at the middle of the Bottom of the Tube, is not seen; the foreside of the Metal rests against these Buttons in three Points of it's Circumference, nearly equidistant from each other, and is held in them by three Screws, one of which appears at *b*, which run through the octangular Board at the End of the Tube, and bear against the Back of the Metal, in three Points, which directly answer those three on the foreside, with just so much Force as is requisite to keep it steady in it's place; they must not be screwed harder against the Metal for fear of bending it, which, though it is half an Inch in Thickness, a very little Force is sufficient to do: when the Instrument is not used, these Screws are loosened, and the Object-Metal is taken out and laid by to prevent it's tarnishing.

Fig. 19. The oval Plane (Fig. 19.) is composed of a Plate of the same Metal with the great Speculum, about $\frac{1}{15}$ or $\frac{1}{10}$ of an Inch in Thickness, soldered on the Back to another of Brass; it's

it's Breadth is something less than half an Inch, and is in the Ratio to it's Length, As 1 is to the square Root of 2, or as the Side of a Square is to it's Diagonal; at one End of the Oval, the Brass Plate projects a little beyond the other, and has a Screw cut through it in that part, as also another directly against the Centre of the foreside; the other End is cyphered away on the backside, that it may intercept as few of the Rays in their Passage towards the Object-Metal as possible. The two Screw-Holes in the back (Figure) serve to fix this Oval A, to a Brass Arm B, which is fastened at the other End into a Slider E E (Fig. 18, 19.); this Slider is of an equal Thickness with the Side of the Tube, and has a Groove G G (Fig. 18.) cut for it in that side parallel to the Axis, and long enough to give room for it's Motion, to set the Speculums at the different Distances, which the several Eye-Glasses require; it rests on the Inside against two thin Ledges, fastened within the Tube along the Sides of the Groove; on the Outside, it is kept in it's place, by a sliding Shutter not expressed in the Figure. In the middle (Fig. 19.) it has a cylindrical Cavity D, whose Axis is perpendicular to it's inner and outer Surfaces; each of the Boxes in which the Eye-Glasses are contained, is fitted to this Cavity; the beforementioned Brass Arm is fixed into the Inside of this Slider, towards the End farthest from the Object-Metal; it rises perpendicularly for about two Inches, and is made flat, so as to turn one Edge to the Rays which come from the Object; about *b* it is bent forwards, and flatted the other way, so that when the back of the oval Plane is held flat to it by the two Screws *c c*, the Axis of the cylindrick Cavity may fall on the Centre of it's foreside, inclined to it's Surface in an Angle something less than 45 Degrees; this Angle is brought to be exact by two very small Screws *i i*, whose Threads take hold in the flatted End of the Brass Arm, and their Points bearing against the back of the Oval, raise one End of it a little from the flat of the Arm. The Speculums are set at their due Distance by turning a long Screw C C, for which there is a Nut lodged in the Slider at *g*; the Screw is kept from moving backwards or forwards, when it is turned, by a Brass Plate F, which is to be fixed to the flat End of the Side of the Tube, and taken off at Pleasure. Each of the Eye-Glass Boxes H, has a Screw on the outward End to fasten to it a Bowl or Dish I, to receive the Ball of the Eye, and guard it from external Light. Fig. 18, 19. Fig. 19.

On the Top of the Tube a common Dioptrick Telescope H is fixed upon two small Pillars. about 18 Inches long, with it's Axis parallel to that of the Tube; with two small Hairs in the common Focus of the Object and Eye Glasses; crossing one another in it's Axis.

There are three convex Eye-Glasses belonging to the Instrument; the first, or shallowest, has it's focal Distance of about $\frac{1}{3}$ of an Inch; the second of $\frac{3}{10}$; and the deepest of $\frac{11}{40}$; or something less. When the first of these is used with the Instrument, it magnifies about 188, or 190 Times in Diameter; with the second, about 208, and with the third about 228, or 230: Each of these Glasses has placed in that Focus nearest the Oval, a Circle to determine the part of the Object seen at one View; and in the other Focus towards the Eye, a Brass Plate with a little Hole in the middle, to let no Light pass to the Eye from the Inside of the Tube, but what comes from the Oval: besides these three convex, there are two concave Eye-Glasses with which it magnifies about 200, or 220 times; and also a Set of three convexes, which turn it into a Day Telescope, magnifying about 125 times. The Aperture is limitted by a Circle of Card or Paste-board placed before the Object-Metal in the Tube. To vary the Apertures, there are three of these Circles, and the Apertures altered by them are five Inches and a half, five Inches, and four and an half, though for some Objects, the whole Tube may be left open.

Thus far is the particular Description of the Body and main Parts of the Telescope, as is to be found in the *Philosophical Transactions*, Numb. 376. As to the particular Description of the wooden Support to direct the Tube to any Object which is there given I shall omit, the Figure thereof being sufficient to satisfy all those who have not such a Telescope, and have no Desire of making it; besides, it is in any one's Power almost to invent a Support for the Body of the Instrument, and that, perhaps, different from this here, of Mr *Hadley*'s; there is no great ingenuity in doing it.

Mr *Hadley* tells us, this Instrument when tried at an Object seen in the Day-Time, seems to bear an Aperture of five Inches and an half, with the deepest of the aforementioned Eye-Glasses, as well as the common Telescopes do the usual Charge and Aperture given to them, except that in these the Objects appear a little brighter.

Drs *Pound* and *Bradley* say, that this Telescope will bear such a Charge as to make it magnify the Object as many Times as Mr *Huygens*'s dioptrick Telescope, the focal Length of whose Object-Glass is 123 Feet, and that it represents Objects as distinct, though not altogether so clear and bright.

I have always thought, that these catadioptrick Telescopes of both Kinds, do represent Objects more obscurely than refracting Telescopes, though indeed in this the *Newtonian* one has the Advantage of Dr *Gregory*'s; and that they will always do so, unless a reflecting Substance for a concave can be found, that will reflect as much Light as Glass transmits, which I take to be impossible. For I ever saw the Image of an Object in the Focus of a concave Speculum more indistinct and obscure than the Object itself, (which argues the Loss of Rays by the Reflection) or than an Object appearing by the Reflection of a plane Speculum; whereas an Object seen through a Glass-Lens will appear more enlightened and bright, than even it does with the naked Eye.

Mr *Huygens* says, that he had never found any Speculums that had so good a Polish as Glass; and so says Sir *Isaac Newton*, and accordingly he recommends glass Speculums quicksilvered over, as preferable to metalline ones.

In the second Volume of Dr *Smith*'s *Opticks*, beginning at *pag.* 363. there is a Reflecting *Telescope* of Sir *Isaac Newton*'s Form, made, and described by *Samuel Molyneux*, Esq; and sent by him as a Present to his Majesty *John* V. *King* of *Portugal*, being of a much simpler Construction than that of Mr *Hadley*'s, and magnifies as much, and as distinctly, as a Refracting Telescope of 35 or 40 Feet. So says Dr *Smith*.

And at Numb. 918, *pag.* 365. there follows a very strange and singular Assertion of Dr *Smith*'s, *viz.* he says, that one's Imagination is always misled in comparing the Effects of this Instrument with Refracting ones of that Length. For although (says he) by the Minuteness of the Parts visible in the Object observed; as also, by the Proportions between the focal Distance of the Eye-Glass made use of, and the focal Distance of the great Speculum, it is demonstrable, that the Distinctness and magnifying Power thereof exceeds that of a Refracting Telescope of 35 Feet, yet the Spectator will always fancy that the Refractor has the Advantage of this Instrument. And again, that it will be certainly found to be but a Deception, and probably an universal one to all Persons. Now, is not the Doctor here rather deceived himself. For to prefer optical Demonstrations, founded upon dubious Principles (which I think his are, about the apparent Distinction, and brightness of Objects) rather than to the Experience of all Men,. is too bold an Assertion. And one would be almost led to believe, that the Doctor would have Reflecting Telescopes, right or wrong, to be better than Refracting ones, as to the Distinctness of the Objects appearing through them.

CHAP. VIII.

A Description of the Reflecting Telescope, made by Sir Isaac Newton *himself.*

HE made two small ones, with an Object metal spherically concave, the second being better than the other; the worst of which he describes in the *Philosophical Transactions*, (at Numb. 80.) and the other he sent to the *Royal Society*. This worst was not long ago to be seen at Mr *Heath*'s, the Mathematical Instrument Maker in the *Strand*, having upon it, wrote with his own Hand, *Isaac Newton*.

Fig. 20. A B (Fig. 20.) is the concave Speculum, of which the Radius or Semidiameter is 12 $\frac{2}{3}$ or 13 Inches. C D is another metalline Speculum, whose Surface is flat, and the Circumference oval. G D is an iron Wire, holding a Ring of Brass, in which the Speculum C D is fixed. F is a small Eye-Glass, flat above, and convex below, of the twelfth Part of an Inch Radius, if not less; forasmuch as the Metal collects the Sun's Rays at 6 $\frac{1}{3}$ Inches distance from it's vertex. G G G the fore-part of the Tube is fastened to a brass Ring H I to keep it immoveable. P Q K L is the hind-part of the Tube fastened to another brass Ring P Q. O is an iron Hook fastened to the Ring P Q, and furnished with a Screw N. Thereby to advance or draw Back the hind-part of the Tube, and so by that means to put the Speculums to their due Distance. M Q G is a crooked Iron sustaining the Tube, and fastened by the Nail R to the Ball and Socket S, whereby the Tube may be turned every way. The Centre of the flat Speculum C D must be placed in the same Point of the Tube's Axis V T where falls the Perpendicular to this Axis drawn to the same, from the Centre of the Eye-Glass, which Point is here marked at T.

The Tube of this Telescope is open at the End which respects the Object, the other End is close, where the said Concave is laid. The flat oval Speculum is near the open End, made as small as may be, the less to obstruct the Entrance of the Rays of Light, where is a little Hole furnished with the said small plano-convex Eye-Glass. So that the Rays coming from the Object do first fall on the Concave placed at the Bottom of the Tube, and are thence reflected towards the other End of it, where they meet with the flat Speculum obliquely posited, by the Reflection of which they are directed to the little plano-convex Glass; and so to the Spectator's Eye, who looking downward sees the Object which the Telescope is turned to.

This Telescope will magnify as to Surface about 38 Times, *viz.* as much as a refracting Telescope of 2 Feet, and by it you may read in the *Philosophical Transactions*, at the Distance of 100 or 120 Feet.

Sir

Sir *Isaac Newton*, in his Opticks, *page* 95, gives another Description of a Reflecting Telescope, which see there.

A little after Sir *Isaac Newton* had sent his Telescope to the *Royal Society*, Mr *Oldenburgh*, the Secretary, wrote him a Letter of Thanks, to which Sir *Isaac Newton* made Answer in 167$\frac{1}{2}$, giving a farther Account of the Instrument.

About this Time Dr *James Gregory* having an Account of Sir *Isaac Newton*'s Telescope, wrote his Thoughts about it to Mr *Collins*, in a Letter from *Aberdeen*, dated *August* 6. 1672. In which he gives the Preference to Sir *Isaac*'s Telescope, above that which he described in the *Optica Promota*, in one respect, but thinks his own better in another. At the same Time one Mr *Cassegraine*, a *French* Man, published a Description of a *Catadioptrick Telescope*, as his own Invention, which he pretended had been prior to Sir *Isaac*'s Telescope. Sir *Isaac* contrived his in the Year 1666, and executed it in the Year 1670, or 1671. And indeed Dr *James Gregory* his before either of them, *viz.* before 1663. This Telescope of Mr *Cassegraine*'s differs nothing from the *Doctor*'s, excepting that he would have the small Metal to be convex, whereas in that of the other it is concave; but I never heard that an Instrument of this Kind was ever yet made. See an Account of it, in the *Philosophical Transactions*, at Numb. 83. In the second Volume of Dr *Smith*'s Opticks, amongst the Remarks, *page* 105. he says, that what Trials have been made of Mr *Cassegraine*'s Form he knows not; but it appears by a Table there set down, for the Parts of this Telescope, compared to another there set down for the Parts of *Gregory*'s Telescope, that the former has the Advantage, being shorter by twice the focal Distance of the lesser Speculum, and yet magnifies more.

Sir *Isaac Newton* says, the Telescope he made, was but 6 Inches long, bears something more than an Inch Aperture, with a plano-convex Eye-Glass, which is $\frac{1}{6}$ or $\frac{1}{8}$ Part of an Inch in depth, so that it magnifies about 40 Times in Diameter, which he thinks is more than any 6 Foot Tube can do with Distinctness; but (says he) by reason of bad Materials, and for want of a good Polish, it represents Things not so distinctly as a 6 Foot Tube will do; yet he thinks it will discover as much as any 3 or 4 Foot Tube, especially if those Objects be luminous; and says, he has seen with it *Jupiter* distinctly round, and his Satellites and *Venus* horned. He doubts not but in Time a 6 Foot Reflecting Telescope of this sort may be made, which will perform as much as any 60 or 100 Feet Refracting Telescope. That a Refracting Telescope, made of the purest Glass, exquisitely polished with the best Figure any Geometrician has or can design, would scarce perform as much more as an ordinary good Telescope of the same Length, *&c.* This is part of a Letter written by Sir *Isaac Newton*, from *Trinity College, Cambridge*, dated *Feb.* 23. 166$\frac{8}{9}$. to a Friend. See *page* 259. of Dr *Desagulier*'s *Appendix* to Dr *David Gregory*'s Elements of Opticks.

CHAP. IX.

Of MICROSCOPES.

THESE most pleasing and useful Instruments for enlarging and viewing very small Objects, and their Parts, not perceivable by the naked Eye (by reason of their Smallness, and Deficiency in reflecting Light from them to the Eye) and thereby discovering many surprizing Kinds of minute Animals, wonderful in Shape and Motion, as well as some of the Texture, Contrivance, and Structure of natural Bodies, and their Parts, are of two Sorts, *viz.* single, which consist of one very small Lens, or Globe of Glass or Water, put near to the Eye, or compound ones, consisting of two or three Lenses.

Mr *Huygens*, in his little Tract upon Microscopes, in his *Dioptricks*, beginning at *page* 221, shews, how to make the small Glass Globes for single Microscopes, by melting powdered Glass in the Flame of a Lamp, which will run into various small Globes, and making Choice of such as he found best, which he put into small round Holes, made in a very thin Copper Plate, punched with a Needle, and those he liked best he fixed therein; thus easily making several Microscopes. The well figuring, and polishing such very small Globes, being much more owing to Chance than Design.

Mr *Stephen Gray* used Drops of Water, placed near the Eye, for single Microscopes. See the *Philosophical Transactions*, Numb. 221, 223.

Mr *Leewenhoeck* of *Delft* in *Holland*, was doubtless the greatest Judge of the best Kind of Microscopes, as well as the greatest Master in the Use of the Instrument, and has obliged the World with the greatest Discoveries by it of any Man that I know; yet he prefers single Microscopes to compound ones, as may be seen in the Account of *Martin Folkes*, Esq; given of Mr *Leewenhoeck*'s Glasses, which by his Will he left to the *Royal Society*. See Numb. 380. Also a further Account of Mr *Leewenhoeck*'s Microscopes, by the ingenious Mr *Henry Barker*, is to be seen at Numb. 458. of the *Philosophical Transactions* for the Year 1740. This great

Man

Man preferred Diſtinction to great Degrees of magnifying, in which he moſt certainly was right; for what ſignifies augmenting the Bulk of an Object, if it and it's Parts cannot be diſtinctly and clearly ſeen; conſequently, thoſe are the beſt Microſcopes, whether ſingle or double, that do this the beſt. And within theſe 100 Years there have been a vaſt Variety of theſe Inſtruments made (ſome ſingle, ſome double) by ingenious Workmen in *England*, *Italy*, *Holland*, &c. ſo many, and much diſperſed, that few Perſons can really tell which are beſt. Many of them may be good enough, as to the Perfection of the Glaſſes, but they differ in the leſſer eſſential Contrivances, *viz.* to move them to the Object, to caſt Light upon, and to hold the Object, with the greateſt Eaſe and beſt Convenience.

The Lenſes of ſingle Microſcopes are from $\frac{1}{20}$ of an Inch, to in Diameter; the manner of making them by one Mr *Butterfield*, of the Bigneſs of great Pins Heads, or leſs, is to be ſeen in the *Philoſophical Tranſactions*, Numb. 141. Dr *Hook* uſed to take a clear Piece of Glaſs, and draw it out into long Threads in a Lamp, then he held theſe Threads in the Flame 'till they run into round Globules hanging to the End of the Threads; and having fixed the Globules with Wax to the End of a Stick, ſo that the Threads ſtood upwards, he ground the Ends of the Threads upon a Whetſtone, and poliſhed them upon a ſmooth metal Plate, with a little Putty. Mr *Gray* makes them ſomewhat otherwiſe, See the *Philoſophical Tranſactions*, Numb. 221, 223, who ſays, That ſmall round Globules magnified and ſhewed Objects clearer than Hemiſpheres. Dr *Hook* ſays, he made uſe of a double Microſcope in moſt of his Obſervations. See the Preface to his *Micrography*: and to enlighten his Objects as much as poſſible, he made choice of a Room with but one Window towards the South, where at the Diſtance of 3 or 4 Feet from the Window he placed his double Microſcope upon a Table, and by a Glaſs Globe full of Water, and a thick plano-convex Lens he threw the Light upon the Object; or when the Sun ſhone, he placed a Piece of oiled Paper very near the Object, and with a very large Burning-Glaſs he threw the Sun's Rays upon the Paper, ſo as that a great Quantity of them might be tranſmitted through it to the Object; but the Paper being ſubject to take Fire, if the Focus of the Rays fell upon it, inſtead thereof he put a Piece of plane Glaſs, not poliſhed, but only rough ground with fine Sand, which when warmed gradually would endure much more Heat than the oiled Paper; and ſo would ſuffer more Light to paſs through it to the Object; and by this means ſays he, the Sun's Light was more equally diffuſed upon the Parts of the Object than the Sun's direct Light. In the Night-Time he illuminated his Object with the Light of a Lamp, firſt refracted through his Globe full of Water, or clear Brine, which refracts more than Water, and then collected into a ſmaller Spot upon the Object by a plano-convex Lens. He alſo placed a poliſhed concave Metal on the Side of the Lamp oppoſite to the Globe, to reflect part of the Rays upon the Globe; and thus he ſays he could illuminate an Object with a ſmall Lamp as much as it would well bear, and he drew moſt of the Repreſentations of Objects by the Light of his Lamp.

Note, A double Microſcope contrived for viewing Objects by Sun-ſhine is called by ſome of the Moderns, a ſolar *Microſcope*; and without diſpute, Objects viewed in Sun-ſhine will be beſt perceived and diſtinguiſhed, provided the Sun-ſhine be not too great, or the Eye dazzled by the Glare.

I have already hinted, that the World is now full of Microſcopes, ſome better, ſome worſe. Mr *Wilſon*'s Pocket Microſcope deſcribed in the *Philoſophical Tranſactions*, Numb. 281. Mr *Campani*'s and *Divini*'s. Mr *Leuwenhoeck*'s. Dr *Hook*'s. Mr *Marſhal*'s double Microſcope, diſcribed in Dr *Harris*'s *Lexicon*. Mr *Culpepper*'s and *Scarlat*'s, Mr *Muſſchenbroeck*'s, &c. have been reckoned to be very good. And ſome of the beſt Writings upon the Uſe of the Microſcope are, 1. Dr *Hook*'s *Micrography*, in Folio, printed at *London*, in the Year 1667.

2. Mr *Leuwenhoeck*'s various Tracts, *viz.* his *Arcana Naturæ detecta*, in Quarto, printed at *Delft*, in the Year 1695; his *Continuatio Arcanorum Naturæ detectorum*, in Quarto, in the Year 1697; his *Arcana ope & beneficio exquiſitiſſimorum Microſcopiorum detecta*, in Quarto, in the Year 1696; his *Continuatio epiſtolarum datarum ad longe celeberrimam regiam Societatem Londinenſem*, printed at *Leyden*, in the Year 1696, in Quarto.

3. *John Grendel*'s *Micrographia Curioſa*, in Quarto, printed at *Noremburgh*, in the Year 1687.

4. *Franciſcus Fontana*'s *Obſervationes Celeſtium Terreſtriumque Rerum.*

5. *Malpighius*'s *Anatomia Plantarum*; his *Tractatus de ovo incubato*, *de Bombyce*, *de Viscerum Structura.*

6. *Bonnani*'s *Micrographia Curioſa*, in Quarto, printed at *Rome*, in the Year 1691.

7. Mr *Baker*'s Treatiſe of the *Microſcope*, printed at *London*, in the Year 1741. There are many others who have given microſcopical Obſervations; in the *Philoſophical Tranſactions*, and other Tracts, too long to mention. Sir *Iſaac Newton*, in the *Philoſophical Tranſactions*, Numb. 80, hints at a Reflecting Microſcope, and Dr *Smith* in his Opticks, gives the Dimenſions for making one; but I have heard the Contrivance will not do well.

CHAP. X.

A short Account of some Instruments either of less general Use, whose full Descriptions are too long to insert in this Appendix, or such as with some Alterations or Additions of Parts are already described in this Book.

I. *A Meridian Telescope.*

THIS is a Telescope fixed at right Angles to an Axis, and turned about it in the Plane of the Meridian. Dr *Halley* had one of these; another of them was contrived by Mr *Cotes*, and the present Duke of *Argyle* has a very good one. See their Description in Dr *Smith*'s *Opticks*, Vol. II. Book iii. Chap. 6. The Use of these Instruments is for finding Time, by observing when the Sun or any Star has equal Altitudes on each Side of the Meridian; as also for finding the Time of their Appulses to the Meridian. And thence by means of a Clock, and the computed right Ascension of the Sun at that Time by the Clock, or that of a Star, we have the Difference between a Day reckoned by the Clock, and that of the Sun or Stars. Dr *Halley* at first made all his Observations of the right Ascensions of the Moon by such an Instrument; his Telescope was $5\frac{1}{2}$ Feet long, and the transverse Axis above three Feet.

There are other Instruments that will do this, one of which is described in a little Folio Tract of Mr *Molyneux*'s, printed at *London* in the Year 1700, entituled *Sciothericum Telescopicum:* being a large horizontal Dial with a Telescope adapted to it. Such an Instrument was made in *London*, by Mr *Richard Whitehead*, in the Year 1685. This Instrument gives the Hour, almost by Inspection, by a Star, as well as by the Sun; it has also this Advantage, that when the Sun is over-clouded so as just to be seen faintly, and casts no Shadow at all upon a Dial, yet if the Sun be in the least perceivable by the Eye, the Time may be exactly told by him. The Contrivance in brief is thus. It is a very large Brass horizontal Dial made to the Latitude of the Place it is to be used in, with a strong double Gnomon, casting the Morning Shadow from it's Western Edge, and the Afternoon Shadow from it's Eastern Edge, and the Noon Shadow by it's Thickness; it has two Pair of Sights, or Rulers, one Pair serving for the Morning, or for Stars that are to the East of the Meridian; and the other Pair to serve in the Afternoon, or for Stars on the Western Side of the Meridian; each of these Pair of Sights consists of two moveable Rulers, called the horizontal Ruler, and the Gnomonick one, by Mr *Molyneux*; which Rulers are so adapted, as that their two Edges which are next to the Gnomon, are always in the same Plane with each other; and at the Time of Observation, that they both may be in the same Plane with their correspondent Edge of the Gnomon; and on the Style-Ruler are fixed Telescopick Sights with the Cross-Hairs in their proper Place. The horizontal Ruler turns exactly about the Centre of the Hour-Lines; and to the other End of it, the Style-Ruler is adapted by means of a Joint, so as to have two Motions, the one upwards or downwards, and another Eastward and Westard, according as it follows the horizontal Ruler.

II. *An Equatorial Telescope.*

The ingenious Mr *Short* (in the *Philosophical Transactions*, at Numb. 493, for the Year 1749.) gives the Description and Uses of an Instrument consisting of a Combination of several Circles, with two Spirit-Levels, and a Reflecting Telescope, which will serve for a Dial; an equal Altitude Instrument, a Transit Instrument, a Theodolite, a Quadrant, an Azimuth Instrument, and a Level. He says, he has made three of these Instruments, one of which he sold to Count *Bentinck*, for the Use of the late Prince of *Orange*. That he had the other two by him. That he does not pretend to any thing new in the Combinations of the Circles of which the Instrument consists. The same Combinations having several Times before been made by way of Dial; but believes the putting so large a Telescope upon the Machinery, and applying it to the Uses he has done, is somewhat new.

III. *The Micrometer.*

Any Micrometer, of which there are several Sorts, well made, may be sufficient to measure small Angles subtended by remote Objects at the naked Eye; to do which is the chief Business of this Instrument.

 The

The old Micrometer of Mr *Townley*'s, improved by Dr *Hook* (and long since described in the *Philosophical Transactions*, Numb. 29.) and it's Uses in Numb. 25. appears to have been good, or else Sir *Isaac Newton* would not, at the Beginning of the third Book of his *Mathematical Principles of Natural Philosophy*, have ventured to set down the Measures of the Distances of the Satellites of *Jupiter*, from *Jupiter*'s Center, as taken by Mr *Townley* with that Micrometer. This to me is a sufficient Proof of the Exactness of that old Instrument. As to Facility of Use, and some other lesser Advantages, other more modern Micrometers have been made and described. And Dr *Smith*, *in his Opticks*, Vol. II. Chap. 8. *Book* 3. describes a Micrometer, which he calls one of the best Sort. The *Micrometer*, and it's Use, is also treated of by Mr *Auzont*, a *Frenchman*; by Mr *Hevelius*, in the *Acta Eruditorum*, *Anno* 1708; by Mr *Balthasaris*, and others. Scarcely, in my Opinion, deserving a particular Account here. What our Author and myself having already said upon this Instrument appearing to me to be enough.

IV. *Of Mr.* Graham's *Astronomical Sector.*

This Instrument is for observing the Place of a Planet or Comet, by taking the Differences of it's right Ascension, and Declination, from those of a known fixed Star; when it cannot be done at all by a Micrometer, or not conveniently by a large Quadrant or Sextant, the Contriver in brief is thus.

Plate 2. Fig 21.

Let A B (Plate 2. Fig. 21.) represent an Arch of a Circle containing 10 or 12 Degrees, having a long Plate C D for its Radius fixed to the middle of the Arch at D. Let this Radius be applied to the side of an Axis H F I, and be moveable about a Joint fixed to it at F, so as the Plane of the Sector may be always parallel to the Axis H I, which being parallel to the Axis of the Earth, the Plane of the Sector will always be parallel to the Plane of some Hour Circle. Let a Telescope C E be moveable about the Centre C of the Arch A B, from one end of it to the other, by turning a Screw at G, and let the Line of Sight be parallel to the Plane of the Sector; then by turning the whole Instrument about the Axis H I, 'till the Plane of it successively directed, first to one of the Stars, and then to another, it is easy to move the Sector about the Joint F into such a Position, that the Arch A B when fixed, shall take in both Stars in their Passage by the Plane of it, if the Difference of their Declination does not exceed the Arch A B. Then having fixed the Plane of the Sector a little to the Westward of both the Stars, move the Telescope C E by the Screw G, and observe by a Clock the Time of each Transit over the cross Hairs; and also the Degrees and Minutes upon the Arch A B, cut by the Index at each Transit. Then is the Difference of the Arches, the Difference of the Declinations, and by the Difference of the Times, we have the Difference of the right Ascensions of the Stars. See a more particular Description of this Instrument in the ninth Chapter of the third Book of Dr *Smith*'s Opticks, Vol. II.

V. *Of Mr* Sisson's *Theodolite.*

This is certainly the best, most complete, handsome, and well designed Instrument possible, not only serving as a Theodolite to take horizontal Angles, but likewise to take vertical Angles as a Quadrant; and besides, may be used to take the Levels of Places, all with Speed, Ease, and sufficient Exactness, for most of the common Purposes for which it is dedesigned. See more particularly, concerning the same, in Mr *Lawrence*'s *Surveying*.

There is likewise another very good *Theodolite*, made by Mr *Heath* the *Mathematical Instrument* Maker, whose Uses are to be seen in *Hammond*'s *Surveying*, wrote in reality by the late very ingenious Mr *Samuel Cunn* (who being a Butcher, that kept a Butcher's Shop in *Newport Market*) was also a very great *Mathematician:* One of the best Measurers of Artificers Works, Surveyors of Land, and Expounders of *Euclid*, and *Apollonius*, in the World.

VI. *Mr* Barston's *Universal Astronomical Quadrant*, (as he calls it) is an Instrument contrived to take the Altitudes of the Sun, Moon, and Stars, without a visible Horizon, with the greatest Accuracy, as well as Speed, as the Author himself will have it. The Instrument consists of a Semicircle, with half its Arch, divided into 90 Degrees, having a Telescope upon the Diameter, and Wheel-work inclosed within the Instrument, which turns two small Hands about, pointing at the Divisions of two circular Arches, on that Radius of the Quadrant, or rather Semicircle at right Angles to the Diameter of the Semicircle, which Divisions give the Degrees and Minutes of Altitude. The Contrivance of such a Quadrant to take Altitudes at once is not new. I heard of one, it is now almost fifty Years ago, which was then called by the Name of a Ketch-Quadrant. They were well known, and commonly talked of amongst a Society of ingenious Mathematicians, that met together once a Week in *London*, from the Year 1710 to 1724. But I won't say these Ketch-Quadrants were exactly the same as to Structure, with those of Mr *Barston*. His may possibly be better. He got a Patent for the sole making and selling them; and they were made and sold by Mr *Joseph Turner*, a Watch-Maker in *Hatton Garden*, *London*, with the Names of *John Barston* and *Joseph Turner* engraved upon the Limb.

VII.

VII. Dr *Hook*, *Samuel Molyneux*, Esq; and Dr *Bradley*, our present Royal Professor of Astronomy, had each an Instrument for trying to discover the annual Parallax of the fixed Stars. Mr *Molyneux*'s Instrument was a Telescope of 25 Feet long, with a square Tube, contrived to swing in a vertical Position by two polished Cylinders fixed near the top of the Tube, so that their common Axis, if produced through the Tube, would pass at right Angles to it's Axis through a Point near the Centre of the Object-Glass. When the Telescope turned upon those Cylinders, it's Axis of Vision moved like a Pendulum in the Plane of the Meridian, while a fine long Wire hung down by the side of the Tube, whose Loop was put over one of the Cylinders, was gently stretched by a Plummet immersed in a Vessel of Water, designed to retard it's Vibrations. The lower End of this Wire played gently against the smooth side of a slender Brass Plate fixed cross-ways to the side of the Tube, so as to point Northwards and Southwards; and in the middle of this Plate was punched a very fine round Hole, a little broader than the Thickness of the Wire. The Telescope was gradually moved upon it's Axis of Suspension, by the Pressure of a long Screw, divided into fine Threads, like that of a Micrometer, which worked in the Hole of a Plate fixed to the Wall of the House, and the Tube was made to bear against the end of this Screw by a Weight fixed to the end of a String passing over a Pulley having it's other end tied to a Hook fixed in the side of the Tube. The opposite end of that long Screw was fixed like an Axis in the Centre of a Brass Wheel, whose Circumference was divided into a great Number of equal Parts, while an Index fixed to an upright Board pointed to the Divisions of the Wheel.

Now while the Wheel was gently turned, the part of the Wire which played against the Cross-Plate was viewed through a double Microscope, 'till the little Hole in this Plate appeared to be bisected by the Wire. The Telescope being by this means placed verticle, and rectified immediately before the beginning of every Observation of the Transit of the bright Star in the Head of the Dragon, which passes very near the Zenith of *Kew*, (for viewing of which this Instrument was contrived and set up in the Year 1725, at *Kew* near *Richmond*) and the Division of the Wheel over against the Index being then noted, the Wheel was turned again 'till the Intersection of the Wires in the Focus was brought to touch the Star at the instant of it's Transit. Then by the Number of the Revolutions of the Wheel and it's parts that had passed by the fixed Index, the angular Motion of the Axis of the Telescope was easily found by a Table of Minutes and Seconds, answering to those Revolutions; then the Differences of those Angles found at different Observations, are the Differences of the Star's Declinations.

Dr *Hook*'s Instrument, which he fixed up at *Gresham College*, described by himself, was much such another, with a Telescope of 36 Feet long.

Dr *Bradley*'s Instrument, as he himself says in the *Philosophical Transactions*, Numb. 406. was erected at *Wanstead* in *Essex*, in the Year 1727. It's Telescope was but 12 $\frac{1}{2}$ Feet long. That it was upon the same Principles, and for the same Purpose, as those of Dr *Hook* and Mr *Molyneux*; that it's Structure was contrived, and chiefly directed by Mr *Graham*. That Mr *Molyneux*'s Instrument being originally designed to try, whether the bright Star in the Head of the Dragon had any sensible Parallax, could not be altered, as to it's Direction, more than seven or eight Minutes, and so could not well be used to make trial with other Stars. That his own Instrument had a divided Arch of 12 $\frac{1}{2}$ Degrees, reaching to 6 $\frac{1}{4}$ ° on each side the Zenith. And that he could be secure of it's Situation to half a Second, *viz.* he could take by it a Zenith Distance of a Star to half a Second. Which I think to be scarcely possible in the actual Practice, however it may be made out by Theory. For an Arch of half a Second to a Radius of 12 $\frac{1}{2}$ Feet will be but about the 2600th part of an Inch, which I take to be too small an Interval to be distinguished by the Sight, especially when it is liable to be disturbed by Refraction, (even in the Zenith, as it has been asserted) as well as other Impediments. Be this as it will, Dr *Bradley* could not find any annual Parallax of the fixed Stars. Nor could Mr *Molyneux* with his Instrument, although Dr *Hook* says he did with his. Dr *Bradley* suspects the Accuracy of Dr *Hook*'s Instrument, as well as the Truth of the Conclusions deduced from the Observations made with it; that is, that the fixed Stars have really no annual Parallax, though Dr *Hook*, by his Observations makes it out, that they have one of about 27 or 30″. See Dr *Hook*'s Treatise, called, *An Attempt to prove the Motion of the Earth*. So also Mr *Flamstead* will have it, that the Distances of the Pole Star from the Zenith varies, as he found by Observations made for seven Years with a Mural Quadrant of almost seven Feet Radius. His Letter to Dr *Wallis*, *Anno* 1698. *Dec.* 20. is to be seen at *page* 701. of the Third Volume of Dr *Wallis*'s Mathematical Works.

VIII. *Artificial Magnets.*

This is a Name given to pieces of Steel, which being properly touched with Loadstones, do thereby acquire the same Magnetical Qualities which those Loadstones have themselves; that is, will have attractive and directive Powers; some in a greater, and some in a less Degree, according to the size, bigness, sort, hardness or softness, of the Steel, and manner of preparing and touching it. The way of making these artificial Magnets is not new, several did it in some Degree many Years ago. But some of the best that have been made of late Years

Years are those of *Servington Savery*, Esq; who, in the *Philosophical Transactions*, Numb. 414. for the Year 1730. gives a very honest, elegant, and plain Account of them, and how to make them. Afterwards, in the Year 1744. the very ingenious Dr *Knight* (see the *Philosophical Transactions*, Numb. 474.) produced several artificial Magnets of his own Contrivance before the *Royal Society*, some of which consisted of plain Bars of Steel naked, and others of Bars or Blocks of the same Substance, armed with Iron, after the common Manner of natural Loadstones, with much greater lifting Powers than those of Mr. *Savery*. But I do not find the *Doctor* has any where discovered his way of making them, which it is pity he did not, in order for one to see whether his was only an Improvement of Mr *Savery*'s, or some other of the Doctor's own, not thought of by Mr *Savery*, or others before. However, in the Year 1751. there was printed at *Cambridge* a small Tract by *J. Mitchell*, B. A. shewing how to make artificial Magnets in an easy and expeditious Manner, and which (as the Author himself says) are superior to the best natural Magnets, *&c.* Now I shall only take Notice here of two Things he says in this Treatise. He tells us at *page* 19. that the Attraction and Repulsion of Magnets decrease as the Squares of the Distances of the respective Poles increase, but gives no Proof of it by undoubted Experiments of his own, or those of any body else. Sir *Isaac Newton*, Dr *Brook Taylor*, and Dr *Muschenbroeck*, all differ from him about this Matter. Sir *Isaac* says, in the 5th Corollary to the 6th Proposition of the 3d Book of his Mathematical Principles of Natural Philosophy, that *vis magnetica in recessu a magnete decrescit in ratione distantiæ non duplicata*, sed ferè triplicata, *quantum* ex crassis quibusdam *observationibus animadvertere* potui; that is, the magnetical Attraction, as you go from the Magnet decreases, not in the duplicate Ratio of the Distance, but nearly in the triplicate Ratio, as far as I was able to gather from some gross Experiments. Dr *Brook Taylor* (in the *Philosophical Transactions*, Numb. 368.) says from Experiments, which he made, it appears that the Power of Magnetism does not alter according to any particular Power of the Distances, but decreases much faster in the greater Distances than it does in the near ones. And Dr *Muschenbroeck*, at Numb. 390. of the *Philosophical Transactions*, has confirmed by Experiments the same thing, *viz.* that there is no Proportion between the attractive Force of the Magnet and the Distance. His way of doing this being very convincing and ingenious, with two Terrellas or spherical Magnets, whose Poles were in the Extremes of their Axes. He suspended one of these from a Thread over the other, at different Distances from each other, and with Balance put to the end of the Thread, he could weigh the Quantity of the Forces wherewith the Terrella's acted upon one another at different Distances, *&c.* Mr *Mitchell* says, at *page* 74. that most Brass, and much burned Bricks, *&c.* are magnetical, but has produced no Proof by Experiments, without which, in these Matters, no Man can go one Step, or truly affirm or deny any Thing. In a Word, I shall have a better Opinion of these artificial Magnets, and be a better Judge of their Goodness, when the following Queries are answered by Experiments, sufficient in Number and Quality.

1. Whether artificial Magnets in general, have such strong and lasting lifting, and north-pointing Powers, as good natural Magnets, and accordingly whether they are so proper to give the Needles of Sea-Compasses so strong and lasting directive Powers as natural Magnets?

Why I make a Question of this may be gathered from the following Passages of Mr *Savery*'s and Dr *Knight*'s Accounts, in those Transactions of the *Royal Society*, mentioned above. Mr *Savery* says, he touched 37 large Steel Wires, 2.74 Inches long one by one, made a hexagonal Bundle of them, and bound them together with Armour, which then would lift a Key weighing 1125 Grains *Troy* by the South Pole; but, says he, when after each of those Wires had been placed separately in a magnetical Line for two Days, each of them had lost some Virtue; and again, that seven round Bars of Steel, all of a Size, $\frac{3}{8}$ of an Inch in Diameter, and Length above 12 Inches, being prepared, touched, and armed, the same as those Wires, lifted up about half a Year afterwards, more than one Pound; by which it should seem that Mr *Savery* meant they lifted more than one Pound at first.

Dr *Knight* produced, at a Meeting of the *Royal Society*, a compound artificial Magnet, consisting of 12 Steel Bars, which had, in an Experiment before the President, lifted up twenty-three Pound 2 $\frac{1}{2}$ Ounces *Troy*, did here, under all the Inconveniences and Disadvantages of a crowded Room, still lift up 21 Pounds and 11 Ounces *Troy*. That a single armed Block of Steel, which had before lifted 14 Pounds and two Ounces, did here, under the same Disadvantages, as the former, lift thirteen Pounds and seven Ounces *Troy* Weight. Now I confess, that I cannot comprehend how a crowded Room should cause the Magnets to lift up less Weight than they did at first; but rather think they might have lost some of their Virtue.

2. Whether those artificial Magnets that have the greatest lifting Powers, do always give to touched Needles the greatest north-pointing Powers?

3. Whether the Conclusions from Experiments drawn from one of the Ways of measuring the Strength of the north-pointing Power of a touched Needle, by bringing a Magnet, *&c.* towards it, 'till the Magnet just begins to move the Needle from it's fixed Place of pointing, be not truer and more to be depended upon than the other way, by the

greater

greater or less Number of horizontal Vibrations of the Needle, and that of their Times of Vibration?

From what has been said, it is evident to me, that artificial Magnets lose some of their Power in some Time; and if they are found in a longer Time to lose more, they may at last be so weak, as not to communicate to a Needle a North-pointing Power of sufficient Strength; that is, would be good for nothing. It might likewise be proper to observe, whether the Virtue of natural Magnets may not decrease in time, as well as that of artificial Magnets; and which do this the least, for these last are certainly the best.

The natural Magnets of the greatest attractive Powers, that I ever heard of, are three. One is an armed *Terrella*, belonging to the *Royal Society*, presented to them by the Earl of *Abercorn*, which, when his Lordship had it, is said to have lifted 40 Pounds weight. Another of the late Duke of *Devonshire*'s, some Years ago, at the House of the *Royal Society*, did at a Meeting of the Society lift up near 100 weight, if I remember right. Lastly, I have been told, that the King of *Portugal* has got a Magnet that will lift up 6000 Ounces (but what Ounces I do not know). A Son of mine saw a Print and Description of it in the Hands of a Tradesman at *Cork*.

Mr *Muschenbroeck*, (see the *Philosophical Transactions*, Numb. 390.) had a Magnet that would act upon a touch'd Needle at the Distance of 14 *Rhinland* Feet.

The best Philosophers, who have endeavoured to explain the Manner in which the magnetical Virtue of the Loadstone acts, say, that the magnetick Matter of a Loadstone moves in a Stream from one Pole to the other internally, and is then carried back in curve Lines externally, 'till it arrives again at the Pole where it first entered, to be again admitted. The immediate Cause why two or more magnetical Bodies attract each other, is the Flux of one and the same Stream of magnetical Matter through them; and the immediate Cause of magnetical Repulsion, is the Conflux and Accumulation of the magnetical Matter. This is tolerably well made appear by *Des Cartes*, in his Treatise upon the Magnet. See *page* 270, 283. Part 4. of his *Principles* of *Philosophy*; and better by others after him.

I shall now proceed to mention a few Things in general, about the Variation of the magnetical Needle.

1. It has been found by Observations, both at Sea and Land, for near 200 Years last, that the touched Needle of a Sea-Compass does not point exactly to the North or South, but in a few Places, which all lie nearly in the same Meridian. At all other Places, in which Experiments have been made, the pointing of the Needle is not North, but to some other Degree of the Horizon, either towards the North-West, North-East, South-West, or South-East, more or less. It has also been found, that the Variation of the Needle from the North at the same Place alters, more and more, by a very slow Motion. For Example, Mr *Burrows*, in the Year 1580, at *Limehouse* near *London*, observed the Variation to be 11° 11′ easterly. Mr *Gunter*, in 1612, 6° easterly. Mr *Gellibrand*, in 1633. 4° easterly. In the Year 1657. there was no Variation. In 1672. the Variation was 2° 30′ westerly. In 1692. it was 6° westerly. In 1723. 14°. 17′. And Mr *George Graham* (in the *Philosophical Transactions*, at Numb. 487. for the Year 1748.) concludes, that during the Course of 167 Years, *viz.* from 1580, to the End of the Year 1747. the magnetical Needle has moved westwards at *London* 28° 55′.

Hence, if the magnetical Needle at *London* moved equally towards the West, it would go westerly about 10′. 2″ every Year, and perform one Revolution in about 2079 Years. But it does not move equally to the West, according to these Observations; for because in the Year 1580. the Variation was E. 11°. 15′. and in the Year 1657. there was no Variation; in 77 Years, it moved 11°. 15′. And in the Year 1723. it was observed to be 14°. 17′. westerly; and so had moved in 66 Years (from 1657 to 1723.) 3°. 2′. more than it did before in 77 Years; consequently it's Motion was slower from 1580 to 1657, than it was from 1657 to 1723.

Again, in the *Philosophical Transactions*, Numb. 64. Mr *Hevelius* at *Dantzick*, observed the Variation of the Needle to be westward. In the Year 1620, 1°. In the Year 1642, 3°. 5′. In the Year 1670, 7°. 20′. Hence, according to him, the Variation in 50 Years, *viz.* from 1620 to 1670. was 6°. 20′. wherefore, if the Motion were equable, a compleat Revolution by Mr *Hevelius*'s Observations, would be performed in about 2857 Years. He makes the Variation at *Dantzick* to increase every Year 9′. 6″. But how true this is I know not; it can only be found by Experience, whether it be so or not; and if it be true, the Variation at any given Time, past or to come, might easily be known without Observation.

2. The Variation of the Needle in the high Latitudes has generally been found to be much greater than in other Places, as in *Hudson*'s *Streights*, from 68 Degrees of Longitude to 81 Degrees, and Latitude 64°. The Variation was observed by Captain *Middleton* (see Numb. 393. of the *Philosophical Transactions*) to be 46°; and in *Baffin*'s *Bay*, in the Latitude of 80°. the Variation (see a Table in the *Philosophical Transactions*, Numb. 148.) was observed to be 57°. westerly in the Year 1668. Capt. *Middleton* says, the Compass in *Hudson*'s *Streights*, would scarcely traverse at all. He says also (at Numb. 449. of the said Transactions for the Year 1735.) when he was in the Ice in *Hudson*'s *Bay*, the Needle of his Com-

pass would oftentimes lose so much of it's magnetick Virtue, as that it would not traverse at all, any longer than the Quarter-Master kept touching it, but by bringing the Compass to the Fire he recovered it's Virtue. The accurate Mr *Joseph Harris* (see the *Philosophial Transactions*, Numb. 428. for the Year 1733.) says, that being at Sea about *Jamaica*, and the *Havannah*, in the Year 1733. he thought the Virtue of the Needle of the Compass was not always of equal Strength. Sometimes several Observations would agree very well, at other Times the Card would stand indifferently any where, within a Degree or more of it's Meridian; and he observed this several Times. He also says, the Card would differ from itself about 2°. sometimes between the Morning and the Evening of the same Day; and this Difference would continue regular, as it were, for several Days, and then vanish for a Week and more, and then would return as before. This ingenious Person also says (see Numb. 401. of the *Philosophical Transactions*) that in his Voyage from *England* to *Vera Cruz* they observed the Variation with a good Azimuth Compass; but he always found the best Observations they could make, when compared together, differed so much, that they could not be depended upon, to much less than three or four Degrees, and sometimes to half a Point of the Compass. Moreover, the ingenious Mr *George Graham* (see the *Philosophical Transactions*, Numb. 383.) from Observations he made at *London*, of the Variation of the magnetick Needle, found that three touched Needles would exactly settle and point to the same Place, after being removed, by a great Number of Trials made immediately one after another, but yet he found them at different Times of removal to differ considerably from their former Directions. After many Trials he found all the Needles he used would not only vary in their Directions upon different Days, but frequently upon different Times of the same Day; and this Difference would sometimes amount to more than half a Degree in the same Day, sometimes in a few Hours. He also found the Variation has generally been greatest for the same Day, between the Hours of twelve and four in the Afternoon, and the least about six or seven in the Evening, *&c.* This ingenious Person also, in the *Philosophical Transactions*, Numb. 389. for the Year 1723. gives an Account of a Dipping Needle of his, with Observations to try if the Dip and Vibrations were constant and regular.

All these Inequalities and Irregularities of the Variation of the magnetick Needle (and many others to be found in Authors) may in part be true, and proceed from true Causes assigned, and partly be not so, but set down as true from fallable, uncertain, erroneous Observations of those that made them, which might proceed, 1. From the Nearness of Iron. 2. The Weakness of the magnetick Virtue of the Needle disposing it to stand indifferently at any Point of the Compass it is set to. 3. The Friction of the Cap. 4. The Motion of the Ship. 5. The Resistance of the Air, which in high Latitudes may be so great in the cold Weather, as to considerably affect the true pointing of the Needle. 6. The great Refraction, in high Latitudes, causing an Error in finding the Meridian Line, which Line, in high Latitudes, cannot be determined with any great Certainty, by any Method whatsoever, in my Opinion; because in these Latitudes all the Points of the Compass begin almost to be nearly Meridians, (for at the Pole itself, every Point of the Horizon is South at the North Pole, and North at the South Pole,) and because the Amplitudes and Azimuths of the Sun or Stars, cannot be taken in those high Latitudes, without Error, if at all, by reason of the too great obliquity of the Arches of the diurnal Motion in those Latitudes; and since the meridian Lines cannot be found, but by an Amplitude or Azimuth observed, it is evident the Variation cannot be had to any great Exactness, if at all, in such high Latitudes.

Mr *Wright*, in his Treatise of the Errors of Navigation, first published before the Year 1600, gives a Table of the Variation of the Compass, as taken by himself, both at Sea and at Shore, in a Voyage of the Right Honourable the Earl of *Cumberland*, in the Year 1598. He also gives two other Tables of the Variation at the End of that Book, the one pretty long and extensive, with regard to the principal Places of the Earth or Sea; and the other a lesser Table, of *Peter Plancius*'s. Dr *Halley* (in the *Philosophical Transactions*, Numb. 148. for the Year 1683.) gives another Table of the Variation in different Places, at different Times. But, in my Opinion, this Table is much too short and imperfect for a Foundation to build a true Theory of the Variation upon, though the Doctor himself did take it to be sufficient in that Transaction, where he supposes the whole Globe of the Earth to be one great Magnet with four fixed magnetical Poles, near each Pole of the Equator two, and that in those Parts of the World which lie near adjacent to any of those magnetick Poles, the nearest Pole being always predominant over the more remote, *&c.* Upon this Supposition the Doctor ingeniously enough accounts in general for the Variations and Alterations, when they are always the same at the same Place; but since they have been observed to be different at the same Place. The Doctor had laid aside, for some Years, this Theory of his of four fixed magnetical Poles, and (in the *Philosophical Transactions*, Numb. 195.) resumed it again with Alterations and Amendments, making two of the magnetical Poles moveable, *&c.* and conjectures, that these two moveable Poles perform one Revolution in about 700 Years, at the end of which Time all the Variations will be the same again in the same Places, *&c.* Mr *Bond*, in his *Treatise of the Longitude found*, pag. 7. says this Revolution will be once in 600 Years, and that the Earth has but two magnetick Poles, at 80 $\frac{1}{2}$ ° Distance from the Poles of the Equator, *&c.* The *Frenchmen* who, in the Year 1737, went to measure a Degree of the Earth, at *Tornea*, in the

the Latitude 65°. 50′. found the Variation to be there 5°. 5′. weſtwardly. And in the Year 1695. it was found to be there 7° weſtwardly; hence, in 42 Years the Variation has moved 1°. 55′. and ſo one Revolution, ſuppoſing the Variation to alter equally, will not be performed 'till the end of about 8062 Years. Now neither Dr *Halley*'s, or any Body's elſe Suppoſitions to account for the Cauſes of the Variation, can reconcile theſe ſeveral Differences of the Times of the Revolutions of the Variation that I have given above, and therefore their Hypotheſes cannot be true.

In ſhort, this Affair of the Variation is very uncertain, it's true Cauſe not rightly known; nor have we any true and fixed Rules how to compute what it is or will be at a given Place, at a given Time. Tables of the true Quantity thereof for ſeveral Years, at many different Places, under the ſame Meridians and Parallels, being wanted to build a true Syſtem upon. From ſuch Tables only it can be gathered how the Variation at given Places alters, and at what Rate, and according to what Regularity, and whether any of theſe Things can be diſcerned or not.

Dr *Halley*'s Chart of the Variation is very ingenious, but how true it is I cannot ſay. If true for the Time he made it, it cannot be ſo in future, becauſe of the Alteration of the Variation at a given Place.

There are ſeveral Authors, our own Countrymen, who have wrote upon the attractive Virtue of the Magnet. As *Gilbert de Magnete*, *Robert Norman*, and Mr *Burroughs*'s new Attractive, wherein is an Account of the Variation. The Lord *Paiſley*'s Treatiſe about the attractive Virtue of the Loadſtone, with Calculations and Tables, *&c.* And Mr *Muſchenbroeck*'s *Diſſertatio de Magnete.* This laſt by an ingenious *Dutchman*, who has wrote well upon the Subject.

IX. *A New contrived Mariner's Compaſs.*

This Inſtrument is deſcribed in the *Philoſophical Tranſactions*, Numb. 495. it differs from the others that are commonly made, in having it's Needle in ſhape of a Parallalepipedon. It's Card, conſiſting of unſtiffened Paper, and a light thin Circle of Braſs, divided into Degrees, *&c.* An ivory Cap, turned ſo as to receive a ſmall bit of Agate at the Top. The Point ſupporting the Card is a common ſewing Needle. The very ingenious Dr *Knight*, who exhibited one of theſe Compaſſes before the *Royal Society* in the Year 1750. made by Mr *Smeaton*, will have theſe Alterations to be for the better, both for the true and well pointing of the Needle, and it's long Continuance to do ſo. But although the great Number of Experiments the Doctor has tryed and made, many of which are to be ſeen in the *Philoſophical Tranſactions*, at Numb. 474, in the Year 1744. is a plain Proof of his Skill in Magnetiſm; yet I am not quite ſatisfied with what he ſays of the Faults in the Compaſs Cards, and his Remedies. His Experiments about this to me ſeem to be too few, to deduce from them a general Rejection of the Make and Faſhion of the old Cards, as being leſs perfect than thoſe he recommends. Skilful Mariners (of which we have enough) who have uſed Compaſſes at Sea for many Years, may at leaſt be allowed to be as competent Judges of the Goodneſs of their Compaſſes, as the Doctor; and I have always been apt to think, that real Improvements of Inſtruments that have been generally uſed for more than two Hundred Years, would have been thought of before the Year 1750. The Plants and Trees of the Gardens, of the Arts and Sciences, cultivated by the Dung of Ambition, and nouriſhed with the Waters of Intereſt, are very ſubject to be blaſted by the Winds of Error, and ſometimes ſtunted by the Weeds of Impoſition.

X. *The Uſe of an* Azimuth Compaſs, *of a new Contrivance, by Capt.* Middleton, *is to be found in the* Philoſophical Tranſactions, *Numb.* 450. *for the Year* 1738.

The Captain ſays, that the Variation of the magnetical Needle at Sea may be found by it, with greater Eaſe and Exactneſs, than by any other Azimuth-Compaſs contrived before that Time, *viz.* the Year 1738. But as he conceals the Deſcription of his Inſtrument, we don't well know how to judge of it's Excellency, he telling us, that thoſe who have the Inſtrument before them, have no Occaſion for a Deſcription of it. That he himſelf found by Experience it was effectual. He only ſhews the Manner of uſing it; ſays that one Perſon may manage it, whereas the old Compaſs requires ſeveral Perſons, which alſo makes it ſubject to many great Errors, as he will have it.

The Main of this new contrived Compaſs of the Captain's, gathered from the ſhort Account he gives of it's Uſe, is the taking the Sun's Altitude by Reflection, *viz.* bringing it down to the Horizon, in ſome ſuch Manner as is done by Mr *Hadley*'s Sea-Quadrant. It carries a Teleſcope with a vertical Hair within it. It is very poſſible this is a very good Compaſs, but it's dearneſs may be ſome Objection to it's general Uſe.

XI. *Celeſtial Globes ſomewhat improved.*

The late ingenious Mr *Senex*, at Numb. 447. of the *Philoſophical Tranſactions* for the Year 1738. ſhews how to make the Poles of the Diurnal Motion in a Celeſtial Globe paſs round the Poles of the Ecliptic. His Contrivance is this; the Poles of the diurnal Motion do not

enter

enter into the Globe, but are affixed at one End to two Shoulders or Arms of Brass, at the Distance of 23 ½ Degrees from the Poles of the Ecliptick. These Shoulders at the other end are strongly fastened on to an iron Axis, passing through the Poles of the Ecliptick, and is made to move round, but with a very stiff Motion; so that when it is adjusted to any Point of the Ecliptick which you desire the Equator may intersect, the diurnal Motion of the Globe about it's Axis will not be able to disturb it.

The Reason of such a Contrivance is to render the Use of the celestial Globes more perfect and lasting, than they can be without it; because the Places of the fixed Stars, put down upon celestial Globes, made at any given Time, will not be true but for some Years, by reason of the Procession of the equinoctial Points, *viz.* the Intersection of the Planes of the Ecliptick and Equator, causing the Distances of the fixed Stars from those Points continually to alter; this was observed by the antient Astronomers, who made the Sphere of the fixed Stars to move about the Poles of the Ecliptick with a slow Motion, so that all the fixed Stars in the Ecliptick or its Parallels, will go once round in the Space of 25920 Years, after which Time the Stars will again return to their former Places.

Hence the Longitude of the fixed Stars alter one Degree at the End of every 72 Years, or 50″ every Year. Therefore celestial Globes made many Years ago, or such as are new made, many Years hence, cannot be used with Certainty, the Places of the Stars set down upon them altering too much, at the Ends of those Times, and to remedy this in some measure is what Mr *Senex* proposes to do, by his Contrivance abovementioned.

Note, There is another Way to do the same Thing by Dr *Latham*, at Numb. 460. of the *Philosophical Transactions*, for the Year 1741.

The very ingenious Mr *James Ferguson*, (at Numb. 483. in the Year 1747. of the *Philosophical Transactions*,) has added another Improvement to the celestial Globe, chiefly for finding the Times of the Rising and Setting of the Moon, with her Time of coming to the South, tolerably exact, without an *Ephemeris*; but nearer the Truth, by having given the Moon's Latitude for the Day by an *Ephemeris*.

Plate 4. Fig. 24.

His Contrivance is as follows: On the Axis of the Globe above the Hour circle is fixed the Arch A at one End by the Screw D (see Fig. 24.) so as to leave sufficient Room for turning the Hour Index. The other End at B, being always over the Pole of the Ecliptick, has a Pin fixed into it, whereon the Collets C and B, are moveable by their Wires F and G, when the Screw E is slackened, and may be made fast at Pleasure by this Screw; so that the turning the Globe round will carry the Wires round with it, shewing thereby the apparent Motions of the Sun and Moon by the little Balls on their Ends at H and I. On the Collet C, in which the Sun's Wire is fixed, there is also fixed the circular Plate L, whereon the 29 ½ Days of the Moon's Age are engraven, which have their beginning just below the Sun's Wire, consequently the Plate L cannot be turned without carrying the Sun's Wire along with it, by which means the Moon's Age is always counted from the Sun; and the Moon's Wire being turned so as to be under the Day of her Age on this Plate, will set her at her due Distance from the Sun at that Time. These Wires being Quadrants from C to H, and from B to I, must still keep the Sun and Moon directly over the Ecliptick, because the Centre of their Motions at C and B is directly over the Pole of the Ecliptick, in the artick Circle. But because the Moon does not keep her Course in the Ecliptick, as the Sun does, having a Declination of 5 ½ Degrees on each side of it in every Lunation, she is made to screw on her Wire as far on both sides, as this her Declination or Latitude amounts to; for this purpose K is a small Piece of Paste-board to be applied over the Ecliptick at right Angles, the middle Line oo standing perpendicularly thereon; from this Line there is maked 5 ½ Degrees on each side upon the outward Limb, which reaching to the Moon makes her to be easily adjusted to her Latitude at any Time. *Note*, The Horizon of the Globe should be supported by two semicircular Arches, instead of the usual Way of doing it by Pillars, because the Arches will not stop the Progress of the Balls, when they go below the Horizon in an oblique Sphere.

The U S E.

To rectify the Globe. Elevate the Pole to the Latitude of the Place, then bring the Sun's Place in the Ecliptick to the brazen Meridian, and set the Hour Index to XII at Noon. Keeping the Globe in this Position, slacken the Screw E, and set the Sun directly over his Place in the Meridian; which done, set the Moon's Wire under the Day of her Age, for that Time, on the Plate D, and she will stand over her Place in the Ecliptick, for that Time, and you will see in what Constellation she then is. Lastly, fasten the Wires by the Screw E, and the Globe will be rectified; this done, turn it round in the usual Way, and you will see the Sun and Moon rise and set for that Day, on the same Point of the Horizon, as they do in the Heavens; the Times of their rising and setting are shewn by the Hour Index, which also shews the Time of the Moon's passing over the Meridian. If you want to find the Times of the rising and setting of the Moon to greater Exactness, find her Latitude for that Day by the *Ephemeris*; and as it is South or North, screw her so many Degrees from the Ecliptick, measuring them by the Pasteboard K, applying it to the Ecliptick, as abovementioned; and then turning the Globe round, you will see the Time of the Moon's rising and setting by the Hour Index, and her Amplitude on the Horizon

Horizon for that Time, as it is affected by her Latitude, which will sometimes be very considerable. *Note*, All the Phænomena of the Harvest-Moon become very plain by this additional Contrivance, which is curious enough. But some People may think the common way of finding the Time of the Moon's coming to the South, (given in what is called the *Julian* Calendar, to be found in some Books of Navigation) which is easy and well known, and may be exact enough for many Purposes, may be sufficient, without such an additional Contrivance as this, to do the same Thing.

XII. *Clocks, and other Time-Keepers.*

The Instruments for measuring Time, are Sun-Dials, Water Dials, Sand-Dials, a single Pendulum, or Clocks and Watches. Sun-Dials have been enough treated of by Mr *Bion*. Water-Dials and Sand-Dials are now in no very great Esteem, although some of them are ingenious enough; they are treated of in several Writings, *viz.* in *Vitruvius*'s *Architecture*, *Gesner*'s *Pandecticks*, *Martinelli* of *Elementary Clocks*, *Archangelo Maria Radi.* of *Sand-Dials*, *Schottus* in his *Technica Curiosa*, &c.

The old indented Wheel-Clocks that were regulated by horizontal Balances, and generally used 'till the Year 1660, are now in no great Use, being far more imperfect than those which have Pendulums adapted to them to regulate their Motion; nor do Watches measure Time so equable and true as Pendulum-Clocks. *Galileo*'s Son was the first who applied a Pendulum to a Clock in the Year 1649. And the great Mr *Huygens*, in the Year 1673, published his famous Book, called *Horologium Oscillatorium*, explaining the Manner of doing it with a Figure of the Clock, being indeed the very same as that Clock described by Mr *Bion*.

There are several more Things about Clocks, treated of by Mr *Huygens*, in his Book abovementioned, the chief of which are what follow.

Mr *Huygens*, at *page* 13. of his *Horologium*, gives two Ways to regulate the Motion of a Clock. The first is by the Observations of the Transits or Passages of some of the known fixed Stars, and the other by means of the Times given by the Shadow of the Sun upon a Dial, together with an Equation Table.

The Substance of what he says is this, choose some proper Place for the Eye, from which you may see several of the fixed Stars instantly disappear in passing behind high Buildings, and in that Place fix a Plate with a Hole in it to look through, of the Magnitude of the Pupil of the Eye, in order to bring it always to the same Point; this done, observe the Time shewn by the Clock the Moment you see a known fixed Star vanishing from your Sight behind the Building, the Eye looking through that fixed Hole, and doing the same Thing the next Night, or rather some Nights after, if the Interval of Time between two such Observations of the same Star be only one Day, and the Time given by the Clock in the later Observation be less than that given in the former, by three Minutes and 56 Seconds, then it is evident the Clock goes right, because every Revolution of the fixed Stars is less than a mean solar Day by that Quantity. I say a mean solar Day, because solar Days, or the Intervals of the Sun's Appulses to the Meridian are unequal. But if the same Observations be repeated some Nights after, the Computation of that Difference must be made at each. For Example, suppose at the first Observation a Star vanishes at $9^h\ 30'\ 18''$ by the Clock, and seven Days after the same Star appears to vanish by the Clock at $8^h.\ 50'.\ 24''$. the Difference of these Time is $39'.\ 54''$. and a seventh Part of it is $5'.\ 42''$ which shews, that the diurnal Period of the Clock is $5'\ 54''$. longer than a sidereal Day, whereas a solar Day of a mean Length is but $3'.\ 56''$. longer, and by Consequence the diurnal Period of the Clock is $1'.\ 46''$. too long. *Note*, The Intervals of Time between the successive Appulses of a fixed Star to the same vertical Circle, are equal to the Intervals between it's Appulses to the Meridian. But to avoid the Uncertainty of the Air's Refraction, as much as may be, the Face of the Building should be as near the Plane of the Meridian as possible. *Note also*, The Stars made Choice of should be those near the Equinoctial, by reason of their quick Passages.

The Motion of a Clock may be examined another way by the Sun. But we must here have regard to the Inequality of the Natural Days; for as has been said, these are not all equal between themselves, and though the Difference be but small, yet it often happens, that in the Space of several Days it becomes so considerable, as it must not be disregarded; for although we should have the most perfect Description of the solar Days, and the Motion of the Clock most exactly agreed with the true Measure of those Days; yet it would necessarily happen that at certain Times of the Year they would differ from one another a Quarter of an Hour, or even Half an Hour, and then again at stated Times return to their Agreement; as is to be seen in the Tables of the Equation of Days.

Now, to compare the Motion of a Clock by that of the Sun on a Sun-Dial, take the Equation of any Day, when the Clock is set to go with the Sun-Dial, and the Equation of that Day, you want to know how true the Clock goes. If the former Equation be greater than the latter, the Time by the Clock will exceed that of the Sun-Dial, by the Difference of those Equations; but if the Equation of the latter Day be found to be greater, the Time by the Sun Dial will be the greatest. For Example, If on the Fifth of *March* the Clock and Sun going together, the Equation of that Day be found in the Table to be $3'.\ 11''$. and I want to know, whether or not, on the 20th Day of the same Month, the Clock truly meaſures equal Time, the Equation for this Day in the Table will be $7'.\ 27''$. which becauſe it is greater than that for the 5th, by $4 . 16'$. the Time ſhewn by the Sun-Dial will be ſlower than that given by the Clock by the ſaid Difference; wherefore if it be found different, it is eaſy to gather from thence how much every Day the Sun Dial goes faſter or ſlower than the Clock.

Note, The Conſtruction of an Equation Table of this Kind, is founded upon a twofold Cauſe, both known to Aſtronomers, *viz.* upon the Obliquity of the Ecliptick, and the Irregularity of the Sun's Motion, becauſe it is deduced as well by Reaſon as Experiments founded upon theſe very Clocks, without which it could not be inferred; for Obſervations of the Time of the Sun's coming to the Meridian every Day for many Months have moſt evidently been found to agree.

Now, if upon Trial by both Methods, but rather by the firſt, the daily Error of the Clocks going ſhould be ſo great as to amount to $3'$. or $4'$. this muſt be remedied by lengthening or ſhortening it's Pendulum, in doing of which the following Rule ſhould be obſerved, *viz.* the daily Motion of the Clock will be accelerated or retarded by ſo many Minutes, as the Pendulum is ſhortened or lengthened about $\frac{7}{120}$ Parts of an Inch taken ſo many times, as there are Minutes. The going of the Clock being thus almoſt corrected, the reſt of the Correction is performed by moving upwards or downwards upon the Rod V V a ſmall Weight D, as I have ſaid before. (See the Figure in the 17th Plate of the Book of *Mathematical Inſtruments.*)

Mr *Huygens* ſays, that two ſuch Clocks as theſe were carried to Sea in the Year 1664, in order to find the Longitude by them, being moved by a Spring inſtead of a Weight; and to avoid their being affected by the Motion of the Ship, the Clocks incloſed in a braſs Cylinder, were ſuſpended from a ſteel Ball, and the Stern, which continued the Motion of the Pendulum, (which Pendulum was half a Foot long) produced downwards, and governed it, was in Shape of the Letter F inverted; for fear leaſt the Motion of the Pendulum ſhould become circular, and thereby the Ceſſation of it's Motion might ariſe. The Captain of the Ship that had theſe two Clocks being in Company at Sea with three others, on the Coaſt of *Guinea*, and ſailing from thence near the Equinoctial, had a Conſultation with the other Captains to know where they were, they produced their Journals, the Captain with the Clocks uſing them to correct his Journal, by comparing their going with the Motion of the Sun. The Event was, that they all differed very much in their Reckoning from this Captain's, *viz.* one ſaid he was 80 Leagues, another 100, and another ſtill more, from the Land, but this Captain ſaid he was but about 30 Leagues, which was found to be true, and the next Day they arrived at one of the *African* Iſlands, called *Del Fuego.*

Mr *Huygens*, at *page* 17. of his *Horologium Oſcil.* gives a more particular Account of this, as well as other Obſervations of other People finding the Longitude at Sea by other of theſe Clocks, whoſe Pendulums were half a Pound weight, and about nine Inches long, the Motion of whoſe Wheels was by a Weight, the whole Clock being put into a Caſe four Foot long, and at the Bottom of the Caſe there was hung an additional Weight of more than 100 Pounds, thereby to keep the Inſtrument better ſuſpended in the perpendicular Poſture.

This great Man Mr *Huygens*, in his *Horologium Oſcil.* gives two other Contrivances for the equable going of his Clock, and for the Preſervation of it's Motion at Sea, in blowing Weather, both of which, not being long, are as follow: To the Crown Wheel, in which the Pallets of the Pendulum play, is hung a ſmall Weight from a ſlender Chain, by which it is only moved, all the reſt of the Machinery acting no otherwiſe than that in every half Second, this little leaden Weight will be reſtored to it's firſt Altitude, almoſt after the ſame Manner as is to be ſeen in the Conſtruction of our Clock already given; where one Weight is raiſed up by the Line, while the other (*viz.* that of the Pendulum) by it's Gravity preſerves the Motion of the Clock. Now by this Contrivance, every Thing being reduced, as it were, to one Wheel only, the Clock will appear to have a greater Equality in it's Motion than it had before, *viz* according to the former Conſtruction. And ſomething happened here very remarkable. When two ſuch Clocks were ſuſpended from the ſame Beam, ſupported upon two Props, the Motion of the Pendulum of each agreed ſo well by reciprocal Beats, as not in the leaſt to vary, the Noiſe of each being always heard at the ſame Moment. And if by ſome means this Harmony was diſturbed, it would be reſtored again of itſelf in a ſhort time; I at firſt wondered at this uncommon Event, but afterwards, upon ſtrict Enquiry, I found the Cauſe thereof to exiſt in the Motion of the Beam itſelf, although this was ſcarcely ſenſible. For the alternate Vibrations of the Pendulums, by any the leaſt Weight communicate ſome Motion to the Clocks, and this Motion is impreſſed upon the Beam itſelf, and by that means if each Pendulum did not moſt exquiſitely move with reciprocal Vibrations, it would at length happen, that the Motion of the Beam would altogether ceaſe, which Cauſe nevertheleſs will not take place but when the Motion of the Clocks from thence are moſt equable, and conſentaneous between themſelves.

The other Contrivance of Mr *Huygens*'s, to preſerve the Motion of the Clock at Sea in
Plate 2. great Agitations of the Ship, is as follows. (See Plate 2. Fig. 22.) The Pendulum is of a
Fig. 22. triangular Form as A B C, at the Bottom of which the leaden Weight B is hung, and at the Angles A and C of it the ſame is alſo ſuſpended from the cycloidal Plates or Cheeks E D, F G. The middle

middle of the Base A C receives the perpendicular Axis H K of the horizontal Crown Wheel N, which turns at K by means of the Teeth of that Wheel, during which Time the Weight B moves the Base A C of the triangular Pendulum A C B. The Motion of all the Wheels, is not by a Weight, but by a steel Spring enclosed in a Barrel, and the Crown Wheel is below the other Wheels of the Clock. L L are the small lenticular Weights to regulate the Motion of the Pendulum.

The Manner of the Clock's Suspension is expressed in the 23d Figure, Plate 2. where the Case AB is set upon two Axes, (one of which, as C, can only be seen) fitted to an iron Rectangle D E, which Rectangle likewise moves about the Axes F G, going into the Ends F, G of the iron Gnomon F H K G, which Gnomon is firmly fixed to the Top of that part of the Ship wherein the Clock is, and to the Bottom of the Case is hung a Weight of fifty Pounds. By this Contrivance the Clock will preserve it's upright Posture in all Inclinations of the Ship. The Axis C, together with that opposite to it, is so placed, as to be in a right Line with the two Points of Suspension of the Pendulum, that we have already spoken off, by which means it's oscillatory Motion cannot move the Clock, than which there is nothing more liable to destroy the Motion of the Pendulum. Moreover, the Thickness of the Axes C C, of about an Inch, and the Bigness of the Gravity of the Weight hung on at the Bottom, are a Check upon the too great Motion of the Clock, and make it continually return to it's upright Posture, when the Ship should happen to be in a great deal of Agitation.

Now one of these Clocks, thus managed, being tried at Sea, seems more promising than the others that have been already tried, to avoid being affected with the different Motions of a Ship. Thus far from Mr *Huygens*.

One Mr *Harrison*, several Years ago, made a Clock to go at Sea of a different Construction from those of Mr *Huygens*'s, whose Pendulum consisted of several horizontal Rods that see-sawed, or vibrated up and down. The Rods were partly of Wood, and partly of Brass, in order to check one another's lengthening or shortening by heat or cold, and thereby keeping the Motion of the Clock equable. I have not seen the Clock, and have been only told this, by a Person who saw it many Years ago. The Inventor himself now lives in *London*. One of them was shewn to the Lords of the Admiralty, and approved of by them.

That great *Mechanick*, the late Mr *George Graham*, has made some of the best Clocks in the World for the true measuring Time; they are partly described in the *Philosophical Transactions*, at Numb. 432. There is also another curious Clock of his, described in Dr *Desaguliers*'s Course of Experiments to measure Time to quarter Seconds.

Gesner, in his *Pandecticks*, says, that *Richard Wallingford*, an Abbot of *St Albans*, who lived in the Year 1326, made a Clock with wonderful Art, that had not it's Fellow in all the World. Some Clocks have been made to shew apparent Time, *viz.* to go with the Sun at all Times of the Year. These are Curiosities indeed, but little else. Some Clocks shew the Sun's annual Motion by the Addition of a Wheel going once round in 365 Days 5 Hours and 49 Minutes; and others shew the Moon's synodical Revolution of 29 Days 12 Hours, and 44 Minutes, being the mean Time from one Conjunction to the next. There are other Clocks that shew the Motions of the Planets particularly; and that Instrument shewing the Motions and Phases of them all, of which there are many amongst us, made by our Mathematical Instrument-Makers, is called a *Planetarium* or *Orrery*. And indeed that described by me at *page* 189, is not so perfect, as some that have been made since, the superiors Planets not being in it.

Gasper Scottus the Jesuit, in his *Technica Curiosa, seu Mirabilia Artis*, printed in the Year 1664, in Quarto, says, that Father *Schirley de Reita*, in a Book called, *The Eye of Enoch and Elias*, gives the Construction of a Planetarium, representing all the Motions of the Planets, both true and mean, their Stations, Retrogradations, and Directions, without Epicycles or Equations, and that with a few Wheels, endless Screws and Pullies. The first Wheel of this Instrument that gives Motion to all the rest, is moved round by the Fall of Water. The Sun's Motion in it is only 365 Days, and the Disks of the Planets are too great. See a particular Account of all the Parts of this Instrument in the aforesaid Book of *Scottus*'s; wherein likewise he treats of the different Machines that were in use to measure Time. The Book is curious, full of Figures; and although he sometimes fails in the Exactness to what we are now arrived, yet in my Opinion the Book is valuable enough.

At the End of Mr *Huygens*'s *Opuscula posthuma*, printed in Quarto at *Leyden*, in the Year 1703, there is a masterly Description of the Motion of the Planets by Clock-Work, at least upon the Astronomical Principles assumed by him. He indeed herein makes the annual Motion of the Earth to be but 365 Days, when it should have been near 6 Hours more, but this he did to avoid increasing the Number of Wheels. He gives the best way of finding the Number of Teeth, that will make all the Motions the nearest the Truth possible, by the Method of finding the nearest Ratio in lesser Numbers, that shall approach to a given Ratio in greater Numbers. The Number of the Teeth of one of his Wheels is 300, of another 206, of another 219, of another 166, of another 158, of another 137, *&c.* He gives two very large Figures of his Machine, which is 2 Feet over.

At the End of the first Volume of Dr *Desaguliers*'s Course of Experimental Philosophy, printed in the Year 1745, is a Description of a *Planetariam* of the Doctor's, with Figures, which

which I take to be one of the best of these Instruments that has yet appeared, and is 3 Feet over. The *Doctor* says, these sort of Machines had the Name of *Orrery* given to them by Sir *Richard Steele*, and the first that was made in *England* was by the late Mr *George Graham* about the Year 1712, which was sent over to Prince *Eugene*. But that it only shewed the Motion of the Moon round the Earth, and that of the Earth and Moon round the Sun. That this Instrument being in the Hands of Mr *Rowley*, a Mathematical Instrument-Maker, he copied it, adding Improvements of his own, and thereby got all the Praise due to Mr *Graham*, though Sir *Richard Steele* did not know Mr *Graham* to be the first Inventor.

In the *Philosophical Transactions*, at Numb. 479. for the Year 1746. the ingenious Mr *Ferguson* describes the *Phænomena* of *Venus*, as represented in an Orrery of his, agreeable to the Observations of S. *Bianchini*, who will have her Axis to be inclined 75°. from a Line, supposed to be drawn perpendicular to the Plane of the Ecliptick, and that her diurnal Motion is performed in 24 Days and 8 Hours. Whereas in other Orrerys her Axis is perpendicular to the Plane of the Ecliptick, and her diurnal Motion about it is only 23 Hours. This *Gentleman*'s Improvement and Discoveries are ingenious enough, but it's Usefulness will be best perceived by the Inhabitants of *Venus*.

I shall here say no more of these very elaborate, expensive and artificial Clock-Work Representations of the Motions, and Appearances of the heavenly Bodies, which at best are only amusive, and apt to afford more Honour to the Inventors, and Interest to the Makers of them, than any ways really promote the most useful Parts of Astronomy. The next Things I shall mention are some more concerning the useful Instruments Clocks.

Mr *Huygens*'s second Pendulum Clock, described by Mr *Bion*, has five Wheels of 15, 24, 48, 48, and 80 Teeth, and 2 Pinions of 8 Teeth each. Which are too many. For three Wheels of 25, 72, and 80 Teeth, and 2 Pinions of 8 and 10 Teeth, or else 3 Wheels of 25, 64 and 90 Teeth, and those 2 Pinions, or 3 Wheels of 30, 60, and 80 Teeth, and those two Pinions, or three Wheels of 21, 75, and 80 Teeth, and 2 Pinions of 7 and 10 Teeth, will cause one Revolution of Wheels, with the greatest Number of Teeth, to be performed in one Hour, or 3600 Seconds; and thence to get a Revolution of 12 Hours, two Wheels of an equal Number of Teeth, with a Pinion of 6, and another Wheel of 72 Teeth, or a Pinion of 7, and a Wheel of 84, or a Pinion of 8, and a Wheel of 96, or a Pinion of 9, and a Wheel of 108 Teeth, will cause one Revolution in 12 Hours. *Note*, The way of finding the Number of Teeth of the principal Wheels and Pinions of a Clock is founded upon this Theorem, *viz.* that the Product of the Teeth of the Pinions multiplied by half the Number of Vibrations, is equal to the Product of the Teeth of all the Wheels; and hence, by the Method of finding all the Divisors of a Number, may be obtained all the several different Numbers of Teeth of the Wheels and Pinions that will produce one Revolution in the same given Time.

The Length of a Pendulum is the Distance from it's Point of Suspension to it's Centre of Oscillation. A Pendulum, whose Length is 39.2 Inches will vibrate one of it's smallest Arches in one Second of Time. The Distance of the Centre of Oscillation of a simple Pendulum from it's Point of Suspension, may be found by saying, As the Sum of the Weight of half the Rod, and the Weight of the Bob, is to the Sum of $\frac{1}{3}$ of the Weight of the Rod, and the Weight of the Bob, So is the Distance from the Point of Suspension, to the Centre of Gravity of the Bob, to the Distance of the Centre of Oscillation from the Point of Suspension. Hence the Centre of Oscillation lies above the Centre of the Bob. But when the Weight of the Rod is exceeding small, the Centre of Oscillation will be so near the Centre of Gravity of the Bob, or its Centre of Magnitude (supposing it to consist of uniform Matter of the same Density) that all three may be taken for one another, without any sensible Error. Twice the Time of one Vibration of a Pendulum, whose Centre of Oscillation describes the shortest circular Arch possible, is equal to the Time of a Body's perpendicular fall by the Force of Gravity, through a Space twice the Length of the Pendulum. The Times of the Vibrations of two Pendulums of different Lengths, when their Centres of Oscillation describe the smallest circular Arches possible, are in the subduplicate Ratio of the Lengths of the Pendulums. Hence it is easy to find the Length of a Pendulum that shall perform one of it's least Vibrations in a given Time: Suppose any given Number of Seconds. For it is but saying, As the Square of 60 Seconds, is to the Square of the given Number of Seconds, So is 39.2 Inches to the Length of the Pendulum wanted. The Time of the Vibration of a Pendulum, whose Centre of Oscillation describes any Arch of a Circle, will be to the Time of the Vibration of the least Arch possible, nearly as the vibrating Arch is to twice it's Chord. Hence because the Times of the perpendicular Fall of all Bodies, through any Chords of the same Circle, are all equal between themselves; the Times of the Vibration of a Pendulum, describing small Arches of a Circle, not much different from their Chords, may be taken as equal between themselves, *viz.* equal to the Time of one Vibration of it's smallest Arch. But since these Times of the Vibrations of the small Arches are not exactly and geometrically equal to one another, the longer Arches being really a longer Time describing than the shorter; and as many small Errors, separately considered, do not deserve Notice, yet a great many put together, must be taken Notice of and avoided; and for this Mr *Huygens*, in his *Horologium Oscill.* proposes a Remedy, *viz.* by making the Centre of Oscillation of the

Pendulum describe the Curve of the Cycloid, instead of the Arch of a Circle by means of two other equal Cycloids, by which Contrivance all the Vibrations, both great and small, will be performed in the same Time; and the Time of any Vibration in this Cycloid is to the Time of the perpendicular Fall through the Length of the Pendulum, as the Circumference of a Circle is to twice the Side of the Square inscribed in that Circle. *Note*, The Demonstrations of the major Part of what is here said about Pendulums are given by Mr *Huygens* and others; and the Pendulum is supposed to move in a Vacuum free from the external Resistance of the Air, or any other Fluid in it.

Sir *Isaac Newton*, in the sixth Section of the second Book of his Mathematical Principles of Natural Philosophy, says, that all the Oscillations in the Cycloid of a Pendulum, moving in a Fluid which resists it's Motion in the Ratio of the Velocity will be isochronal. And if the Medium a Pendulum moves in resists it in the duplicate Ratio of the Velocity, the very short Oscillations are the more isochronal, and the shortest of all are performed nearly in the same Times as in Vacuo, and the Times of them that are performed in greater Arches, are somewhat greater. And also the Times of all Oscillations, both great and small, seem to be prolonged by the Motion of the Medium the Pendulum swings in. This is only Theory. But in the general Scholium Sir *Isaac* deduces from some Experiment he made with Pendulums, that the Resistance of the Air to Globes moving in it, when they move swiftly, is nearly in the duplicate Ratio of the Velocity, but when slowly, a little greater than in that Ratio. He also says, that the Resistance of Globes moving in the Air is nearly in the duplicate Ratio of their Diameters, and makes the Resistance of the Rod of small Pendulums to be so considerable, as to be more than one third Part of the Resistance of the whole Pendulum. He makes the Resistance of a Pendulum vibrating in Water, to it's Resistance when vibrating in Air, to be as 535 is to 1', *&c.* from all which, joined to other Considerations, we may certainly conclude, that the Resistance of a Pendulum with a globular Bob, when swinging in the Air, causes the Times of the Vibrations to be longer than they would be in vacuo, and that the denser the Air is, the longer will be their Times of Description; and the same is true of lenticular Bobs, although in a less Degree, because these Bobs meet with less Resistance from the Air than spherical ones of the same Breadth.

Again, the late Dr *Derham*, at Numb. 480. of the *Philosophical Transactions*, for the Year 1736, recites some Experiments he made on Pendulums vibrating in vacuo, and says, that the Arches of Vibrations in Vacuo were larger than in the open Air, or in the Receiver before it was exhausted. That the Enlargement or Diminution of the Arches of Vibrations were constantly proportional to the Quantity of Air, or Rariety or Density of it, which was left in the Receiver of the Air-Pump; and as the Vibrations were larger or shorter, so the Times were accordingly, *viz.* 2″ in an Hour slower, when the Vibrations were largest, and less and less as the Air was re-admitted, and the Vibrations shortened. But notwithstanding (says he) the Times were slower, as the Vibrations were, he had great Reason to conclude, that the Pendulum moved really quicker in Vacuo than in the Air; because the same Difference or Enlargement of the Vibrations (as $\frac{1}{5}$ of an Inch on each side) would cause the Movement instead of 2″ in an Hour to go 6″ or 7″ slower in the same Time, as he found by nice Experiments. The Doctor also says, that the Length of the Rod of a Pendulum, he found by trial was increased by the Summer Heat $\frac{1}{11}$ part of an Inch. All this from Dr *Derham*.

Hence the Resistance of the Air must certainly be a considerable Obstacle to the equable going of a Clock, although this Cause has been mostly disregarded by our most able Clock-Makers; and accordingly, the Irregularities in the going of a Clock are caused principally. 1. By the Lengthening of the Pendulum, or its shortening by heat and cold. 2. By the Lengthening the Arch described by it's Centre of Oscillation, or it's shortening, caused by the greater or less Cleanness of the Clock, and the greater or less Weight, or Force of the Springs giving Motion to the Clock. 3. By the greater or less Density of the Air, (or other Fluid mixed with it) causing the Times of the Vibrations to be increased or lessened, in proportion to that Density. 4. The Pendulum Clocks greater or less Distance from the Centre of the Earth.

It has been commonly said, that Clocks go faster in cold Weather than they do in warm Weather, because the Pendulum in cold Weather describes less Arches, and is shorter than in warm Weather. But this can only be true, when the Pendulum vibrates in Vacuo, *viz.* free from any external Resistance, or when this Resistance is always unalterable. But in dry hard frosty Weather, when the Air is very dense, and mixed with innumerable small icy Particles, (the Mercury in the Barometer being very high) the Pendulum may be so much resisted as the Times of the Vibrations caused thereby, may exceed the Times arising from the lessening of the small Arches described, and the shortening the Pendulum, both taken together. And then a Clock will go slower in such cold Weather rather than faster. The *Dutch*, who formerly wintered at *Nova Zembla*, could not make any of their Clocks go; and Captain *Middleton* (see the *Philosophical Transactions*, Numb. 465.) who wintered in the Year 1741, at Prince of *Wales*'s *Fort* on *Churchill* River in *Hudson*'s *Bay*, says, the Cold hindered the going of all Watches but one of Mr *Graham*'s, which went always too slow by 15″.

As the swinging of a Pendulum between the two cycloidal Cheeks removes one of the Obstacles to the equable going of a Clock, so does it's swinging in Vacuo another; and the other Obstacle arising from the lengthening or shortening of the Pendulum by heat and cold, has been endeavoured to be overcome by making the Pendulum partly of Wood, and partly of Metal, so that the one may be as much contracted by heat, as the other is lengthened, and thereby the Pendulum preserve the same Length in all Vicissitudes of heat and cold. Mr *George Graham* (in the *Philosophical Transactions*, Numb. 392.) endeavours to do this by a glass or brass Tube of Mercury for a Pendulum, instead of a Rod and Bob. But I think he fails here, as well as in his Method to know the going of a Clock in an Air of the same Temperature by means of a Thermometer. For a Barometer, and perhaps Hygrometer, should be added in order to obtain the principal End of best knowing and avoiding the Irregularities of the going of a Clock.

I shall conclude this Article upon Clocks with only mentioning two uncommon Treatises upon them, the one in *Latin*, and the other in *French*; in the former of which Treatises is a Contrivance to measure Time equably, by a Motion caused by three Weights, two of which form two perpetual Levers, by the means of which the third Weight is balanced, and these two Levers rest upon the Wheel of the Axis, which Axis shews the Minutes, Seconds, and Hour, by means only of an Index. And what is peculiar in this Invention is, that the Time it measures is not really in itself the same as that of common Clocks, nor is it a Pendulum, but another Thing, whose Motions are equable and regulated, as is that of a Pendulum; and besides, it will move in all Situations, either horizontal, perpendicular, and oblique. And therefore such a Contrivance as this has a Prospect of going well at Sea, and thereby being helps towards finding the Longitude. In the *French* Treatise you have seventeen Contrivances of Clocks, some of which are diverting enough, having small brass Balls perpetually dropping through Holes, and out again; others descend upon an inclined Plane (though these are not new Contrivances); others are Hour-Glasses, that turn up of themselves as soon as they are run out; another is a celestial Globe, turning about upon the Shoulders of an Atlas; another has the Hours placed horizontally, and another upright, *&c.* The two Treatises are,

Mat. Campani de alemenis Horologium solo naturæ motu atque ingenio dimetiens & numerans momenta temporis constantissime æqualia. Romæ, 1677. Quarto.

Recuil d' ouvrages curieuse ou Description du cabinet de M. de Servire; *a* Lyons, 1719. Quarto.

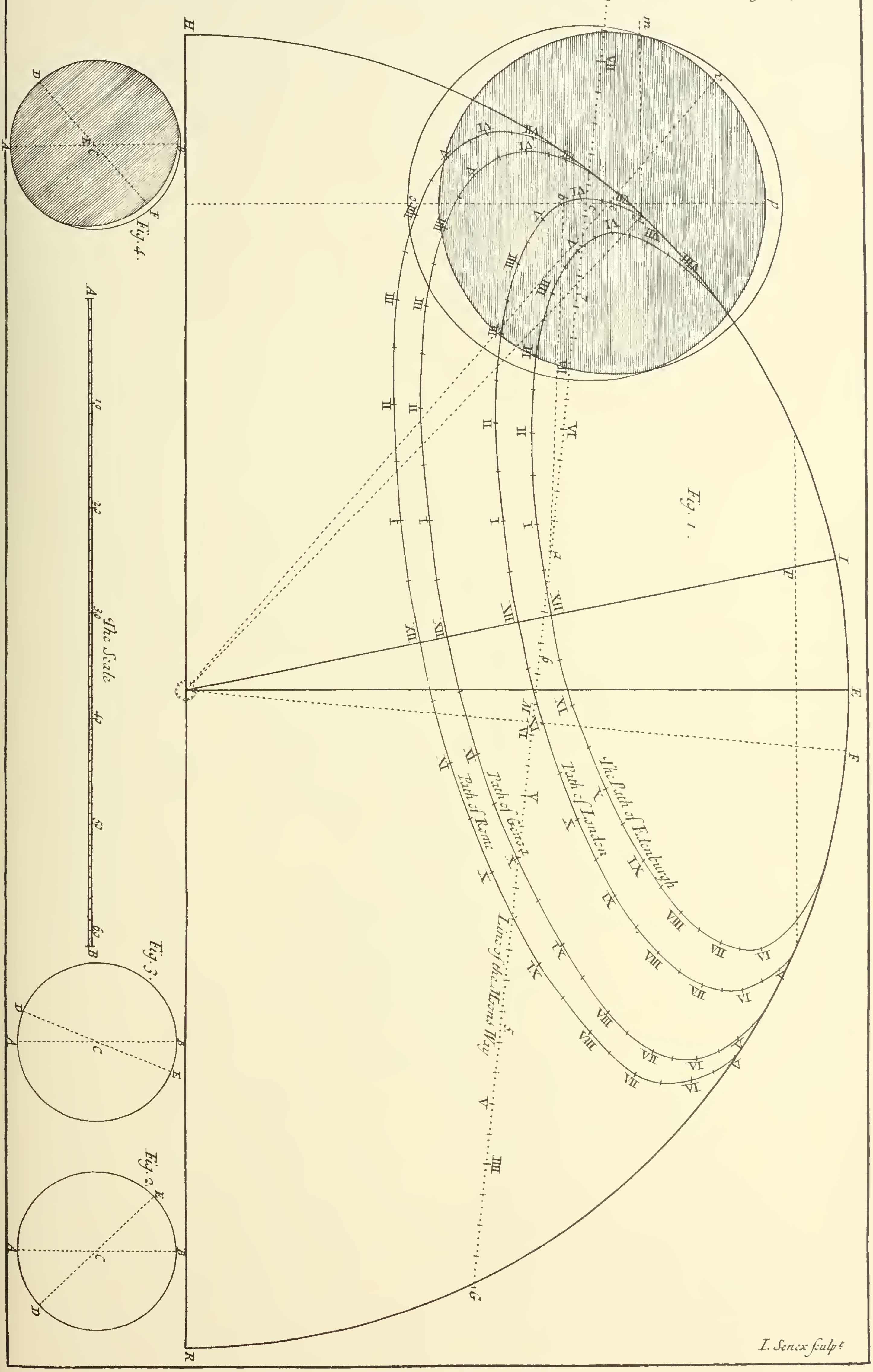
XIII. *Of concave Mirrors or Speculums.*

1. The Focus of parallel Rays is contained between the 4th and 5th Part of the Diameter of the great Circle of the Sphere of which the Speculum is a Segment; and so, 2. The Focus of one of these spherical Speculums is not a Point, but a small round Solid of such a Breadth. 3. The Diameter of the Aperture of one of these concave spherical Speculums, should not be a Chord of more than 18 Degrees of the Arch of the great Circle whose Segment that Speculum is. 4. Metalline Speculums are not so easy to polish as Glass ones quicksilvered over on the back-side, nor do they reflect so much Light. 5. These Instruments burn best when they are cold. 6. So that when they are exposed to the Meridian Sun in clear frosty Weather their Effects are greatest. 7. In the Focus of any of them directed to the Sun at Noon-Day there is not the least Appearance of a lucid Image, unless it falls upon an opake Body, and yet there is in that Place, and in some of the best of them, a Fire so intensely hot, that Stones are instantly melted by it and turned into Glass. 8. If the Back of a concave Glass Speculum be covered over with a very white Composition of Tin and Mercury, the Reflection of the Image of the Sun from the Focus will be so strong, that the Eye will not be able to bear it's Brightness. 9. If a Piece of white Paper be put in the Focus of a large Concave of this kind, so as to receive the contracted Image of the Moon, when shining at full on the Meridian in a clear Winter's Night; you will have so refulgent a Light that the strongest Eyes will not be able to bear it; and yet in the Focus there will be no Heat at all, instead thereof there will be found a very piercing cold. 10. The Heat of the Focus of a Concave-Speculum will be lessened, when acting upon any Thing laid upon a black Body in that Focus. 11. Whether, if the concave Surface of a Speculum were covered with some black polished Substance, the Effect of it's focal Heat would be lessened. 12. The Rays reflected from the yellow Colour of Gold are vastly refulgent, as has been found by a wooden Concave polished, and nicely covered over with Leaf-Gold, which burned with an incredible Power; as did another covered over with Pieces of yellow Straw very accurately fitted together. Hence the different Colours of a Speculum causes different focal Heat.

The most eminent burning Concaves that we are certain have ever as yet been made, are those of *Manfred Septala* at *Milan*; who is said by *Scottus*, to have made a parabolical Speculum, that would burn almost at the Distance of 15 or 16 Paces; those of the *Villets* at *Lyons*, whereof one is of Metal, weighing about 400 Pounds; the concave and convex Sides Sides are spherical, the Diameter of the Aperture 43 Inches; that of the Sphere whereof it is the Segment 14 Feet, the focal Distance $3\frac{1}{2}$ Feet, and the focal Depth is half an Inch. By the focal Heat of this Instrument, Metals, Stone, Bricks, Ashes, *&c.* are melted and turned into Glass. (See our *Philosophical Transactions*, Numb. 6. and the *Paris* Diary of the

Plate XXVI

fronting the End



I. Senex sculp^t

the Learned for the Month of *December*, *Anno* 1675.) And, Laſtly, Thoſe of Mr *Tſchirnhaus*, whoſe burning Effects are deſcribed in the *Acta Eruditorum*, publiſhed at *Leipſick* for *Jan.* 1687. The Diameter of the Aperture of this Speculum was almoſt three *Leipſick* Yards; it was made of Copper Plates not much exceeding the Back of a common Knife in thickneſs, and the Focus was two Yards diſtance from the Speculum. In imitation of theſe Speculums of Mr *Tſchirnhaus*, a certain celebrated Artiſt at *Dreſden*, made larger burning Concaves of Wood, which produced Effects no leſs wonderful. Even ſome have made large Concaves of this ſort, by properly placing 30, 40, or more ſquare Pieces of concave or plain Speculums, on the under Surface of a wooden Concave, whoſe Effects were not much leſs than if the Surface had been covered all over with them; and after this manner may polyhedrous burning Concaves, either ſpherical or parabolical, of a vaſt Size be made.

In the *Philoſophical Tranſactions*, at Numb. 483. in the Year 1747, there is an Account of a Mirror of one Mr *Buffon*, a *Frenchman*, conſiſting of a great Number of ſmall plain Mirrors, each of about 4 by 3 Inches ſquare, fixed at about $\frac{1}{4}$ of an Inch from one another, upon a large wooden Frame about ſix Feet ſquare, ſtrengthened with many croſs Bars of Wood for the mounting theſe Mirrors; each of them has three moveable Screws, which the Operator commands from behind, ſo contrived that the Mirror can be inclined to any Angle in any Direction that meets the Sun; and by this means the ſolar Image of each Mirror is made to coincide with all the reſt. Twenty-four of theſe Mirrors thus placed, in a few Seconds of Time ſet fire to a Compoſition of Pitch and Tow at the Diſtance of 66 *French* Feet. Alſo a ſort of Polyhedron, conſiſting of 168 ſmall Mirrors, or flat Pieces of Glaſs of ſix Inches ſquare each, ſet fire to ſome Beach Boards at the Diſtance of 150 Feet. This was done by the Marquis *Nicolini*. And in another Tranſaction of the *Royal Society*, Numb. 489, for the Year 1748, the ſame Mr *Buffon* ſays, he has made a polyhedron Speculum ſix Feet broad, and as many high, which burns Wood at the Diſtance of 200 Feet; melts Tin and Lead at the Diſtance of about 120 Feet, and Silver at 50 Feet; and beſides, ſays, that Heat is not proportional to Light, nor do the Rays come from the Sun in parallel right Lines.

Whether the burning Speculums of *Archimedes* and *Proclus*, by which they are ſaid to have burned the Enemy's Ships at a Diſtance, (ſee *Zonara*'s *Annal. Tzetza*'s *variarum Hiſtoriarum*, *Chiliad.* 2. *Galen*'s Book *de Temperamentis*; and others of the Antients) were contrived after ſome ſuch manner, or whether they be not rather fabulous, I leave others to judge. As to myſelf, I cannot aſſert, whether it be true or falſe, that *Archimedes* and *Proclus* could have made Speculums to produce ſuch great Effects.

If a Light be ſet in the Focus of a concave ſpherical Speculum, the Rays are parallel after Reflection; ſo that the Light of a Candle placed in the Focus, will be ſtrongly projected to a conſiderable Diſtance, whereby one may be enabled even to ſee to read at the Diſtance of 30 or 40 Yards.

THE

THE CONCLUSION:

Containing some

MISCELLANIES.

I. *The Comparison of the* English, French, Rhynland, *and* Roman *Foot.*

IF the *English* Foot be 1; the *French* Foot will be 1.0654; the *Rhynland* Foot 1.0284; the *Roman* Foot 0.970, expressed in Decimal Fractions. The above Proportion of the *English* and *French* Foot is taken from the *Philosophical Transactions*, Numb. 465.

Mr *Huygens*, at *page* 152. of his *Horolog. Oscill.* says, the *Paris* or *French* Foot is to the *Rhynland* Foot as 144 is to 139; from whence it follows, that the *English* Foot is to the *Rhynland* Foot as 1 is to 1.0284 nearly, as is expressed above; for since the *French* Foot is to the *English* Foot, as 1.0654 is to 1; it will be as 144 is to 1.0654, so is 139 to 1.0284 nearly. The Proportion of the *English* Foot to the *Roman* Foot, *viz.* of 100 to 97, or 1 to .97, is taken from Dr *Bernard*'s Treatise of Weights and Measures. *Mersennus* the *Jesuit* says, the *Roman* Foot that they have in their Library at *Paris*, taken from the Pavement of the Capitol at *Rome* is 14 Lines less than their Royal *French* Foot; that is, the *Paris* Foot Royal is to the *Roman* Foot, as 144 is to 130. He says also, that a *Roman* Palm is $\frac{1}{4}$ of a *Roman* Foot; and the *Rhynland* Foot used in many Places in *Holland*, which he says is thought to be equal to the antient *Roman* Foot, is only six Lines, or half an Inch less than their *French* Foot.

II. The Proportion of the *English* Pound *Troy*, and *French* two Mark 16 Ounce Weight, is as 16 is 21. The *English Troy* Ounce is to the *French* Ounce, as 64 is to 63. The *English Avoirdupois* Pound and Ounce, is to the *French* two Mark Weight and Ounce, as 63 is to 68 nearly. These Proportions are to be found in the aforesaid *Philosophical Transaction*, Numb. 465.

III. A *French* Wine Gallon is to an *English* Wine Gallon as 464.3328 is to 231, wherefore the former is very near double the latter. And a *French* Wine Vessel called a *Muid*, containing 300 *French* Wine Pints, or 37 $\frac{1}{2}$ *French* Gallons of eight Pints each, is 57.89 *English* Gallons of *Winchester* Measure. A *French* Pint contains 48 *French* cubic Inches. As 1.2092 is to 1, so is a *French* cubic Inch to an *English* cubic Inch.

IV. *A short Account of the Measure of the Earth, by various Persons.*

The Length of a Degree of a great Circle of the Earth, actually measured by *Eratosthenes*, an antient Geographer of *Alexandria*, about 200 Years before Christ, (which Measure was approved of, made use of, and confirmed by *Ptolemy* himself) is 500 Stadiums.

Some *Arabian* Mathematicians, at the Command of their King, about the Year 827, by Measurement, found a Degree to be 56 of their Miles, and a little more.

Alhazen,

Alhazen, the *Arabian* Optician, who lived about the Year 1100, says, in his Book *de Crepusculis*, that the Compass of the Earth is 24000000 Paces.

Mr *Snell*, about the Year 1615. a famous Professor of Mathematicks at *Leyden* in *Holland*, found a Degree by Measurement to be 28500 *Rhynland* Perches, of 12 *Rhynland* Foot each. But,

Varenius, in the 4th Chapter of his *General Geography*, has corrected this, and makes it to be only 28300 *Rhynland* Perches. *Varenius* also gives, in that Chapter, the Manner used both by *Eratosthenes* and *Snell*, in the Measurement of their Degrees of a great Circle of the Earth. *Snell*'s Measure by himself is to be seen in his Treatise, entitled, *Eratosthenes Batavus*.

Our Countryman Mr *Norwood*, a Reader of Mathematicks, in a Treatise published at *London*, in the Year 1702. called the *Sea-Man's Practice*, containing a fundamental Problem in Navigation experimentally verified, touching the Compass of the Earth and Sea, and the Quantity of a Degree in our *English* Measure, *&c.* In the Year 1635. with a Circumferentor and a Chain measuring the Distance from *London* to *York*, and taking the Difference of Latitude of those two Places, with a Sextant of more than five Feet Radius, found a Degree to be 3709 Chains, or 367196 *English* Feet.

Mr *Picart*, and other *Frenchmen* of the *Royal* Academy of Sciences, by Order of *Lewis* the XIVth, about the Year 1672, measured and found the Length of a Degree about *Paris* to be 57060 *French* Toises of six *French* Feet each. His Quadrant for finding the Differences of the Latitudes was 10 Feet Radius; and that for finding the horizontal Angles of the Triangles of Verification 3 ½ *French* Feet, for he took his Lengths two Ways, by actual Measurement, and by Trigonometrical Calculations of the Lengths of the Sides of several Triangles, in order for one way to prove the other. See the whole Operation (long and troublesome enough indeed, if compared to Mr *Norwood*'s) in Mr *Picart*'s Book, called, *La Mesure de la Terre*; or, in Dr *Jurin*'s Appendix to *Varenius*'s *Geography*.

Mr *Cassini*, in the Year 1700, by Order of *Lewis* the XIV, repeating the same Labour over again (likewise two Ways), carried a meridian Line of about 7 ½ Degrees, the whole Length of *France* through *Paris*, from *Dunkirk* to the *Pyrenean* Mountains on the Borders of *Spain*, and makes a mean Degree to be 57292 *French* Toises: He used a Sextant of 10 Feet, *&c.* See the History of the *Academy* of Sciences for the 1701, as also for the Year 1718.

The *Frenchmen* who, in the Year 1737, measured a Degree at *Tornea*, under the Arctick Circle, make it to be 57437 *French* Toises. They used a Quadrant of 2 Feet Radius, with a Micrometer upon the Index, by means of which they have taken Angles to Seconds. (But this I am certain, the Nature of an Instrument of that short Radius will not admit.) They had also a Brass Telescope about nine Feet long, forming the Radius of a Limb or Arch of 5 ½ °, made by the late Mr *George Graham*, which they used in their Business. There is a Description of this Instrument at *page* 124. of the *English* Translation of Mr *Maupertuis*'s Treatise of the Figure of the Earth, determined from Observations made at the polar Circle. They measured upon a frozen River, with two Feet of Snow upon it, *&c.* with Rods of 30 *French* Foot long, and different Persons measured a Length of 7406 Toises five Feet to within four Inches, as they say. See particularly in the above Treatise of Mr *Maupertuis*.

Now in order to reduce all these various Measures into our *English* ones, thereby to compare and make them useful, we must get their reputed Proportions to ours. Mr *Norwood*, in his *Sea-Man's Practice*, page 5, tells us, that *Herodotus*, *Suidas*, and others, make a Stadium to be 600 *Alexandrian* Feet. And at *page* 6, he says, that *Here Mechanicus* makes five *Alexandrian* Feet to be equal to six *Roman* Feet. But since above one *Roman* Foot is to an *English* Foot, as 972 is to 1000, therefore will one *Alexandrian* Foot be equal to 1.166 *English* Feet; but as there are 500 Stadiums in a Degree, according to *Eratosthenes* and *Ptolemy*, and as 600 *Alexandrian* Feet is a Stadium, there will be 300000 *Alexandrian* Feet in a Degree, and so according to this $\overline{300000 \times 1.166}$ or 349800 will be the Number of *English* Feet in a Degree, according to *Eratosthenes*, being 66.25 Miles. *Note*, Mr *Norwood*, in his Book abovementioned, has mistook in saying, (*page* 6.) that the antient *Roman* Foot was greater than the *English* Foot; and so has *Snell*, and *Varenius* too.

Mr *Norwood*, in his Sea-Man's Practice (at *page* 29.) says, that according to *Alphraganus*, the Quantity of an *Arabian* Mile was 4000 Cubits, or 6000 *Arabian* Feet; and so the Quantity of a Degree, according to the *Arabian* Mathematicians of 56 of their Miles, as mentioned above, will be 336000 *Arabian* Feet; but, says Mr *Norwood*, the Length of their Foot is something uncertain; they only say, it is the Length of 96 Barley-Corns laid side by side. Now this being granted, I myself found upon Trial, that 23 Barley-Corns, laid side by side, extended nearly three Inches; and so, according to this, an *Arabian* Foot is to our *English* Foot, nearly as 96 is to 92, or as 24 is to 23. Wherefore, as 23 is to 24, so is 336000 *Arabian* Feet, to 350608 *English* Feet, in a Degree, or about 66.39 *English* Miles. Now Mr *Norwood* reckons a Degree of this Measurement to be much more, *viz.* 370222 *English* Feet. But he was misled by Mr *Snell*, who says, that by Trial he found one *Rhynland* Foot to be equal to 90 Barley-Corns laid side by side, and 96 *Rhynland* Feet to be about 91 ½

English Feet; whereas the true Proportion of one *English* Foot to one *Rhynland* one, is very nearly as 1 to 1.0284.

Again, *Alhazen*'s Account of 24000000 Paces in the whole Compaſs of the Earth and Sea may be nearly reduced to our *English* Meaſures thus; there muſt be 66666 $\frac{2}{3}$ of theſe Paces in one Degree. And as a Pace is by all allowed to be five Feet, there muſt be 333333 *Arabian* Feet in a Degree; but ſince above it appears, that an *English* Foot is to an *Arabian* Foot, nearly as 23 is to 24, therefore there will be 347825 *English* Feet in one Degree of *Alhazen*'s Compaſs of the Earth, being about 65.87 *English* Miles. Mr *Norwood* at *page* 29. of his Sea-Man's Practice, makes this Degree to be more, *viz.* 367283 *English* Feet. But herein he is alſo deceived by making 90 *Arabian* Feet equal to 99 $\frac{1}{6}$ *English* ones, when 90 of them ſhould but be equal to about 93.9 *English* Feet.

Again, it is eaſy to reduce Mr *Snell*'s Meaſure of 28500 *Rhynland* Perches to a Degree, into *English* Feet, by having given the Proportion 1 to 1.0284 of an *English* Foot to a *Rhynland* one, for there will be 342000 *Rhynland* Feet in one Degree; and ſo 351712 *English* Feet, or taking *Varenius*'s Corrections of *Snell*'s Meaſure, *viz.* being to *Snell*'s in the Proportion of 235 to 283, a Degree of Mr *Snell*'s will be 349244 *English* Feet, or about 66.1 *English* Miles.

Again, Mr *Picart*'s Meaſure of a Degree reduced to *English* Feet, from the Proportion 1,0654 to 1, of a *French* Foot to an *English* one, will be 364710 *English* Feet, or 69.07 *English* Miles. Mr *Caſſini*'s Meaſure of 57437 Toiſes, will be 367160 *English* Feet, or about 69.5 Miles. And, laſtly, The *Frenchmens* Meaſure of 57437 Toiſes, to a Degree under the Arctick Circle, will be 367656 *English* Feet, or about 69.68 *English* Miles. Now all theſe various Meaſures of a Degree in *English* Feet, when brought together, will ſtand thus:

	Eng. Feet.
1. *Eratoſthenes* or *Ptolomy*'s	349800
2. The *Arabian* Mathematicians, or *Abelfeda*'s	350608
3. *Alhazen*'s	347825
4. *Snell*'s corrected	349244
5. Mr *Norwood*'s	367196
6. Mr *Picart*'s	364710
7. Mr *Caſſini*'s	367160
8. Mr *Maupertuis*'s	367656

Upon which we may obſerve, that *Eratoſthenes* and *Snell*'s Meaſures of a Degree, do not differ from one another, more than 566 *English* Feet, or about 188 Yards, although very differently obtained, which is ſome ſort of a Confirmation of their being both right, whatever others have ſaid to the contrary. Likewiſe *Abelfedas*'s Meaſure does not differ from *Eratoſthenes* but 308 Feet, and from *Snell*'s but 1444 Feet, the former being 266 Yards, and the latter about 481 Yards. Nor does *Alhazen*'s Meaſure, which is the leaſt of all, differ about half a Mile from *Abelfedas*'s; therefore the firſt four, and moſt antient of theſe Meaſures of a Degree, are not ſo unequal, as to be a ſort of Argument for their being rejected. The other four modern Meaſures of a Degree, by Mr *Norwood* and the *Frenchmen*, are indeed greater, but differ from one another, more or leſs. For Example, Mr *Norwood* and *Picart* differ 2446 *English* Feet. *Note*, in *page* 33, of Mr *Norwood*'s *Sea-Man*'s *Practice*, it is ſaid Mr *Picart*'s Meaſure of a Degree is 365184 *English* Feet, as taken from the *Philoſophical Tranſactions*, Numb. 112. But this is 434 *English* Feet too much. Mr *Maupertuis*'s Meaſure is 901 *English* Feet more than *Picart*'s.

Hence, allowing the Earth and Sea to be one Sphere, (excepting all Mountains, and Cavities not exceeding two or three Miles in heighth or depth, and which bare no ſenſible Proportion to the whole Diameter,) all the aboveſaid different Meaſures of a Degree cannot be true; nor is it eaſy to know which is the trueſt, if it be conſidered, 1. That the Antients knew as well how to meaſure Diſtances upon the Ground, as the Moderns. 2. That their Plains being ſo extenſive, upon which they meaſured the Diſtances for this Buſineſs, *viz.* thoſe of *Alexandria* and the Fields of *Meſopotamia*, made their Work eaſier and leſs liable to Error, than the Lengths meaſured for the ſame Purpoſe by Mr *Norwood* and the *French*, in crooked Ways, uneven Ground, or upon Ice covered with Snow. Moſt of theſe uneven Lengths being apt to be ſet down too great, without much Caution, Diligence, proper Correction, and trigonometrical Computation and Verification; and this trigonometrical Meaſure by means of Angles near upon the Horizon is ſubject to Uncertainty, by reaſon of the horizontal Refractions, (ſee Dr *Nettleton*'s Account of this already mentioned by me) it giving the oppoſite Baſes of the Triangles too great in length. 3. I am of Opinion, the Antients could take the Latitude of a Place, and draw a meridian Line by their Gnomons, as exactly as any of the Moderns can do by their Sextants and Quadrants, armed with Teleſcopes. I do not find Mr *Norwood* had any Teleſcope to his Sextant more than of five Feet Radius that he uſed. Moreover, Mr *Hevelius* would not uſe Teleſcopes to his Inſtruments, as thinking they added nothing to the Exactneſs of their Uſe. But I think he was wrong in this. 4. The Abilities of moſt of the Meaſurers, *viz. Ptolemy* (for at leaſt he approved of that of 500 Stadiums to a Degree.)

Degree.) Mr *Snell*, Mr *Norwood*, *Picart*, *Caſſini*, and *Maupertius*, ſufficiently appear by their Works.

From what I have already ſaid, and ſeveral other Conſiderations, I am ſo divided in my Opinion, as not to be certain which of thoſe Meaſures of a Degree is the trueſt; and if it were not for the Uncertainty of the exact Compariſon of an *Alexandrian* Foot to our *Engliſh* one, I ſhould be apt to think *Ptolemy*'s Meaſure to be as true as any of them. Alſo *Albazen*'s Meaſure might in reality come out to be nearer to the reſt, if we had the exact Proportion of an *Arabian* Foot to an *Engliſh* Foot; for the Compariſon by Barley-Corns is a little uncertain. However, as all the eight Meaſures do not differ from one another more than about three Miles in a Degree. The Medium of them all ſeems to be the neareſt the Truth, and I think ought to be uſed as ſuch, 'till it be found by many and good Experiments either to be more confirmed, or to be otherwiſe; and this Medium is about 67.3 Miles to a Degree. There are many other Accounts of the Meaſure of a Degree to be ſeen in Authors of no moment to mention here.

V. *Of the Notes which were promiſed at the Beginning of the Book of Mathematical Inſtruments, together with a few inſtrumental Conſtructions of Geometrical Problems.*

An *Ellipſis* is a curve lined Space, made by cutting a right Cone or Cylinder through, by any Plane, making an acute Angle with the Plane of the Baſe of the Cone or Cylinder.

A ſpiral Line may be thus deſcribed If the infinite right Line A C revolves equally round it's end A, and at the ſame Time a Point B moves from A equably along that right Line, and the Velocities of the Line A C, and that of the Point B along it, be as the Circumference of a Circle is to its Diameter, then will the Point B deſcribe a ſpiral Line, of an infinite Number of Revolutions upon the Plane, whereon the right Line revolves. I have ſeen ſuch a Spiral deſcribed upon the Surface of a round revolving Table, by drawing a Piece of Chalk, or a Pencil, held in the Hand equably along that Surface from the Centre to the Circumference. *Note*, There are many other Species of ſpiral Lines beſides this here; but this is one of the moſt ſimple. Or a ſpiral Line may be obtained thus, by finding a great many Points through which it muſt paſs. Let B D E (Fig. 26.) be a Circle, whoſe Centre is A, and B E, F D, two Diameters croſſing one another at right Angles infinitely continued; find a right Line M N equal to the Circumference of that Circle, either by making as 7 is to 22, ſo is the Diameter to B E to M N or elſe by wrapping a String quite about the Circumference. This done, divide the right Line M N into a great even Number of equal Parts M *p*, and the Circumference of the Circle F B E into the ſame Number of equal Parts by trial; let B Q be one of theſe Parts, draw the Radius A Q, and upon it take A P equal to the part M *p* of the right Line M N, then will P be one Point of the Spiral, and doubling the Arch B Q, and drawing a Radius as before, upon this lay off from A, the double of the Part M *p* of the Line M N, and another Point of the Spiral will be had; and by a Repetition of theſe Operations, with the Triples, Quadruples, &c. of the equal Arches B Q, and the equal Parts M *p*. other Points will be obtained, through which the Spiral muſt paſs. A D continued to 1, ſo that A P 1 be equal to $\frac{1}{4}$ M N, or $\frac{1}{4}$ the Circumference: A E to P 2, ſo that A P 2 be equal to ' M N; A F to P 3, ſo that A P 3 be equal to $\frac{1}{4}$ M N; A B to P 4, ſo that A P 4 be equal to M N, then will P 1, P 2, P 3, P 4, be other Points of the Spiral. If A P be continued downwards, ſo that A *p* be equal to A P 2 + A P, then will *p* be another Point. When Points enough ſufficiently near be found, and properly joined, one Revolution A P 4 of the Spiral will be had; and another Revolution will be obtained, by finding Points through which it muſt paſs thus. Continue out A P to S, ſo that A S be = M N + A P, then will S be one Point of this ſecond Revolution of the Spiral. And thus may any Number of Points be found for this ſecond Revolution. For a third Revolution A S muſt be = 2 M N + A P, and ſo on, then will S be a Point of a third Revolution. Fig. 25. Fig. 26.

Hence the Nature of this Spiral is ſuch, that any Radius A P is equal to the correſpondent circular Arch B Q, when the Spiral has but one Revolution; when it has two, any Radius of it is equal to once the Circumference of the Circle B D E F, together with the correſpondent Arch B Q of the Circle; when three, equal to twice the Circumference, added to the correſpondent circular Arch B Q, and ſo on.

Note, This Spiral may alſo be deſcribed by a continued Motion, by means of a String in length equal to one or more Times the Circumference of the Circle B D E F, having a further additional Length equal to the Radius A B of that Circle; for if one end Q of ſuch a String be faſtened to the end Q, of a moveable Radius A Q, and that String be wrapped about the Circumference of the Circle, when A B and A Q coincide, and the other end A of that String be extended tightly down the fixed Radius A B, from B to A, (and there bent upwards) and that Radius A Q carrying the end Q of the String moves round from B to Q, &c. at the ſame Time that the other end A of the String is perpetually drawn tightly up from A to P along that moveable Radius A Q now in the Situation A Q. By this Motion the Point P, or end of the String, will deſcribe the Spiral, for the part A P of the String is always equal to the Arch B Q, which is diſengaged from as much of the String as is equal to it in length.

The

The Ufe of this Spiral is confiderable in the Solutions of fome difficult Problems relating to the Divifion of the Arches of Circles into any Number of equal Parts, or given Ratio's, finding the fides of all regular Polygons. Dividing a Circle into two Segments that fhall have a given Ratio, proving that the abfolute Quadrature of the Circle is impoffible, *&c.*

Fig 27. An Example of one Ufe is as follows: To draw aright Line G Q (Fig. 27.) from a given Point G, in the Diameter B E, of a given Semicirle B Q E fuch, that it fhall divide the fame into two Segments B Q G, G Q E, having the given Ratio of M to N.

Let A H P I be a part of a Spiral, as above fuch, that any Radius A P of it be equal to the correfpondent Arch B Q of the Semicircle B Q E, then will A E continued out to cut the Spiral in I, be equal to the Length of the Arch B Q E of the Semicircle. Make as M + N : M : : A I : R. From the Centre A of the Semicircle, raife the Perpendicular A K = A G, and upon A K defcribe a Semicircle, take a right Line or Ruler in length equal to R, and always making it pafs through the Centre A, move one end S of it along the Semicircle K S, 'till the other end P of it cuts the Spiral in P, and the Semicircle B Q E in Q, join G Q, and the Bufinefs will be done; for then will the Semicircular Space B Q G be to E Q G, in the given Ratio of M to N. The Demonftration is eafy, for A S is always equal to the Perpendicular G L, drawn from G upon A Q, and the Space B Q G is always as the Arch B Q + G L; that is, as A P [illegible] S = R, *&c.*

Note, This Problem is otherwife conftructed in Sir *Ifaac Newton*'s *Principia Mathem.* Lib. I. by means of an Epicycloid, which he tells you there how to defcribe.

Fig. 28. *To trifect a given Angle* A C B About the angular Point C, with any Semidiameter F C or C G, defcribe a Semicircle F E G, cutting the fide C A of the given Angle in E; continue out G F, and laying the Edge of a Ruler N E to the Point E, fo turn it that the Part N M be equal to the Radius of the Circle, then will the Arch F M be $\frac{1}{3}$ of the given Arch E G, or the Angle F C M = [illegible] A C B.

Fig. 29. *To find two continual mean Proportionals between two given right Lines* X *and* Y. Make the rightangled Parallelogram A B C D, having its fides A B, A C, equal to the given right Lines X and Y, and defcribe a Circle about this Parallelogram, whofe fides A C, A B, let be continued out, and laying the Edge of a Ruler E F to the Point D, turn the Ruler about 'till it's Edge fo cuts the Circle in G, and the Continuations of A C, A B in E F, that the Parts E D, G F of that Edge be equal between themfelves, then will the four right Lines X, C E, B F, Y be continual Proportionals. Both thefe Problems are folid ones, as they are called, and cannot be conftructed geometrically by the Interfection of right Lines and Circles. There are many other Ways of conftructing them by the conic Sections, well known to Geometricians; but none eafier in the Practice than thefe two above given, being in effect by the Interfection of a Circle, and the *Conchoid* of *Nicomedes*, a famous curve Line, efteemed both by *Archimedes* and Sir *Ifaac Newton*, as the beft for the Conftruction of folid Problems, with regard to the fhortnefs and facility of the Practice, the Curve itfelf being almoft as eafily defcribed as the Circle; and of fuch a Nature, that the part of a right Line drawn from any part of the Curve to the fixed Pole intercepted by a given right Line, *viz.* it's Afymptote, is of a given Magnitude.

A *Parabola* is a curve-lined plane Space, or Section, made by cutting a Cone by any plain Parallel, to any Plane touching the fide of a Cone in a right Line. An *Hyperbola* is a Section or curve-lined plain Space, made by cutting a Cone, by a Plane not parallel to a Plane touching the fide of that Cone in a right Line, nor paffing through the Vertex of the Cone. *Note*, It is common to call the curve-lined Terminations of the Ellipfis, Parabola, and Hyperbola, by thefe Names, inftead of thefe Planes of Section, which they really are. A Parabola opens wider and wider infinitely, according as the Cone whereof it is the Section is more continued out, and fo does an Hyperbola too. Although fuch a Section of one Cone only, as has been defcribed above, is ufually called an Hyperbola, yet it is but one part of an Hyperbola, ftrictly fpeaking, for there is another fuch fimilar Section made by cutting the oppofite Cone by the fame Plane, that goes to make up one compleat Hyperbola. That is, an Hyperbola confifts of two oppofite fimilar Parts, having their Convexities next to one another, and at fome Diftance from each other. The middle Point of which Diftance is the Centre through which thofe two right Lines drawn to touch the Hyperbola at an infinite Diftance are called *Afymptotes*, and that Hyperbola whofe Afymptotes are at right Angles to one another, is called an *Equilateral Hyperbola*, which of all the different Species is the moft fimple and ufeful.

The Ellipfis, Parabola, and Hyperbola, are called Conic Sections. If two parallel right Lines terminating in a Conic Section be bifected, and a right Line be drawn through the Points of Bifection meeting the Curve, either in one or two Points, this bifecting right Line is called a *Diameter*, which will bifect all Parallels to thofe firft mentioned ones. Thefe Parallels are called *Ordinates* to that Diameter, and in the Ellipfis and Hyperbola, the middle Point of any Diameter is called the *Centre*, and that Diameter drawn through the Centre parallel to an Ordinate to any Diameter, is called a *conjugate Diameter*. That Diameter whofe Ordinates are at right Angles to it, is called an Axis, and the Point wherein it meets the Curve, is called the Vertex. In the Parabola each Diameter meets the Curve in one Point only, and they are all parallel to the Axis. If any two parallel right Lines terminating in a conic Section be cut by another right Line terminating in that Section, the Ratio of the two Rectangles under the Segments of those two Parallels, will be equal to the Ratio of the two Rectangles under the two Segments of the cutting Line and one Parallel, and the Segments of that cutting Line and the other Parallel. And so if this cutting Line be a Diameter, passing through the Centre, the Rectangle under the two Parts of it, cut by any Ordinate, will be to the Square of that Ordinate in a constant Ratio, *viz.* as the Square of that Diameter is to the Square of the conjugate Diameter, or as that Diameter is to a third Proportional to those two conjugate Diameters, which third Proportional is called the *Latus rectum* of those Diameters. But in the Parabola, the Square of any Ordinate is equal to the Rectangle under the Segment of the Diameter intercepted between the Vertex and that Ordinate, and a right Line of a constant Magnitude, called the *Latus rectum*, or Parameter of that Diameter. Moreover, in the Hyperbola and Ellipsis, the Tangent at the Extremity of any Semi-ordinate to a Diameter, and that Semi-ordinate will so cut that Diameter, as that the Rectangle under the Distances of the two Points of Section from the Centre, will be always equal to the Square of half that Diameter. And in the Parabola any Tangent cuts a Diameter continued out, as far beyond the Vertex of that Diameter, as the Semi-ordinate of that Diameter, drawn from the Point of Contact is distant from the said Vertex. In the Hyperbola two infinite Tangents, *viz.* the Asymptotes cross one another in the Centre; and if it be made as the Semi-axis is to the Distance from the Centre of any Semi-ordinate, so is that Semi-ordinate to a fourth Proportional, and a Perpendicular raised at one Vertex of the Axis be made equal to that fourth Proportional; a right Line drawn through the Centre and the Extremity of that Perpendicular will be one Asymptote, and the other Asymptote will be had by drawing a right Line through the Centre, making the same Angle with the Axis as the first Asymptote does. The *Foci* of an Ellipsis are two fixed Points in the great Axis equally distant from the Centre, by the Length of a mean Proportional between the Sum and Difference of $\frac{1}{2}$, the great and $\frac{1}{2}$ the lesser Axis. And two right Lines drawn from those two Foci to any Point of the Curve will be equal to the greater Axis. But in the Hyperbola, the Foci are two Points in the transverse Axis equally distant from the Centre, such that the Difference of two right Lines drawn from them to any Point of the Curve will be equal to the traverse Axis. In the Parabola, the Focus is a Point in the Axis distant from the Vertex by $\frac{1}{4}$ of the *Latus rectum.* In the Ellipsis and Hyperbola, two right Lines drawn from the Foci to any Point of the Curve, make equal Angles with a Tangent drawn through that Point. In the Parabola, a Line drawn from the Focus to the Point of Contact of a Tangent, will make the same Angle with that Tangent as a Diameter drawn through that Point of Contact does. In the Hyperbola, if any right Line be drawn cutting the Asymptotes, as well as the Curve in two Points, the two Parts of this Line intercepted, between the Curve and one Asymptote, and the Curve and the other Asymptote, will be equal to one another. So that if this Line cuts the Curve in two Points infinitely near to one another, *viz.* is a Tangent to the Curve; the two Parts of that Tangent between the Point of Contact, and the Asymptotes will be equal to one another. And all Triangles made by such a Tangent, and the two Segments (from the Centre) of the Asymptotes cut off by that Tangent will be equal to one another. Wherefore every Triangle made by a Semi-diameter drawn to the Point of Contact, by half of that Tangent, and by the Segment of one Asymptote cut off by that half of the Tangent, will be of a constant Magnitude. And if from any Point of an Hyperbola a right Line be drawn upon one Asymptote parallel to the other Asymptote, the Parallelogram under that right Line, and the Segment (from the Centre) of the Asymptote cut by it, will be of a constant Magnitude.

I have collected together these few Definitions, and fundamental Properties of the Conic Sections, for the Use of those who may have some Occasion for them, either in the Constructions of Problems, or for other Purposes, and have not the Books at hand, which treat of them fully, and as they should be; nor even know, perhaps, that the Conic Sections have such Properties.

The easiest Way of describing by one Operation a Parabola and Equilateral Hyperbola by means of Points, is thus,

Upon the infinite right Line C E (Fig. 30. Numb. 1, 2.) assume two Points A and F, and through A draw the right Line A B perpendicular to C E. In C E from A take any Point P on the contrary side of A that F is, for the Parabola, but on the same side for the Equilateral Hyperbola; and in this last Case greater, at such a Distance from A, that F P be greater than A F, and from P (Numb. 1.) take the Point Q in A B distant from P by the Length F P; but in (Numb. 2.) distant from F by the Length F P. This done, about the Points Q F (Numb. 1.) and Q P (Numb. 2.) with the Distance F P describe two Arches intersecting one another at M, and M will be one Point of a Parabola G A H, or equilateral Hyperbola G F H. The former passing through the Point A, and the latter through the Point F. About the Centre A with the Distance A M describe a Circle cutting the right Line C E, or Axis of the Parabola or Hyperbola in the Points I I, and make one Arch I *m* (Numb. 1.) and three Arches I *m* (Numb. 2.) equal to the Arch I M, then will *m* be another Point of the Parabola, and *m*, *m*, *m*, three other Points of the opposite Hyperbola's G F H, *g f h.* and by a Repetition of many more such Operations, a Number of Points M, *m*, may be Fig. 30.

 found

found fo near each other, as that being properly joined, a Parabola, or equilateral Hyperbola fufficiently exact, will be formed.

A Parabola, Hyperbola, and Ellipfis, may be defcribed by a continued Motion. All three by the fame way of Operation varied, by means of a moveable Square and moveable Ruler. For if the Angle of a Square be faftened to a given Point, fo that the Square be moveable about the fame, and the Interfection of one fide of that Square and a Ruler moving parallel to itfelf, be carried along a given right Line; the Interfection of the other fide of that Square, with that Ruler, will defcribe part of the Curve of the Parabola. But if the moveable Ruler, inftead of moving parallel to itfelf, moves about a given fixed Point, and the fixed right Line be between this Point and the angular Point of the moveable Square, fuch an Interfection of one fide of that Square, and this moveable Ruler, will defcribe part of the Curve of an Hyperbola. If the two fixed Points aforefaid be both on the fame fide, the fixed right Line, part of the Curve of the Ellipfis will be defcribed by fuch a Motion. This Manner of defcribing the Curves of the Conic Sections is well known to Geometricians. The following Way of defcribing any Hyperbola, by a continued Motion (being that of Mr *De Witt*'s in his *Elementa Linearum Curvarum*) by having the Afymptotes A E, A F (Fig.
Fig. 31. 31.) given, as alfo a Point B, through which the Curve is to pafs is thus; from B draw B G parallel to A E cutting A F in G, take a Bevel D C F, whofe Angle C is equal to the given Angle A. In the fide C Q of the Bevel take the Point Q at the fame Diftance from C, as G is from A, and at Q faften the end Q of a Ruler Q B always paffing through the fixed Point B. This done, move the fide C F of the Bevel along the Afymptote A F, and the Interfection P of the Ruler Q B, and the other fide C D of the Bevel, will defcribe the part I B K of an Hyperbola paffing through the Point B; and whofe Afymptotes are A E, A F.

Prob. I. *The Point* B *being given, as alfo the Angle* E A F, *and the Space* S, *to draw a*
Fig. 32. *right Line* B Q, *to form the Triangle* A P Q *equal to the given Space* S. (Fig. 32.) *This* is a noted plain Problem, but I take the eafieft Way of conftructing it to be by a parallel Ruler, thus: Make fome Triangle A C D equal to S, then take a parallel Ruler, and letting it's two oppofite Sides P C, D Q, always pafs through the given Points D, C, fo move it, that a Ruler laid from B fhall interfect thofe it's two oppofite fides at P and Q where thefe fides cut the fides A E, A F of the given Angle, then will the Triangle A P Q be equal to the Space S.

Note, The Hyperbola within it's Afymptotes is the only curve Line that is natural to the Conftruction of the Problem above, and a fmall Knowledge in the Nature of this Curve, is alone fufficient for any one of himfelf, to find out feveral different Conftructions of the Problem, as well Mechanical as Geometrical, without any other Affiftance, the Problem being in effect only to draw a Tangent to a given Hyperbola within the Afymptotes from a given Point.

Fig. 33. Prob. II. *The Lengths of the three contiguous Chords* A B, B C, C D (Fig. 33.) *infcribed in a Semicirle being given: to find the Diameter of that Semicircle.*

This is an old folid Problem, and it may be conftructed thus: Draw two right Lines (Numb. 2.) B H, B F, at right Angles to one another, and make A B equal to one of the given Chords A B. About the Point B with a Diftance B C equal to the fecond given Chord B C defcribe a Circle G C H. This done take a Square A C D, one fide C D of which is equal to the third given Chord C D. Let the other fide A C of this Square pafs through the given Point A, while the end D of the fide C D moves along the right Line B F, 'till that Square becomes fo fituate, that it's angular Point C falls upon the Circumference of the Circle G C H; then will the right Line A D, joining the given Point A, and the end D of the Square thus fituate, be the Diameter required of the Semicircle.

Fig. 34. Prob. III. *Given the Bafe* AB (Fig. 34. Numb. 1.) *of a Triangle* A B C, *the Aggregate of the fides* A C, B C, *and the Length of a Perpendicular* B D *to the fide* B C *drawn from one of the Angles* B *to the other fide* A C, *to conftruct the Triangle.*

This is an old hyperfolid Problem of Mr *Ward*'s, in his Compendium of *Algebra*, and is thus conftructed. Draw (Fig. 34. Numb. 2.) two right Lines F B, B G, at right Angles to one another. Make B D equal to the given Perpendicular B D, and about the Point B with the Diftance B F equal to the given Bafe A B defcribe a Circle A F H; this done, take a Thread in length equal to the given fides A C, B C taken together, and faftening one end of it to the Point B, partly extend it along the right Line B C from B to C, and partly from C through the given Point D, 'till it's other end A falls upon the Circumference of the Circle A F H. Join the Points A and B, and the Triangle A B C required will be formed.

Fig. 36. Prob. IV. *Given the Area of a Triangle* A B C, (Fig. 36.) *as alfo it's Perimeter, when it is infcribed in a given Circle, to conftruct fuch a Triangle.*

This is an old folid Problem to be feen in Mr *Ozanam*'s Mathematical Dictionary, and is thus conftructed. Let a Line P be equal to the given Perimeter of the Triangle, and a Line Q the fide of a Square equal to the given Area; take three double Squares A E B E H, A D C D H, and C F B F H, having three equal fides E H, F H, D H, each being a third Proportional to $\frac{1}{2}$ P and Q, and faften their three ends H together, fo as to be moveable about H like a Joint. And at any Point A of the Periphery of the given Circle, place two of the other fides A B, A C of thofe Squares, fo as to be moveable about the faid fixed Point A. This done,

done, fo move the three Squares about, as that the Interfections of the fides A B, A C, with B C always fall in the Circumference of the given Circle, at the fame Time that a Thread equal to P may be extended tightly from A to B, B to C, and from C to A again. When this happens, the three fides A B, B C, A C of the three double Squares will be fo fituate, as to form an infcribed Triangle A B C in the given Circle, whofe Circuit fhall be equal to the given right Line P, and Area equal to the Square of the given right Line Q.

Note, Mr *Ozanam* firft obtains an algebraic cubic Equation, in order to conftruct this Problem by the Interfection of a Parabola, and the given Circle, but fuch a cubic Equation may much eafier be obtained after the following Manner. (See Numb. 2. Fig. 36.) Let the Fig. 36. unknown fide A C of the Triangle required be called $2x$, and A C be bifected in G. Let G D be called z, and fuppofe D E F to be a Circle infcribed in the Triangle A B C fought. Let A B + B C + A C given be $= 4a$, and $2ab =$ to the given Area of the Triangle A B C, alfo let d be equal to the Diameter of the given Circle wherein the Triangle A B C is to be infcribed; then, (by the Nature of the infcribed Circle G E F) will A E = A D, E B = B F, C D = C F, and fo E B + B F $= 4a - 4x$, and F B = E B $= 2a - 2x$. Wherefore A C $= 2x$, C B $= 2a - x - z$, A B $= 2a - x + z$. Therefore by the old Rule of finding the Area of a Triangle by having the three fides, it will be $\sqrt{\overline{2a-2x}\times\overline{x+z}\times\overline{x-z}\times 2a} = 2ab$, or $\overline{xx-zz}\times\overline{a-x} = abb$, or $zz = \frac{ax^3 - x^3 - abb}{a-x}$. Again, becaufe from the Nature of the Circle A B C, the Rectangle under A C and C B = a Rectangle under the Diameter, and the Perpendicular drawn from C upon the oppofite Side A B of the Triangle A B C; we fhall have $\frac{2x\times\overline{2a-x-z}}{d} =$ that Perpendicular; and therefore the Area of the Triangle A B C, will be $x\times\overline{2a-x-z}\times\overline{2a-x+z} = 2abd$, or $4aa - 4ax + xx - zz = \frac{2abd}{x}$, and fo $zz = 4aa - 4ax + xx - \frac{2abd}{x}$. But before $zz = \frac{axx - x^3 - abb}{a-x}$, wherefore $\frac{axx - x^3 - abb}{a-x} = 4aa - 4ax + xx - \frac{2abd}{x}$. And at length $x^3 - 2axx + a^2 + \frac{bb}{4} + \frac{bd}{2}x = \frac{abd}{2}$.

Prob. V. *To conftitute a Triangle* F P G (Fig. 35.) *fuch that the three fides* F P, F G, P G, *and the Perpendicular* F H *drawn from the Angle* F *upon the third fide* P G, *fhall be four arithmetical Progreffionals* Fig. 35.

This is the 14th Problem of Sir *Ifaac Newton*'s univerfal Arithmetick, and a folid Problem.

Affume any right Line F G for one of the fides of the Triangle required, and with the ends F and G of it, as Foci, with an Axis A B equal to twice that fide, defcribe a Semi-Ellipfis A D B whofe Centre is C, erect the Perpendicular G M upon the Focus G, equal to C F or C G, and upon M G defcribe a Semicircle M N G; this done, take a Ruler G N P always moveable about the Point G, and fo turn it about, that the part N P of it beyond the Point N, where it cuts the Semicircle M N G intercepted between that Semicircle and the Ellipfis, be equal to the given focal Diftance C G, or the Diameter G M of the Semicircle. Join the right Line F P, then will F P G be the Triangle to be conftructed.

Upon F G defcribe the Semicircle F H G. Now it eafily appears from the Conftruction, and the Nature of the Ellipfis, that F P, F G, and G P, are continual arithmetical Progreffionals, for F P + P G = A B = 2 F G, and from the Nature of the two Semicircles F H G, M N G, it is eafily proved that the Chord G N of the Semicircle G M H is equal to one half of the Chord F H, (or perpendicular to G P) for becaufe of the right Angles F H G, G N M, as alfo C G M, the right angled Triangles G N M, E F H will be equiangular. Wherefore F G : G M : : F H : G N : : 2 : 1, and fo G N = $\frac{1}{2}$ F H; and becaufe P N = C G, it will be G P = $\frac{1}{2}$ F H + C G; that is, 2 G P = F H + 2 C G, or 2 G P = F H + F G; wherefore F G, G P, F H, are alfo arithmetical Progreffionals. But F P, F G, G P, have been fhewn to be fuch already. Wherefore now it appears, that F P, F G, G P, and F H, are four continual Progreffionals.

Prob. VI. *To conftruct a Trapezium* A B C D (Fig. 37.) *fuch, that its four fides fhall be equal to four given right Lines* M, N, O, P, *any three of which are greater than the fourth, and whofe Area fhall be equal to a given Square whofe fide is* Q. Fig. 37.

This plain Problem is to be feen in *Ozanam*'s Mathematical Dictionary, where it is refolved in the worft manner poffible.

Upon the indefinite right Line A Z, take A D equal to one of the given right Lines M. About the Centre D, with the Diftance D C equal to the given right Line P, defcribe a Circle K C R. This done let N + O, be called r; N — O, s; and the Hypothenufe of a right angled Triangle, whofe Bafe is A D and Perpendicular D C, be called h. Upon A D take D G a fourth Proportional to 2 A D, the Sum of h and r, and the Difference between h and r. Alfo take D F a fourth Proportional to 2 A D, the Sum of h and s, and the Difference of h and s. Upon F G defcribe a Semicircle F H G. Let K R be a Quadrant of the Circle K C R.

K C R. And making C H a third Proportional to $\frac{1}{2}$ A D and Q, from R take R S equal to C H. Continue out R S to V, so that S V be equal to R D, and about the Centre V with the Semidiameter V S describe a Circle T H S, whose Centre falls at V in R D continued, cutting the aforesaid Semicircle F H G in H, from H draw H C perpenicular to A D cutting the Circle K C R in C. Join D C, and about the Centre A with a Distance equal to the right Line N describe an Arch of a Circle. Also about the Centre C, with a Distance equal to the right Line O, describe an Arch of a Circle cutting the former Arch in B. Join A B, C B, then will the Trapezium A B C D, be that required.

The Reason of the Construction will be sufficiently evident from what follows. Join A C. Let us call A D, d; A C, z; D L, x. Then, by a well known Property of a Triangle, the Area of the Triangle A B C, will be $= \frac{1}{4}\sqrt{\overline{rr - zz} \times \overline{zz - ss}}$ $= \frac{1}{4}\sqrt{\overline{rr - hh + 2dx} \times \overline{hh - ss - 2dx}}$. Since $zz = hh - 2dx$. [by 13. 2. Euclid]. Let $hh - rr = mm$, and $hh - ss = nn$, then will the Area of the Triangle A B C be $\frac{1}{4}\sqrt{\overline{2dx - mm} \times \overline{nn - 2dx}}$. Let $mm = 2dp$, and $nn = 2dq$, then will the Area of the Triangle A B C be $= \frac{d}{2}\sqrt{-xx + \overline{p + q} \times x - pq}$. Now let $\sqrt{-xx + \overline{p + q} \times x - pq}$ be $= u$. And so $-xx + \overline{p + q} \times x - pq = uu$, the Locus of which Equation will be a Circle, and by Construction D G will be equal to p, since it is made as $2d : h + r :: h - r : p$. Also D F will be equal to q, since, by Construction, $2d : h + s :: h - s : q$, wherefore the Semicircle F H G will be the Locus of the Equation $-xx + \overline{p + q} \times x - pq = uu$, any perpendicular ordinate L H being u; and therefore if A B C D be any Trapezium whose sides A B, B C, C D, A D, are equal to the given right Lines M, N, O, P, and from the Angle C of it be drawn the right Line C L H cutting the side A D in L, and the Semicircle F H G in H, the Rectangle under $\frac{1}{2}$ A D and H L will be equal to the Triangle A B C. But the Rectangle under $\frac{1}{2}$ A D and C L is also equal to the Triangle A C D, wherefore the Area of the Trapezium A B C D, will always be equal to the Rectangle under $\frac{1}{2}$ A D and C L + L H; that is, equal to the Rectangle under $\frac{1}{2}$ A D and C H. But because the Area is given, *viz.* equal to $\overline{Q}^2$, therefore $\overline{Q}^2 = \frac{1}{2}$ A D $\times$ C H; that is, C H is a third Proportional to $\frac{1}{2}$ A D and Q, wherefore C H is given. Consequently the Problem will be resolved by so drawing the right Line C H of a given Length perpendicular to A D, that one of it's Extremes C falls on the Circle K C R, and the other H on the Semicircle F H G; and this may be done by describing the Locus of the Point H, or end of a right Line C H always moving parallel to itself, while the beginning C thereof moves along the Circumference of the Circle K C R; which Locus will be the Circle T H S equal to the Circle K C R, the Centre being V, and Semi-diameter V S = D R. Where this Circle intersects the Semicircle F H G, will give the Point H such, that C H being drawn will be of the given Magnitude required. The rest is sufficiently evident.

Note, I have given the instrumental Constructions of the few Problems above, as a Specimen of the most easy, natural, and obvious Way possible of performing the Business, in order to invite others to proceed in this Way in the Resolution of difficult geometrical Problems, rather than by that usual one so long in vogue, of first obtaining an algebraic Equation by means of the given Conditions of the Problem; and then finding the linear Roots of that Equation, which in almost all Cases is troublesome, unelegant, and unnatural, and in many other Cases is intolerable, and almost impossible; for Example, how difficult would it be, by the common Methods, to find the Diameter of a Semicircle, wherein 5, 7, 9, *&c.* given unequal and contiguous Chords may be inscribed; and how easy is it to do this by 5, 7, 9, *&c.* squares with the respective Number (less by two) of given Chords fastened by their Extremes to each of the Angles of those Squares, so as to be moveable about. Moreover, as every geometrical Problem points out the best and most simple Way to its own Construction, by the Intersections of some one Line (straight or curve) of a given Order, with some other Line of the same, or a different Order; those Constructions are best, when these Lines are easiest described, either by the continual Motion of Rulers, Squares, Bevels, particular Adaptions of Figures, Threads, *&c.* or by finding a few Points of those intersecting Lines, so many as to determine the Intersections, and when the preparatory right Lines, Perpendiculars, Parallels, Proportionals, *&c.* are fewest or none at all. Again, all plain Problems which have two Answers, may be constructed by the Intersections of a right Line and a Circle, or by the Intersections of two Circles, but oftentimes the Construction will be much easier and simpler done by the Intersection of a right Line and a conic Section. For Example, *Vieta* shews how to construct a Triangle, whose Base, Perpendicular, and Sum of the sides, are each given, by describing one Circle through two given Points to touch another Circle. But this is not the best Construction; for, if with the Extremes of the given Base, as Foci, with a Thread in length equal to the given Sum of the sides, part of an Ellipsis be described; then a Parallel to the Base at the given perpendicular Distance from it being drawn, will intersect the Curve of the Ellipsis in two Points, either of which will be the Vertex of the Triangle required; and hence it appears, that this Problem of *Vieta*'s is no more than

than drawing that Semi-Ordinate to the Axis of a given Ellipſis, which ſhall have a given Magnitude. And even this may be eaſily done without the actual Deſcription of any Ellipſis at all differently from *Vietas*'s Way, upon the Conſideration of the Manner of deſcribing an Ellipſis by an elliptick Compaſs.

Pappus tells us, the Antients preferred Mechanical Conſtructions to uſeleſs Geometrical ones; and the great Dr *Barrow*, in his Optical Lectures, has given ſeveral moſt eaſy and natural Conſtructions of difficult Problems after this way, by means of the Curve-Lines peculiar to them; and whoever delights in Conſtructions, will have abundance of Reaſon to be highly pleaſ'd with the Simplicity and Eaſineſs of theſe uncommon ſort of Conſtructions. I myſelf, in Imitation of Dr *Barrow*, have conſtructed many Problems after this way.

Algebra has really nothing to do in the Conſtruction of Geometrical Problems, it's Equations being Expreſſions of an Arithmetical Computation, and accordingly whatever is done by means of them is rather an Arithmetical Reſolution of a Problem than a Geometrical one. Beſides, when Equations of this ſort ariſe to above two Dimenſions there is no obtaining their Roots in moſt Caſes, unleſs by a troubleſome Approximation; and to find the firſt Figures of theſe Roots, in order to get the other Figures, Sir *Iſaac Newton* long ago contrived an Inſtrument for this purpoſe. A ſhort Deſcription of which is to be found in the *Commercium Epiſtolicum* (publiſhed by Order of the *Royal Society*, in the Year 1725.) at *page* 123, in a Letter from Mr *Oldenburgh*, their Secretary, to Mr *Leibnitz*, dated the 24th of *June*, in the Year 1675. He ſays, Sir *Iſaac*'s Inſtrument conſiſts of the Logarithms laid off upon parallel Rulers, or concentrick Circles, placed at equal Diſtances from one another, three Rulers being ſufficient for cubic Equations, and four for biquadratick ones; they are ſo diſpoſed, as that the reſpective Co-efficients all lie in the ſame right Line, from a Point of which ſo remote from the firſt Ruler, as the divided ones are from each other; a right Line is extended over them, with Directions conformable to the Nature of the Equation, by which the pure Power of a Root ſought is given in one of thoſe Rulers.

The End of the APPENDIX.

ERRATA.

PAGE 266. line 58. *for* drawing Paper; *p q read* drawing Paper *p q*; Page 286. line 23. dele I Page 291. line 63. *for* Θ *read* θ. Page 312. line 27. *for* Wheels *read* of the Wheel.

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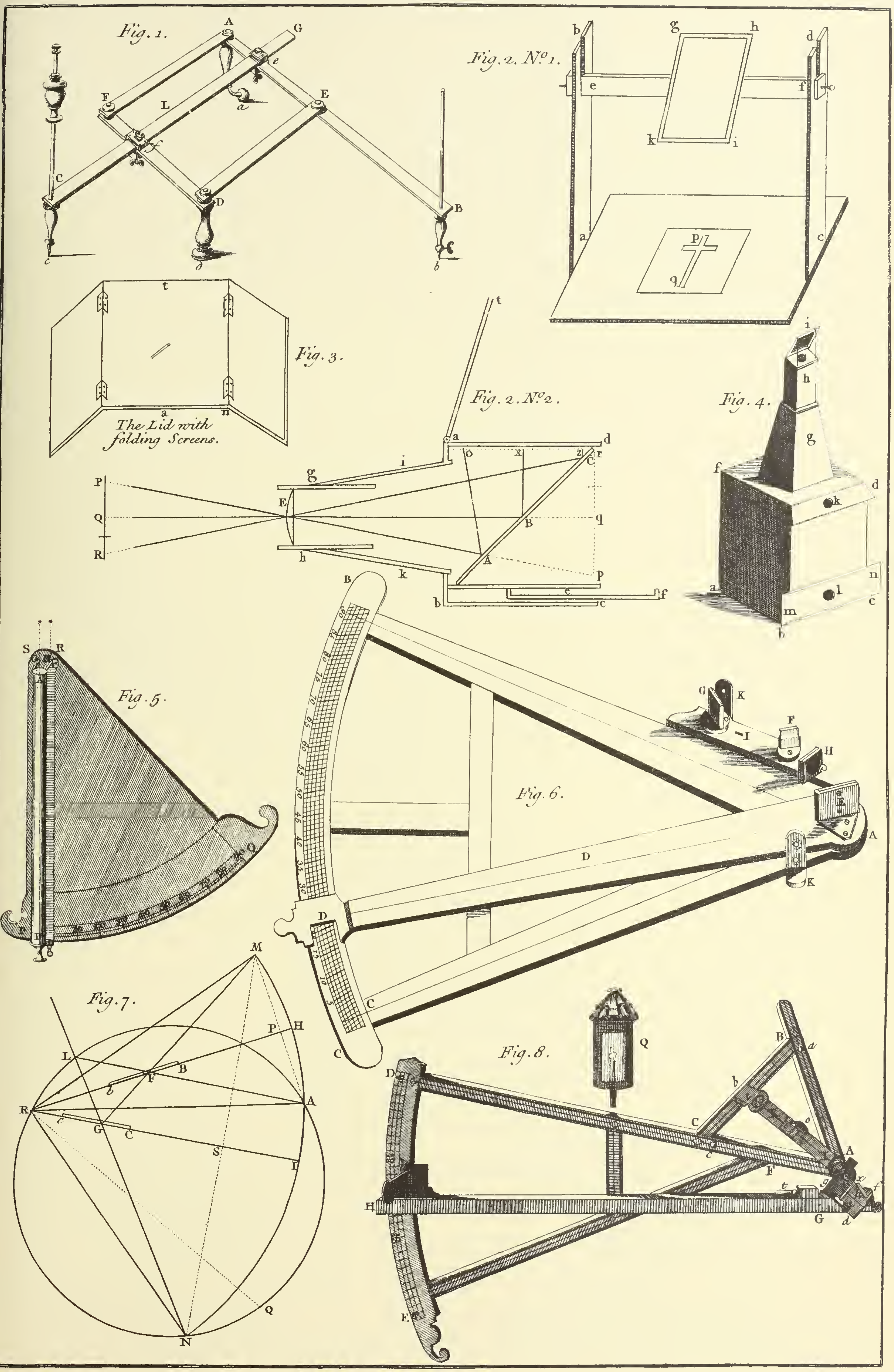
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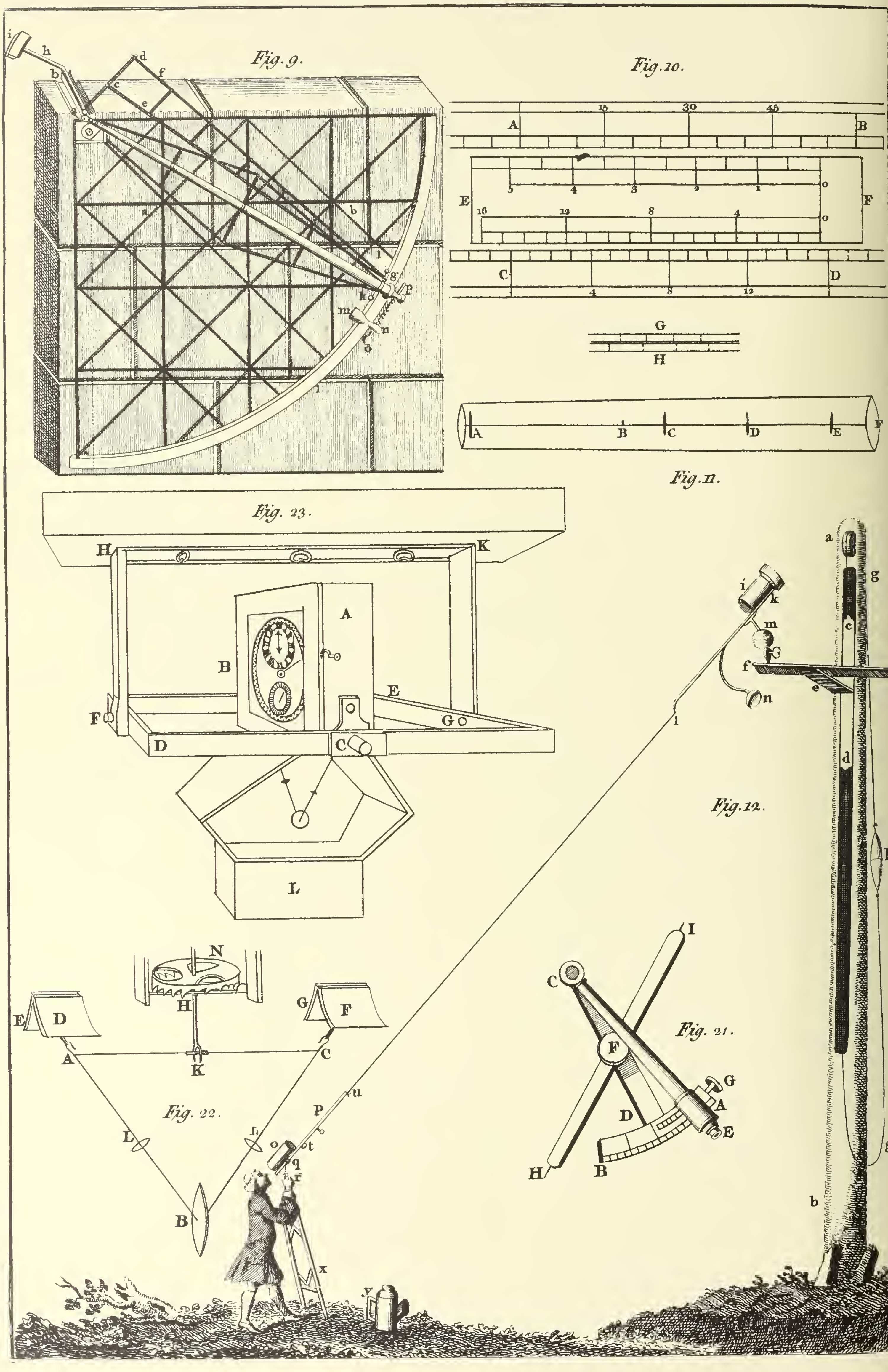
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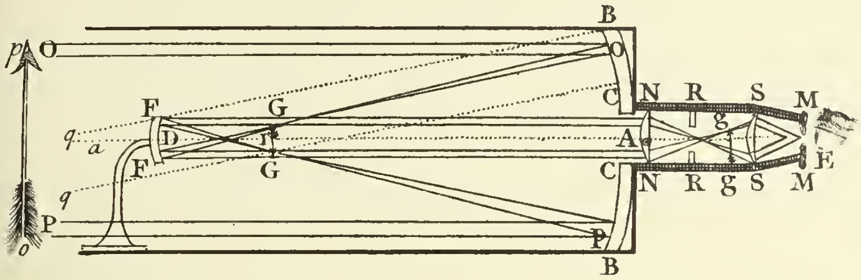

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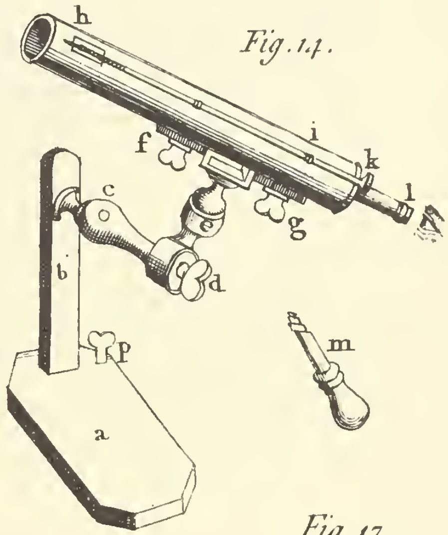

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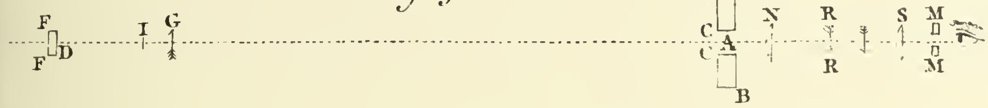

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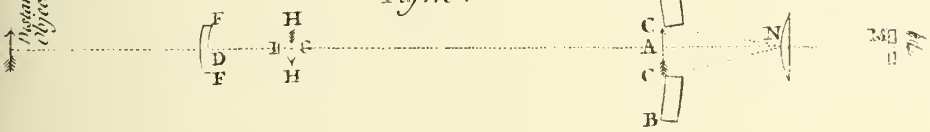

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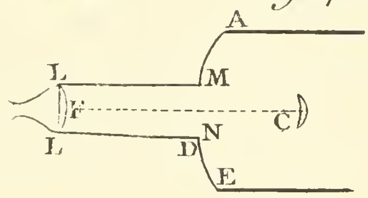

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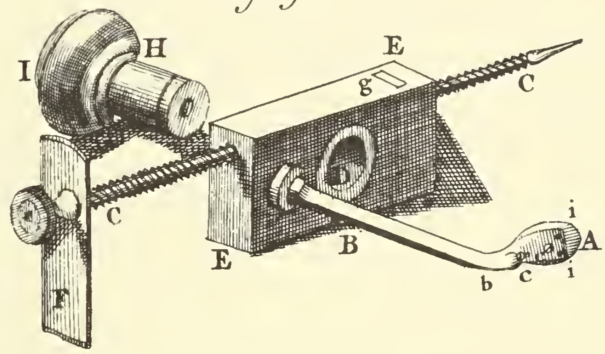

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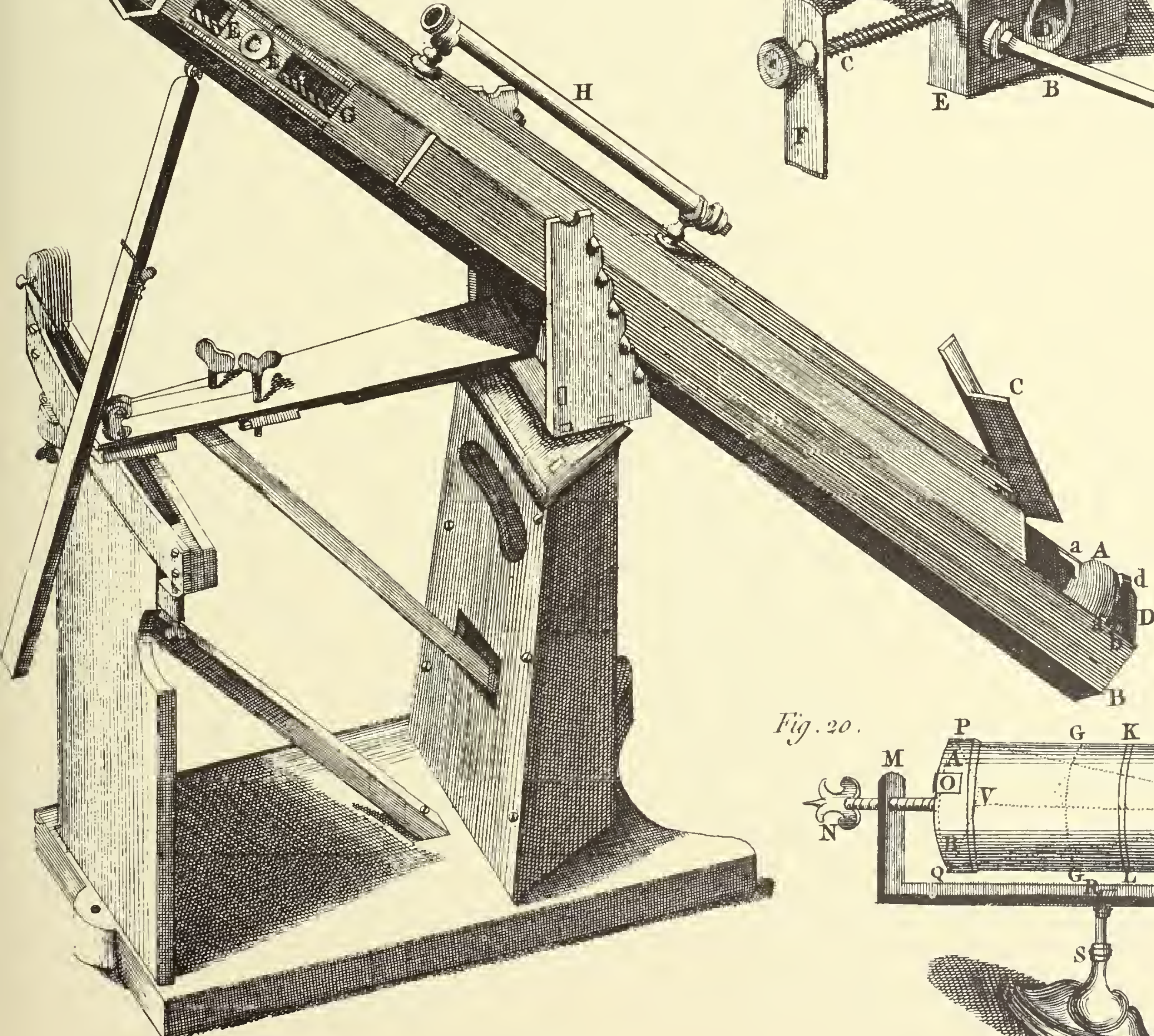

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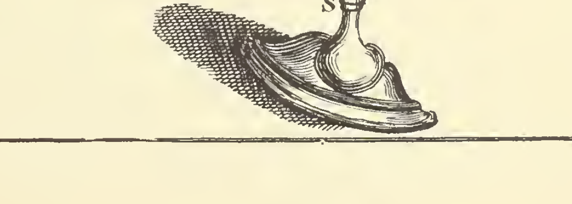

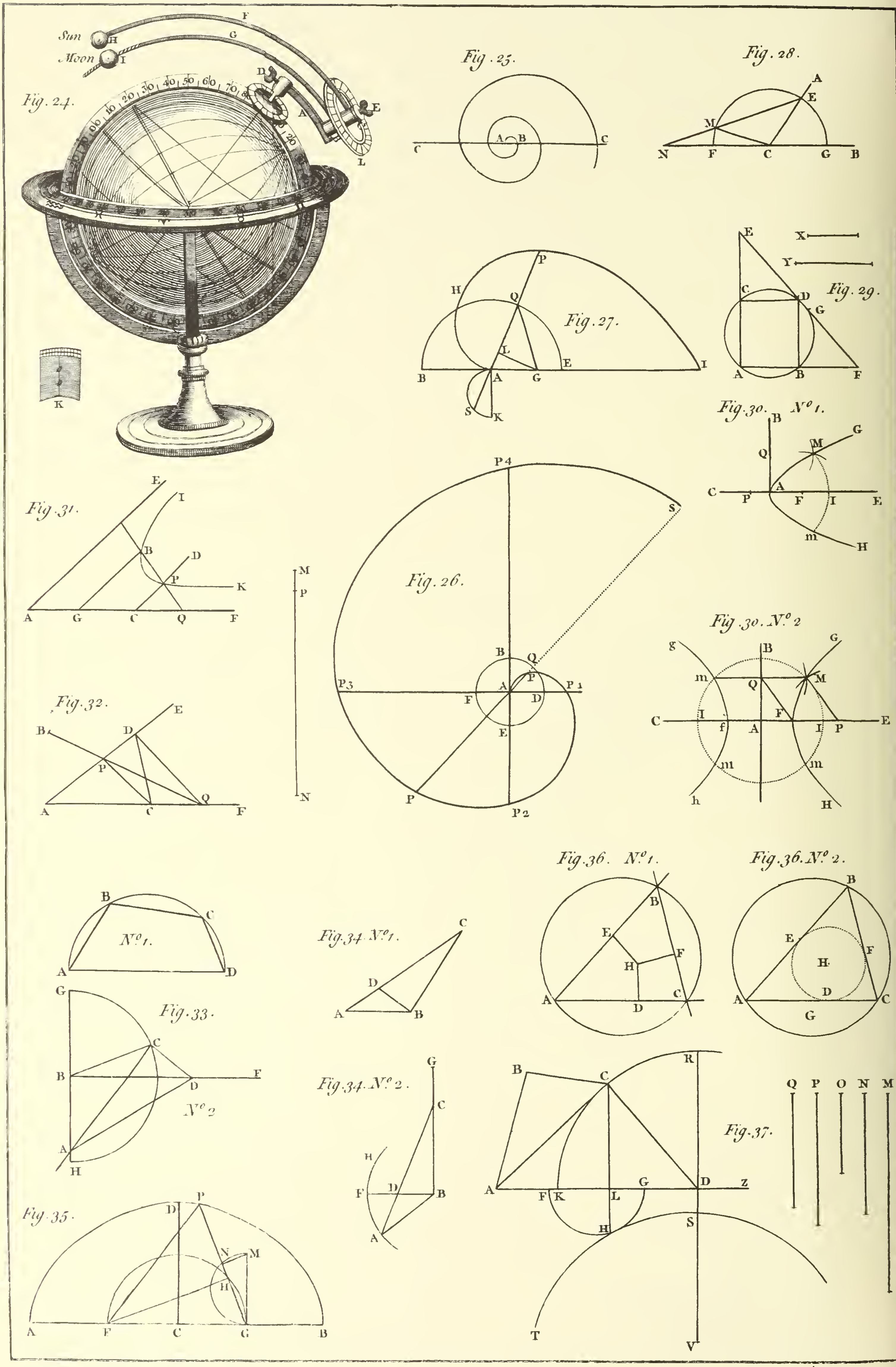
Sun
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Fig. 24.
Fig. 25.
Fig. 28.
Fig. 27.
Fig. 29.
Fig. 30. N.º 1.
Fig. 31.
Fig. 26.
Fig. 30. N.º 2
Fig. 32.
N.º 1.
Fig. 33.
N.º 2
Fig. 34. N.º 1.
Fig. 34. N.º 2.
Fig. 36. N.º 1.
Fig. 36. N.º 2.
Fig. 37.
Fig. 35.

agrees with the next nearest fixed Hair of the other Frame; or, which is the same thing, move the Frame E F G H the Length of four Lines, or one third of an Inch, which may be easily known by means of the Object-Glass, which magnifies Objects, and count the Revolutions and Parts of the Screw, compleated in moving the said Frame that Length. Finally, make a Table, shewing how many Revolutions, and Parts of a Revolution of the said Screw, are answerable to every Minute and Second, by having the Angle subtended by the two black Lines on the Board given, and taking the Revolutions proportional to the Angles; that is, if a certain Number of Revolutions give a certain Angle, half this Number will give half the Angle, *&c.* And this Proportion is exact enough in these small Angles.

Now the Manner of taking the apparent Diameters of the Planets, is thus: Having directed the Telescope, and it's Micrometer, towards a Planet, dispose the Hairs, by the Motion of the Telescope, in such a Manner, that one of the fixed parallel Hairs do just touch one Edge of the Planet, and turn the Screw 'till the moveable Hair just touches the opposite Edge of the said Planet. Then, by means of the Table, you will know how many Minutes or Seconds correspond to the Number of Revolutions or Parts, reckoning from the Point of the Plate over which the Index stood when the fixed Hair touched one Edge of the Planet, to the Point it stands over when the moveable Hair touches the opposite Edge; and consequently, the apparent Diameter of the said Planet will be had. And in this manner may small Angles on Earth be taken, which may be easier done than those of the Celestial Bodies, because of their Immobility.

This Method is convenient enough for measuring the apparent Diameters of the Planets, if the Body of any one of them moves between the parallel Hairs. Yet it ought to be observed, that the Sun and Moon's Diameters appear very unequal upon the account of Refraction; for in small Elevations above the Horizon, by the Space of 30 Minutes, the vertical Diameters appear something lesser than they really are in the Horizon, and the horizontal Diameters cannot be found, unless with much Trouble, and several repeated Observations; as likewise the Distance between two Stars, or the Horns of the Moon, because of their Diurnal Motions, which appear thro' the Telescope very swift.

If two Stars of different Altitudes pass by the Meridian at different Times, the Difference of their Altitudes will be the Difference of their Distances from the Equator towards either of the Poles, which is called their Difference of Declination; and by their Difference of Time in coming to the Meridian, the Difference of their Distance from a determinate Point of the Equator, that is, the first Degree of *Aries* will be had; and this is their Difference of Right Ascension.

If the two Stars are distant from each other, we have Time enough, in the Interval of their Passage by the Meridian and Micrometer, to finish the Operations regarding the first, before proceeding to those of the second; but if they be very near each other, it is extremely difficult to make both the Observations at the same Time, that so the two Stars may be precisely catched in the Meridian. But M. *de la Hire* shews how to remedy this Inconveniency, by only using the common Micrometer: for the Observation of the Passage of Stars between, or upon the Hairs of the Micrometer, will give, by easy Consequences, their Difference of Right Ascension and Declination, without even supposing a Meridian known or drawn.

But if the Difference of Declination and Right Ascension of two Stars that cannot be taken in between the Hairs of the Micrometer be required, this may be found in the following Manner.

We adjust a Cross-Hair to the Micrometer, cutting the parallel ones at Right Angles, Fig. 10. which we fasten with Wax to the Middle of the Sides A C and B D. Then the Telescope, and it's Micrometer, being fixed in a convenient Position, so that the Stars may successively pass by the parallel Hairs, as the Stars A and S, in Figure 10; we observe, by a second Pendulum Clock, the Time wherein the first Star A touches the Point in which the aforementioned Cross-Hair A S crosses some one of the parallel Hairs, as A *d*. The Micrometer being disposed for this Observation, which is not difficult to do, reckon the Seconds of Time elapsed between the Observations made in the Point A, and the arrival of the said Star to the Point B, being the Concourse of another parallel Hair B D. We likewise observe the Time wherein the other Star S meets the Cross-Hair at the Point S, and then at the Point D of the parallel Hair B D. *Note*, It is the same thing if the Star S first meets the parallel Hair in D, and afterwards the Cross-Hair in S.

Now as the Number of Seconds the Star A is moving through the Space A B, is to the Number of Seconds the Star S is moving through the Space S D; so is the Distance A C, known in Minutes and Seconds of a Degree in the Micrometer, to the Distance C S, in Minutes and Seconds of a Degree. But the Horary Seconds of the Motion through the Space A B, must be converted into Minutes and Seconds of a great Circle, by the Rule of Proportion.

Having first converted the Seconds of the Time of the said Motion from A to B, which may be here esteemed as a Right Line, or an Arc of a great Circle, into Minutes and Seconds of a Circle, in allowing 15 Minutes of a Circle to every Minute of an Hour, and